On New Classes of Supra Soft Connectedness via m-Polar Neutrosophic Topological Spaces

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Abstract: We presented the idea of new class for supra soft (resp; supra soft pre, supra soft β) via m-polar Neutrosophic topological spaces and their relationship in this paper. Additionally, we provide the idea of supra soft connected (resp; supra soft pre- connected, supra soft $\beta-$connected). Furthermore, provided the opposing examples of similar ideas. Lastly, we investigate and examined some properties related them.

Keywords: Supra soft pre-connected, supra soft β-connected, m-polar neutrosophic topology, supra soft m-polar-connected.

1. Introduction

Uncertainties in corporate management, mathematics, economics, the environment, psychological sciences, even medical research are present in many real-world problems. Addressing the ambiguities within these data presents challenges for traditional mathematical modeling. Theories like intuitionist fuzzy set [1], fuzzy set [2], as well as rough set [3] are employed to avoid challenges in handling
ambiguity. However, there are certain issues with all of these theories when it comes to dealing with contradicting or unclear data. According to Smarandache [4], the neutrosophic set is a mathematical approach to solving ambiguous and imprecise natural situations. Numerous domains, including information technology, decision support systems, and data systems, make extensive use of it.

Abdel-Basset [5] has created a novel smart medical guidance model that uses neutrosophic models to choice-making and depends on soft technology and the Internet of Things. A neutrosophic multi-criterion approach was created by the researchers in [6] to aid medical professionals in sickness prediction. The Neutrosophic Linear Fraction Programming Issue (NLFI) is addressed in [7] where the objective function, capital, and construction coefficients possess trapezoidal neutrosophic quantity cost. The scientists in [8-9] propose a technique called Neutrosophic multi-criteria decision making (NMCDM) to assist both the patient and physician in determining whether the patient is experiencing cardiac failure.

The neutrosophic topological structure theory was first suggested in [10]. [11] examined Neutrosophic topological space in more detail. The sets were then added in a manner akin to that of the Neutrosophic open and closed sets. In [12], Iswaraya et al. presented the Neutrosophic semi-open set [NSO] and the Neutrosophic semi-closed set [NSC]. Types of weakly neutrosophic crisp open mappings have been suggested and their fundamental features were examined by Al-Obaidi et al. [13]. Arokiarani et al. [14] investigated the relationships between Neutrosophic semi-open functions (also known as pre-open and α-open functions). Neutrosophic pre-open sets have been suggested by Rao et al. [15].

Numerous extensions and generalizations of neutrosophic topological ideas have been thoroughly studied in the past. A foundation for new types of separating and closure operations was laid by studies like Imran et al. [16-19] with Abdulkadhim et al. [20-21], which developed and examined extended semi-generalized closed sets with alpha-generalized separation axiom with neutrosophic crisp settings. New insights into function performance under uncertainty were also provided by the investigation of the continuity of these kinds of sets in the setting for neutrosophic crisp spaces [19]. The mathematical exploration of neutrosophic topology has been greatly enhanced by these contributions, which have also influenced the development of the instances and organizational approach used in this study.

In order to address vulnerability, Molodstov [22] presented the soft set principle as a computational approach. Maji [23] introduced the present conceptual structure known as Neutrosophic soft set, that combines the concepts of soft set with Neutrosophic set. The Neutrosophic soft set utilized in [24] to make the decision. Numerous researchers have used the idea for neutrosophic soft sets in a variety of mathematical structures [2-29 5]. Neutrosophic soft topological spaces were first shown by Bera [30].

Supra topological spaces were initially introduced by Mashhour et al. [31]. Following that, some new procedures for soft set theory were introduced by Ali et al. [32]. Abd-El-Latif and Alqahtani expanded the supra-soft topology framework in 2024 by presenting new SS-sd (supra soft somewhere dense) operators: exterior, boundary, interior, and closure. They also examined how these operators interacted with homeomorphism, continuity, openness, and closedness in supra-soft settings. Their findings include decomposition theorems for soft sets via SD operators, as well as sufficient and necessary circumstances for SS-sd-continuity, SS-sd-irresoluteness, and SS-sd-homeomorphism [33].

