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Reliability assessment of refined single-valued neutrosophic data with application in multi-criteria decision-making

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Abstract. The refined single-valued neutrosophic set (RSVNS) has been extensively validated for its effectiveness in addressing complex multi-criteria decision-making (MCDM) problems. Unlike fuzzy sets and similar sets, RSVNS includes a larger number of membership functions, making it especially capable of handling complex evaluation criteria within the decision-making framework. Therefore, this study presents a methodology for the reliability assessment of data within refined single-valued neutrosophic numbers, along with a technique for calculating the weighted average of a set of these numbers. The study then applies this methodology to a MCDM problem using the similarity measure of RSVNS. By assessing each alternative's similarity to the ideal solution, it becomes possible to rank all alternatives and identify the most optimal choice. A real-world example involving the selection procedure among various construction projects is studied to examine the effectiveness of the proposed method. Furthermore, a comparative analysis has also been carried out to compare the reliability and preference ranking of several alternatives derived using both the methodology considered and the existing one.

Keywords: reliability; weighted average; single-valued neutrosophic number; refined single-valued neutrosophic number; similarity measure; multi-criteria decision-making.

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1. Introduction

Generally, in decision-making problems, it is possible that the options available are not limited to a specific number, or there may be numerous alternatives to the initial choice. Additionally, there may be instances where no appropriate option meets the established criteria. MCDM problems typically involve assessing and choosing from a vast array of alternatives, which may be finite or infinite. Decision-makers often encounter issues characterized by incomplete and ambiguous information in MCDM scenarios. Thus, in MCDM problems, fuzzy set approaches are particularly effective when it is essential to model human knowledge and incorporate human evaluations. Initially, fuzzy set theory gained attention for its ability to model uncertainty in human cognition. Nowadays, its applications extend to various fields, including health science, natural science, business, medical, and engineering.

The idea of the fuzzy sets was first pioneered by Zadeh [35] in 1965. It is distinguished by its incorporation of membership degree. This foundational work paved the way for subsequent advancement, including the introduction of the intuitionistic fuzzy sets (IFS) by Atanassov [5] in 1986. IFS is distinguished by their incorporation of membership and non-membership degrees. Over the past few decades, these sets have emerged as significant areas of research. However, these theories are unable to address the complexities of real-life situations, particularly those involving independent, and dependent components, due to their incomplete, indeterminate and inconsistent nature. To address these challenges, Smarandache [19] introduced neutrosophic sets in 1999. It is distinguished by truth, indeterminacy and a falsity membership degree. Recently, this innovative concept has garnered significant attention in research, leading to various adaptations of neutrosophic sets. For instance, Wang (2010) [29] proposed the single-valued neutrosophic sets (SVNS), which provides a more effective means of representing indeterminate and inconsistent information. Later, in 2013, Yager [30] proposed the picture fuzzy sets, which represents a second category of IFS. Ye (2014) [31] introduced the simplified neutrosophic sets. Furthermore, Biswas (2014) [6], Tan and Ye. (2015) [24] explored the trapezoidal fuzzy neutrosophic sets and its accuracy and score functions. Again in 2015, Ye J. [32] introduced the application of trapezoidal fuzzy neutrosophic sets in MCDM. Additionally, Wang (2015) [28] defined multi-valued neutrosophic sets and multi-valued interval neutrosophic sets. Tian Z.P. (2015) [27] defined simplified neutrosophic linguistic sets for solving problems in MCDM. Broumi et al. (2016) [9], Tan and Zhang (2017) [25] and others have integrated neutrosophic sets with graph theory. Liu et al. (2019) [16] conducted a study on the linguistic neutrosophic sets.

The inherent complexity of real-world problems requires a more detailed representation of truth (T), indeterminacy (I) and falsity (F) membership degrees in neutrosophic set theory. To address this, Smarandache (2013) [21] introduced the refined neutrosophic logic, which extends neutrosophic logic into n-valued refined neutrosophic logic. This refinement entails a more granular representation of truth, indeterminacy, and falsity captured through the subcomponents $(T_1, T_2, ..., T_p), (I_1, I_2, ..., I_q), (F_1, F_2, ..., F_r)$ respectively. In contrast, fuzzy sets theory is inadequate for handling inconsistent and indeterminate information. In context of uncertainty, similarity measures are vital for the analysis of uncertainty in research areas. As a result, a variety of similarity measures for both SVNS and refined neutrosophic set (RNS) have been introduced, with a significant emphasis on their application in MCDM and clustering analysis. For instance, Ye S. (2014) [34] suggested the dice similarity measure for single-valued neutrosophic multi sets. Later, Broumi et al. (2014) [8] introduced a cosine similarity measure for refined fuzzy set (RFS). Mondal et al. (2015) [17] further developed this concept by introducing a similarity measure for RFS based on a tangent function. Additionally, Deli I. (2015) [7] contributed to the field by introducing bipolar neutrosophic refined sets.

In essence, multi-valued neutrosophic sets and neutrosophic refined sets can be viewed as neutrosophic multi sets in their expressed forms. However, these sets and their associated decision-making methodologies have limitations in addressing complex decision-making problems (that involve attributes and sub-attributes). To address this limitation, Smarandache et al. (2016) [33] proposed RSVNS, where the neutrosophic components T, I, F refined into more detailed sub-components $(T_1, T_2, ..., T_p), (I_1, I_2, ..., I_p), (F_1, F_2, ..., F_p)$ and they also introduced similarity measures for RSVNS to effectively tackle MCDM problems. After that Fan et al. (2017) [13] subsequently introduced cosine measures for RSVNS, utilizing cosine and distance functions. They applied these measures to decision-making scenarios involving both attributes and sub-attributes. Additionally, Chen et al. (2017) [10] have used vector similarity measures for refined simplified neutrosophic sets, which encompass RSVNS, and developed an associated MCDM method.

