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Complex Pentapartitioned Neutrosophic Aggregation Operators with an Application in the Selection of Millets for Cultivation

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Abstract. The Complex Pentapartitioned Neutrosophic Set (C-PNS) is an advanced generalization of Fuzzy Sets, designed to model complex uncertainties characterized by periodic, vague and indeterminate information. In this study, we introduce two novel aggregation operators tailored for C-PNS environments: the Complex Pentapartitioned Neutrosophic Weighted Arithmetic Average (CPNWAA) and the Complex Pentapartitioned Neutrosophic Weighted Geometric Average (CPNWGA) operators. We establish their mathematical properties, including idempotency, boundedness and monotonicity, ensuring their theoretical soundness and applicability in decision-making scenarios. To demonstrate the utility of the proposed operators, we apply them to a Multi-Criteria Decision-Making (MCDM) problem focused on selecting the most suitable millet variety for cultivation under uncertain conditions. Evaluations were conducted with inputs from three domain experts, considering multiple cultivation-related criteria. The analysis reveals that Ragi (finger millet) emerges as the most optimal choice among the alternatives. Furthermore, comparative studies with existing MCDM methods; WSM, WPM, N-TOPSIS, N-WASPAS and N-MOORA along side statistical validation, confirm the robustness, consistency and effectiveness of our proposed methodology. This research not only enriches the C-PNS framework with new tools but also provides a reliable and flexible approach to decision-making in complex agricultural contexts.

Keywords: C-PNS, Aggregation Operator, Millets Cultivation, MCDM.

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1. Introduction

Solving complex problems with vague and uncertain information is a significant challenge in decision-making. The theory of fuzzy sets, introduced by Zadeh in 1965, overcame the limitations of classical sets by providing a mathematical framework for handling imprecise and ambiguous data. This breakthrough enabled solutions for decision-making problems involving uncertainty. Building on this foundation, Atanassov extended fuzzy sets by associating membership and non-membership degrees, ensuring their sum does not exceed 1. This extension, called the Intuitionistic Fuzzy Set (IFS), provided a more nuanced representation of uncertainty. Smarandache [22] in the late 1990s, extends classical and fuzzy set theories to better address uncertainty, imprecision and indeterminacy. Unlike traditional fuzzy sets that only measure degrees of membership or intuitionistic fuzzy sets that incorporate membership and non-membership, Neutrosophic sets introduce three distinct parameters: truth (T), indeterminacy (I) and falsity (F). These parameters operate within the range [0,1] but are not necessarily constrained by a predefined sum, providing enhanced flexibility in uncertainty modeling. Neutrosophic sets have been applied in various domains, including decision-making, image processing, artificial intelligence, medical diagnosis, and engineering. To address the constraints of Fuzzy Sets (FS), Intuitionistic Fuzzy Sets (IFS), and Neutrosophic Sets (NS), Ramot et al. [20] proposed the Complex Fuzzy Set (CFS). Unlike traditional fuzzy membership functions, the membership values in CFS are represented as complex numbers on the unit circle in the complex plane. This extension offered a sophisticated framework for describing membership with both magnitude and phase, thereby handling periodicity and additional complexities more effectively. The concept of CFS has since evolved into various extensions, including Complex Neutrosophic Sets (CNS) [5], Complex Spherical Fuzzy Sets (CSFS) [3], Complex Picture Fuzzy Sets (CPFS) [14], and Complex Fermatean Fuzzy Sets (CFFS) [11], to address specific decision-making needs. Recently, Complex Pentapartitioned Neutrosophic Sets (CPNS) [15] were introduced by Porchudar and Ajay to incorporate five membership degreestruth, falsity, contradiction, ignorance, and unknownproviding a comprehensive framework for tackling uncertainty and periodicity.

1.1. *Objectives:*

This research aims to enhance decision-making under complex and uncertain conditions by leveraging the Complex Pentapartitioned Neutrosophic Set (CPNS) framework. The specific objectives are:

- To extend the mathematical foundation of CPNS: Develop and define essential operations such as union, intersection, complement, algebraic sum, and product to establish a robust theoretical base for CPNS-based reasoning.
- To introduce two novel aggregation operators; Complex Pentapartitioned Neutrosophic Weighted Arithmetic Average (CPNWAA), Complex Pentapartitioned Neutrosophic Weighted Geometric Average (CPNWGA). These operators are designed to aggregate complex, uncertain, and periodic information from multiple sources.

- To design a CPNS-based Multi-Criteria Decision-Making (MCDM) model. Apply this model to a real-world agricultural problemselecting the best millet varietyconsidering multiple conflicting criteria and expert judgments.
- To evaluate the performance of the proposed model: Compare results with existing MCDM methods such as WSM, WPM, TOPSIS, WASPAS and MOORA to demonstrate its effectiveness.
- To validate the results statistically, use correlation measures like Pearson and Spearman coefficients to assess the reliability and consistency of the proposed approach.
- To explore the potential of CPNS in broader applications, lay the foundation for extending CPNS-based decision-making models to other domains such as healthcare, environmental sustainability, and smart agriculture.

1.2. *Literature Review:*

Fuzzy set theory has inspired a wide range of extensions and applications in decision-making and uncertainty analysis. Key contributions include:

- Zadeh (1965): Introduced fuzzy sets to address vague and ambiguous data in decision-making.
- Atanassov (1986): Proposed Intuitionistic Fuzzy Sets (IFS) to extend fuzzy sets by incorporating membership and non-membership degrees.
- Smarandache (1999): Introduced Neutrosophic Sets (NS), a robust mathematical tool for handling indeterminate and inconsistent data.
- Ramot et al. (2003): Proposed Complex Fuzzy Sets (CFS), where membership functions extend to the complex plane, addressing periodicity and complex relationships.
- Porchudar and Ajay (2024): Developed Complex Pentapartitioned Neutrosophic Sets (CPNS), incorporating five membership degrees for enhanced uncertainty modeling.

Recent studies highlight the growing importance of advanced fuzzy and neutrosophic methods in decision-making are given in Table 1.

1.3. *Research Gap:*

While advancements such as Complex Fuzzy Sets (CFS) and their derivativeslike Complex Neutrosophic Sets (CNS) and Complex Picture Fuzzy Sets (CPFS)have improved the capacity to represent uncertainty and periodicity, they remain limited in capturing multi-layered indeterminacy, contradictory evidence, and unknown or missing data simultaneously. The recently proposed Complex Pentapartitioned Neutrosophic Sets (CPNS), introduced by Porchudar and Ajay [15], offer five distinct membership degreestruth, falsity, contradiction, ignorance, and unknown. This framework addresses the deficiencies in existing models by offering a more

Table 1. Detailed Review of Literature

Authors	Contributions
Tan [23]	Developed entropy-based MCDM approach for the SVNS model
Choudhary and De [10]	Integrated Intuitionistic framework for sustainable supply chain
Ajay et.al [2]	Extended Geometric Bonferroni Mean Operator under Neutrosophic Cubic fuzzy sets
Ajay et.al [1]	Developed MCDM model using Complex q-Rung Orthopair Fuzzy
Aldring and Ajay [3]	Extended projection measures on CPFSs
Chatterjee et al. [9]	Identified Quadripartitioned SVNS
Smarandache [21]	Suggested Five Valued Neutrosophic Set
Mallick and Pramanik [13]	Introduced Pentapartitioned Neutrosophic Set
Radha et.al [18, 19]	Extended Bipolar Pentapartitioned Neutrosophic Set with topological space
Turan and Yasrah [16]	Incorporated Intuitionistic fuzzy entrophy and variable weight theory
Anjum et. al [8]	Integrated IFS with MCDM and addressed challenges like carbon footprints
Ali et. al [6]	Developed Bonferroni Mean Operators on CSFSs
Quek SG et al. [17]	Introduced Pentapartitioned Neutrosophic Graphs
Hussain and Ullah [12]	Developed Sugeno-Weber Aggregation Operators
Asif et. al [7]	Extended Hamacher Aggregation Operators in Pythagorean Fuzzy Set

detailed structure for representing real-world uncertainties. However, there remains a lack of practical decision-making tools and aggregation operators based on CPNS, especially in MCDM scenarios involving periodic data.

1.4. *Motiovation:*

Decision making in complex environments such as agriculture, healthcare and environmental management often involves conflicting criteria, periodic fluctuations, and vague expert opinions. Traditional fuzzy based models like Fuzzy Sets (FS), Intuitionistic Fuzzy Sets (IFS), and Neutrosophic Sets (NS) have contributed significantly to uncertainty modeling. However, these models are typically insufficient when dealing with contradictions, incomplete information, or periodic data behavior, which are crucial in dynamic systems.

In the agricultural context, for instance, selecting suitable millet varieties requires consideration of multiple factors such as soil compatibility, climate resilience, market trends and health benefits. These factors are inherently uncertain and often exhibit periodic or seasonal variations. Thus, a more expressive mathematical tool is needed to handle these complexities effectively. Millets, once a staple crop in India, have been marginalized since the Green Revolution. However, their nutritional superiority, resilience to climate variability, and adaptability

TABLE 2. Advantages of CPNS for Millet Selection

Factor	Traditional Fuzzy Sets	Complex Pentapartitioned Neutrosophic Set (CPNS)
Handles Missig Data(Ignoranace)	NO	YES
Captures Contradictory Information	NO	YES
Accounts Periodic Changes	NO	YES
Models Uncertainty in Expert Judgment	Limited	Comprehensive
Distinguishes between unknown and contradictory information	NO	YES

to rainfed conditions make them an essential crop for sustainable agriculture.

With the United Nations declaring 2023 the "International Year of Millets," there is renewed interest in promoting millet cultivation. Yet, many farmers hesitate to adopt millet farming due to uncertainty about the suitability of different millet varieties. CPNS provides an advanced mathematical structure that integrates multiple dimensions of uncertainty, contradiction and periodicity, making it the ideal tool for ranking and selecting millet varieties. Unlike static decision models, CPNS accommodates periodic variations in environmental conditions.

Millet growth and yield depend on: Seasonal weather changes, Soil moisture variations and Market demand fluctuations. The CPNS-based approach enhances decision making in agriculture by integrating multiple forms of uncertainty and periodic behavior. It outperforms traditional models in scenarios where data incompleteness, contradiction, and cyclic variation are significant. Despite certain computational demands, its flexibility, realism, and robustness make it a strong candidate for applications beyond agriculture, including healthcare and environmental systems.

Research Contributions:

Our major contributions of this study include:

(1) We extend the Complex Pentapartitioned Neutrosophic Set (CPNS) to better handle real-world uncertaintiescapturing incomplete, conflicting, periodic, and multi source data with greater precision.

(2) We define essential operations on CPNS such as union, intersection, complement, algebraic sum and product, ensuring a solid foundation for further analysis and reasoning.

(3) Two novel operators are introduced:

   (a) CPNWAA: A weighted arithmetic average that gives balanced importance to all criteria.

   (b) CPNWGA: A weighted geometric average that emphasizes more critical criteria. These operators support flexible and context-aware decision-making.

(4) A CPNS-based Multi-Criteria Decision-Making (MCDM) model is developed and applied to millet cultivation. It helps policymakers and farmers make informed, uncertainty-aware decisions.

(5) We compare our approach with popular MCDM methods like WSM, WPM, N-TOPSIS, N-WASPAS, and N-MOORA. Using Pearson and Spearman correlations, we confirm the robustness and effectiveness of our model.

(6) This work sets the stage for applying CPNS in diverse fields such as healthcare, environmental sustainability, smart agriculture, and supply chain management, wherever complex uncertainty is present.

2. Preliminaries

Definition: [13]

Let P be a non-empty set. A Pentapartitioned Neutrosophic Set A over P characterizes each element p in P by the truth T_A, the contradiction C_A, the ignorance G_A, the unknown U_A and the falsity F_A membership functions such that for each p $\in$ P, $T_A, C_A, I_A, U_A, F_A \in [0,1]$ and $0 \leq T_A(p) + C_A(p) + I_A(p) + U_A(p) + F_A(p) \leq 5$.

