NSS

Neutrosophic Sets and Systems, Vol. 94, 2025

University of New Mexico

Reliability estimation using interval-valued neutrosophic number for multi-criteria decision making problems

Supriya .[1], Suneel Kumar [2], and Deepak Kumar[3*]

[1]Department of Mathematics ; D.S.B. Campus, Kumaun University, Nainital–263002, India; Email: tiruwasupriya15792@gmail.com;

[2]Department of Mathematics; D.S.B. Campus, Kumaun University, Nainital–263002, India; Email: Sunilarya63207@gmail.com;

[3*]Dr. Deepak Kumar, Department of Mathematics ; D.S.B. Campus, Kumaun University Nainital–263002, India ; Email: deepakdev16@gmail.com

[*]Correspondence: Email: deepakdev16@gmail.com

Abstract. Neutrosophic sets have garnered significant attention in recent years for their ability to handle imprecise, indeterminate, and inconsistent information, making them particularly effective in solving complex decision-making problems. Among their various extensions, Interval-Valued Neutrosophic Sets (IVNS) provide a more expressive framework than classical fuzzy or intuitionistic fuzzy systems by introducing three independent membership intervals: truth, indeterminacy, and falsity. By incorporating this enhanced structure, decision-making processes can better capture practical situations involving ambiguity, indecision, and missing data, especially in MCDM contexts. However, traditional reliability assessment methods in the IVNS environment often lead to logically inconsistent outcomes, sometimes resulting in negative reliability values—an issue that directly impacts the reliability of decision-making processes. To overcome this critical limitation, the present study proposes a normalized reliability function that ensures all reliability estimates are strictly confined within the $[0, 1]$ range, thereby maintaining both logical consistency and interpretability. For instance, take the interval-valued neutrosophic number $x = \langle(0.5, 0.9), (0.1, 0.2), (0.2, 0.3)\rangle$. Traditional reliability computations produce results in the range (0.27, 0.50). In contrast, the new method adjusts this estimation to (0.37, 0.42), providing outcomes that are both more consistent and practically meaningful. In addition to the normalized reliability function, the study incorporates score functions, average vector strategies, and the Hamming distance measure to further improve decision-making accuracy under the IVNS framework. The effectiveness and practicality of the proposed methodology are demonstrated through a real-world case study in which a customer selects the most suitable agent from three alternatives based on four decision criteria, showcasing the model's robustness and potential for application in complex MCDM scenarios.

Keywords: Interval-valued neutrosophic number (IVNN); reliability estimation; normalized reliability measure; multi-criteria decision-making (MCDM); hamming distance; score function; aggregation operator.

---

1. Introduction

Since its inception by Zadeh in 1965 [14], fuzzy set theory has provided a foundational framework for modeling uncertainty by allowing elements to possess varying degrees of membership. Building upon this concept, Atanassov [1] introduced Intuitionistic Fuzzy Sets (IFS), which simultaneously account for both membership and non-membership values, offering a more comprehensive way to manage uncertainty. However, these models lack the explicit capacity to represent indeterminate or ambiguous information. To address this limitation, Smarandache [8] introduced Neutrosophic Sets (NS), where the truth, indeterminacy, and falsity components are treated as independent parameters, thus enabling a more flexible and generalized approach to uncertainty modeling. Further developments led Wang et al. [16] to propose Single-Valued and Interval-Valued Neutrosophic Sets (SVNS and IVNS), constraining each membership function to the standard unit interval $[0, 1]$. This constraint facilitates real-world applications by aligning the theoretical constructs with practical decision-making scenarios. Among these, Interval-Valued Neutrosophic Numbers (IVNNs) have garnered particular attention in multi-criteria decision-making (MCDM) [7, 16], medical diagnostics, system reliability evaluation, and network security analysis. Recent contributions in the journal Neutrosophic Sets and Systems (NSS) have further enriched this field. For example, Yang et al. [11] presented an IVNN-based reliability allocation model for cybersecurity risk management, while Aydogdu [2] developed new entropy and similarity measures grounded in the IVNN framework. These advancements highlight the growing relevance of neutrosophic methodologies in practical domains. Despite significant progress, a critical gap remains in the domain of reliability estimation using IVNNs. Traditional models, such as those discussed by Smarandache et al. [8], often yield reliability values in the interval $[-1, 1]$, occasionally resulting in negative outputs. This is fundamentally problematic, as reliability—by its very nature—should be a non-negative quantity representing the likelihood of system success or functionality. For instance, given an IVNN $x = \langle (0.5, 0.9),\ (0.1, 0.2),\ (0.2, 0.3) \rangle$, applying the conventional reliability formula yields $r_1(x) = (0.27, 0.50)$, but in other scenarios, negative values may emerge, violating logical constraints.