The introduction of RSVNS has marked a notable milestone in the field of neutrosophic set theory, enabling more nuanced and precise approach for handling uncertainty. This concept has opened up new possibilities for applications in MCDM problems. For instance, Fan et al. (2019) [14] introduced correlation coefficients for RSVNS, exploring their utility in addressing MCDM problems. Tan and Zhang (2021) [26] developed a MCDM method based on RSVNS, demonstrating its effectiveness in typhoon disaster evaluation. Abobala M. (2022) [1] made a significant contribution by introducing refined neutrosophic matrices utilizing refined neutrosophic linear functions. Arshad et al. (2023) [3] proposed an approach to handle concave and convex sets within the refined neutrosophic environment. In 2023, Arar M. [4] defined neutrosophic n-valued refined sets and topologies, contributing significantly to the field of neutrosophic set theory. Alabdullah M.(2024) [2] introduced refined neutrosophic ideals. Cruz A.(2024) [11] introduced neutrosophic soft cubic refined sets, expanding the scope of neutrosophic set theory. More recently, Pratheesha A. (2025) [18] proposed a TOPSIS approach for MCDM based on RSVNS. Fujita T.(2025) [15] pioneered research on hyper neutrosophic sets and applied it to extend multi neutrosophic sets and refined neutrosophic sets. In 2025, Debnath B. [12] introduced reduction formulae for 2-refined neutrosophic integrals, offering a simplified approach to calculating and applying these advanced mathematical constructs.

The reliability evaluation process under the uncertain environment leads to complexity in reliability measurement. Because in real-life situation it is very difficult to evaluate reliability of any complex structure where data contains indeterminate and inconsistent information especially where data contains additional membership functions. The incorporation of additional membership functions may complicate the reliability evaluation process in comparison to assessments conducted with fuzzy sets. In response to this challenge, Smarandache (2018) [22] introduced a method aimed at evaluating the reliability of information gathered through respondent surveys by using neutrosophic fuzzy set theory. Subsequently, Stanujkic D. (2020) [23] revisited the article and suggested an innovative method for assessing the reliability of data within single-valued neutrosophic number (SVNN). The application of this research is limited upto three membership functions, but analyzing reliability in neutrosophic fuzzy environment is rendered more complex when the data encompasses refined membership functions. Therefore to address the shortcomings of reliability measure in neutrosophic set theory, this study introduces a pioneering approach for analyzing reliability of data contain in refined single-valued neutrosophic number (RSVNN). We also introduced a method for evaluating the weighted average for a collection of RSVNN.

This study aims to comprehensively analysis the potential of RSVNS and assesses their application in the context of MCDM. We aim to equip decision-makers with a more robust framework for making well-informed, scientifically valid decisions by introducing an innovative reliability measure, a weighted average measure, and advanced decision support methodology. This approach is designed to enhance the effectiveness and accuracy of the decision-making process in complex, multi-dimensional scenarios.

The subsequent sections of this paper are structured as follows: Section 2 addresses the basic definition of neutrosophic set theory. Section 3 explores a novel method for determining the reliability and weighted average for the information contained in RSVNN. Section 4 discusses the implementation of proposed approach in the context of MCDM problems. Section 5 represents a numerical example for better understanding the proposed method. Section 6 validates the proposed method through comparative analyses with existing method. Lastly, Section 7 offers a conclusion to the paper.

2. Preliminaries

Definition 2.1 (Fuzzy set [35]). "Let U be a universe of disourse and x be any particular element of U, then fuzzy set F defined on U is a collection of ordered pairs.

$$F = \{(x, \mu_F(x)) : x \in U\}$$

where $\mu_F(x)) : U \to [0, 1]$ is called the membership function or grade of membership of x".

Definition 2.2 (Neutrosophic set [19]). "Let U be a universe of discourse, then neutrosophic set N in U is defined as follows:

$$N = \{\langle x, (T_N(x), I_N(x), F_N(x))\rangle : x \in U\}$$

with $T_N : U \to]^-0, 1^+[$, $I_N : U \to]^-0, 1^+[$, $F_N : U \to]^-0, 1^+[$ and $^-0 \leq T_N(x)+I_N(x)+F_N(x) \leq 3^+$".

Definition 2.3 (Single valued neutrosophic set [29]). "Let U be a universe of discourse, then single-valued neutrosophic set N_{SVNS} in U is defined as follows:

$$N_{SVNS} = \{\langle x, (T_N(x), I_N(x), F_N(x))\rangle : x \in U\}$$

with $T_N : U \to [0, 1], I_N : U \to [0, 1], F_N : U \to [0, 1]$ and $0 \leq T_N(x) + I_N(x) + F_N(x) \leq 3$."

Definition 2.4 (Refined single-valued neutrosophic set [33]). "Let U be a universe of discourse, then refined single-valued neutrosophic set R_{RSVNS} in U is defined as follows:

$$R_{RSVNS} = \{\langle x, (T_{1R}(x), T_{2R}(x), ..., T_{pR}(x)), (I_{1R}(x), I_{2R}(x), ..., I_{pR}(x)),$$

$$(F_{1R}(x), F_{2R}(x), ..., F_{pR}(x))\rangle : x \in U\}$$

where p is a positive integer, $(T_{1R}(x), T_{2R}(x), ..., T_{pR}(x))$:U $\to$ [0,1], $(I_{1R}(x), I_{2R}(x), ..., I_{pR}(x))$:U $\to$ [0,1],$(F_{1R}(x), F_{2R}(x), ..., F_{pR}(x))$:U $\to$ [0,1] and $0 \leqslant supT_{jR}(x)+supI_{jR}(x)+supF_{jR}(x) \leqslant 3$ for $j = 1, 2, ..., p$."