Definition: [15]

A Complex Pentapartitioned Neutrosophic Set $\check{P}$ defined on the universe of discourse $\tilde{\wp}$ is represented by the degree of truth $\tilde{\xi P}(\wp)$, the degree of contradiction $\tilde{\psi P}(\wp)$, the degree of ignorance $G_{\check{P}}(\wp)$, the degree of unknown $U_{\check{P}}(\wp)$ and the degree of falsity $F_{\check{P}}(\wp)$ membership functions for any $\wp \in \tilde{\wp}$ and is explained as

$$\check{P} = \left\{ \left\langle \wp, \tilde{\xi P}(\wp), \tilde{\psi P}(\wp), \tilde{\chi P}(\wp), \tilde{\zeta P}(\wp), \tilde{\omega P}(\wp) \right\rangle : \wp \in \tilde{\wp} \right\}$$

where $\tilde{\xi P}(\wp) : \tilde{\wp} \to \left\{ \mathfrak{z}_1 : \mathfrak{z}_1 \in \check{P}, |\mathfrak{z}_1| \right\}$, $\tilde{\psi P}(\wp) : \tilde{\wp} \to \left\{ \mathfrak{z}_2 : \mathfrak{z}_2 \in \check{P}, |\mathfrak{z}_2| \right\}$, $\tilde{\chi P}(\wp) : \tilde{\wp} \to \left\{ \mathfrak{z}_3 : \mathfrak{z}_3 \in \check{P}, |\mathfrak{z}_3| \right\}$, $U_{\check{P}}(\wp) : \tilde{\wp} \to \left\{ \mathfrak{z}_4 : \mathfrak{z}_4 \in \check{P}, |z_4| \right\}$ and $F_{\check{P}}(\wp) : \tilde{\wp} \to \left\{ \mathfrak{z}_5 : \mathfrak{z}_5 \in \check{P}, |\mathfrak{z}_5| \right\}$. Such that $\tilde{\xi P}(\wp) = \mathfrak{z}_1 = a_1 + ib_1$, $\tilde{\psi P}(\wp) = \mathfrak{z}_2 = a_2 + ib_2$, $\tilde{\chi P}(\wp) = \mathfrak{z}_3 = a_3 + ib_3$, $\tilde{\zeta P}(\wp) = \mathfrak{z}_4 = a_4 + ib_4$ and $\tilde{\omega P}(\wp) = \mathfrak{z}_5 = a_5 + ib_5$ provided that $0 \leq \tilde{\xi P}(\wp) + \tilde{\psi P}(\wp) + \tilde{\chi P}(\wp) + \tilde{\zeta P}(\wp) + \tilde{\omega P}(\wp) \leq 5$. Other simple form of CPNSs is defined as

$$\check{P} = \left\{ \left\langle \wp, \tilde{\xi P}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi P}(\wp)}}, \tilde{\psi P}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi P}(\wp)}}, \tilde{\chi P}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi P}(\wp)}}, \tilde{\zeta P}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta P}(\wp)}}, \tilde{\omega P}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega P}(\wp)}} \right\rangle : \wp \in \tilde{\wp} \right\}$$

3. 3|Operations on Complex Pentapartitioned Neutrosophic Sets

Let $\tilde{\flat_1} = \left\langle \tilde{\xi_1}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_1}(\wp)}}, \tilde{\psi_1}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_1}(\wp)}}, \tilde{\chi_1}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_1}(\wp)}}, \tilde{\zeta_1}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_1}(\wp)}}, \tilde{\omega_1}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_1}(\wp)}} \right\rangle$ and $\tilde{\flat_2} = \left\langle \tilde{\xi_2}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_2}(\wp)}}, \tilde{\psi_2}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_2}(\wp)}}, \tilde{\chi_2}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_2}(\wp)}}, \tilde{\zeta_2}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_2}(\wp)}}, \tilde{\omega_2}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_2}(\wp)}} \right\rangle$ be two CPNSs, then their operations are defined as follows:

i. Minimum of two CPNSs

$$\tilde{\flat_1} \cap \tilde{\flat_2} = \left\{ \begin{array}{l} \left(min\left\{\tilde{\xi_1}(\wp), \tilde{\xi_2}(\wp)\right\}\right).\mathfrak{e}^{j2\pi\left(min\left\{\Theta_{\tilde{\xi_1}(\wp)}, \Theta_{\tilde{\xi_2}(\wp)}\right\}\right)}, \\ \left(min\left\{\tilde{\psi_1}(\wp), \tilde{\psi_2}(\wp)\right\}\right).\mathfrak{e}^{j2\pi\left(min\left\{\Theta_{\tilde{\psi_1}(\wp)}, \Theta_{\tilde{\psi_2}(\wp)}\right\}\right)}, \\ \left(max\left\{\tilde{\chi_1}(\wp), \tilde{\chi_2}(\wp)\right\}\right).\mathfrak{e}^{j2\pi\left(max\left\{\Theta_{\tilde{\chi_1}(\wp)}, \Theta_{\tilde{\chi_2}(\wp)}\right\}\right)}, \\ \left(max\left\{\tilde{\zeta_1}(\wp), \tilde{\zeta_2}(\wp)\right\}\right).\mathfrak{e}^{j2\pi\left(max\left\{\Theta_{\tilde{\zeta_1}(\wp)}, \Theta_{\tilde{\zeta_2}(\wp)}\right\}\right)}, \\ \left(max\left\{\tilde{\omega_1}(\wp), \tilde{\omega_2}(\wp)\right\}\right).\mathfrak{e}^{j2\pi\left(max\left\{\Theta_{\tilde{\omega_1}(\wp)}, \Theta_{\tilde{\omega_2}(\wp)}\right\}\right)} \end{array} \right\}$$

ii. Maximum of two CPNSs

$$\tilde{\flat_1} \cup \tilde{\flat_2} = \left\{ \begin{array}{l} \left(max\left\{\tilde{\xi_1}(\wp), \tilde{\xi_2}(\wp)\right\}\right).\mathfrak{e}^{j2\pi\left(max\left\{\Theta_{\tilde{\xi_1}(\wp)}, \Theta_{\tilde{\xi_2}(\wp)}\right\}\right)}, \\ \left(max\left\{\tilde{\psi_1}(\wp), \tilde{\psi_2}(\wp)\right\}\right).\mathfrak{e}^{j2\pi\left(max\left\{\Theta_{\tilde{\psi_1}(\wp)}, \Theta_{\tilde{\psi_2}(\wp)}\right\}\right)}, \\ \left(min\left\{\tilde{\chi_1}(\wp), \tilde{\chi_2}(\wp)\right\}\right).\mathfrak{e}^{j2\pi\left(min\left\{\Theta_{\tilde{\chi_1}(\wp)}, \Theta_{\tilde{\chi_2}(\wp)}\right\}\right)}, \\ \left(min\left\{\tilde{\zeta_1}(\wp), \tilde{\zeta_2}(\wp)\right\}\right).\mathfrak{e}^{j2\pi\left(min\left\{\Theta_{\tilde{\zeta_1}(\wp)}, \Theta_{\tilde{\zeta_2}(\wp)}\right\}\right)}, \\ \left(min\left\{\tilde{\omega_1}(\wp), \tilde{\omega_2}(\wp)\right\}\right).\mathfrak{e}^{j2\pi\left(min\left\{\Theta_{\tilde{\omega_1}(\wp)}, \Theta_{\tilde{\omega_2}(\wp)}\right\}\right)} \end{array} \right\}$$

iii. Complement of CPNS

$$\flat_a^{\tilde{\psi}} = \left\{\left\langle \tilde{\omega_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}(\wp)}}, \tilde{\zeta_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}(\wp)}}, 1-\tilde{\chi_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}(\wp)}}, \tilde{\psi_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}(\wp)}}, \tilde{\xi_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}(\wp)}} \right\rangle\right\}$$

Remarks:

- Intersection (Minimum) and Union (Maximum) define the lower and upper bounds of uncertainty.
- The minimum operation ensures conservativeness, preserving the lowest truth membership while increasing ignorance and falsity.
- The maximum operation provides an optimistic approach, allowing for alternatives that have higher truth degrees to be preferred.
- The complement operation reverses the truth and falsity degrees, helping to evaluate opposite perspectives in decision-making.

iv. Algrbraic sum of CPNS

$$\tilde{\flat_1} \oplus \tilde{\flat_2} = \left\{ \begin{array}{l} \left(\tilde{\xi_1}(\wp) + \tilde{\xi_2}(\wp) - \tilde{\xi_1}(\wp)\cdot\tilde{\xi_2}(\wp)\right).\mathfrak{e}^{j2\pi}\left(\Theta_{\tilde{\xi_1}(\wp)} + \Theta_{\tilde{\xi_2}(\wp)} - \Theta_{\xi_{\underset{\approx}{\tilde{\xi}}1}(\wp)}\cdot\Theta_{\tilde{\xi_2}(\wp)}\right), \\ \left(\tilde{\psi_1}(\wp) + \tilde{\psi_2}(\wp) - \tilde{\psi_1}(\wp)\cdot\tilde{\psi_2}(\wp)\right).\mathfrak{e}^{j2\pi}\left(\Theta_{\tilde{\psi_1}(\wp)} + \Theta_{\tilde{\psi_{a_2}}(\wp)} - \Theta_{\tilde{\psi_1}(\wp)}\cdot\Theta_{\tilde{\psi_2}(\wp)}\right), \\ \left(\tilde{\chi_1}(\wp)\cdot\tilde{\chi_2}(\wp)\right).\mathfrak{e}^{j2\pi}\left(\Theta_{\tilde{\chi_1}(\wp)}\cdot\Theta_{\tilde{\chi_2}(\wp)}\right), \\ \left(\tilde{\zeta_1}(\wp)\cdot\tilde{\zeta_2}(\wp)\right).\mathfrak{e}^{j2\pi}\left(\Theta_{\tilde{\zeta_1}(\wp)}\cdot\Theta_{\tilde{\zeta_2}(\wp)}\right), \\ \left(\tilde{\omega_1}(\wp)\cdot\tilde{\omega_2}(\wp)\right).\mathfrak{e}^{j2\pi}\left(\Theta_{\tilde{\omega_1}(\wp)}\cdot\Theta_{\tilde{\omega_2}(\wp)}\right) \end{array} \right\}$$

v. Algebraic product of CPNS

$$\tilde{\flat_1} \otimes \tilde{\flat_2} = \left\{ \begin{array}{l} \left(\tilde{\xi_1}(\wp) \cdot \tilde{\xi_2}(\wp)\right) .\mathfrak{e}^{j2\pi} \left(\Theta_{\tilde{\xi_1}(\wp)} \cdot \Theta_{\tilde{\xi_2}(\wp)}\right), \\ \left(\tilde{\psi_1}(\wp) \cdot \tilde{\psi_2}(\wp)\right) .\mathfrak{e}^{j2\pi} \left(\Theta_{\tilde{\psi_1}(\wp)} \cdot \Theta_{\tilde{\psi_2}(\wp)}\right), \\ \left(\tilde{\chi_1}(\wp) + \tilde{\chi_2}(\wp) - \tilde{\chi_1}(\wp) \cdot \tilde{\chi_2}(\wp)\right) .\mathfrak{e}^{j2\pi} \left(\Theta_{\tilde{\chi_1}(\wp)} + \Theta_{\tilde{\chi_2}(\wp)} - \Theta_{\tilde{\chi_1}(\wp)} \cdot \Theta_{\tilde{\chi_2}(\wp)}\right), \\ \left(\tilde{\zeta_1}(\wp) + \tilde{\zeta_2}(\wp) - \tilde{\zeta_1}(\wp) \cdot \tilde{\zeta_2}(\wp)\right) .\mathfrak{e}^{j2\pi} \left(\Theta_{\tilde{\zeta_1}(\wp)} + \Theta_{\tilde{\zeta_2}(\wp)} - \Theta_{\tilde{\zeta_1}(\wp)} \cdot \Theta_{\tilde{\zeta_2}(\wp)}\right), \\ \left(\tilde{\omega_1}(\wp) + \tilde{\omega_2}(\wp) - \tilde{\omega_1}(\wp) \cdot \tilde{\omega_2}(\wp)\right) .\mathfrak{e}^{j2\pi} \left(\Theta_{\tilde{\omega_1}(\wp)} + \Theta_{\tilde{\omega_2}(\wp)} - \Theta_{\tilde{\omega_1}(\wp)} \cdot \Theta_{\tilde{\omega_2}(\wp)}\right) \end{array} \right\}$$

vi. Scalar multiplication of CPNS

$$k\tilde{\flat_a} = \left\{ \begin{array}{l} \left[1 - \left(1 - \tilde{\xi_a}(\wp)\right)^k\right] .\mathfrak{e}^{j2\pi} \left[1 - \left(1 - \Theta_{\tilde{\xi_a}(\wp)}\right)^k\right], \\ \left[1 - \left(1 - \tilde{\psi_a}(\wp)\right)^k\right] .\mathfrak{e}^{j2\pi} \left[1 - \left(1 - \Theta_{\tilde{\psi_a}(\wp)}\right)^k\right], \\ \left[\tilde{\chi}_a^k\right] .\mathfrak{e}^{j2\pi} \left[\Theta^k_{\tilde{\chi_a}(\wp)}\right], \\ \left[\tilde{\zeta}_a^k\right] .\mathfrak{e}^{j2\pi} \left[\Theta^k_{\tilde{\zeta_a}(\wp)}\right], \\ \left[\tilde{\omega}_a^k\right] .\mathfrak{e}^{j2\pi} \left[\Theta^k_{\tilde{\omega_a}(\wp)}\right] \end{array} \right\} \forall_{\mathfrak{a}} = 1, 2$$

vii. Power of an CPNS

$$\left[\tilde{\flat_a}\right]^k = \left\{ \begin{array}{l} \left[\tilde{\xi}_a^k\right] .\mathfrak{e}^{j2\pi} \left[\Theta^k_{\tilde{\xi_a}(\wp)}\right], \\ \left[\tilde{\psi}_a^k\right] .\mathfrak{e}^{j2\pi} \left[\Theta^k_{\tilde{\psi_a}(\wp)}\right], \\ \left[1 - \left(1 - \tilde{\chi_a}(\wp)\right)^k\right] .\mathfrak{e}^{j2\pi} \left[1 - \left(1 - \Theta_{\tilde{\chi_a}(\wp)}\right)^k\right], \\ \left[1 - \left(1 - \tilde{\zeta_a}(\wp)\right)^k\right] .\mathfrak{e}^{j2\pi} \left[1 - \left(1 - \Theta_{\tilde{\zeta_a}(\wp)}\right)^k\right], \\ \left[1 - \left(1 - \tilde{\omega_a}(\wp)\right)^k\right] .\mathfrak{e}^{j2\pi} \left[1 - \left(1 - \Theta_{\tilde{\omega_a}(\wp)}\right)^k\right], \end{array} \right\} \forall_{\mathfrak{a}} = 1, 2$$

Remarks:

- Algebraic Sum ensures that truth and contradiction accumulate, while ignorance and falsity reduce the final outcome.
- Algebraic Product enforces a multiplicative effect, amplifying the most significant decision factors while minimizing contradictions.
- Scalar multiplication models progressive decision weighting, where increasing the weight shifts focus toward stronger alternatives.
- Power operation influences dominance, allowing repeated evaluation of an alternatives impact across different conditions.