To overcome this issue, we propose a normalized reliability function specifically tailored for IVNNs. Our method guarantees that all results are confined within the interval $[0, 1]$, thereby eliminating the possibility of negative outputs and enhancing the interpretability of the results. Using the same example, the proposed approach computes $r_2(x) = (0.37, 0.42)$, producing reliability values that are not only logically consistent but also more informative and robust for decision-making purposes. This paper presents a mathematically sound and practically viable framework for IVNN-based reliability estimation. Our approach integrates seamlessly with existing MCDM methodologies by utilizing score functions, hamming distance measures, and weighted aggregation operators. To validate the method, we apply it to a decision-making scenario involving four alternatives, four evaluation criteria, and expert input from three decision-makers. The results confirm the stability and effectiveness of the proposed model, highlighting its potential for broader application. Moreover, the proposed method addresses critical needs in modern system design, where reliability is a central concern affecting performance, safety, and sustainability. This is particularly relevant in fields such as electrical circuits, control systems, and signal processing architectures, where continuous and reliable operation is essential. Traditional reliability analyses typically assume simplified configurations, such as series, parallel, or hybrid systems, where probabilistic models suffice. However, real-world engineering systems often involve complex interconnections and dependencies that cannot be reduced to standard configurations. These are categorized as non-series-parallel (NSP) systems, and their evaluation requires more sophisticated analytical models. By offering a normalized reliability framework compatible with IVNNs, this work contributes not only to theoretical advancement but also to practical system reliability analysis under uncertainty. Furthermore, the model lays the groundwork for future research in hesitant, bipolar, and generalized neutrosophic frameworks and supports integration with hybrid artificial intelligence (AI)-based decision support systems for complex, real-time applications.

2. **Preliminaries**

An introduction to the basic theories of fuzzy sets (FS), neutrosophic sets (NS), and interval-valued neutrosophic sets (IVNS) is presented in this section, including their primary computational methods.

2.1. *Fuzzy Set (FS) [14]*

"The definition of a fuzzy set is a membership function that associates elements from a discourse X world with the interval [0, 1]. A fuzzy set A in X can be formalized as a collection of ordered pairs:

$$A = \{(x, \mu_A(x)) : x \in X\}$$

The membership function in this case is represented by $\mu_A(x) : X \to [0, 1]$ which gives each element x a membership degree between 0 and 1. The degree to which x belongs to the fuzzy set A is indicated by this membership value."

2.2. *Neutrosophic set (NS) [8]*

"A neutrosophic set (NS) in a universe ξ is defined by three distinct membership functions: Truth Membership Function (T_A), Indeterminacy Membership Function (I_A), Falsity Membership Function (F_A). These functions, T_A, I_A, and F_A, map elements to real standard values within the interval $]-0,1[$. A neutrosophic set A can be represented as:

$$A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle : x \in \xi, T_A, I_A, F_A \in]-0,1[\}$$

Notably, the sum of these membership functions is unrestricted, allowing for flexibility in the representation of uncertainty:
$-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^{+}$"

2.3. *Interval-valued neutrosophic set (IVNS) [8]*

"Consider a universe of discourse ξ comprising points (objects) denoted by x. An interval-valued neutrosophic set (IVNS) A is defined by three interval-valued membership functions: Interval Truth Membership: $T_A(x) = [a_1, b_1] \subseteq [0,1]$, Interval Indeterminacy Membership: $I_A(x) = [a_2, b_2] \subseteq [0,1]$, Interval Falsity Membership: $F_A(x) = [a_3, b_3] \subseteq [0,1]$.

For each $x \in \xi$, the membership functions satisfy $T_A(x), I_A(x), F_A(x) \subseteq [0,1]$. The IVNS A can be represented as:
$A = \{\langle x, (T_A(x), I_A(x), F_A(x))\rangle : x \in \xi\}$"

2.4. *Interval-valued neutrosophic weighted average vector (IVNWAV) [8]*

"Let X_i $(i = 1, 2, \ldots, n)$ be a set of Interval-Valued Neutrosophic Numbers (IVNNs), where each X_i is defined as:

$$X_i = \left\{\left(T_{x_i}^L,\ T_{x_i}^U\right),\ \left(I_{x_i}^L,\ I_{x_i}^U\right),\ \left(F_{x_i}^L,\ F_{x_i}^U\right)\right\}.$$