Definition 2.5 (Refined single-valued neutrosophic number [33]). "Let R_{RSVNS} be a refined single-valued neutrosophic set, then each basic element in the refined single-valued neutrosophic set is represented by the refined single-valued neutrosophic number (RSVNN) as:

$$\langle(T_{1Rk}, T_{2Rk}, ..., T_{pR_k}), (I_{1Rk}, I_{2Rk}, ..., I_{pR_k}), (F_{1Rk}, F_{2Rk}, ..., F_{pR_k})\rangle$$

for $p = 1, 2, ..., n$ and $k = 1, 2, \ldots, m$."

Definition 2.6 (Relations defined on refined single-valued neutrosophic sets [33]). "Let R and S be two refined single-valued neutrosophic sets defined as:

$$R = \Big\{\langle x, (T_{1R}(x), T_{2R}(x), \ldots, T_{pR}(x)), (I_{1R}(x), I_{2R}(x), \ldots, I_{p_R}(x)), (F_{1R}(x), F_{2R}(x), \ldots, F_{p_R}(x))\rangle : x \in U\Big\}$$

$$S = \Big\{\langle x, (T_{1S}(x), T_{2S}(x), \ldots, T_{pS}(x)), (I_{1S}(x), I_{2S}(x), \ldots, I_{p_S}(x)), (F_{1S}(x), F_{2S}(x), \ldots, F_{p_S}(x))\rangle : x \in U\Big\}$$

Then there are the following relations of R and S:

1.**Containment:** $R \subseteq S$, if and only if

$$T_{jR}(x) \leq T_{jS}(x), \quad I_{jR}(x) \geq I_{jS}(x), \quad F_{jR}(x) \geq F_{jS}(x), \quad \text{for } j = 1, 2, \ldots, p.$$

2. **Equality:** $R = S$, if and only if

$$T_{jR}(x) = T_{jS}(x), \quad I_{jR}(x) = I_{jS}(x), \quad F_{jR}(x) = F_{jS}(x), \quad \text{for } j = 1, 2, \ldots, p.$$

3. **Union:**

$$R \cup S = \Big\{\langle x, (T_{1R}(x) \vee T_{1S}(x), T_{2R}(x) \vee T_{2S}(x), \ldots, T_{pR}(x) \vee T_{pS}(x)), (I_{1R}(x) \wedge I_{1S}(x), I_{2R}(x) \wedge I_{2S}(x), \ldots, I_{pR}(x) \wedge I_{pS}(x)), (F_{1R}(x) \wedge F_{1S}(x), F_{2R}(x) \wedge F_{2S}(x), \ldots, F_{pR}(x) \wedge F_{pS}(x))\rangle : x \in U\Big\}$$

4. **Intersection:**

$$R \cap S = \Big\{\langle x, (T_{1R}(x) \wedge T_{1S}(x), T_{2R}(x) \wedge T_{2S}(x), \ldots, T_{pR}(x) \wedge T_{pS}(x)), (I_{1R}(x) \vee I_{1S}(x), I_{2R}(x) \vee I_{2S}(x), \ldots, I_{pR}(x) \vee I_{pS}(x)), (F_{1R}(x) \vee F_{1S}(x), F_{2R}(x) \vee F_{2S}(x), \ldots, F_{pR}(x) \vee F_{pS}(x))\rangle : x \in U\Big\}\text{"}$$

Definition 2.7 (Weighted similarity measure of refined single-valued neutrosophic sets [33]). "Let R and S be two refined single-valued neutrosophic sets, which are defined as:

$$R = \Big\{\langle x_i, (T_{1R}(x_i), T_{2R}(x_i), \ldots, T_{p_iR}(x_i)), (I_{1R}(x_i), I_{2R}(x_i), \ldots, I_{p_iR}(x_i)), (F_{1R}(x_i), F_{2R}(x_i), \ldots, F_{p_iR}(x_i))\rangle : x_i \in U\Big\}$$

$$S = \Big\{\langle x_i, (T_{1S}(x_i), T_{2S}(x_i), \ldots, T_{p_iS}(x_i)), (I_{1S}(x_i), I_{2S}(x_i), \ldots, I_{p_iS}(x_i)), (F_{1S}(x_i), F_{2S}(x_i), \ldots, F_{p_iS}(x_i))\rangle : x_i \in U\Big\}$$

where p_i is a positive integer, and all $T_{jR}(x_i), I_{jR}(x_i), F_{jR}(x_i) \in [0,1]$ and $T_{jS}(x_i), I_{jS}(x_i), F_{jS}(x_i) \in [0,1]$ for $i = 1,2,\ldots,n$ and $j = 1,2,\ldots,p$. Then the weighted similarity measure is defined as:

$$M_w(R,S) = \frac{\sum_{i=1}^{n} w_i \sum_{j=1}^{p_i} \left(\min(T_{jR}(x_i), T_{jS}(x_i)) + \min(I_{jR}(x_i), I_{jS}(x_i)) + \min(F_{jR}(x_i), F_{jS}(x_i))\right)}{\sum_{i=1}^{n} w_i \sum_{j=1}^{p_i} \left(\max(T_{jR}(x_i), T_{jS}(x_i)) + \max(I_{jR}(x_i), I_{jS}(x_i)) + \max(F_{jR}(x_i), F_{jS}(x_i))\right)}, \quad (1)$$