Definition: A Score function $\mathbb{S}\left(\tilde{\flat_a}\right)$ and Accuracy function $\mathbb{H}\left(\tilde{\flat_a}\right)$ on $\tilde{\flat_a} = \left\langle \tilde{\xi_a}(\wp), \tilde{\psi_a}(\wp), \tilde{\chi_a}(\wp), \tilde{\zeta_a}(\wp), \tilde{\omega_a}(\wp) \right\rangle$ are defined as

$$\mathbb{S}\left(\tilde{\flat_a}\right) = 1/5\left[\left(\tilde{\xi_a}(\wp) + \tilde{\psi_a}(\wp) - \tilde{\chi_a}(\wp) - \tilde{\zeta_a}(\wp) - \tilde{\omega_a}(\wp)\right) + \left(\Theta_{\tilde{\xi_a}(\wp)} + \Theta_{\tilde{\psi_a}(\wp)} - \Theta_{\tilde{\chi_a}(\wp)} - \Theta_{\tilde{\zeta_a}(\wp)} - \Theta_{\tilde{\omega_a}(\wp)}\right)\right]$$

$$\mathbb{H}\left(\tilde{\flat_a}\right) = 1/5\left[\left(\tilde{\xi_a}(\wp) + \tilde{\psi_a}(\wp) + \tilde{\chi_a}(\wp) + \tilde{\zeta_a}(\wp) + \tilde{\omega_a}(\wp)\right) + \left(\Theta_{\tilde{\xi_a}(\wp)} + \Theta_{\tilde{\psi_a}(\wp)} + \Theta_{\tilde{\chi_a}(\wp)} + \Theta_{\tilde{\zeta_a}(\wp)} + \Theta_{\tilde{\omega_a}(\wp)}\right)\right]$$

where $\mathbb{S}\left(\tilde{\flat_a}\right), \mathbb{H}\left(\tilde{\flat_a}\right) \in [0,1]$

Definition: An order relation between two CPNSs $\tilde{\flat_1}$ and $\tilde{\flat_2}$ is of the form

1. If $\mathbb{S}\left(\tilde{\flat_{a_1}}\right) > \mathbb{S}\left(\tilde{\flat_{a_2}}\right)$ then $\tilde{\flat_{a_1}} > \tilde{\flat_{a_2}}$, similarly if $\mathbb{H}\left(\tilde{\flat_{a_1}}\right) > \mathbb{H}\left(\tilde{\flat_{a_2}}\right)$ then $\tilde{\flat_{a_1}} > \tilde{\flat_{a_2}}$

2. If $\mathbb{S}\left(\tilde{\flat_{a_1}}\right) = \mathbb{S}\left(\tilde{\flat_{a_2}}\right)$ then $\tilde{\flat_{a_1}} = \tilde{\flat_{a_2}}$, similarly if $\mathbb{H}\left(\tilde{\flat_{a_2}}\right) = \mathbb{H}\left(\tilde{\flat_{a_2}}\right)$ then $\tilde{\flat_{a_1}} = \tilde{\flat_{a_2}}$

Theorem 3.1. *Let* $\tilde{\flat_a} = \left\{\left\langle \tilde{\xi_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}(\wp)}}, \tilde{\psi_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}(\wp)}}, \tilde{\chi_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}(\wp)}}, \tilde{\zeta_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}(\wp)}}, \tilde{\omega_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}(\wp)}} \right\rangle\right\}$ *be a CPNN.*

Then, the score function $\mathbb{S}\left(\tilde{\flat_a}\right) = 1/5\left[\left(\tilde{\xi_a}(\wp) + \tilde{\psi_a}(\wp) - \tilde{\chi_a}(\wp) - \tilde{\zeta_a}(\wp) - \tilde{\omega_a}(\wp)\right) + \left(\Theta_{\tilde{\xi_a}(\wp)} + \Theta_{\tilde{\psi_a}(\wp)} - \Theta_{\tilde{\chi_a}(\wp)} - \Theta_{\tilde{\zeta_a}(\wp)} - \Theta_{\tilde{\omega_a}(\wp)}\right)\right]$ *is a monotonic increasing function with* $\tilde{\xi_a}(\wp), \Theta_{\tilde{\xi_a}(\wp)}, \tilde{\psi_a}(\wp), \Theta_{\tilde{\psi_a}(\wp)}$ *and monotonic decreasing function with* $\tilde{\chi_a}(\wp), \Theta_{\tilde{\chi_a}(\wp)}, \tilde{\zeta_a}(\wp), \Theta_{\tilde{\zeta_a}(\wp)}, \tilde{\omega_a}(\wp), \Theta_{\tilde{\omega_a}(\wp)}$.

Proof: The proof is readily apparent.

Theorem 3.2. *Let* $\tilde{\flat_a} = \left\{\left\langle \tilde{\xi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}}}, \tilde{\psi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}}}, \tilde{\chi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}}}, \tilde{\zeta_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}}}, \tilde{\omega_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}}} \right\rangle\right\}$ $\forall \mathfrak{v} = 1,2$ *be two CPNNs. Let* $\tilde{\flat'_a} = \left\{\left\langle \tilde{\xi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}}}, \tilde{\psi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}}}, \tilde{\chi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}}}, \tilde{\zeta_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}}}, \tilde{\omega_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}}} \right\rangle\right\}$ $\forall \mathfrak{v} = 1,2$ *be their associated inverse (complement) function. Then,* $\mathbb{S}\left(\tilde{\flat_{a_1}}\right) \leq \mathbb{S}\left(\tilde{\flat_{a_2}}\right)$ *iff* $\mathbb{S}\left(\tilde{\flat'_{a_1}}\right) \leq \mathbb{S}\left(\tilde{\flat'_{a_2}}\right)$.

Proof :

Using score function,

$$\mathbb{S}\left(\tilde{\flat_{a_1}}\right) = \frac{1}{5}\left[\left(\tilde{\xi_{a_1}}(\wp) + \tilde{\psi_{a_1}}(\wp) - \tilde{\chi_{a_1}}(\wp) - \tilde{\zeta_{a_1}}(\wp) - \tilde{\omega_{a_1}}(\wp)\right) + j\frac{1}{5}\left(\Theta_{\tilde{\xi_{a_1}}(\wp)} + \Theta_{\tilde{\psi_{a_1}}(\wp)} - \Theta_{\tilde{\chi_{a_1}}(\wp)} - \Theta_{\tilde{\zeta_{a_1}}(\wp)} - \Theta_{\tilde{\omega_{a_1}}(\wp)}\right)\right],$$

$$\mathbb{S}\left(\tilde{\flat_{a_2}}\right) = \frac{1}{5}\left[\left(\tilde{\xi_{a_2}}(\wp) + \tilde{\psi_{a_2}}(\wp) - \tilde{\chi_{a_2}}(\wp) - \tilde{\zeta_{a_2}}(\wp) - \tilde{\omega_{a_2}}(\wp)\right) + j\frac{1}{5}\left(\Theta_{\tilde{\xi_{a_2}}(\wp)} + \Theta_{\tilde{\psi_{a_2}}(\wp)} - \Theta_{\tilde{\chi_{a_2}}(\wp)} - \Theta_{\tilde{\zeta_{a_2}}(\wp)} - \Theta_{\tilde{\omega_{a_2}}(\wp)}\right)\right]$$

The inverse(complement) function for two CPNNs,

$$\mathbb{S}\left(\tilde{\flat'_{a_1}}\right) = \frac{1}{5}\left[\left(-\tilde{\chi_{a_1}}(\wp) - \tilde{\zeta_{a_1}}(\wp) - \tilde{\omega_{a_1}}(\wp) + \tilde{\xi_{a_1}}(\wp) + \tilde{\psi_{a_1}}(\wp)\right) + j\frac{1}{5}\left(-\Theta_{\tilde{\chi_{a_1}}(\wp)} - \Theta_{\tilde{\zeta_{a_1}}(\wp)} - \Theta_{\tilde{\omega_{a_1}}(\wp)} + \Theta_{\tilde{\xi_{a_1}}(\wp)} + \Theta_{\tilde{\psi_{a_1}}(\wp)}\right)\right]$$

$$\mathbb{S}\left(\tilde{\flat'_{a_2}}\right) = \frac{1}{5}\left[\left(-\tilde{\chi_{a_2}}(\wp) - \tilde{\zeta_{a_2}}(\wp) - \tilde{\omega_{a_2}}(\wp) + \tilde{\xi_{a_2}}(\wp) + \tilde{\psi_{a_2}}(\wp)\right) + j\frac{1}{5}\left(-\Theta_{\tilde{\chi_{a_2}}(\wp)} - \Theta_{\tilde{\zeta_{a_2}}(\wp)} - \Theta_{\tilde{\omega_{a_2}}(\wp)} + \Theta_{\tilde{\xi_{a_2}}(\wp)} + \Theta_{\tilde{\psi_{a_2}}(\wp)}\right)\right]$$

To prove the equivalence:

$\mathbb{S}\left(\tilde{\flat_{a_1}}\right) \leq \mathbb{S}\left(\tilde{\flat_{a_2}}\right)$ *iff* $\mathbb{S}\left(\tilde{\flat'_{a_1}}\right) \leq \mathbb{S}\left(\tilde{\flat'_{a_2}}\right)$.

We analyse each term in both real and imaginary part, since it follows the same mathematical structure, their inequalities are preserved under negation and transformation. Thus, the theorem holds.

Theorem 3.3. *Let* $\tilde{\flat_a} = \left\{\left\langle \tilde{\xi_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}(\wp)}}, \tilde{\psi_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}(\wp)}}, \tilde{\chi_a}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}(\wp)}}, \tilde{\zeta_\psi}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_\psi}(\wp)}}, \tilde{\omega_\psi}(\wp).\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}(\wp)}} \right\rangle\right\}$ *be a CPNNs. Then, the accuracy function* $\mathbb{H}\left(\tilde{\flat_a}\right) = 1/5\left[\left(\tilde{\xi_a}(\wp) + \tilde{\psi_a}(\wp) + \tilde{\chi_\psi}(\wp) + \tilde{\zeta_a}(\wp) + \tilde{\omega_a}(\wp)\right) + \left(\Theta_{\tilde{\xi_a}(\wp)} + \Theta_{\tilde{\psi_a}(\wp)} + \Theta_{\tilde{\chi_a}(\wp)} + \Theta_{\tilde{\zeta_a}(\wp)} + \Theta_{\tilde{\omega_a}(\wp)}\right)\right]$ *is a monotonic increasing function with* $\tilde{\xi_a}(\wp), \Theta_{\tilde{\xi_a}(\wp)}, \tilde{\psi_a}(\wp), \Theta_{\tilde{\psi_a}(\wp)}$ *and monotonic decreasing function with* $\tilde{\chi_a}(\wp), \Theta_{\tilde{\chi_a}(\wp)}, \tilde{\zeta_a}(\wp), \Theta_{\tilde{\zeta_a}(\wp)}, \tilde{\omega_a}(\wp), \Theta_{\tilde{\omega_a}(\wp)}$.

Proof: The proof is obvious.

Theorem 3.4. *Let* $\tilde{\flat_a} = \left\{\left\langle \tilde{\xi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}}}, \tilde{\psi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}}}, \tilde{\chi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}}}, \tilde{\zeta_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}}}, \tilde{\omega_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}}} \right\rangle\right\}$ $\forall \mathfrak{v} = 1, 2$ *be two CPNNs. Then,* $\tilde{\flat'_a} = \left\{\left\langle \tilde{\xi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}}}, \tilde{\psi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}}}, \tilde{\chi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}}}, \tilde{\zeta_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}}}, \tilde{\omega_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}}} \right\rangle\right\}$ $\forall \mathfrak{v} = 1, 2$ *be their associated inverse (complement) function respectively. Then we have* $\mathbb{H}\left(\tilde{\flat_{a_1}}\right) = \mathbb{H}\left(\tilde{\flat_{a_2}}\right)$.

Proof: The proof is readily apparent.

4. 4|Complex Pentapartitioned Neutrosophic Operators

In this section, we present the Complex Pentapartitioned Neutrosophic operators with their properties.