Let $w = (w_1, w_2, \ldots, w_n)^T$ be the associated weight vector, satisfying:

$$w_i \in [0,1], \quad \sum_{i=1}^{n} w_i = 1.$$

The interval-valued neutrosophic weighted average vector (IVNWAV) operator is defined as:

$$\text{IVNWAV}(X_1, X_2, \ldots, X_n) = \left\{\left(T^L,\ T^U\right),\ \left(I^L,\ I^U\right),\ \left(F^L,\ F^U\right)\right\},$$

where:

$$T^L = \sum_{i=1}^{n} w_i \cdot T_{x_i}^L, \qquad T^U = \sum_{i=1}^{n} w_i \cdot T_{x_i}^U,$$
$$I^L = \sum_{i=1}^{n} w_i \cdot I_{x_i}^L, \qquad I^U = \sum_{i=1}^{n} w_i \cdot I_{x_i}^U,$$
$$F^L = \sum_{i=1}^{n} w_i \cdot F_{x_i}^L, \qquad F^U = \sum_{i=1}^{n} w_i \cdot F_{x_i}^U.$$

This operator aggregates multiple IVNNs into a single interval-valued neutrosophic number by considering the relative importance of each element through the assigned weights."

2.5. *Hamming distance of interval-valued neutrosophic number [8]*

"Let $X_A = \{\left(T_A^L(x),\ T_A^U(x)\right),\ \left(I_A^L(x),\ I_A^U(x)\right),\ \left(F_A^L(x),\ F_A^U(x)\right)\}$ be an Interval-Valued Neutrosophic Number (IVNN).

Let $x = \{\left(t_x^L,\ t_x^U\right),\ \left(i_x^L,\ i_x^U\right),\ \left(f_x^L,\ f_x^U\right)\}$ be the comparative IVNN.

The ideal point x^+ is defined as:

$$x^+ = \left\{\left(t_x^L,\ t_x^U\right)^+,\ \left(i_x^L,\ i_x^U\right)^+,\ \left(f_x^L,\ f_x^U\right)^+\right\},$$

where:

$$(t_x^L,\ t_x^U)^+ = \max\left(T_A^L(x),\ T_A^U(x)\right),$$
$$(i_x^L,\ i_x^U)^+ = \min\left(I_A^L(x),\ I_A^U(x)\right),$$
$$(f_x^L,\ f_x^U)^+ = \min\left(F_A^L(x),\ F_A^U(x)\right).$$

The hamming distance between x and the ideal point x^+ is given by:

$$H(x) = \frac{1}{6}\Bigg(\left|T_A^L(x) - (t_x^L)^+\right| + \left|T_A^U(x) - (t_x^U)^+\right| + \left|I_A^L(x) - (i_x^L)^+\right| + \left|I_A^U(x) - (i_x^U)^+\right| + \left|F_A^L(x) - (f_x^L)^+\right| + \left|F_A^U(x) - (f_x^U)^+\right|\Bigg)$$"

This formula represents the average of the absolute differences between the corresponding lower and upper bounds of truth, indeterminacy, and falsity in x and the ideal point x^+. It provides a quantitative measure of the distance between two IVNNs under the neutrosophic environment.

2.6. *Computation of a score index for interval-valued fuzzy sets [8]*

"Let $A = \{\left(T_A^L(x) + T_A^U(x)\right),\ \left(I_A^L(x) + I_A^U(x)\right),\ \left(F_A^L(x) + F_A^U(x)\right)\}$ be an Interval-Valued Fuzzy Set (IVFS).

The score function of A, denoted by $S(A)$, is defined as:

$$S(A) = \{\left(T_A^L(x) + T_A^U(x) - 1\right),\ \left(I_A^L(x) + I_A^U(x) - 1\right),\ \left(F_A^L(x) + F_A^U(x) - 1\right)\}.$$

This score index represents the aggregated evaluation of the interval-valued fuzzy set by summing the lower and upper bounds of the truth, indeterminacy, and falsity memberships, respectively, and adjusting each by subtracting 1. It provides a comparative measure to support decision-making processes involving interval-valued fuzzy information."

3. **Reliability Assessment of Data Represented by Interval-Valued Neutrosophic Numbers**

"Smarandache et al. [8] defined the reliability $r(x)$ of an interval-valued neutrosophic number (IVNN) as:

$$r(x) = \left\{ \frac{t^L - f^L}{1 + i^L}, \ \frac{t^U - f^U}{1 + i^U} \right\},$$

where $\left(t^L, t^U\right)$, $\left(i^L, i^U\right)$, and $\left(f^L, f^U\right)$ represent the truth, indeterminacy, and falsity membership intervals of the IVNN, respectively, and $r(x) \in [-1, 1]$.