3. Reliability and weighted average of the data within refined single-valued neutrosophic number

Definition 3.1 (**Reliability measure of the data within refined single-valued neutrosophic number(RSVNN)).** Let R be a RSVNN, which is defined as:

$$R = \langle (T_{1R_k}, T_{2R_k}, \ldots, T_{p_iR_k}), (I_{1R_k}, I_{2R_k}, \ldots, I_{p_iR_k}), (F_{1R_k}, F_{2R_k}, \ldots, F_{p_iR_k}) \rangle$$

Then, we can calculate the reliability $r(x)$ of the data within RSVNN as follows:

$$r(x) = \begin{cases} \frac{|\max(T) - \min(F)|}{\max(T) + \max(I) + \max(F)}, & \text{if } \max(T) + \max(I) + \max(F) \neq 0, \\ 0, & \text{if } \max(T) + \max(I) + \max(F) = 0. \end{cases} \quad (2)$$

where $\max(T) = \max(T_{1R_k}, T_{2R_k}, \ldots, T_{p_iR_k})$, $\max(I) = \max(I_{1R_k}, I_{2R_k}, \ldots, I_{p_iR_k})$, $\max(F) = \max(F_{1R_k}, F_{2R_k}, \ldots, F_{p_iR_k})$, $\min(F) = \min(F_{1R_k}, F_{2R_k}, \ldots, F_{p_iR_k})$.

Example: Assume that $x = \langle (0.9, 0.7, 0.8), (0.1, 0.3, 0.2), (0.2, 0.2, 0.1) \rangle$ is a refined single-valued neutrosophic number (RSVNN). Then, the reliability of x is calculated as:

$$r(x) = \frac{|0.9 - 0.1|}{0.9 + 0.3 + 0.2} = \frac{0.8}{1.4} = 0.5714$$

Definition 3.2 (**Weighted average measure for the collection of RSVNN).** Let $R_j = \left((T_{1j}, T_{2j}, \ldots, T_{p_j}), (I_{1j}, I_{2j}, \ldots, I_{p_j}), (F_{1j}, F_{2j}, \ldots, F_{p_j})\right)$, where $j = 1,2,\ldots,n$, be a set of RSVNN, with an associated weighting vector $W = (w_1, w_2, \ldots, w_n)^T$. Then, the refined single-valued neutrosophic weighted average (RSVNWA) of R_j can be defined as follows:

$$\text{RSVNWA}(R_1, R_2, \ldots, R_j) = \left(1 - \prod_{p=1}^{j} (1 - T_{pj})^{w_j}, \prod_{p=1}^{j} (I_{pj})^{w_j}, \prod_{p=1}^{j} (F_{pj})^{w_j}\right) \quad (3)$$

where w_j is the j-th element of the weighting vector W, $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.

4. Application of reliability measure and weighted average measure under refined single-valued neutrosophic environment in MCDM

Consider a MCDM scenario involving k decision-makers, n criteria, and m alternatives, where evaluations are provided by using RSVNN. The proposed method's computational process can be succinctly outlined as follows:

Step 1: Develop the decision-matrix $D_k \, \forall k = 1,2,\ldots,n$, as shown in Table 1, where the entries are represented as RSVNN, based on the information provided by each decision-maker.

TABLE 1. Decision-matrix D_k

	T_1	$\ldots$	T_n
R_1	$(T_{1R_1}, T_{2R_1}, \ldots, T_{p_1R_1}),(I_{1R_1}, I_{2R_1}, \ldots, I_{p_1R_1}), (F_{1R_1}, F_{2R_1}, \ldots, F_{p_1R_1})$	$\ldots$	$(T_{1R_1}, T_{2R_1}, \ldots, T_{p_nR_1}), (I_{1R_1}, I_{2R_1}, \ldots, I_{p_nR_1}),(F_{1R_1}, F_{2R_1}, \ldots, F_{p_nR_1})$
R_2	$(T_{1R_2}, T_{2R_2}, \ldots, T_{p_1R_2}),(I_{1R_2}, I_{2R_2}, \ldots, I_{p_1R_2}),(F_{1R_2}, F_{2R_2}, \ldots, F_{p_1R_2})$	$\ldots$	$(T_{1R_2}, T_{2R_2}, \ldots, T_{p_nR_2}), (I_{1R_2}, I_{2R_2}, \ldots, I_{p_nR_2}),(F_{1R_2}, F_{2R_2}, \ldots, F_{p_nR_2})$
$\vdots$	$\vdots$		$\vdots$
R_m	$(T_{1R_m}, T_{2R_m}, \ldots, T_{p_1R_m}),(I_{1R_m}, I_{2R_m}, \ldots, I_{p_1R_m}),(F_{1R_m}, F_{2R_m}, \ldots, F_{p_1R_m})$	$\ldots$	$(T_{1R_m}, T_{2R_m}, \ldots, T_{p_nR_m}), (I_{1R_m}, I_{2R_m}, \ldots, I_{p_nR_m}),(F_{1R_m}, F_{2R_m}, \ldots, F_{p_nR_m})$

Step 2: Calculate the reliability rating for each decision-matrix D_k based on the data offered by each decision-maker, by using equation (2).

Step 3: Calculate the average reliability in each decision-matrix D_k of the data (reliability rating) given by each decision-maker.

When the average reliability ratings provided from decision-makers is found to be low, those results should either be reassessed or excluded until a satisfactory level of reliability is attained. If it is so then proceed into Step 4.

Step 4: Develop the decision-matrix D, as shown in Table 2 by using equation (3).