Definition: Let $\tilde{\flat_a} = \left\{\left\langle \tilde{\xi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}}}, \tilde{\psi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}}}, \tilde{\chi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}}}, \tilde{\zeta_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}}}, \tilde{\omega_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}}} \right\rangle\right\}$ $\forall \mathfrak{v} = 1, 2...n$ be CPN values. Then the three unitary operators of CPNSs which consist of power, rotation and reflection are stated as follows:

- Power of $\tilde{\flat_a}$

$$\tilde{\flat_a^k} = \left\langle \tilde{\xi_a^k}.\mathfrak{e}^{j2\pi\Theta^k_{\tilde{\xi_a}}}, \tilde{\psi_a^k}.\mathfrak{e}^{j2\pi\Theta^k_{\tilde{\psi_a}}}, \tilde{\chi_a^k}.\mathfrak{e}^{j2\pi\Theta^k_{\tilde{\chi_a}}}, \tilde{\zeta_a^k}.\mathfrak{e}^{j2\pi\Theta^k_{\tilde{\zeta_a}}}, \tilde{\omega_a^k}.\mathfrak{e}^{j2\pi\Theta^k_{\tilde{\omega_a}}} \right\rangle, k \neq 0.$$

- Rotation of $\tilde{\flat_a}$ by an angle θ

$$\mathrm{s}Rot_\theta\left(\tilde{\flat_a^k}\right) = \left\langle \tilde{\xi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}}+\theta}, \tilde{\psi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}}+\theta}, \tilde{\chi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}}+\theta}, \tilde{\zeta_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}}+\theta}, \tilde{\omega_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}}+\theta} \right\rangle.$$

- Reflection of $\tilde{\flat_a}$

$$Ref\left(\tilde{\flat_a^k}\right) = \left\langle \tilde{\xi_a}.\mathfrak{e}^{-j2\pi\Theta_{\tilde{\xi_a}}}, \tilde{\psi_a}.\mathfrak{e}^{-j2\pi\Theta_{\tilde{\psi_a}}}, \tilde{\chi_a}.\mathfrak{e}^{-j2\pi\Theta_{\tilde{\chi_a}}}, \tilde{\zeta_a}.\mathfrak{e}^{-j2\pi\Theta_{\tilde{\zeta_\psi}}}, \tilde{\omega_a}.\mathfrak{e}^{-j2\pi\Theta_{\tilde{\omega_a}}} \right\rangle.$$

Let $\tilde{\flat_a} = \left\langle \tilde{\xi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}}}, \tilde{\psi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}}}, \tilde{\chi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}}}, \tilde{\zeta_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}}}, \tilde{\omega_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}}} \right\rangle$ $\forall \mathfrak{v} = 1, 2$ be two CPNNs, then the following are frequently utilized binary operations:

(i). Algebric sum

$\tilde{\flat_{a_1}} + \tilde{\flat_{a_2}} =$

$$\left\{\begin{array}{l}\left(\tilde{\xi_{a_1}}\cos\left(2\pi\Theta_{\tilde{\xi_{a_1}}}\right)+\tilde{\xi_{a_2}}\cos\left(2\pi\Theta_{\tilde{\xi_{a_2}}}\right)\right)+j\left(\tilde{\xi_{a_1}}\sin\left(2\pi\Theta_{\tilde{\xi_{a_1}}}\right)+\tilde{\xi_{a_2}}\sin\left(2\pi\Theta_{\tilde{\xi_{a_2}}}\right)\right),\\ \left(\tilde{\psi_{a_1}}\cos\left(2\pi\Theta_{\tilde{\psi_{a_1}}}\right)+\tilde{\psi_{a_2}}\cos\left(2\pi\Theta_{\tilde{\psi_{a_2}}}\right)\right)+j\left(\tilde{\psi_{a_1}}\sin\left(2\pi\Theta_{\tilde{\psi_{a_1}}}\right)+\tilde{\psi_{a_2}}\sin\left(2\pi\Theta_{\tilde{\psi_{a_2}}}\right)\right),\\ \left(\tilde{\chi_{a_1}}\cos\left(2\pi\Theta_{\chi\tilde{a}_1}\right)+\tilde{\chi_{a_2}}\cos\left(2\pi\Theta_{\chi\tilde{a}_2}\right)\right)+j\left(\tilde{\chi_{a_1}}\sin\left(2\pi\Theta_{\chi\tilde{a}_1}\right)+\tilde{\chi_{a_2}}\sin\left(2\pi\Theta_{\chi\tilde{a}_2}\right)\right),\\ \left(\tilde{\zeta_{a_1}}\cos\left(2\pi\Theta_{\tilde{\zeta_{a_1}}}\right)+\tilde{\zeta_{a_2}}\cos\left(2\pi\Theta_{\tilde{\zeta_{a_2}}}\right)\right)+j\left(\tilde{\zeta_{a_1}}\sin\left(2\pi\Theta_{\tilde{\zeta_{\mathfrak{v}_1}}}\right)+\tilde{\zeta_{a_2}}\sin\left(2\pi\Theta_{\tilde{\zeta_{a_2}}}\right)\right),\\ \left(\tilde{\omega_{a_1}}\cos\left(2\pi\Theta_{\omega\tilde{a}_1}\right)+\tilde{\omega_{a_2}}\cos\left(2\pi\Theta_{\omega\tilde{a}_2}\right)\right)+j\left(\tilde{\omega_{a_1}}\sin\left(2\pi\Theta_{\omega\tilde{a}_1}\right)+\tilde{\omega_{a_2}}\sin\left(2\pi\Theta_{\omega\tilde{a}_2}\right)\right)\end{array}\right\}$$

(ii). Algebric product

$$\tilde{\flat_{a_1}}*\tilde{\flat_{a_2}}=\left\{\begin{array}{l}\left\langle\tilde{\xi_{a_1}}\tilde{\xi_{a_2}}.\mathfrak{e}^{j2\pi\left(\Theta_{\xi\tilde{a}_1}+\Theta_{\xi\tilde{a}_2}\right)}\right\rangle,\\ \left\langle\tilde{\psi_{a_1}}\tilde{\psi_{a_2}}.\mathfrak{e}^{j2\pi\left(\Theta_{\psi\tilde{a}_1}+\Theta_{\psi\tilde{a}_2}\right)}\right\rangle,\\ \left\langle\tilde{\chi_{a_1}}\tilde{\chi_{a_2}}.\mathfrak{e}^{j2\pi\left(\Theta_{\chi\tilde{a}_1}+\Theta_{\chi\tilde{a}_2}\right)}\right\rangle,\\ \left\langle\tilde{\zeta_{a_1}}\tilde{\zeta_{a_2}}.\mathfrak{e}^{j2\pi\left(\Theta_{\zeta\tilde{a}_1}+\Theta_{\zeta\tilde{a}_2}\right)}\right\rangle,\\ \left\langle\tilde{\omega_{a_1}}\tilde{\omega_{a_2}}.\mathfrak{e}^{j2\pi\left(\Theta_{\omega\tilde{a}_1}+\Theta_{\omega\tilde{a}_2}\right)}\right\rangle\end{array}\right\}$$

Definition: The Complex Pentapartitioned Neutrosophic Operators (CPNOs) are explained on the complex plane and the geometric properties such as rotational invariance and reflectional invariance is stated as follows:

(i). A CPNOs $\Delta:\tilde{\flat}_a^n\rightarrow\tilde{\flat_a}$ is rotationally invariant iff, for any θ,

$$\Delta\left(Rot_\theta\left(\tilde{\flat_{a_1}}\right),Rot_\theta\left(\tilde{\flat_{a_2}}\right),...,Rot_\theta\left(\tilde{\flat_{a_n}}\right)\right)=Rot_\theta\left(\Delta\left(\tilde{\flat_{a_1}},\tilde{\flat_{a_2}},...,\tilde{\flat_{a_n}}\right)\right)\tag{1}$$

(ii). A CPNOs $\Delta:\tilde{\flat}_a^n\rightarrow\tilde{\flat_a}$ is reflectionally invariant iff,

$$\Delta\left(Ref\left(\tilde{\flat_{a_1}}\right),Ref\left(\tilde{\flat_{a_2}}\right),...,Ref\left(\tilde{\flat_{a_n}}\right)\right)=Ref\left(\Delta\left(\tilde{\flat_{a_1}},\tilde{\flat_{a_2}},...,\tilde{\flat_{a_n}}\right)\right)\tag{2}$$

5. 4.1|Aggregating Operators on CPNs

Definition: Let $\tilde{\flat_a}=\left\{\left\langle\tilde{\xi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}}},\tilde{\psi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}}},\tilde{\chi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}}},\tilde{\zeta_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}}},\tilde{\omega_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}}}\right\rangle\right\}$ $\forall\mathfrak{a}=1,2...n$ be a number of CPNNs and $w=(w_1,w_2,w_3,...,w_n)^T$ be a real valued weight vector of $\tilde{\flat_a}$ with $\sum_{\mathfrak{a}=n}^n w_\mathfrak{a}=1$. Then the Complex Pentapartitioned Neutrosophic Weighted Arithmetic Average (CPNWAA) operator is a function CPNWAA: $\tilde{\flat}_a^n\rightarrow\tilde{\flat_a}$, where

$$CPNWAA\left(\tilde{\flat_{a_1}},\tilde{\flat_{a_2}},\tilde{\flat_{a_3}},.....,\tilde{\flat_{a_n}}\right)=\left\{\begin{array}{l}\left(\sum_{\mathfrak{v}=1}^n w_\mathfrak{a}\tilde{\xi_a}\right).\mathfrak{e}^{j2\pi\left(\sum_{\mathfrak{a}=1}^n w_\mathfrak{a}\Theta_{\tilde{\xi_a}}\right)},\\ \left(\sum_{\mathfrak{a}=1}^n w_\mathfrak{a}\tilde{\psi_a}\right).\mathfrak{e}^{j2\pi\left(\sum_{\mathfrak{a}=1}^n w_\mathfrak{a}\Theta_{\tilde{\psi_a}}\right)},\\ \left(\sum_{\mathfrak{a}=1}^n w_\mathfrak{a}\tilde{\chi_a}\right).\mathfrak{e}^{j2\pi\left(\sum_{\mathfrak{a}=1}^n w_\mathfrak{a}\Theta_{\tilde{\chi_a}}\right)},\\ \left(\sum_{\mathfrak{a}=1}^n w_\mathfrak{a}\tilde{\zeta_a}\right).\mathfrak{e}^{j2\pi\left(\sum_{\mathfrak{a}=1}^n w_\mathfrak{a}\Theta_{\tilde{\zeta_a}}\right)},\\ \left(\sum_{\mathfrak{a}=1}^n w_\mathfrak{a}\tilde{\omega_a}\right).\mathfrak{e}^{j2\pi\left(\sum_{\mathfrak{a}=1}^n w_\mathfrak{a}\Theta_{\tilde{\omega_a}}\right)}\end{array}\right\}\forall_\mathfrak{a}=1,2,...,n$$

Definition: Let $\tilde{\flat_a}=\left\{\left\langle\tilde{\xi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\xi_a}}},\tilde{\psi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\psi_a}}},\tilde{\chi_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\chi_a}}},\tilde{\zeta_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\zeta_a}}},\tilde{\omega_a}.\mathfrak{e}^{j2\pi\Theta_{\tilde{\omega_a}}}\right\rangle\right\}$ $\forall\mathfrak{a}=1,2...n$ be a number of CPNNs and $w=(w_1,w_2,w_3,...,w_n)^\xi$ be a real valued weight vector of $\tilde{\flat}_a$ with $\sum_{\mathfrak{a}=n}^{n} w_{\mathfrak{a}} = 1$, then a Complex Pentapartitioned Neutrosophic Weighted Geometric Average (CPNWGA) operator is a function CPNWGA: $\tilde{\flat}_a^n \to \tilde{\flat}_a$, where

$$CPNWGA\left(\tilde{\flat}_{a_1}, \tilde{\flat}_{a_2}, \tilde{\flat}_{a_3}, \ldots.., \tilde{\flat}_{a_n}\right) = \left\{ \begin{array}{l} \left(\prod_{\mathfrak{a}=1}^{n} w_{\mathfrak{a}} \tilde{\xi_a}\right).\mathfrak{e}^{j2\pi\left(\prod_{\mathfrak{a}=1}^{n} w_{\mathfrak{a}} \Theta_{\tilde{\xi_a}}\right)}, \\ \left(\prod_{\mathfrak{a}=1}^{n} w_{\mathfrak{a}} \tilde{\psi_a}\right).\mathfrak{e}^{j2\pi\left(\prod_{\mathfrak{a}=1}^{n} w_{\mathfrak{a}} \Theta_{\tilde{\psi_a}}\right)}, \\ \left(\prod_{\mathfrak{a}=1}^{n} w_{\mathfrak{a}} \tilde{\chi_a}\right).\mathfrak{e}^{j2\pi\left(\prod_{\mathfrak{a}=1}^{n} w_{\mathfrak{a}} \Theta_{\tilde{\chi_a}}\right)}, \\ \left(\prod_{\mathfrak{a}=1}^{n} w_{\mathfrak{a}} \tilde{\zeta_a}\right).\mathfrak{e}^{j2\pi\left(\prod_{\mathfrak{a}=1}^{n} w_{\mathfrak{a}} \Theta_{\tilde{\zeta_a}}\right)}, \\ \left(\prod_{\mathfrak{a}=1}^{n} w_{\mathfrak{a}} \tilde{\omega_a}\right).\mathfrak{e}^{j2\pi\left(\prod_{\mathfrak{a}=1}^{n} w_{\mathfrak{a}} \Theta_{\tilde{\omega_a}}\right)} \end{array} \right\} \forall_{\mathfrak{a}} = 1, 2, ..., n$$

Remarks:

- The CPNWAA operator is a linear combination of CPNS membership values, allowing for flexible trade-offs among different criteria.
- This operator is suitable for moderate decision-making, where alternatives that perform well on some criteria but not all can still be considered.
- The CPNWGA operator considers weighted multiplicative aggregation, which ensures a balance between expert judgments and individual membership functions.
- This operator is suitable for risk-averse decision-making, where only strong alternatives with high truth membership across multiple criteria are favored.