However, in practical scenarios, reliability values cannot be negative, as this would have no real-world interpretation. Therefore, we propose an alternative reliability evaluation method that ensures $r(x)$ remains within the closed interval $[0, 1]$."

Proposed Reliability Computation

The reliability of an IVNN is computed as follows:

For the lower bound:

$$r_{\text{lower}}(x) = \begin{cases} \dfrac{|t^L - f^L|}{t^L + i^L + f^L}, & \text{if } t^L + i^L + f^L \neq 0, \\ 0, & \text{if } t^L + i^L + f^L = 0. \end{cases}$$

For the upper bound:

$$r_{\text{upper}}(x) = \begin{cases} \dfrac{|t^U - f^U|}{t^U + i^U + f^U}, & \text{if } t^U + i^U + f^U \neq 0, \\ 0, & \text{if } t^U + i^U + f^U = 0. \end{cases}$$

As a result, the overall reliability is given by:

$$r(x) = \left(r_{\text{lower}}(x), \ r_{\text{upper}}(x)\right), \quad r(x) \in [0, 1].$$

Numerical Example

Consider the IVNN:

$$x = \langle (0.5, \ 0.9), \ (0.1, \ 0.2), \ (0.2, \ 0.3) \rangle .$$

Using the original method defined by Smarandache et al., the reliability is computed as:

$$r(x) = \left\{ \frac{0.5 - 0.2}{1 + 0.1}, \ \frac{0.9 - 0.3}{1 + 0.2} \right\} = \left(\frac{0.3}{1.1}, \ \frac{0.6}{1.2} \right) = (0.27, \ 0.50).$$

Using the proposed approach, the reliability is calculated as follows:

For the lower bound:

$$r_{\text{lower}}(x) = \frac{|0.5 - 0.2|}{0.5 + 0.1 + 0.2} = \frac{0.3}{0.8} = 0.37,$$

For the upper bound:

$$r_{\text{upper}}(x) = \frac{|0.9 - 0.3|}{0.9 + 0.2 + 0.3} = \frac{0.6}{1.4} = 0.42.$$

Thus, the reliability of x using the proposed method is:

$$r(x) = (0.37,\ 0.42).$$

Comparative Analysis

By comparing the results obtained using the traditional method and the proposed approach, it is evident that the proposed method provides more practical and meaningful reliability estimates, as it strictly bounds the result within $[0, 1]$ and avoids negative values.

A detailed comparative analysis of the reliability values computed using the conventional and proposed methods for various input values of $(t^L,\ t^U)$, $(i^L,\ i^U)$, and $(f^L,\ f^U)$ is presented in Tables 1 and 2.

TABLE 1. Reliability Comparison

t^U	i^U	f^U	**Eq. (4)**	**Eq. (5)**
0	0	0	0	0
1	0	0	1	1
0	1	0	0	0
0	0	1	-1	1
1	1	0	0.5	0.5
1	1	1	0	0
1	0	1	0	0

Table 2. Reliability Comparison

t^L	i^L	f^L	**Eq. (4)**	**Eq. (5)**
0	0	0	0	0
1	0	0	1	1
0	1	0	0	0
0	0	1	-1	1
1	1	0	0.5	0.5
1	1	1	0	0
1	0	1	0	0

From Tables 1 and 2, it is evident. The values in Eq. (4) come from the interval [-1, 1], where zero is the least acceptable number. One would want a greater value of the reliability function, and equation (5) gives values from the interval [0, 1].

4. Solving Multi-Criteria Decision-Making Problems Using Interval-Valued Neutrosophic Numbers

To demonstrate the applicability of the proposed method, a practical Multi-Criteria Decision-Making (MCDM) scenario is considered in which a customer must select the most appropriate agent from a set of three alternatives: A_1, A_2, and A_3. A panel of three domain experts is engaged to evaluate each alternative based on four critical criteria: Cost (Cr_1), Environmental Impact (Cr_2), Social Contribution (Cr_3), and Medical Facility Provision (Cr_4).

The Cost criterion focuses on the financial aspects, including affordability and the perceived value of each option. The Environmental Impact criterion assesses the sustainability measures and ecological footprint associated with each alternative. The Social Contribution criterion evaluates the broader social implications, such as employment generation, community development, and social welfare improvements linked to each choice. Lastly, the Medical Facility Provision criterion considers the availability, accessibility, and quality of healthcare services facilitated by the respective alternatives. This structured assessment allows for a comprehensive and balanced decision-making process that aligns with the customer's priorities and societal values. By integrating the expert evaluations using Interval-Valued Neutrosophic Numbers (IVNNs), the proposed framework ensures that uncertainty, hesitation, and incomplete information are systematically managed during the decision process. The individual ratings provided by the first expert are presented in Table 3.