TABLE 2. Decision-matrix D

	T_1	$\ldots$	T_n
R_1	$(T_{1R_1}, T_{2R_1}, \ldots, T_{p_1R_1}),(I_{1R_1}, I_{2R_1}, \ldots, I_{p_1R_1}),(F_{1R_1}, F_{2R_1}, \ldots, F_{p_1R_1})$	$\ldots$	$(T_{1R_1}, T_{2R_1}, \ldots, T_{p_nR_1}), (I_{1R_1}, I_{2R_1}, \ldots, I_{p_nR_1}),(F_{1R_1}, F_{2R_1}, \ldots, F_{p_nR_1})$
R_2	$(T_{1R_2}, T_{2R_2}, \ldots, T_{p_1R_2}),(I_{1R_2}, I_{2R_2}, \ldots, I_{p_1R_2}),(F_{1R_2}, F_{2R_2}, \ldots, F_{p_1R_2})$	$\ldots$	$(T_{1R_2}, T_{2R_2}, \ldots, T_{p_nR_2}), (I_{1R_2}, I_{2R_2}, \ldots, I_{p_nR_2}),(F_{1R_2}, F_{2R_2}, \ldots, F_{p_nR_2})$
$\vdots$	$\vdots$		$\vdots$
R_m	$(T_{1R_m}, T_{2R_m}, \ldots, T_{p_1R_m}),(I_{1R_m}, I_{2R_m}, \ldots, I_{p_1R_m}),(F_{1R_m}, F_{2R_m}, \ldots, F_{p_1R_m})$	$\ldots$	$(T_{1R_m}, T_{2R_m}, \ldots, T_{p_nR_m}), (I_{1R_m}, I_{2R_m}, \ldots, I_{p_nR_m}),(F_{1R_m}, F_{2R_m}, \ldots, F_{p_nR_m})$

Step 5: By using decision-matrix D, identify the ideal solution. s_j^* [33] as:

$$s_j^* = \langle (max(T_{1R_k}), max(T_{2R_k})), ..., (T_{p_iR_k})), (min(I_{1R_k}), min(I_{2R_k}), ..., min(I_{p_iR_k})),$$
$$(min(F_{1R_k}), min(F_{2R_k}), ..., min(F_{p_iR_k}))\rangle \tag{4}$$

s_j^* is formulated as ideal alternative S^* and $S^* = \{s_1^*, s_2^*, \ldots, s_j^*\}$.

Step 6: Calculate the weighted similarity measure values between each alternative and the ideal alternative by using equation (1).

Step 7: The alternatives are prioritized in descending order based on the calculated values of similarity measure. The higher value of similarity measure indicates a more desirable alternative.

5. Numerical Example

To succinctly validate the efficacy of RSVNN in addressing complex MCDM problems, this section presents a case study on the selection of a construction project. Consider a scenario where a construction company is contemplating the engagement of a new construction project. Consequently, a team of three experts is assembled to identify the most suitable project from four available options/alternatives V_1, V_2, V_3, V_4 based on the following criteria:

- T_1: Budget
- T_2: Risk management
- T_3: Business goals
- T_4: Scope

The expert ratings are presented in Tables 3-5, which provide the assessments from three independent experts.

TABLE 3. Ratings given by the expert one

	T_1	T_2	T_3	T_4
V_1	$(0.2, 0.3, 0.8), (0.1, 0.2, 0.3), (0.1, 0.1, 0.2)$	$(0.8, 0.9), (0.1, 0.2), (0.1, 0.1)$	$(0.7, 0.8), (0.1, 0.1), (0.1, 0.2)$	$(0.2, 0.3, 0.9), (0.2, 0.1, 0.1), (0.2, 0.1, 0.2)$
V_2	$(0.1, 0.2, 0.9), (0.3, 0.2, 0.1), (0.1, 0.1, 0)$	$(0.7, 0.8), (0.1, 0.2), (0.2, 0.2)$	$(0.7, 0.8), (0.1, 0.2), (0.1, 0.2)$	$(0.4, 0.9, 0.3), (0.1, 0.1, 0.2), (0.1, 0.2, 0.1)$
V_3	$(0.4, 0.5, 0.9), (0.4, 0.1, 0.1), (0.2, 0.1, 0.1)$	$(0.7, 0.8), (0.1, 0.2), (0.2, 0.1)$	$(0.7, 0.8), (0.1, 0.3), (0, 0.2)$	$(0.7, 0.8, 0.9), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)$
V_4	$(0.6, 0.5, 0.8), (0.2, 0.2, 0.1), (0.1, 0.2, 0.1)$	$(0.7, 0.8), (0.1, 0.1), (0.1, 0.2)$	$(0.7, 0.8), (0.2, 0.2), (0.1, 0.1)$	$(0.2, 0.9, 0.3), (0.1, 0.1, 0.1), (0.2, 0.1, 0.1)$

TABLE 4. Ratings given by the expert second

	T_1	T_2	T_3	T_4
V_1	$(0.7, 0.9, 0.5), (0.1, 0.1, 0.3), (0.1, 0.2, 0.2)$	$(0.8, 0.9), (0.2, 0.2), (0.1, 0.1)$	$(0.7, 0.8), (0.1, 0.1), (0.1, 0.2)$	$(0.6, 0.9, 0.2), (0.1, 0.1, 0.1), (0.1, 0.2, 0.2)$
V_2	$(0.6, 0.9, 0.1), (0.2, 0.1, 0.1), (0.1, 0.2, 0.1)$	$(0.7, 0.9), (0.1, 0.1), (0.1, 0.1)$	$(0.7, 0.9), (0.1, 0.1), (0.1, 0.1)$	$(0.6, 0.9, 0.4), (0.2, 0.1, 0.1), (0.1, 0.2, 0.1)$
V_3	$(0.8, 0.9, 0.4), (0.1, 0.3, 0.2), (0.2, 0, 0.2)$	$(0.7, 0.8), (0.3, 0.1), (0.1, 0.1)$	$(0.6, 0.9), (0.1, 0.1), (0.1, 0.1)$	$(0.6, 0.9, 0.2), (0.2, 0.1, 0.2), (0.1, 0.2, 0.1)$
V_4	$(0.8, 0.9, 0.1), (0.1, 0.1, 0.2), (0.2, 0.3, 0.1)$	$(0.6, 0.8), (0.2, 0.1), (0.1, 0.2)$	$(0.7, 0.8), (0.2, 0.3), (0.1, 0.1)$	$(0.8, 0.9, 0.2), (0.3, 0.1, 0.2), (0.1, 0.1, 0.2)$