The m-polar fuzzy set (MPFS) idea was introduced by Chen et al. [34] in 2014 and can handle data containing uncertainty and ambiguity with multicriteria, multi-source, multi-sensor, as well as multi-polar information. The neutrosophic set was expanded by Smarandache [35].

Specifically, to neutrosophic offset (when some neutrosophic parts are off the interval [0, 1], that is, some neutrosophic portion > 1 and other neutrosophic component < 0), neutrosophic underset (when some neutrosophic portion is < 0), and neutrosophic overset (such as when some neutrosophic portion is > 1). The neutrosophic tripolar set, neutrosophic multi-polar set, neutrosophic tripolar graph, and neutrosophic multi-polar graph were all introduced by Smarandache in 2016 [35].

M-polar fuzzy sets' membership grades span the range $[0, 1]^{m}$, which corresponds to the object's m criteria; nonetheless, they are unable to handle the object's falsehood and indeterminacy.

While the neutrophilic set (NS) handles truth, falsehood, and indeterminacy for a single alternative criterion, it is unable to handle the alternatives' multi-criteria, multi-source, and multi-polar information fusion. In order to solve this issue, we combine the ideas of m-polar fuzzy set (MPFS) with neutrosophic set (NS) to create a new model called m-polar neutrosophic set (MPNS). MPNS can handle the truth, falsity, and uncertainty grades for every option in addition to the m criterion. The bipolar neutrosophic set, which was first presented by Deli et al. [36], is really extended by the m-polar neutrosophic set. n-Valued refined neutrosophic crisp sets were initially introduced by AL-Nafee et al. [39]. Center set theory of proximity space were introduced by Abdulsadaet al. [40]. In [41], Al-Tamimi et al. presented the partner sets for generalizations of multineutrosophic sets. Moreover, the applications of nueotrosophic sets are discussed in pure mathematics [42-46].

In this article, we establish new concepts like supra soft (resp; supra soft pre, supra soft β) via m-polar neutrosophic topological spaces. Moreover, through these concepts, we will study and investigate the connection among these concepts, such as supra soft connected (resp; supra soft pre- connected, supra soft $\beta-$ connected), and study the relationships and properties related to these concepts.

2. Preliminaries

In this section, we will introduce some basic concepts that we need in our work.

Definition 2.1. [36] Let $\Im$ be a reference set. An object $\mathcal{M}_{\mathcal{H}}$ is referred to as the m-polar neutrosophic set (MPNS) if it takes the form:

$\mathcal{M}_{\mathcal{H}} = \{(\not{f}, (\Psi_{\not{f}}(\not{f}), \mathcal{T}_{\not{f}}(\not{f}), \mathcal{Y}_{\not{f}}(\not{f}))) : \not{f} \in \Im,\ \alpha = 1, 2, 3, \ldots, m\}$,

where the mappings $\Psi_{\not{f}}$, $\mathcal{T}_{\not{f}}$, and $\mathcal{Y}_{\not{f}}$ are defined from $\Im$ to [0, 1], and they satisfy the constraint:

$0 \leq \Psi_{\not{f}}(\not{f}) + \mathcal{T}_{\not{f}}(\not{f}) + \mathcal{Y}_{\not{f}}(\not{f})) \leq 3$ for all ς ∈ Q and $\not{f}$ = 1, 2, 3, ..., m.

These components respectively represent the degrees of truth, indeterminacy, and falsity related to the object or alternative being evaluated across multiple criteria. The components are considered independent. An m-polar neutrosophic number (MPNN) can thus be represented as:

$\mathcal{T} = \langle(\Psi_{\not{f}}, \mathcal{T}_{\not{f}}, \mathcal{Y}_{\not{f}}\rangle$, with $0 \leq \Psi_{\not{f}} + \mathcal{T}_{\not{f}} + \mathcal{Y}_{\not{f}} \leq 3\ \forall\ \not{f} = 1, 2, \ldots, m$.