Example 5.1. Example:
Let $\tilde{\flat}_{a_1} = \left\langle 0.8\mathfrak{e}^{j2\pi 0.7}, 0.5\mathfrak{e}^{j2\pi 0.7}, 0.4\mathfrak{e}^{j2\pi 0.7}, 0.7\mathfrak{e}^{j2\pi 0.7}, 0.6\mathfrak{e}^{j2\pi 0.7}\right\rangle$,
$\tilde{\flat}_{a_2} = \left\langle 0.3\mathfrak{e}^{j2\pi 0.7}, 0.6\mathfrak{e}^{j2\pi 0.7}, 0.4\mathfrak{e}^{j2\pi 0.7}, 0.3\mathfrak{e}^{j2\pi 0.7}, 0.4\mathfrak{e}^{j2\pi 0.7}\right\rangle$,
$\tilde{\flat}_{a_3} = \left\langle 0.\mathfrak{e}^{j2\pi 0.7}, 0.8\mathfrak{e}^{j2\pi 0.7}, 0.8\mathfrak{e}^{j2\pi 0.7}, 0.8\mathfrak{e}^{j2\pi 0.7}, 0.8\mathfrak{e}^{j2\pi 0.7}\right\rangle$ and
$\tilde{\flat}_{a_4} = \left\langle 0.8\mathfrak{e}^{j2\pi 0.7}, 0.8\mathfrak{e}^{j2\pi 0.7}, 0.8\mathfrak{e}^{j2\pi 0.7}, 0.8\mathfrak{e}^{j2\pi 0.7}, 0.8\mathfrak{e}^{j2\pi 0.7}\right\rangle$ be four CPNNs and the weight vector of $\tilde{\flat}_{a_i}$ $(i = 1, 2, 3, 4)$ is $w = (0.25, 0.2, 0.15, 0.4)^T$; then the complex Pentapartitioned Neutrosophic (CPN) aggregation operator.

$CPNWGA\left(\tilde{\flat}_{a_1}, \tilde{\flat}_{a_2}, \tilde{\flat}_{a_3}, \tilde{\flat}_{a_4}\right)$
$= \left\langle 0.8\mathfrak{e}^{j0.7}, 0.5\mathfrak{e}^{j0.7}, 0.4\mathfrak{e}^{j0.7}, 0.7\mathfrak{e}^{j0.7}, 0.6\mathfrak{e}^{j0.7}\right\rangle$
$= \left\langle 0.8\mathfrak{e}^{j0.7}, 0.5\mathfrak{e}^{j0.7}, 0.4\mathfrak{e}^{j0.7}, 0.7\mathfrak{e}^{j0.7}, 0.6\mathfrak{e}^{j0.7}\right\rangle$
Idempotency, boundedness, and monotonicity are the essential properties of aggregation operators.

Theorem 5.2. *Let $\tilde{\flat}_{a_1}, \tilde{\flat}_{a_2}, \tilde{\flat}_{a_3}, \ldots., \tilde{\flat}_{a_n}$ be CPNVs and let w_a be the weight vectors. Then we have the following axioms:*

(1) *Idempotency: If all CPNVs are equal $\left(\tilde{\flat}_{a_1} = \tilde{\flat}_{a_2} = \ldots = \tilde{\flat}_{a_n}\right)$, then*

$$CPNWAA\left(\tilde{\flat_1} = \tilde{\flat_{a_2}} = ... = \tilde{\flat_{a_n}}\right) = \tilde{\mathbb{S}_1}$$

Proof: Since $\left(\tilde{\flat_{a_1}} = \tilde{\flat_{a_2}} = ... = \tilde{\flat_{a_n}}\right)$ *and* $\sum_{\mathfrak{v}=1}^{n} w_{\mathfrak{v}} = 1$, *then*

$$\begin{aligned} CPNWAA\left(\tilde{\flat_1}, \tilde{\flat_{a_2}}, ..., \tilde{\flat_{a_n}}\right) &= w_1\tilde{\flat_1} + w_2\tilde{\flat_{a_2}} + ... + w_n\tilde{\flat_{a_n}} \\ &= w_1\tilde{\flat_{a_1}} + w_2\tilde{\flat_{a_2}} + ... + w_n\tilde{\flat_{a_n}} \\ &= (w_1 + w_2 + ... + w_n)\,\tilde{\flat_{a_1}} \\ &= \tilde{\flat_{a_1}} \end{aligned}$$

(2) *Boundedness:* $\left|CPNFWAA\left(\tilde{\flat_{a_1}}, \tilde{\flat_{a_2}}, \tilde{\flat_{a_3}}, ..., \tilde{\flat_{a_n}}\right)\right| \leq max_a \left|\tilde{\flat_{\approxeq\mathfrak{v}}}\right|$

Proof: let $\tilde{\flat_a} = max_{\mathfrak{v}} \left|\tilde{\flat_{a_i}}\right|$. *Since* $\sum_{\mathfrak{v}=1}^{n} w_{\mathfrak{v}} = 1$, *then*

$$\begin{aligned} \left|CPNFWAA\left(\tilde{\flat_1}, \tilde{\flat_2}, \tilde{\flat_3}, ..., \tilde{\flat_n}\right)\right| &= \left|w_1\tilde{\flat_1} + w_2\tilde{\flat_2} + ... + w_n\tilde{\flat_n}\right| \\ &\leq \left|w_1\tilde{\flat_1}\right| + \left|w_2\tilde{\flat_2}\right| + ... + \left|w_n\tilde{\flat_n}\right| \\ &= w_1\left|\tilde{\flat_1}\right| + w_2\left|\tilde{\flat_{a_2}}\right| + ... + w_n\left|\tilde{\flat_n}\right| \\ &\leq w_1\tilde{\flat_1} + w_2\tilde{\flat_2} + ... + w_n\tilde{\flat_n} \\ &= (w_1 + w_2 + ... + w_{\mathfrak{n}})\,\tilde{\flat_a} \\ &= \tilde{\flat_a} \end{aligned}$$

Theorem 5.3. *Let* $\tilde{\flat_a} = \left\langle \tilde{\xi_{\flat a}}.\mathfrak{e}^{j2\pi\Theta_{\xi_{\tilde{\flat_a}}}}, \tilde{\psi_{\flat a}}.\mathfrak{e}^{j2\pi\Theta_{\psi_{\tilde{\flat_a}}}}, \tilde{\chi_{\flat a}}.\mathfrak{e}^{j2\pi\Theta_{\chi_{\tilde{\flat_a}}}}, \tilde{\zeta_{\flat a}}.\mathfrak{e}^{j2\pi\Theta_{\zeta_{\tilde{\flat_a}}}}, \tilde{\omega_{\flat a}}.\mathfrak{e}^{j2\pi\Theta_{\omega_{\tilde{\flat_a}}}} \right\rangle$ *and* $\tilde{\chi_a} = \left\langle \tilde{\xi_{\chi a}}.\mathfrak{e}^{j2\pi\Theta_{\xi_{\tilde{\chi_a}}}}, \tilde{\psi_{\chi a}}.\mathfrak{e}^{j2\pi\Theta_{\psi_{\tilde{\chi_a}}}}, \tilde{\chi_{\chi a}}.\mathfrak{e}^{j2\pi\Theta_{\chi_{\tilde{\chi_a}}}}, \tilde{\zeta_{\chi a}}.\mathfrak{e}^{j2\pi\Theta_{\zeta_{\tilde{\chi_a}}}}, \tilde{\omega_{\chi a}}.\mathfrak{e}^{j2\pi\Theta_{\omega_{\tilde{\chi_a}}}} \right\rangle$ $\forall a = 1, 2, ..., n$ *be two families of CPNVs. The weight vectors of CPNWGA operator be real valued i.e.,* $w_a \in [0, 1]\,\forall a$ *and* $\sum_{a=1}^{n} w_a = 1$. *Then we have the following axioms:*

- *Idempotency: If all CPNVs are equal i.e.* $\left(\tilde{\flat_{a_1}} = \tilde{\flat_{a_2}} = ... = \tilde{\flat_{a_n}}\right)$, *then* $CPNWGA\left(\tilde{\flat_{a_1}}, \tilde{\flat_{a_2}}, \tilde{\flat_{a_3}}, ..., \tilde{\flat_{a_n}}\right) = \tilde{\flat_a}$
- *Boundedness: Let* $\tilde{\flat_{a_1}}, \tilde{\flat_{a_2}}, ..., \tilde{\flat_{a_n}}$ *be a CPNVs.* $\left|CPNWGA\left(\tilde{\flat_{a_1}}, \tilde{\flat_{a_2}}, \tilde{\flat_{a_3}}, ..., \tilde{\flat_{a_n}}\right)\right| \leq max_a \left|\tilde{\flat_a}\right|$
- *Monotonicity: If* $\tilde{\xi_{\flat_a}} \geq \tilde{\xi_{\chi_a}}, \Theta_{\xi_{\tilde{\flat_a}}} \geq \Theta_{\xi_{\tilde{\chi_a}}}, \tilde{\psi_{\flat_a}} \leq \tilde{\psi_{\chi_a}}, \Theta_{\psi_{\tilde{\flat_a}}} \leq \Theta_{\psi_{\tilde{\chi_a}}}, \tilde{\chi_{\flat_a}} \leq \tilde{\chi_{\chi_a}}, \Theta_{\chi_{\tilde{\flat_a}}} \leq \Theta_{\chi_{\tilde{\chi_a}}}, \tilde{\zeta_{\flat_a}} \leq \tilde{\zeta_{\chi_a}}, \Theta_{\zeta_{\tilde{\flat_a}}} \leq \Theta_{\zeta_{\tilde{\chi_a}}}, \tilde{\omega_{\flat_a}} \leq \tilde{\omega_{\chi_a}}$ *and* $\Theta_{\omega_{\tilde{\flat_a}}} \leq \Theta_{\omega_{\tilde{\chi_a}}}$ *then* $\left|CPNWGA\left(\tilde{\flat_{a_1}}, \tilde{\flat_{a_2}}, ..., \tilde{\flat_{a_n}}\right)\right| \leq \left|CPNWGA\left(\tilde{\chi_{a_1}}, \tilde{\chi_{a_2}}, ..., \tilde{\chi_{a_n}}\right)\right|$

Proof: The proof of the theorem follows from Theorem 4.4

From the above theorems 5.5 and 5.6, we have proved that the axioms of CPNWAA and CPNWGA operators hold good.

Remarks:

- Idempotency: Ensures that if all inputs are identical, the result remains unchanged, maintaining consistency in decision-making.

- Boundedness: Guarantees that the aggregation result remains within a reasonable range, preventing extreme bias.
- Monotonicity: Preserves ranking consistency, ensuring that an improvement in any criterion leads to a better overall ranking.
- These properties ensure that the proposed aggregation operators are mathematically sound and practically applicable, making them robust tools for decision-making in agriculture and beyond.

6. **Application in MCDM**

Our objective in this section is to develop a new algorithm that includes CPN data for solving an MCDM problem, which has the given steps:

STEP 1: The gathered information has been transformed into CPNNs, Experts provide evaluations for alternatives using CPNS.

$$\mathbb{D} = (d_{i\tilde{\iota}})_{\substack{1\leq i\leq \phi \\ 1\leq \tilde{\iota}\leq \varphi}} =^{(} \tilde{\xi}_{ai\tilde{\iota}}, \tilde{\psi}_{ai\tilde{\iota}}, \tilde{\chi}_{ai\tilde{\iota}}, \tilde{\zeta}_{ai\tilde{\iota}}, \tilde{\omega}_{ai\tilde{\iota}})_{\substack{1\leq i\leq \phi \\ 1\leq \tilde{\iota}\leq \varphi}} \tag{3}$$

where, $\tilde{\xi}_a(\wp)$ denotes the degree of Truth, $\tilde{\psi}_a(\wp)$ the degree of Contradiction, $\tilde{\chi}_a(\wp)$ the degree of Ignorance, $\tilde{\zeta}_a(\wp)$ the degree of Unknown and $\tilde{\omega}_a(\wp)$ the degree of Falsity membership.

$$\mathbb{D} = (d_{i\tilde{\iota}}) = \begin{pmatrix} d_{11}, & d_{12}, & \cdots, & d_{1\phi} \\ d_{21}, & d_{22}, & \cdots, & d_{2\phi} \\ \vdots, & \vdots, & \vdots, & \vdots \\ d_{\varphi 1}, & d_{\varphi 2}, & \cdots, & d_{\varphi\phi} \end{pmatrix} \tag{4}$$

STEP 2: Build the normalized Complex Pentapartitioned Neutrosophic Decision Matrix.