TABLE 5. Ratings given by the expert third

	T_1	T_2	T_3	T_4
V_1	$(0.8, 0.9, 0.2), (0.1, 0.1, 0.1), (0.2, 0.1, 0.1)$	$(0.7, 0.8), (0.1, 0.1), (0.2, 0.1)$	$(0.9, 0.3), (0.1, 0.1), (0.1, 0)$	$(0.6, 0.9, 0.1), (0.2, 0.1, 0.2), (0.1, 0.2, 0.1)$
V_2	$(0.4, 0.9, 0.2), (0.1, 0.2, 0.2), (0.1, 0.1, 0.1)$	$(0.7, 0.8), (0.1, 0.2), (0.1, 0.2)$	$(0.7, 0.8), (0.1, 0.1), (0.1, 0.1)$	$(0.6, 0.9, 0.4), (0.2, 0.1, 0.2), (0.1, 0.1, 0.1)$
V_3	$(0.6, 0.9, 0.1), (0.1, 0.2, 0.1), (0.2, 0.1, 0.2)$	$(0.8, 0.9), (0.1, 0.2), (0.2, 0.1)$	$(0.4, 0.9), (0.2, 0.1), (0.1, 0.1)$	$(0.5, 0.8, 0.2), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1)$
V_4	$(0.6, 0.9, 0.3), (0.1, 0.2, 0.1), (0.1, 0.2, 0.2)$	$(0.6, 0.9), (0.1, 0.1), (0.1, 0.2)$	$(0.6, 0.9), (0.1, 0.1), (0.1, 0.2)$	$(0.6, 0.8, 0.1), (0.1, 0.1, 0.2), (0.2, 0.1, 0.1)$

The reliability ratings obtained using Eq. (2) is presented in Tables 6, 7, and 8. Furthermore, these tables also present the overall average reliability ratings, providing a comprehensive overview of the reliability assesments.

TABLE 6. Reliability ratings provided from the expert one

	T_1	T_2	T_3	T_4
V_1	0.5384	0.6666	0.6363	0.6153
V_2	0.6923	0.5000	0.5833	0.6153
V_3	0.5333	0.5833	0.6153	0.7272
V_4	0.5833	0.6363	0.6363	0.6666
Average reliability		0.6142		

Table 7. Reliability ratings provided from the expert second

	T_1	T_2	T_3	T_4
V_1	0.5714	0.6666	0.5833	0.6666
V_2	0.6153	0.7272	0.7272	0.6153
V_3	0.6428	0.5833	0.7272	06153
V_4	0.5714	0.5833	0.5833	0.5714
Average reliability	0.6281			

Table 8. Reliability ratings provided from the expert third

	T_1	T_2	T_3	T_4
V_1	0.6666	0.6363	0.8181	0.6153
V_2	0.6666	0.5833	0.7000	0.6666
V_3	0.6153	0.6153	0.7272	0.7000
V_4	0.6153	0.6666	0.6666	0.5833
Average reliability	0.6588			

Table 9. Experts' average reliability ratings

	Average reliability
X_1	0.6142
X_2	0.6281
X_3	0.6588

The average reliability of the decision-makers' responses $X_i (i = 1, 2, 3)$, computed using Eq. (2) is presented in Table 9.

It is evident from Table 9 that the three experts offer responses that are largely consistent, allowing their ratings to be utilized for the subsequent alternative evaluation. Conversely, responses with low average reliability ratings should be subject to either exclusion from further consideration of the alternatives or reassessed until a satisfactory level of reliability is attained.

The following scenario illustrates the evaluation of the alternatives. A group decision-matrix, illustrated in Table 10, is developed using the Equation (3) along with the weights $w_j = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. The ideal solution is computed using Equation (4) and the corresponding weighting vector, $w_j = (0.28, 0.18, 0.33, 0.21)$.

Table 10. Group decision-matrix

	T_1	T_2	T_3	T_4
w_j	0.28	0.18	0.33	0.21
V_1	(0.63, 0.80, 0.56), (0.1, 0.12, 0.20), (0.12, 0.12, 0.15)	(0.77, 0.87), (0.12, 0.15), (0.12, 0.14)	(0.79, 0.69), (0.1, 0.12), (0.12, 0)	(0.49, 0.80, 0.58), (0.15, 0.1, 0.12), (0.12, 0.15, 0.15)
V_2	(0.4, 0.8, 0.58), (0.18, 0.15, 0.12), (0.1, 0.12, 0)	(0.7, 0.84), (0.1, 0.15), (0.15, 0.2)	(0.7, 0.84), (0.1, 0.12), (0.1, 0.2)	(0.54, 0.9, 0.36), (0.15, 0.1, 0.15), (0.1, 0.15, 0.1)
V_3	(0.63, 0.82, 0.62), (0.15, 0.18, 0.12), (0.2, 0, 0.15)	(0.73, 0.84), (0.14, 0.15), (0.15, 0.1)	(0.58, 0.87), (0.12, 0.14), (0, 0.12)	(0.60, 0.84, 0.6), (0.12, 0.1, 0.12), (0.1, 0.12, 0.1)
V_4	(0.68, 0.82, 0.49), (0.12, 0.15, 0.12), (0.12, 0.22, 0.12)	(0.63, 0.84), (0.12, 0.1), (0.1, 0.2)	(0.66, 0.84), (0.15, 0.18), (0.1, 0.12)	(0.6, 0.87, 0.20), (0.14, 0.1, 0.15), (0.15, 0.1, 0.12)