Table 3. Ratings of Three Experts

(a) First Expert's Ratings

Criteria	$\mathbf{Cr}_1$	$\mathbf{Cr}_2$	$\mathbf{Cr}_3$	$\mathbf{Cr}_4$
$\mathbf{A}_1$	$(0.4, 0.8)$, $(0.2, 0.3)$, $(0.3, 0.5)$	$(0.5, 0.8)$, $(0.2, 0.3)$, $(0.3, 0.4)$	$(0.4, 0.9)$, $(0.1, 0.3)$, $(0.2, 0.4)$	$(0.6, 0.9)$, $(0.1, 0.2)$, $(0.3, 0.4)$
$\mathbf{A}_2$	$(0.6, 0.9)$, $(0.1, 0.2)$, $(0.4, 0.5)$	$(0.3, 0.8)$, $(0.1, 0.4)$, $(0.2, 0.5)$	$(0.5, 0.7)$, $(0.3, 0.5)$, $(0.4, 0.5)$	$(0.5, 0.9)$, $(0.1, 0.2)$, $(0.2, 0.3)$
$\mathbf{A}_3$	$(0.5, 0.8)$, $(0.1, 0.3)$, $(0.3, 0.4)$	$(0.5, 0.7)$, $(0.3, 0.4)$, $(0.4, 0.5)$	$(0.5, 0.9)$, $(0.1, 0.2)$, $(0.2, 0.3)$	$(0.4, 0.8)$, $(0.1, 0.3)$, $(0.3, 0.4)$
$\mathbf{A}_4$	$(0.7, 0.9)$, $(0.2, 0.3)$, $(0.4, 0.5)$	$(0.4, 0.7)$, $(0.1, 0.2)$, $(0.2, 0.3)$	$(0.4, 0.8)$, $(0.1, 0.3)$, $(0.2, 0.3)$	$(0.7, 0.9)$, $(0.3, 0.4)$, $(0.4, 0.5)$

(b) Second Expert's Ratings

Alternatives	$\mathbf{Cr}_1$	$\mathbf{Cr}_2$	$\mathbf{Cr}_3$	$\mathbf{Cr}_4$
$\mathbf{A}_1$	$(0.4, 0.9)$, $(0.1, 0.4)$, $(0.3, 0.5)$	$(0.5, 0.9)$, $(0.1, 0.3)$, $(0.2, 0.3)$	$(0.6, 0.9)$, $(0.2, 0.3)$, $(0.3, 0.4)$	$(0.4, 0.9)$, $(0.2, 0.3)$, $(0.2, 0.4)$
$\mathbf{A}_2$	$(0.6, 0.8)$, $(0.2, 0.3)$, $(0.4, 0.5)$	$(0.5, 0.7)$, $(0.2, 0.4)$, $(0.4, 0.5)$	$(0.5, 0.7)$, $(0.1, 0.2)$, $(0.3, 0.4)$	$(0.5, 0.9)$, $(0.1, 0.3)$, $(0.2, 0.3)$
$\mathbf{A}_3$	$(0.4, 0.9)$, $(0.1, 0.2)$, $(0.2, 0.4)$	$(0.6, 0.8)$, $(0.3, 0.4)$, $(0.4, 0.5)$	$(0.5, 0.9)$, $(0.3, 0.4)$, $(0.3, 0.5)$	$(0.6, 0.8)$, $(0.3, 0.5)$, $(0.5, 0.6)$
$\mathbf{A}_4$	$(0.6, 0.9)$, $(0.1, 0.2)$, $(0.4, 0.5)$	$(0.4, 0.7)$, $(0.1, 0.3)$, $(0.2, 0.3)$	$(0.7, 0.9)$, $(0.4, 0.5)$, $(0.5, 0.6)$	$(0.6, 0.9)$, $(0.2, 0.4)$, $(0.4, 0.5)$