$\mathbb{D}^* = [d^*_{i\tilde{\iota}}]$ from $\mathbb{D} = [d_{i\tilde{\iota}}]$

In order to create a collective decision from individual decisions, it is necessary to derive an aggregated Complex Pentapartitioned Neutrosophic decision matrix $\mathbb{D}^*$.

STEP 3: From the normalized decision matrix $\mathbb{D}^* = [d^*_{i\tilde{\iota}}]$ and the criteria weight vector w_i. The CPNWAA and CPNWGA operator is used to compute the aggregated value of multiple alternatives in a CPNS environment, ensuring a balanced contribution of all pentapartitioned membership degrees.

STEP 4: Determine each aggregated value's score value using score function, go with accuracy function only when scores are equal to resolve ties.

STEP 5: Rank the alternatives based on the absolute values of their scores.

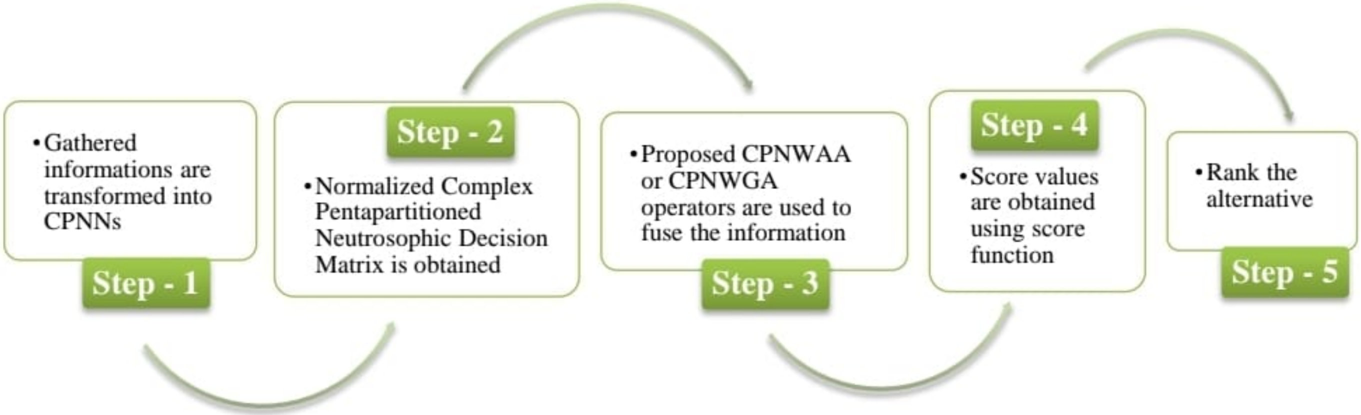

FIGURE 1. **Flowchart of the Algorithm**

7. 5.1 Numerical Illustration

In this section, we applied our proposed aggregation operators to a real-world agricultural issue: selecting the most suitable millet variety for farmers. The methodology involved the following steps:

- **Selection of Decision Makers (DMs):** Three experts were chosen to ensure a diverse and informed perspective: DM_1: An agronomy student with expertise in crop yield and agricultural science. DM_2: An organic farmer with practical experience in millet cultivation. DM_3: A naturopathic doctor knowledgeable about the medicinal properties of millets.
- **Definition of Alternatives and Criteria:** Alternatives: Five millet varieties were selected for evaluation: $\mathfrak{V}_1$ - Foxtail Millet, $\mathfrak{V}_2$ - Kodo Millet, $\mathfrak{V}_3$ - Pearl Millet, $\mathfrak{V}_4$ - Ragi and $\mathfrak{V}_5$ - Little Millet. Criteria: Seven critical factors influencing millet cultivation were identified, including: λ_1 : Yield Rate λ_2 : Seed Cost λ_3 : Stability Over Season λ_4 : Medicinal Value λ_5 : Market Demand
- **Data Collection Using Linguistic Variables:** Each DM used linguistic variables (e.g., Very High, High, Moderate, Low, Very Low) to evaluate the performance of each alternative against the criteria. These linguistic evaluations were converted into complex pentapartitioned neutrosophic values using a predefined scale.
- **Application of Aggregation Operators:** The Complex Pentapartitioned Neutrosophic Weighted Arithmetic Average (CPNWAA) and Weighted Geometric Average (CPNWGA) operators were applied to aggregate the evaluations provided by the DMs.

These operators accounted for the weights assigned to each criterion by the DMs and incorporated the inherent uncertainty in their judgments.

- **Ranking of Alternatives:** Using the aggregated neutrosophic values, a ranking of millet varieties was derived. The rankings were computed based on the aggregated scores for each alternative, with higher scores indicating better suitability for cultivation under the given criteria.

Table3 [15] explains the transformation of linguistic variables with their Complex-Pentapartitioned Neutrosophic Numbers.

TABLE 3. **Linguistic Variables**

Linguistic Terms	**Complex-PNNs**
Extremely Productive (EP)	$(.9.\mathfrak{e}^{i2\pi(.5)}, .8.\mathfrak{e}^{i2\pi(.1)}, .2.\mathfrak{e}^{i2\pi(.2)}, .1.\mathfrak{e}^{i2\pi(.1)}, .1.\mathfrak{e}^{i2\pi(.1)})$
Highly Productive(HP)	$(.8.\mathfrak{e}^{i2\pi(.4)}, .7.\mathfrak{e}^{i2\pi(.2)}, .3.\mathfrak{e}^{i2\pi(.3)}, .2.\mathfrak{e}^{i2\pi(.2)}, .2.\mathfrak{e}^{i2\pi(.3)})$
Moderately Productive(MP)	$(.6.\mathfrak{e}^{i2\pi(.35)}, .6.\mathfrak{e}^{i2\pi(.35)}, .3.\mathfrak{e}^{i2\pi(.3)}, .4.\mathfrak{e}^{i2\pi(.3)}, .4.\mathfrak{e}^{i2\pi(.35)})$
Productive(P)	$(.5.\mathfrak{e}^{i2\pi(.35)}, .5.\mathfrak{e}^{i2\pi(.35)}, .5.\mathfrak{e}^{i2\pi(.35)}, .5.\mathfrak{e}^{i2\pi(.35)}, .5.\mathfrak{e}^{i2\pi(.35)})$
Slightly Productive(SP)	$(.4.\mathfrak{e}^{i2\pi(.6)}, .4.\mathfrak{e}^{i2\pi(.35)}, .5.\mathfrak{e}^{i2\pi(.35)}, .6.\mathfrak{e}^{i2\pi(.35)}, .6.\mathfrak{e}^{i2\pi(.35)})$
Lowly Productive(LP)	$(.3.\mathfrak{e}^{i2\pi(.3)}, .3.\mathfrak{e}^{i2\pi(.35)}, .7.\mathfrak{e}^{i2\pi(.4)}, .8.\mathfrak{e}^{i2\pi(.35)}, .8.\mathfrak{e}^{i2\pi(.4)})$
Extremely low Productive(ELP)	$(.1.\mathfrak{e}^{i2\pi(.3)}, .1.\mathfrak{e}^{i2\pi(.35)}, .8.\mathfrak{e}^{i2\pi(.4)}, .9.\mathfrak{e}^{i2\pi(.35)}, .9.\mathfrak{e}^{i2\pi(.4)})$

TABLE 4. Linguistic decision matrix D_{ij} by the DM_1

D_{ij}	λ_1	λ_2	λ_3	λ_4	λ_5
$\tilde{\mathfrak{V}}_1$	MP	P	SP	HP	HP
$\tilde{\mathfrak{V}}_2$	HP	MP	LP	MP	MP
$\tilde{\mathfrak{V}}_3$	HP	MP	LP	HP	HP
$\tilde{\mathfrak{V}}_4$	EP	MP	SP	HP	HP
$\tilde{\mathfrak{V}}_5$	MP	MP	LP	P	P

TABLE 5. Linguistic decision matrix D_{ij} by the DM_2

D_{ij}	λ_1	λ_2	λ_3	λ_4	λ_5
$\tilde{\mathfrak{V}}_1$	MP	EP	MP	EP	EP
$\tilde{\mathfrak{V}}_2$	MP	HP	P	EP	HP
$\tilde{\mathfrak{V}}_3$	HP	LP	EP	EP	MP
$\tilde{\mathfrak{V}}_4$	HP	HP	EP	HP	SP
$\tilde{\mathfrak{V}}_5$	MP	EP	EP	EP	EP

TABLE 6. Linguistic decision matrix D_{ij} by the DM_3

D_{ij}	λ_1	λ_2	λ_3	λ_4	λ_5
$\tilde{\mathfrak{V}}_1$	MP	HP	P	HP	HP
$\tilde{\mathfrak{V}}_2$	MP	HP	MP	MP	P
$\tilde{\mathfrak{V}}_3$	HP	EP	SP	HP	EP
$\tilde{\mathfrak{V}}_4$	HP	HP	P	EP	EP
$\tilde{\mathfrak{V}}_5$	MP	HP	P	P	SP

By Step 1, Table4 explains the information gathered from DM_1 - Agronomy Student is transformed to CPNNs as shown in Table7.

TABLE 7. Complex Pentapartitioned Neutrosophic decision matrix D_{ij} by the DM_1

	$\tilde{\mathfrak{V}}_1$
λ_1	$(.6.\mathfrak{e}^{i2\pi(.35)},.6.\mathfrak{e}^{i2\pi(.35)},.3.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.35)})$
λ_2	$(.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)})$
λ_3	$(.4.\mathfrak{e}^{i2\pi(.6)},.4.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)},.6.\mathfrak{e}^{i2\pi(.35)},.6.\mathfrak{e}^{i2\pi(.35)})$
λ_4	$(.8.\mathfrak{e}^{i2\pi(.4)},.7.\mathfrak{e}^{i2\pi(.2)},.3.\mathfrak{e}^{i2\pi(.3)},.2.\mathfrak{e}^{i2\pi(.2)},.2.\mathfrak{e}^{i2\pi(.3)})$
λ_5	$(.8.\mathfrak{e}^{i2\pi(.4)},.7.\mathfrak{e}^{i2\pi(.2)},.3.\mathfrak{e}^{i2\pi(.3)},.2.\mathfrak{e}^{i2\pi(.2)},.2.\mathfrak{e}^{i2\pi(.3)})$
	$\tilde{\mathfrak{V}}_2$
λ_1	$(.8.\mathfrak{e}^{i2\pi(.4)},.7.\mathfrak{e}^{i2\pi(.2)},.3.\mathfrak{e}^{i2\pi(.3)},.2.\mathfrak{e}^{i2\pi(.2)},.2.\mathfrak{e}^{i2\pi(.3)})$
λ_2	$(.6.\mathfrak{e}^{i2\pi(.35)},.6.\mathfrak{e}^{i2\pi(.35)},.3.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.35)})$
λ_3	$(.3.\mathfrak{e}^{i2\pi(.3)},.3.\mathfrak{e}^{i2\pi(.35)},.7.\mathfrak{e}^{i2\pi(.4)},.8.\mathfrak{e}^{i2\pi(.35)},.8.\mathfrak{e}^{i2\pi(.4)})$
λ_4	$(.6.\mathfrak{e}^{i2\pi(.35)},.6.\mathfrak{e}^{i2\pi(.35)},.3.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.35)})$
λ_5	$(.6.\mathfrak{e}^{i2\pi(.35)},.6.\mathfrak{e}^{i2\pi(.35)},.3.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.35)})$
	$\tilde{\mathfrak{V}}_3$
λ_1	$(.8.\mathfrak{e}^{i2\pi(.83)},.7.\mathfrak{e}^{i2\pi(.2)},.3.\mathfrak{e}^{i2\pi(.3)},.2.\mathfrak{e}^{i2\pi(.2)},.2.\mathfrak{e}^{i2\pi(.3)})$
λ_2	$(.6.\mathfrak{e}^{i2\pi(.35)},.6.\mathfrak{e}^{i2\pi(.35)},.3.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.35)})$
λ_3	$(.3.\mathfrak{e}^{i2\pi(.3)},.3.\mathfrak{e}^{i2\pi(.35)},.7.\mathfrak{e}^{i2\pi(.4)},.8.\mathfrak{e}^{i2\pi(.35)},.8.\mathfrak{e}^{i2\pi(.4)})$
λ_4	$(.8.\mathfrak{e}^{i2\pi(.4)},.7.\mathfrak{e}^{i2\pi(.2)},.3.\mathfrak{e}^{i2\pi(.3)},.2.\mathfrak{e}^{i2\pi(.2)},.2.\mathfrak{e}^{i2\pi(.3)})$
λ_5	$(.8.\mathfrak{e}^{i2\pi(.4)},.7.\mathfrak{e}^{i2\pi(.2)},.3.\mathfrak{e}^{i2\pi(.3)},.2.\mathfrak{e}^{i2\pi(.2)},.2.\mathfrak{e}^{i2\pi(.3)})$
	$\tilde{\mathfrak{V}}_4$
λ_1	$(.9.\mathfrak{e}^{i2\pi(.5)},.8.\mathfrak{e}^{i2\pi(.1)},.2.\mathfrak{e}^{i2\pi(.2)},.1.\mathfrak{e}^{i2\pi(.1)},.1.\mathfrak{e}^{i2\pi(.1)})$
λ_2	$(.6.\mathfrak{e}^{i2\pi(.35)},.6.\mathfrak{e}^{i2\pi(.35)},.3.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.35)})$
λ_3	$(.3.\mathfrak{e}^{i2\pi(.3)},.3.\mathfrak{e}^{i2\pi(.35)},.7.\mathfrak{e}^{i2\pi(.4)},.8.\mathfrak{e}^{i2\pi(.35)},.8.\mathfrak{e}^{i2\pi(.4)})$
λ_4	$(.8.\mathfrak{e}^{i2\pi(.4)},.7.\mathfrak{e}^{i2\pi(.2)},.3.\mathfrak{e}^{i2\pi(.3)},.2.\mathfrak{e}^{i2\pi(.2)},.2.\mathfrak{e}^{i2\pi(.3)})$
λ_5	$(.8.\mathfrak{e}^{i2\pi(.4)},.7.\mathfrak{e}^{i2\pi(.2)},.3.\mathfrak{e}^{i2\pi(.3)},.2.\mathfrak{e}^{i2\pi(.2)},.2.\mathfrak{e}^{i2\pi(.3)})$
	$\tilde{\mathfrak{V}}_5$
λ_1	$(.6.\mathfrak{e}^{i2\pi(.35)},.6.\mathfrak{e}^{i2\pi(.35)},.3.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.35)})$
λ_2	$(.6.\mathfrak{e}^{i2\pi(.35)},.6.\mathfrak{e}^{i2\pi(.35)},.3.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.3)},.4.\mathfrak{e}^{i2\pi(.35)})$
λ_3	$(.3.\mathfrak{e}^{i2\pi(.3)},.3.\mathfrak{e}^{i2\pi(.35)},.7.\mathfrak{e}^{i2\pi(.4)},.8.\mathfrak{e}^{i2\pi(.35)},.8.\mathfrak{e}^{i2\pi(.4)})$
λ_4	$(.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)})$
λ_5	$(.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)},.5.\mathfrak{e}^{i2\pi(.35)})$