Definition 2.2. [36] Let $\Im$ be a non-empty reference set, and let mpn ($\Im$) denote the collection of all m-polar neutrosophic sets in $\Im$. A collection $\mathcal{T}_{M_{\Im}}$ of such sets is defined as an m-polar neutrosophic topology (MPNT) if it satisfies the following properties:

(i) $o_{M_{\Im}}$ and $1_{M_{\Im}}$ belong to $\mathcal{T}_{M_{\Im}}$;

(ii) If $\{\mathcal{M}_{\Im,\wp}\} \subseteq \mathcal{T}_{M_{\Im}}$ for all $\wp$ in an index set Δ, then the union $\bigcup_{\wp\in\Delta} \mathcal{M}_{\Im,\wp}$ also belongs to $\mathcal{T}_{M_{\Im}}$;

(iii) If $\mathcal{M}_{\Im,1}$, $\mathcal{M}_{\Im,2} \in \mathcal{T}_{M_{\Im}}$, then their intersection $\mathcal{M}_{\Im,1} \cap \mathcal{M}_{\Im,2}$ also belongs to $\mathcal{T}_{M_{\Im}}$.

The pair $(\Im, \mathcal{T}_{M_{\Im}})$ is then referred to as an m-polar neutrosophic topological space (MPNTS). Elements of $\mathcal{T}_{M_{\Im}}$ are called open MPNSs, while their complements are termed closed MPNSs.

Theorem 2.3. [36] If $(\Im, \mathcal{T}_{M_{\Im}})$ is an MPNTS, then the following properties hold:

(i) $o_{M_{\Im}}$ and $1_{M_{\Im}}$are open MPNSs;

(ii) The union of any collection of open MPNSs is open.

Theorem 2.4. [36] Let $(\Im, \mathcal{T}_{M_{\Im}})$ be an MPNTS. Then, the following conditions are satisfied:

(i) $o_{M_{\Im}}$ and $1_{M_{\Im}}$are closed MPNSs;

(ii) The intersection of any family of closed MPNSs is closed;

(iii) The union of a finite number of closed MPNSs is closed.

Definition 2.5. [36] Let $(\Im, \mathcal{T}_{M_{\Im}})$ be an MPNTS ,and let $\mathcal{H}_{\Im} \in$ mpn ($\Im$). The interior of $\mathcal{H}_{\Im}$, denoted by $\mathcal{H}_{\Im}{}^{\circ}$, is defined as the union of all open m-polar neutrosophic subsets contained within $\mathcal{H}_{\Im}$. That is,

$$\mathcal{H}_{\Im}{}^{\circ} = \cup\{\mathcal{G} \in \mathcal{T}_{M_{\Im}} \mid \mathcal{G} \subseteq \mathcal{H}_{\Im}\}.$$

This set $\mathcal{H}_{\Im}{}^{\circ}$ represents the largest open m-polar neutrosophic set that is fully contained in $\mathcal{H}_{\Im}$.

Definition 2.6. [36] Let $(\Im, \mathcal{T}_{M_{\Im}})$ be an MPNTS, and let $\mathcal{H}_{\Im} \in$ mpn $(\Im)$. The closure of $\mathcal{H}_{\Im}$, denoted by $\overline{\mathcal{H}_{\Im}}$, is defined as the intersection of all closed m-polar neutrosophic supersets of $\mathcal{H}_{\Im}$. That is,

$$\mathcal{H}_{\Im} = \cap\{\mathcal{K} \subseteq Q \mid \mathcal{K} \text{ is closed in } \mathcal{T}_{M_{\Im}} \text{ and } \mathcal{H}_{\Im} \subseteq \mathcal{K}\}.$$

This closure $\overline{\mathcal{H}_{\Im}}$ represents the smallest closed m-polar neutrosophic superset that contains $\mathcal{H}_{\Im}$.