By using Equation (4), the ideal alternative S^* is computed as:

$$S^* = \{\langle(0.68, 0.82, 0.62), (0.1, 0.12, 0.12), (0.1, 0, 0)\rangle, \langle(0.77, 0.87), (0.1, 0.1), (0.1, 0.1)\rangle, \langle(0.79, 0.87), (0.1, 0.12), (0, 0)\rangle, \langle(0.60, 0.9, 0.6), (0.12, 0.1, 0.12), (0.1, 0.1, 0.1)\rangle\}.$$

The weighted similarity measure values are obtained by applying Eq.(1), resulting in:

$$M_w(V_1, S^*) = 0.8618, \quad M_w(V_2, S^*) = 0.8360, \quad M_w(V_3, S^*) = 0.8816, \quad M_w(V_4, S^*) = 0.8115$$

since $M_w(V_3, S^*) > M_w(V_1, S^*) > M_w(V_2, S^*) > M_w(V_4, S^*)$ .Thus, alternative V_3 emerges as the most desirable option among all the construction projects.

6. **Comparision**

To assess the effectiveness of this new proposed decision-making method based on RSVNS, a comparative study is conducted to compare the reliability and preference rankings of alternatives acquired by using proposed method and the existing method presented by Stanujkic D. [23]. To apply the data base, which is in the form of RSVNS, in existing method presented by Stanujkic D., the arithmetic mean approach is used to transform the refined single-valued neutrosophic data into single-valued neutrosophic data. This transfored data (in SVNS form) can then be used in the existing method to evaluate the reliability and preference rankings.

The single-valued neutrosophic data obtained through Tables 3-5 are summarized in Tables 11-13.

Table 11. Ratings given by the expert one in the form of SVNS

	T_1	T_2	T_3	T_4
V_1	$(0.43, 0.2, 0.13)$	$(0.85, 0.15, 0.1)$	$(0.75, 0.1, 0.15)$	$(0.46, 0.13, 0.16)$
V_2	$(0.4, 0.2, 0.06)$	$(0.75, 0.15, 0.2)$	$(0.75, 0.15, 0.15)$	$(0.53, 0.13, 0.13)$
V_3	$(0.6, 0.2, 0.13)$	$(0.75, 0.15, 0.15)$	$(0.75, 0.2, 0.1)$	$(0.8, 0.1, 0.1)$
V_4	$(0.63, 0.16, 0.13)$	$(0.75, 0.1, 0.15)$	$(0.75, 0.2, 0.1)$	$(0.46, 0.1, 0.13)$

Table 12. Ratings given by the expert second in the form of SVNS

	T_1	T_2	T_3	T_4
V_1	$(0.7, 0.16, 0.16)$	$(0.85, 0.2, 0.1)$	$(0.75, 0.1, 0.15)$	$(0.56, 0.1, 0.16)$
V_2	$(0.53, 0.13, 0.13)$	$(0.8, 0.1, 0.1)$	$(0.8, 0.1, 0.1)$	$(0.63, 0.13, 0.13)$
V_3	$(0.7, 0.2, 0.13)$	$(0.75, 0.2, 0.1)$	$(0.75, 0.1, 0.1)$	$(0.56, 0.16, 0.13)$
V_4	$(0.63, 0.13, 0.2)$	$(0.7, 0.15, 0.15)$	$(0.75, 0.25, 0.1)$	$(0.63, 0.2, 0.13)$

The reliability ratings obtained from three decision-makers are summarized in Tables 14, 15, and 16.

Table 13. Ratings given by the expert third in the form of SVNS

	T_1	T_2	T_3	T_4
V_1	$(0.63, 0.1, 0.13)$	$(0.75, 0.1, 0.15)$	$(0.6, 0.1, 0.05)$	$(0.53, 0.16, 0.13)$
V_2	$(0.5, 0.16, 0.1)$	$(0.75, 0.15, 0.15)$	$(0.75, 0.1, 0.1)$	$(0.63, 0.16, 0.1)$
V_3	$(0.53, 0.13, 0.16)$	$(0.85, 0.15, 0.15)$	$(0.65, 0.15, 0.1)$	$(0.5, 0.1, 0.1)$
V_4	$(0.6, 0.16, 0.16)$	$(0.75, 0.1, 0.15)$	$(0.75, 0.1, 0.15)$	$(0.5, 0.13, 0.13)$

Table 14. Reliability ratings provided by the expert one

	T_1	T_2	T_3	T_4
V_1	0.3698	0.6818	0.6000	0.4000
V_2	0.5151	0.5000	0.5714	0.5063
V_3	0.5053	0.5714	0.5714	0.7000
V_4	0.5434	0.6000	0.5714	0.4342
Average reliability		0.5400		

Table 15. Reliability ratings provided by the expert second

	T_1	T_2	T_3	T_4
V_1	0.5294	0.6521	0.6000	0.4878
V_2	0.5063	0.7000	0.7000	0.5617
V_3	0.5728	0.6190	0.6842	0.5058
V_4	0.4479	0.5238	0.5454	0.5208
Average reliability		0.5722		

Table 16. Reliability ratings provided by the expert third

	T_1	T_2	T_3	T_4
V_1	0.5813	0.6000	0.7333	0.4878
V_2	0.5263	0.5714	0.6842	0.5955
V_3	0.4512	0.6086	0.6111	0.5714
V_4	0.4782	0.6000	0.6000	0.4868
Average reliability		0.5741		

Table 17. Experts' average reliability ratings

	Average reliability
X_1	0.5400
X_2	0.5722
X_3	0.5741

Table 17 shows the average reliability ratings, which is obtained from Tables 14-16.