And Table5 explains the information gathered from DM_2 - Organic Farmer is transformed to CPNNs as shown in table8.

TABLE 8. Complex Pentapartitioned Neutrosophic decision matrix D_{ij} by the DM_2

	$\tilde{\mathfrak{V}}_1$
λ_1	$(.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.3.e^{i2\pi(.5)},.4.e^{i2\pi(.3)},.4.e^{i2\pi(.35)})$
λ_2	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_3	$(.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.3.e^{i2\pi(.3)},.4.e^{i2\pi(.3)},.4.e^{i2\pi(.35)})$
λ_4	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_5	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
	$\tilde{\mathfrak{V}}_2$
λ_1	$(.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.3.e^{i2\pi(.3)},.4.e^{i2\pi(.3)},.4.e^{i2\pi(.35)})$
λ_2	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_3	$(.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)})$
λ_4	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_5	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
	$\tilde{\mathfrak{V}}_3$
λ_1	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_2	$(.3.e^{i2\pi(.3)},.3.e^{i2\pi(.35)},.6.e^{i2\pi(.4)},.7.e^{i2\pi(.35)},.7.e^{i2\pi(.4)})$
λ_3	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_4	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_5	$(.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.3.e^{i2\pi(.3)},.4.e^{i2\pi(.3)},.4.e^{i2\pi(.35)})$
	$\tilde{\mathfrak{V}}_4$
λ_1	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_2	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_3	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_4	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_5	$(.4.e^{i2\pi(.6)},.4.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)})$
	$\tilde{\mathfrak{V}}_5$
λ_1	$(.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.3.e^{i2\pi(.3)},.4.e^{i2\pi(.3)},.4.e^{i2\pi(.35)})$
λ_2	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_3	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_4	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_5	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$

Simillarly, Table6 the information gathered from DM_3 - Naturopathic Doctor is transformed to CPNNs as shown in table9.

TABLE 9. Complex Pentapartitioned Neutrosophic decision matrix D_{ij} by the DM_3

	$\tilde{\mathfrak{V}}_1$
λ_1	$(.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.3.e^{i2\pi(.3)},.4.e^{i2\pi(.3)},.4.e^{i2\pi(.35)})$
λ_2	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_3	$(.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)})$
λ_4	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_5	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
	$\tilde{\mathfrak{V}}_2$
λ_1	$(.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_2	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_3	$(.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.3.e^{i2\pi(.3)},.4.e^{i2\pi(.3)},.4.e^{i2\pi(.35)})$
λ_4	$(.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.3.e^{i2\pi(.3)},.4.e^{i2\pi(.3)},.4.e^{i2\pi(.35)})$
λ_5	$(.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)})$
	$\tilde{\mathfrak{V}}_3$
λ_1	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_2	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_3	$(.4.e^{i2\pi(.6)},.4.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)})$
λ_4	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_5	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
	$\tilde{\mathfrak{V}}_4$
λ_1	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_2	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_3	$(.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)})$
λ_4	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
λ_5	$(.9.e^{i2\pi(.5)},.8.e^{i2\pi(.1)},.2.e^{i2\pi(.2)},.1.e^{i2\pi(.1)},.1.e^{i2\pi(.1)})$
	$\tilde{\mathfrak{V}}_5$
λ_1	$(.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.3.e^{i2\pi(.3)},.4.e^{i2\pi(.3)},.4.e^{i2\pi(.35)})$
λ_2	$(.8.e^{i2\pi(.4)},.7.e^{i2\pi(.2)},.3.e^{i2\pi(.3)},.2.e^{i2\pi(.2)},.2.e^{i2\pi(.3)})$
λ_3	$(.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)})$
λ_4	$(.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.5.e^{i2\pi(.35)})$
λ_5	$(.4.e^{i2\pi(.35)},.4.e^{i2\pi(.35)},.5.e^{i2\pi(.35)},.6.e^{i2\pi(.35)},.6.e^{i2\pi(.35)})$

By Step 2, we now normalize the CPN Decision Matrix to get $\mathbb{D}^*$ and it is shown in Table10.

TABLE 10. Combined Complex Pentapartitioned Neutrosophic decision matrix D_{ij}

	$\tilde{\mathfrak{V}}_1$
λ_1	$(.306.\mathfrak{e}^{i2\pi(.367)},.306.\mathfrak{e}^{i2\pi(.367)},.153.\mathfrak{e}^{i2\pi(.255)},.204.\mathfrak{e}^{i2\pi(.194)},.204.\mathfrak{e}^{i2\pi(.255)})$
λ_2	$(.396.\mathfrak{e}^{i2\pi(.428)},.360.\mathfrak{e}^{i2\pi(.428)},.180.\mathfrak{e}^{i2\pi(.227)},.144.\mathfrak{e}^{i2\pi(.166)},.144.\mathfrak{e}^{i2\pi(.187)})$
λ_3	$(.300.\mathfrak{e}^{i2\pi(.366)},.300.\mathfrak{e}^{i2\pi(.366)},.260.\mathfrak{e}^{i2\pi(.344)},.300.\mathfrak{e}^{i2\pi(.274)},.300.\mathfrak{e}^{i2\pi(.322)})$
λ_4	$(.625.\mathfrak{e}^{i2\pi(.650)},.550.\mathfrak{e}^{i2\pi(.650)},.200.\mathfrak{e}^{i2\pi(.285)},.125.\mathfrak{e}^{i2\pi(.203)},.125.\mathfrak{e}^{i2\pi(.203)})$
λ_5	$(.500.\mathfrak{e}^{i2\pi(.520)},.440.\mathfrak{e}^{i2\pi(.520)},.160.\mathfrak{e}^{i2\pi(.228)},.100.\mathfrak{e}^{i2\pi(.162)},.100.\mathfrak{e}^{i2\pi(.162)})$
	$\tilde{\mathfrak{V}}_2$
λ_1	$(.340.\mathfrak{e}^{i2\pi(.386)},.323.\mathfrak{e}^{i2\pi(.386)},.153.\mathfrak{e}^{i2\pi(.235)},.170.\mathfrak{e}^{i2\pi(.175)},.170.\mathfrak{e}^{i2\pi(.216)})$
λ_2	$(.396.\mathfrak{e}^{i2\pi(.428)},.360.\mathfrak{e}^{i2\pi(.428)},.162.\mathfrak{e}^{i2\pi(.227)},.144.\mathfrak{e}^{i2\pi(.166)},.144.\mathfrak{e}^{i2\pi(.187)})$
λ_3	$(.280.\mathfrak{e}^{i2\pi(.342)},.280.\mathfrak{e}^{i2\pi(.342)},.280.\mathfrak{e}^{i2\pi(.366)},.320.\mathfrak{e}^{i2\pi(.274)},.320.\mathfrak{e}^{i2\pi(.344)})$
λ_4	$(.525.\mathfrak{e}^{i2\pi(.595)},.500.\mathfrak{e}^{i2\pi(.595)},.200.\mathfrak{e}^{i2\pi(.345)},.225.\mathfrak{e}^{i2\pi(.258)},.225.\mathfrak{e}^{i2\pi(.318)})$
λ_5	$(.380.\mathfrak{e}^{i2\pi(.432)},.360.\mathfrak{e}^{i2\pi(.432)},.220.\mathfrak{e}^{i2\pi(.276)},.220.\mathfrak{e}^{i2\pi(.206)},.220.\mathfrak{e}^{i2\pi(.254)})$
	$\tilde{\mathfrak{V}}_3$
λ_1	$(.408.\mathfrak{e}^{i2\pi(.423)},.272.\mathfrak{e}^{i2\pi(.423)},.153.\mathfrak{e}^{i2\pi(.193)},.102.\mathfrak{e}^{i2\pi(.137)},.102.\mathfrak{e}^{i2\pi(.137)})$
λ_2	$(.324.\mathfrak{e}^{i2\pi(.367)},.284.\mathfrak{e}^{i2\pi(.367)},.198.\mathfrak{e}^{i2\pi(.307)},.198.\mathfrak{e}^{i2\pi(.226)},.216.\mathfrak{e}^{i2\pi(.268)})$
λ_3	$(.320.\mathfrak{e}^{i2\pi(.364)},.430.\mathfrak{e}^{i2\pi(.364)},.280.\mathfrak{e}^{i2\pi(.386)},.260.\mathfrak{e}^{i2\pi(.298)},.280.\mathfrak{e}^{i2\pi(.320)})$
λ_4	$(.625.\mathfrak{e}^{i2\pi(.650)},.375.\mathfrak{e}^{i2\pi(.650)},.200.\mathfrak{e}^{i2\pi(.285)},.125.\mathfrak{e}^{i2\pi(.202)},.125.\mathfrak{e}^{i2\pi(.202)})$
λ_5	$(.460.\mathfrak{e}^{i2\pi(.498)},.160.\mathfrak{e}^{i2\pi(.498)},.160.\mathfrak{e}^{i2\pi(.252)},.140.\mathfrak{e}^{i2\pi(.184)},.140.\mathfrak{e}^{i2\pi(.208)})$
	$\tilde{\mathfrak{V}}_4$
λ_1	$(.425.\mathfrak{e}^{i2\pi(.442)},.374.\mathfrak{e}^{i2\pi(.442)},.136.\mathfrak{e}^{i2\pi(.193)},.085.\mathfrak{e}^{i2\pi(.137)},.085.\mathfrak{e}^{i2\pi(.137)})$
λ_2	$(.396.\mathfrak{e}^{i2\pi(.428)},.360.\mathfrak{e}^{i2\pi(.428)},.162.\mathfrak{e}^{i2\pi(.226)},.108.\mathfrak{e}^{i2\pi(.165)},.108.\mathfrak{e}^{i2\pi(.187)})$
λ_3	$(.340.\mathfrak{e}^{i2\pi(.410)},.320.\mathfrak{e}^{i2\pi(.410)},.280.\mathfrak{e}^{i2\pi(.320)},.280.\mathfrak{e}^{i2\pi(.252)},.280.\mathfrak{e}^{i2\pi(.276)})$
λ_4	$(.625.\mathfrak{e}^{i2\pi(.650)},.550.\mathfrak{e}^{i2\pi(.650)},.200.\mathfrak{e}^{i2\pi(.285)},.125.\mathfrak{e}^{i2\pi(.203)},.125.\mathfrak{e}^{i2\pi(.203)})$
λ_5	$(.420.\mathfrak{e}^{i2\pi(.454)},.380.\mathfrak{e}^{i2\pi(.454)},.200.\mathfrak{e}^{i2\pi(.296)},.180.\mathfrak{e}^{i2\pi(.230)},.180.\mathfrak{e}^{i2\pi(.230)})$
	$\tilde{\mathfrak{V}}_5$
λ_1	$(.306.\mathfrak{e}^{i2\pi(.367)},.306.\mathfrak{e}^{i2\pi(.367)},.153.\mathfrak{e}^{i2\pi(.840)},.204.\mathfrak{e}^{i2\pi(.193)},.204.\mathfrak{e}^{i2\pi(.255)})$
λ_2	$(.414.\mathfrak{e}^{i2\pi(.448)},.378.\mathfrak{e}^{i2\pi(.448)},.144.\mathfrak{e}^{i2\pi(.816)},.126.\mathfrak{e}^{i2\pi(.165)},.126.\mathfrak{e}^{i2\pi(.187)})$
λ_3	$(.340.\mathfrak{e}^{i2\pi(.386)},.320.\mathfrak{e}^{i2\pi(.386)},.280.\mathfrak{e}^{i2\pi(.206)},.280.\mathfrak{e}^{i2\pi(.252)},.280.\mathfrak{e}^{i2\pi(.298)})$
λ_4	$(.475.\mathfrak{e}^{i2\pi(.540)},.450.\mathfrak{e}^{i2\pi(.540)},.300.\mathfrak{e}^{i2\pi(.970)},.275.\mathfrak{e}^{i2\pi(.257)},.275.\mathfrak{e}^{i2\pi(.317)})$
λ_5	$(.360.\mathfrak{e}^{i2\pi(.410)},.340.\mathfrak{e}^{i2\pi(.410)},.240.\mathfrak{e}^{i2\pi(.920)},.240.\mathfrak{e}^{i2\pi(.252)},.240.\mathfrak{e}^{i2\pi(.276)})$