Definition 2.7. [37] Let NSS$(\mathcal{P}, \mathcal{F})$ represent the set of all neutrosophic soft sets over $\mathcal{P}$, and let $\widetilde{\tau_{\mathrm{N}}} \subseteq NSS\,(\mathcal{P}, \mathcal{F})$. The collection $\widetilde{\tau_{\mathrm{N}}}$ represent a neutrosophic soft topology (NST) on $(\mathcal{P}, \mathcal{F})$ if the following conditions hold:

i. $\widetilde{\Phi_N}$ and $\widetilde{\mathcal{P}_{\mathrm{N}}}$ belong to $\widetilde{\tau_{\mathrm{N}}}$,

ii. $\widetilde{\tau_{\mathrm{N}}}$ is closed under arbitrary unions.

iii. $\widetilde{\tau_{\mathrm{N}}}$ is closed under finite intersections.

When these conditions are satisfied, the triple $(\mathcal{P}, \widetilde{\tau_{\mathrm{N}}}, \mathcal{F})$ is referred to as a neutrosophic soft topological space (NSTS).

Each member of $\widetilde{\tau_{\mathrm{N}}}$ is called a NSOS in $(\mathcal{P}, \widetilde{\tau_{\mathrm{N}}}, \mathcal{F})$.

A neutrosophic soft set $\widetilde{\mathcal{H}_{\mathrm{N}}}$, in NSS$(\mathcal{P}, \mathcal{F})$ is said to be soft closed in $(\mathcal{P}, \widetilde{\tau_{\mathrm{N}}}, \mathcal{F})$ if its complement $(\widetilde{\mathcal{H}_{\mathrm{N}}})^c$ represent a neutrosophic soft open set in the same space.

The neutrosophic soft closure of a neutrosophic soft set $\widetilde{\mathcal{H}_{\mathrm{N}}}$ is denoted by $Nscl(\widetilde{\mathcal{H}_{\mathrm{N}}})$ and is defined as: $Nscl(\widetilde{\mathcal{H}_{\mathrm{N}}}) = \cap\{\widetilde{\mathcal{D}_{\mathrm{N}}} : \widetilde{\mathcal{D}_{\mathrm{N}}} \text{ is NSCS and } \widetilde{\mathcal{H}_{\mathrm{N}}} \subseteq \widetilde{\mathcal{D}_{\mathrm{N}}}\}$.

Similarly, the neutrosophic soft interior of $\widetilde{\mathcal{H}_{\mathrm{N}}}$ is denoted by $Nsint(\widetilde{\mathcal{H}_{\mathrm{N}}})$ and is given by: $Nsint(\widetilde{\mathcal{H}_{\mathrm{N}}}) = \cup\{\widetilde{\mathcal{D}_{\mathrm{N}}} : \widetilde{\mathcal{D}_{\mathrm{N}}} \text{ is NSOS and } \widetilde{\mathcal{D}_{\mathrm{N}}} \subseteq \widetilde{\mathcal{H}_{\mathrm{N}}}\}$.

It is easy to see that $\tilde{F}_N$ is neutrosophic soft open if and only if $\tilde{F}_N = N\mathrm{sint}\,(\tilde{F}_N)$ and neutrosophic soft closed if and only if $\tilde{F}_N = \mathrm{Nscl}\,(\tilde{F}_N)$.

Theorem 2.8. [37] Assume $(\mathcal{P}, \widetilde{\tau_{\mathrm{N}}}, \mathcal{F})$ be a NSTS over $(\mathcal{P}, \mathcal{F})$and $\widetilde{\mathcal{H}_{\mathrm{N}}}$ and $\widetilde{\mathcal{W}}_N \in NSS(\mathcal{P}, \mathcal{F})$ thus:

(i) $N\mathrm{sint}\,(\widetilde{\Phi_N}) = \widetilde{\Phi_N}$ and $N\mathrm{sint}\,(\widetilde{\mathcal{P}_N}) = \widetilde{\mathcal{P}_N}$.