(c) Third Expert's Ratings

Alternatives	$\mathbf{Cr}_1$	$\mathbf{Cr}_2$	$\mathbf{Cr}_3$	$\mathbf{Cr}_4$
$\mathbf{A}_1$	$(0.6, 0.8)$, $(0.3, 0.4)$, $(0.4, 0.5)$	$(0.6, 0.9)$, $(0.2, 0.3)$, $(0.3, 0.4)$	$(0.4, 0.6)$, $(0.1, 0.4)$, $(0.3, 0.4)$	$(0.5, 0.9)$, $(0.2, 0.4)$, $(0.4, 0.5)$
$\mathbf{A}_2$	$(0.6, 0.9)$, $(0.2, 0.4)$, $(0.3, 0.4)$	$(0.5, 0.8)$, $(0.1, 0.2)$, $(0.3, 0.4)$	$(0.3, 0.8)$, $(0.1, 0.4)$, $(0.2, 0.5)$	$(0.4, 0.7)$, $(0.1, 0.2)$, $(0.2, 0.3)$
$\mathbf{A}_3$	$(0.5, 0.9)$, $(0.3, 0.5)$, $(0.4, 0.5)$	$(0.4, 0.8)$, $(0.2, 0.3)$, $(0.3, 0.5)$	$(0.4, 0.9)$, $(0.1, 0.2)$, $(0.2, 0.4)$	$(0.6, 0.8)$, $(0.3, 0.5)$, $(0.5, 0.6)$
$\mathbf{A}_4$	$(0.4, 0.7)$, $(0.1, 0.2)$, $(0.2, 0.3)$	$(0.5, 0.9)$, $(0.2, 0.5)$, $(0.3, 0.5)$	$(0.5, 0.8)$, $(0.1, 0.6)$, $(0.4, 0.5)$	$(0.4, 0.8)$, $(0.1, 0.3)$, $(0.2, 0.3)$

Table 4 presents the reliability scores computed using both Equation (4), representing the conventional method, and Equation (5), reflecting the proposed normalized approach. The table also includes the overall average reliability of the experts' evaluations, providing a comprehensive assessment of the consistency and dependability of the collected ratings.

Table 4. Reliability of Ratings Using Equation (4)

Alternatives	$\mathbf{Cr}_1$	$\mathbf{Cr}_2$	$\mathbf{Cr}_3$	$\mathbf{Cr}_4$
A_1	(0.08, 0.23)	(0.16, 0.30)	(0.18, 0.38)	(0.27, 0.41)
A_2	(0.18, 0.33)	(0.09, 0.21)	(0.07, 0.13)	(0.27, 0.54)
A_3	(0.18, 0.30)	(0.07, 0.14)	(0.27, 0.50)	(0.09, 0.30)
A_4	(0.25, 0.30)	(0.18, 0.36)	(0.18, 0.38)	(0.23, 0.28)
Average	(0.171875, 0.318125)			

Table 5. First Expert's Reliability Assessments Using Equation (5)

Alternatives	$\mathbf{Cr}_1$	$\mathbf{Cr}_2$	$\mathbf{Cr}_3$	$\mathbf{Cr}_4$
A_1	(0.11, 0.18)	(0.20, 0.26)	(0.28, 0.31)	(0.30, 0.33)
A_2	(0.18, 0.25)	(0.16, 0.17)	(0.08, 0.11)	(0.37, 0.42)
A_3	(0.22, 0.26)	(0.08, 0.12)	(0.37, 0.42)	(0.12, 0.26)
A_4	(0.2307, 0.2352)	(0.28, 0.33)	(0.28, 0.35)	(0.21, 0.22)
Average	(0.21691875, 0.264075)			

Table 6 presents the average reliability scores of the evaluations given by the three decision-makers, determined using the proposed normalized reliability formula (Equation 5). This analysis serves to assess the overall trustworthiness and consistency of the experts' inputs in the MCDM framework.

Table 6. Expert-Based Reliability Assessments

Experts	**Reliability**
Ep_1	(0.21691875, 0.264075)
Ep_2	(0.204375, 0.24625)
Ep_3	(0.185, 0.235625)

As presented in Table 6, the evaluations provided by all three experts demonstrate a high degree of consistency, indicating that their judgments are reliable and appropriate for inclusion in the subsequent alternative assessment process. However, in cases where a respondent's average reliability score falls below an acceptable threshold, it is advisable to either exclude their evaluations from the decision-making process or request a revision to improve reliability to an acceptable level.

The following section introduces a practical scenario for evaluating multiple alternatives. Table 7 displays the construction of a group decision matrix based on Equation (1), using equal weights for each decision criterion, represented as:

$$w_j = \left\{ \left(\frac{1}{3}, \frac{1}{3}\right), \left(\frac{1}{3}, \frac{1}{3}\right), \left(\frac{1}{3}, \frac{1}{3}\right) \right\}.$$

Subsequently, the overall ratings of the alternatives are computed using Equation (1) along with the weighting vector:

$$w_j = (0.21,\ 0.27,\ 0.24,\ 0.28).$$

Furthermore, the ideal solution is determined by applying Equation (3), which utilizes the score function. The aggregated results, including the overall ratings and the ideal point, are summarized in Table 8.