TABLE 11. Aggregated values of Alternatives

$\tilde{\mathfrak{V}}_1$
$(2.127.\mathfrak{e}^{i14.656},1.956.\mathfrak{e}^{i14.656},0.953.\mathfrak{e}^{i8.415},0.873.\mathfrak{e}^{i6.273},0.873.\mathfrak{e}^{i5.492})$
$\tilde{\mathfrak{V}}_2$
$(1.921.\mathfrak{e}^{i13.724},1.823.\mathfrak{e}^{i13.724},1.015.\mathfrak{e}^{i9.104},1.079.\mathfrak{e}^{i6.777},1.079.\mathfrak{e}^{i8.288})$
$\tilde{\mathfrak{V}}_3$
$(2.137.\mathfrak{e}^{i14.473},1.521.\mathfrak{e}^{i14.473},0.991.\mathfrak{e}^{i8.955},0.825.\mathfrak{e}^{i6.594},0..863.\mathfrak{e}^{i7.143})$
$\tilde{\mathfrak{V}}_4$
$(2.206.\mathfrak{e}^{i14.988},1.984.\mathfrak{e}^{i14.988},0.978.\mathfrak{e}^{i8.307},0.778.\mathfrak{e}^{i6.209},0.778.\mathfrak{e}^{i6.496})$
$\tilde{\mathfrak{V}}_5$
$(1.895.\mathfrak{e}^{i13.523},1.794.\mathfrak{e}^{i13.523},1.117.\mathfrak{e}^{i29.874},1.125.\mathfrak{e}^{i7.045},8.383.\mathfrak{e}^{i8.383})$

As said in Step 3, we fuse the information of each alternative using either CPNWAA or CPNWGA operator as shown in Table11.

Followed by Step 4 and 5, We obtain the score value to rank each alternatives as given in table12.

Table 12. Score Values

Alternative	Score Value	Ranking
$\tilde{\mathfrak{V}}_1$	2.1032	II
$\tilde{\mathfrak{V}}_2$	0.7696	IV
$\tilde{\mathfrak{V}}_3$	1.4466	III
$\tilde{\mathfrak{V}}_4$	2.1238	I
$\tilde{\mathfrak{V}}_5$	-3.5871	V

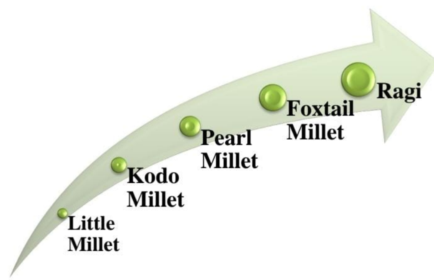

Figure 2. **Pictorial Representation of Results**

7.1. *Discussion:*

The calculation results highlight the practicality and effectiveness of the proposed CPNWAA and CPNWGA operators in addressing uncertainty in decision-making. By incorporating pentapartitioned membership degrees (truth, falsity, contradiction, ignorance, and unknown), the proposed method provides a comprehensive framework for evaluating alternatives in complex scenarios. The application of the CPNWA operators in the millet cultivation problem revealed Ragi $\mathfrak{V}_4$ as the most suitable alternative, followed by Foxtail Millet $\mathfrak{V}_1$, Pearl Millet $\mathfrak{V}_3$, Kodo Millet $\mathfrak{V}_2$, and Little Millet $\mathfrak{V}_5$. This ranking reflects the ability of the operators to capture nuanced relationships among criteria, such as yield rate, seed cost, stability across seasons, medicinal value, and market demand.

8. **Comparitive Analysis**

To validate the scientific accuracy and robustness of the proposed CPNWAA operator, a comparative study was conducted against well-established MCDM methods, including Weighted Sum Method (WSM), Weighted Product Method (WPM), Neutrosophic Technique for Order Preference by Similarity to Ideal Solution (N-TOPSIS), Neutrosophic Weighted Aggregated Sum Product Assessment (N-WASPAS), and Neutrosophic Multi-Objective Optimization on the basis of Ratio Analysis (N-MOORA).

Ranking Comparison:

The ranking results of alternatives $\mathfrak{V}_1$ to $\mathfrak{V}_5$ were compared across all methods. While the proposed method and N-TOPSIS ranked Ragi $\mathfrak{V}_4$ as the best alternative, other methods such as WSM and WPM consistently ranked Pearl Millet $\mathfrak{V}_3$ as optimal.

Correlation Analysis:

To statistically validate the differences in ranking, Spearman's and Pearson's correlation coefficients were calculated between the rankings produced by CPNWA and those of the other methods:

- Spearman's correlation ranged from 0.6 to 0.7, indicating moderate agreement.
- Pearson's correlation ranged from 0.6 to 0.7, further validating the reliability of the proposed method.

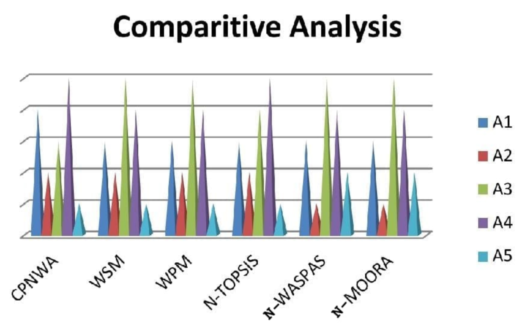

FIGURE 3. **Graphical Representation of Different MCDM Methods**

TABLE 13. Tabulation of Various MCDM Ranking Order

Methods	**Ranking Order**
CPNWA	$\tilde{\mathfrak{V}}_4 > \tilde{\mathfrak{V}}_1 > \tilde{\mathfrak{V}}_3 > \tilde{\mathfrak{V}}_2 > \tilde{\mathfrak{V}}_5$
WSM	$\tilde{\mathfrak{V}}_3 > \tilde{\mathfrak{V}}_4 > \tilde{\mathfrak{V}}_1 > \tilde{\mathfrak{V}}_2 > \tilde{\mathfrak{V}}_5$
WPM	$\tilde{\mathfrak{V}}_3 > \tilde{\mathfrak{V}}_4 > \tilde{\mathfrak{V}}_1 > \tilde{\mathfrak{V}}_2 > \tilde{\mathfrak{V}}_5$
N-TOPSIS	$\tilde{\mathfrak{V}}_4 > \tilde{\mathfrak{V}}_3 > \tilde{\mathfrak{V}}_1 > \tilde{\mathfrak{V}}_5 > \tilde{\mathfrak{V}}_2$
N-WASPAS	$\tilde{\mathfrak{V}}_3 > \tilde{\mathfrak{V}}_4 > \tilde{\mathfrak{V}}_1 > \tilde{\mathfrak{V}}_2 > \tilde{\mathfrak{V}}_5$
N-MOORA	$\tilde{\mathfrak{V}}_3 > \tilde{\mathfrak{V}}_4 > \tilde{\mathfrak{V}}_1 > \tilde{\mathfrak{V}}_5 > \tilde{\mathfrak{V}}_2$

The proposed method ranked alternatives based on a nuanced consideration of truth, falsity, contradiction, ignorance, and unknown membership degrees, which are uniquely captured by the CPNWA operator. The divergence in rankings (e.g., $\mathfrak{V}_4$ ranked higher by CPNWA) reflects the sensitivity of the proposed method to complex interdependencies and uncertainties in the decision-making criteria. Graphical representation of Correlation comparision highlights the correlations between CPNWA and other methods, emphasizing the proposed methods distinct perspective.

Graphical representation in Figure 3 highlights the correlations between CPNWA and other

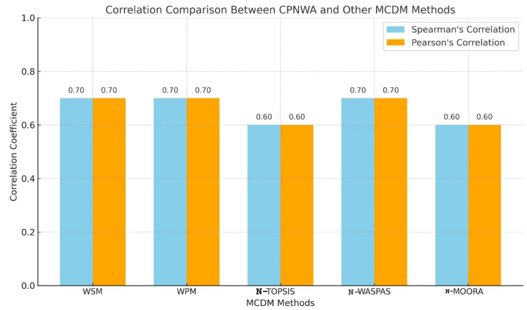

FIGURE 4. **Graphical Representation of Correlation Comparision**

methods, emphasizing the proposed methods distinct perspective. The comparative analysis and statistical validation substantiate the efficacy of the proposed method. The CPNWA operator effectively addresses uncertainty and periodicity in decision-making, offering a reliable alternative for MCDM problems in agriculture and beyond.

9. **Conclusion**

This study utilized the Complex Pentapartitioned Neutrosophic Set (CPNS) as an advanced mathematical framework for handling uncertainty, periodicity, and contradictions in MCDM. The proposed CPNWAA and CPNWGA operators were developed to facilitate decision-making in complex environments where traditional fuzzy sets have limitations with it.

To validate the practical applicability of CPNS-based MCDM, the study applied it to millet selection for cultivation, a problem characterized by uncertain climatic conditions, expert disagreements, and missing data. The results identified Ragi $\mathfrak{V}_4$ as the most suitable millet variety, a ranking that diverged from conventional MCDM methods (WSM, WPM, N-TOPSIS, N-WASPAS and N-MOORA). Statistical validation using Spearmans and Pearsons correlation coefficients (0.60.7) confirmed that while CPNS aligns moderately with existing methods, it offers a more refined and uncertainty-aware decision-making process.

Key Contributions:

- Extended CPNS for MCDM, integrating five membership degrees to capture truth, falsity, contradiction, ignorance, and unknown factors.
- Developed novel aggregation operators (CPNWAA and CPNWGA) for computing alternatives under uncertainty.
- Demonstrated CPNS in agricultural decision-making, providing a framework for millet selection based on multi-expert evaluations.
- Validated the approach with statistical analysis, showing that CPNS captures nuanced uncertainties better than conventional MCDM methods.

9.1. *Limitations:*

- High Computational Load: CPNS involves complex-valued functions, which may be computationally intensive for large datasets.
- Interpretability: The fivefold partition and complex values may be difficult for non-experts to interpret.
- Limited Aggregation Operators: This study only employs CPNWAA and CPNWGA, restricting flexibility in certain decision contexts.
- Reliance on Expert Input: Subjectivity in input weights and values may affect consistency.

9.2. *Future Research Directions:*

- New Aggregation Techniques: Explore operators like CPNS Bonferroni Mean, Aczel-Alsina, and Frank to better handle varied decision behaviors.
- AI Integration: Combine CPNS with machine learning for adaptive weight tuning, pattern detection, and optimization. Real-time & Big
- Data Applications: Implement CPNS in live systems (e.g., precision agriculture, health monitoring) for timely, uncertainty-aware decisions.
- Cross-Domain Use: Extend CPNS models to sectors such as healthcare, supply chain, and environmental planning.
- Empirical Validation: Test the framework on larger, real-world datasets to validate performance and scalability.

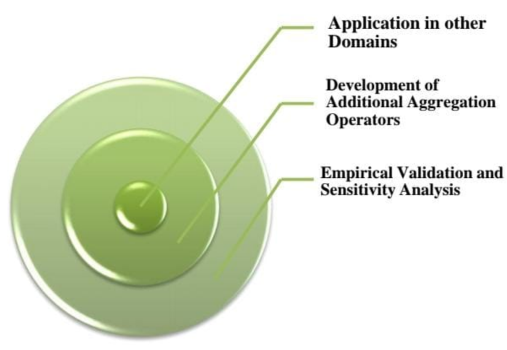

Figure 5. **Pictorial Representation of Future Research Direction**

This study positions CPNS as a powerful tool for handling complex, uncertain decisions. Its ability to model periodicity, contradiction, and multi-source uncertainty offers strong potential in domains requiring nuanced analysisespecially agriculture, healthcare, and sustainability.

Data Availability: The data used to support the findings of the study is available in the article.

Conflicts of Interest: The authors declare no conflicts of interest.

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