(ii) $\widetilde{\mathcal{H}_{\mathrm{N}}} \subset \widetilde{\mathcal{H}_{\mathrm{N}}}$ implies $N\mathrm{sint}\,(\widetilde{\mathcal{H}_{\mathrm{N}}}) \subset N\mathrm{sint}\,(\widetilde{\mathcal{H}_{\mathrm{N}}})$,

(iii) $N\mathrm{sint}\,(\widetilde{\mathcal{H}_{\mathrm{N}}}) \subset \widetilde{\mathcal{H}_{\mathrm{N}}}$ and $N\mathrm{sint}\,(\widetilde{\mathcal{H}_{\mathrm{N}}})$ represent the biggest open set,

(iv) $N\mathrm{sin}t(\widetilde{\mathcal{H}_{\mathrm{N}}})$ represent an NSOS. Thus, $N\mathrm{sint}\,(\widetilde{\mathcal{H}_{\mathrm{N}}}) \in \tilde{\tau}_N$,

(v) $N\mathrm{sin}t\left(N\mathrm{sin}t(\widetilde{\mathcal{H}_{\mathrm{N}}})\right) = N\mathrm{sin}t(\widetilde{\mathcal{H}_{\mathrm{N}}})$,

(vi) $\widetilde{\mathcal{H}_{\mathrm{N}}}$ is NSOS iff $N\mathrm{sin}t(\widetilde{\mathcal{H}_{\mathrm{N}}}) = \widetilde{\mathcal{H}_{\mathrm{N}}}$,

(vii) $N\mathrm{sint}\,(\widetilde{\mathcal{H}_{\mathrm{N}}}) \cup N\mathrm{sint}\,(\widetilde{\mathcal{W}}_N) \subseteq N\mathrm{sint}\,(\widetilde{\mathcal{H}_{\mathrm{N}}} \cup \widetilde{\mathcal{W}}_N)$,

(viii) $N\mathrm{sint}\,(\widetilde{\mathcal{H}_{\mathrm{N}}} \cap \widetilde{\mathcal{W}}_N) = N\mathrm{sint}\,(\widetilde{\mathcal{H}_{\mathrm{N}}}) \cap N\mathrm{sint}\,(\widetilde{\mathcal{W}}_N)$,

Moreover,

(i) $N\mathrm{scl}(\widetilde{\Phi_N}) = \widetilde{\Phi_N}$ and $N\mathrm{scl}(\widetilde{\mathcal{P}_N}) = \widetilde{\mathcal{P}_N}$,

(ii) $\widetilde{\mathcal{H}_{\mathrm{N}}} \subset \widetilde{\mathcal{H}_{\mathrm{N}}}$ implies $N\mathrm{scl}(\widetilde{\mathcal{H}_{\mathrm{N}}}) \subset N\mathrm{scl}(\widetilde{\mathcal{H}_{\mathrm{N}}})$,

(iii) $\widetilde{\mathcal{H}_{\mathrm{N}}} \subset N\mathrm{scl}(\widetilde{\mathcal{H}_{\mathrm{N}}})$ and $N\mathrm{scl}(\widetilde{\mathcal{H}_{\mathrm{N}}})$ represent the smallest closed set,

(iv) $N\mathrm{scl}(\widetilde{\mathcal{H}_{\mathrm{N}}})$ represent an NSCS. Thus, $N\mathrm{sint}\,(\widetilde{\mathcal{H}_{\mathrm{N}}}) \in \tilde{\tau}_N$,

(v) $N\mathrm{scl}\left(N\mathrm{scl}(\widetilde{\mathcal{H}_{\mathrm{N}}})\right) = N\mathrm{scl}(\widetilde{\mathcal{H}_{\mathrm{N}}})$,