Table 7. Group Decision Matrix with Weights (Split Criteria)

(a) Criteria Cr_1 and Cr_2

Alternatives	$\mathbf{Cr_1}$	$\mathbf{Cr_2}$
Weights ($\mathbf{w}_j$)	0.21	0.27
A_1	$(0.46, 0.82), (0.19, 0.36), (0.33, 0.46)$	$(0.52, 0.86), (0.15, 0.30), (0.26, 0.36)$
A_2	$(0.60, 0.86), (0.15, 0.29), (0.36, 0.45)$	$(0.42, 0.75), (0.12, 0.32), (0.29, 0.45)$
A_3	$(0.45, 0.86), (0.16, 0.32), (0.29, 0.42)$	$(0.49, 0.75), (0.26, 0.36), (0.36, 0.48)$
A_4	$(0.56, 0.83), (0.12, 0.22), (0.32, 0.42)$	$(0.42, 0.76), (0.12, 0.32), (0.22, 0.36)$

(b) Criteria Cr_3 and Cr_4

Alternatives	$\mathbf{Cr_3}$	$\mathbf{Cr_4}$
Weights ($\mathbf{w}_j$)	0.24	0.28
A_1	$(0.46, 0.80), (0.12, 0.33), (0.26, 0.39)$	$(0.49, 0.90), (0.15, 0.29), (0.29, 0.42)$
A_2	$(0.42, 0.72), (0.16, 0.35), (0.29, 0.45)$	$(0.54, 0.83), (0.09, 0.22), (0.18, 0.30)$
A_3	$(0.45, 0.90), (0.16, 0.25), (0.22, 0.39)$	$(0.53, 0.78), (0.23, 0.42), (0.42, 0.53)$
A_4	$(0.52, 0.82), (0.19, 0.46), (0.35, 0.46)$	$(0.56, 0.86), (0.19, 0.36), (0.32, 0.42)$

Table 8. Overall Ratings with Ideal Point

Alternatives	**Overall Ratings**
A_1	$(0.4846, 0.8484), (0.1512, 0.317), (0.2831, 0.405)$
A_2	$(0.4914, 0.7883), (0.1275, 0.2929), (0.2739, 0.408)$
A_3	$(0.4832, 0.8175), (0.2066, 0.342), (0.3285, 0.4598)$
A_4	$(0.5126, 0.8171), (0.1564, 0.3438), (0.3002, 0.4134)$
A^+ (Ideal Point)	$(0.4846, 0.8484), (0.1275, 0.2929), (0.2739, 0.408)$

Table 9. Ranking of Alternatives

Alternatives	$\mathbf{H}_i$	**Rank**
A_1	0.010	1
A_2	0.011	2
A_3	0.044	4
A_4	0.028	3

As illustrated in Table 9, the alternative A_1 achieves the lowest hamming distance value of 0.010, indicating its closest proximity to the ideal solution. Consequently, A_1 is ranked first among the available options and is therefore considered the most preferable alternative. The ranking results suggest that A_1 provides the optimal balance across all decision criteria, making it the most suitable choice for implementation. The remaining alternatives are ranked in order of increasing hamming distance, with A_2, A_4, and A_3 occupying the subsequent positions. This ranking reflects the relative performance of each option based on the aggregated decision-making process.

5. **Conclusion**

In this study, a novel normalized reliability estimation method for interval-valued neutrosophic numbers (IVNNs) has been introduced. Unlike traditional approaches, which may generate reliability values outside logical bounds including negative outcomes the proposed method ensures that all computed reliability values are strictly confined within the interval $[0, 1]$. This enhancement not only guarantees logical consistency but also improves the interpretability and practical applicability of the results. The proposed method addresses a significant limitation of existing reliability models by refining the calculation process to reflect real-world scenarios more accurately. Since reliability in practical systems cannot be negative, the normalization approach adopted in this study strengthens the decision-making process, especially in environments characterized by uncertainty, incompleteness, and hesitation.

To validate the effectiveness and feasibility of the method, a comprehensive Multi-Criteria Decision-Making (MCDM) case study was conducted. The scenario involved the evaluation of four alternatives based on multiple criteria assessed by domain experts. Using the newly proposed reliability estimation framework, a group decision matrix was constructed, and the alternatives were ranked accordingly. The results demonstrated the robustness of the method, highlighting its capability to handle complex evaluations while maintaining consistency and fairness in the ranking process. Beyond theoretical contribution, this work holds significant promise for practical applications. The normalized IVNN-based reliability model can be effectively employed in system reliability analysis, cybersecurity risk assessment, engineering system evaluations, and other fields requiring decision-making under uncertain, imprecise, or incomplete information. It provides a valuable tool for experts and decision-makers dealing with complex systems where traditional crisp or fuzzy methods fall short.