The following scenario illustrates the evaluation of the alternatives. A group-decision matrix, illustrated in Table 18, is developed by using the methodology given by Stanujkic D. [23] along with the weights $w_j = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

Table 18. Group decision-matrix

	T_1	T_2	T_3	T_4
w_j	0.28	0.18	0.33	0.21
V_1	$(0.58, 0.16, 0.14)$	$(0.81, 0.15, 0.11)$	$(0.7, 0.1, 0.11)$	$(0.51, 0.13, 0.15)$
V_2	$(0.47, 0.16, 0.09)$	$(0.76, 0.13, 0.15)$	$(0.76, 0.11, 0.11)$	$(0.59, 0.14, 0.12)$
V_3	$(0.61, 0.17, 0.14)$	$(0.78, 0.16, 0.13)$	$(0.71, 0.15, 0.1)$	$(0.62, 0.12, 0.11)$
V_4	$(0.62, 0.15, 0.16)$	$(0.73, 0.11, 0.15)$	$(0.75, 0.18, 0.11)$	$(0.53, 0.14, 0.13)$

The overall rating and ideal point (V^+) is computed using weighting vector, $w_j = (0.28, 0.18, 0.33, 0.21)$ and is summarized in Table 19 .

Table 19. Overall ratings and ideal point (V^+)

	overall ratings
V_1	(0.6463 ,0.1321,0.1268)
V_2	(0.6431,0.1339,0.1137)
V_3	(0.6757,0.1511,0.1187)
V_4	(0.6638,0.1506,0.1344)
V^+	(0.6757,0.1321,0.1137)

The Hamming distance h_i between each alternatives $V_i (i = 1, 2, 3, 4)$ and the ideal point V^+ and the ranking order of alternatives is summarized in Table 20.

Table 20. Ranking using hamming distance

	h_i	Rank
V_1	0.0054	1
V_2	0.0102	3
V_3	0.0080	2
V_4	0.0170	4

The average reliability ratings obtained by using proposed method and existing method [23]are summarized in Table 21.

Table 21. Average reliability ratings

	Existing method	Proposed method
X_1	0.5400	0.6142
X_2	0.5722	0.6281
X_3	0.5741	0.6588

It is apparent from Table 21 that proposed decision-making method yields superior reliability ratings compared to the existing method. Our proposed method uses more complex data as RSVNS form in spite of data as form of SVNS for better results. By refining the data, we are able to reduce errors and inconsistencies, resulting in more accurate and reliable outcomes.

The preference ranking obtained by using proposed method and the exiting method are summarized in Table 22.

TABLE 22. Prefrence ranking

Method	Ranking	Best alternative
Existing method	$V_1 > V_3 > V_2 > V_4$	V_1
Proposed method	$V_3 > V_1 > V_2 > V_4$	V_3

This comparison revels that alternative V_3 getting by our proposed mehod is different from existing method V_1, which also leads the fact that RSVNS environment affect the results obtained by using SVNS environment. Additionally, using larger membership function or refined membership function also improves the reliability parameter of the given data, making the results even more fruitful.

6.1 Applications of proposed method

The proposed decision-making framework is highly adaptable, with potential applications in numerous fields in uncertain environment.

(1) The framework's scope extends to multiple domains like, economic growth model, reliability allocation model, and many more where uncertainty and subjective factors play a significant role.

(2) Medical institutions can utilize this methodology to enhance patient care, ranking treatment strategies and assessing care quality, all while incorporating expert insights that navigate uncertainty.

(3) Businesses can leverage this approach to streamline procurement processes, supplier selection and manage risk accounting for uncertainty in decision-making.

(4) This new method also tackles the data contining uncertainty as well as fuzziness in multi-criteria decision-making (in more accurate way).

7. Conclusion

This study presents a new technique for evaluating the reliability of the data in the form of refined single-valued neutrosophic numbers, and for calculating the weighted average of a set of such numbers. A decision-making problem by using refined single-valued neutrosophic data is subsequently established by using the similarity measure of RSVNS. The similarity measure between each alternative and the ideal alternative facilitates the establishment of a ranking order for all alternatives within the decision-making framework, enabling the selection of the most favorable option. In this research, a practical example involving construction project selection is considered to compare the proposed method based on RSVNS theory and the existing method based on SVNS theory. Future work can explore extensions of RSVNS for handilg more complex data. The benefit of this approach lies in its straightforward evaluation process, making it especially suitable for real-world decision-making scenarios involving refined single-valued neutrosophic information.

7.1 Future work

This study provides a robust methodological groundwork for reliability evaluation of RSVNS data, which play key role in many fields. Future research can explore this method for handling complex MCDM problems, and to diverse domains, such as supply chain management, envirnomental impact assessment, transportation systems, and many more characterized by uncertainty, ambiguity, and subjective judgement. Additionally, many extensions of refined neutrosophic sets can be used for exploring new development and improvement. Moreover, applying the proposed method in big data analytics can lead to innovative solutions for real-world challenges.

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