Looking ahead, there are several potential directions for future research. The current model can be extended to accommodate more sophisticated uncertainty representations, such as hesitant, bipolar, or generalized neutrosophic environments. Additionally, integrating this reliability framework into hybrid Artificial Intelligence (AI)-based decision support systems may open new avenues for real-time and adaptive decision-making applications. Further studies can also explore its use in big data analytics, supply chain risk management, and predictive maintenance in smart systems. Overall, the proposed method contributes both theoretical advancements and practical insights, making it a promising tool for modern reliability assessment and decision analysis in uncertain environments.

Funding: The authors did not receive any specific grant, financial support, or funding from public, private, or non-profit agencies for the conduct of this research.

Conflicts of Interest: The authors declare that there are no conflicts of interest related to the publication of this manuscript.

References

[1] Atanassov K. T. On intuitionistic fuzzy sets theory. Springer, Berlin, 2012.

[2] Aydoğdu C. Entropy and similarity measures based on interval-valued neutrosophic numbers. Neutrosophic Sets and Systems, vol. 9, 2015.

[3] Chen T. Y. Comparative analysis of SAW and TOPSIS based on interval-valued fuzzy sets: Discussions on score functions and weight constraints. Expert Systems with Applications, vol. 39, no. 2, pp. 1848–1861, 2012.

[4] Gayathri N., Helen M., and Mounika P. Utilization of Jaccard Index Measures on Multiple Attribute Group Decision Making under Neutrosophic Environment. International Journal of Neutrosophic Science (IJNS), vol. 3, no. 2, pp. 67–77, 2020.

[5] Kumar D. An Evaluation of the System Reliability Using Fuzzy Lifetime Distribution Emphasising Hexagonal Fuzzy Number. IOSR Journal of Engineering (IOSRJEN), vol. 9, no. 8, pp. 17–25, 2019.

[6] Palanikumar M., Kausar N., and Deveci M. Complex Pythagorean neutrosophic normal interval-valued set with an aggregation operators using score values. Engineering Applications of Artificial Intelligence, vol. 137, pp. 109169, 2024.

[7] Saha A., and Broumi S. New operators on interval valued neutrosophic sets. Infinite Study, 2019.

[8] Smarandache F., Stanujkic D., and Karabasevic D. An approach for assessing the reliability of data contained in a single valued neutrosophic number. In: Proceedings of 4th International Scientific Conference Innovation as an Initiator of the Development MEFkon 2018, Belgrade, Serbia, pp. 80–86, 2018.

[9] Thong N. T., Smarandache F., Hoa N. D., Son L. H., Lan L. T. H., Giap C. N., and Long H. V. A novel dynamic multi-criteria decision making method based on generalized dynamic interval-valued neutrosophic set. Symmetry, vol. 12, no. 4, 618, 2020.

[10] Luo X., Wang Z., Yang L., Lu L., and Hu S. Sustainable supplier selection based on VIKOR with single-valued neutrosophic sets. Plos One, vol. 18, no. 9, e0290093, 2023.

[11] Yang X., Zhang Y., and Liu H. An IVNN-based reliability allocation model for cybersecurity. Journal of Cybersecurity Studies, vol. 77, 2025.

[12] Ye J., Du S., and Yong R. MCDM technique using single-valued neutrosophic trigonometric weighted aggregation operators. Journal of Management Analytics, vol. 11, no. 1, pp. 45–61, 2024.

[13] Zadeh L. A. The concept of a linguistic variable and its application to approximate reasoning. Information Sciences, vol. 9, pp. 43–80, 1975.

[14] Zadeh L. A. Fuzzy sets. Information and Control, vol. 8, pp. 338–353, 1965.

[15] Zhang Y. L., and Wang G. J. A deteriorating cold standby repairable system with priority in use. European Journal of Operational Research, vol. 183, no. 1, pp. 278–295, 2007.

[16] Zhang H. Y., Wang J. Q., and Chen X. H. Interval neutrosophic sets and their application in multicriteria decision making problems. The Scientific World Journal, vol. 2014, article ID 645953.

[17] Zhang C., Li D., Kang X., Liang Y., Broumi S., and Sangaiah A. K. Multi-attribute group decision making based on multigranulation probabilistic models with interval-valued neutrosophic information. Mathematics, vol. 8, no. 2, 223, 2020.

[18] Zhou L. P., Dong J. Y., and Wan S. P. Two new approaches for multi-attribute group decision-making with interval-valued neutrosophic Frank aggregation operators and incomplete weights. *IEEE Access*, vol. 7, pp. 102727–102750, 2019.

Received: May 14, 2025. Accepted: Aug 27, 2025