(vi) $\widetilde{\mathcal{H}_{\mathrm{N}}}$ is NSCS iff $N\mathrm{scl}(\widetilde{\mathcal{H}_{\mathrm{N}}}) = \widetilde{\mathcal{H}_{\mathrm{N}}}$,

(vii) $N\mathrm{scl}(\widetilde{\mathcal{H}_{\mathrm{N}}}) \cup \mathrm{Nscl}(\widetilde{\mathcal{W}_N}) = N\mathrm{scl}\,(\widetilde{\mathcal{H}_{\mathrm{N}}} \cup \widetilde{\mathcal{W}_N})$,

(viii) $N\mathrm{scl}(\widetilde{\mathcal{H}_{\mathrm{N}}} \cap \widetilde{\mathcal{W}_N}) \subseteq N\mathrm{scl}\,(\widetilde{\mathcal{H}_{\mathrm{N}}}) \cap N\mathrm{scl}(\widetilde{\mathcal{W}_N})$.

3. New classes of Supra Soft m-Polar Neutrosophic Topological Spaces

This section presents the new concepts of supra soft connected (resp; supra soft pre- connected, supra soft $\beta-$connected) for supra m-polar Neutrosophic topological spaces also examines a few of its characteristics.

Firstly, we will provide the following main definitions:

Definition 3.1. Assume $\mathcal{P} \neq \emptyset$, and $\mathcal{U}$ a set of parameters, and $\psi = \{\lambda_1, \lambda_2, \dots, \lambda_k\}$ be a finite set of polarity values. A m-polar soft neutrosophic set (MPSNS) $\widetilde{\mathcal{H}_{\mathrm{N}}}$ over $\mathcal{P}$ is a function:

$$\widetilde{\mathcal{H}_{\mathrm{N}}} : \mathcal{U} \to \Omega_N^{\psi}(\mathcal{P}),$$

where $\Omega_N^{\psi}(\mathcal{P})$ represents the set that includes every neutrosophic sets over $\mathcal{P}$ that are indexed using ψ polarization values.

Furthermore, A soft neutrosophic topology that is m-polar $\widetilde{\tau_{\mathrm{N}}}$ on $(\mathcal{P}, \mathcal{U})$ represents a collection for m-polar soft neutrosophic structures that meet the following criteria:

i. $\widetilde{\Phi_N}$, $\widetilde{\mathcal{P}_N} \in \widetilde{\tau_{\mathrm{N}}}$ where $\widetilde{\mathcal{P}_N}$ and $\widetilde{\Phi_N}$, represent the absolute with null m-polar soft neutrosophic sets, respectively,

ii. Any set of compounds in $\widetilde{\tau_{\mathrm{N}}}$ that are united belong to $\widetilde{\tau_{\mathrm{N}}}$,

iii. The elements in $\widetilde{\tau_{\mathrm{N}}}$ that have a finite intersection are part of $\widetilde{\tau_{\mathrm{N}}}$.

A m-polar soft neutrosophic topological space is thus defined as the triple $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{\mathrm{N}}})$.

Furthermore, a set $\widetilde{\mathcal{H}_{\mathrm{N}}} \subseteq \mathcal{P}$ is m-polar closed provided its opposite $\widetilde{\mathcal{H}_{\mathrm{N}}}^{c}$ is m-polar open, since $\widetilde{\mathcal{H}_{\mathrm{N}}}$ denotes m-polar closed $\Leftrightarrow \widetilde{\mathcal{H}_{\mathrm{N}}}^{c} \in \widetilde{\tau_{\mathrm{N}}}$.

As a result, m-polar closed sets are any sets that complement and may be represented as random unions for m-polar sets.

With the help of this structure, one can describe ambiguous, unpredictable, and inconsistent data over large topological domains by taking into account different "polarities" (such as truth-falsehood scales and multi-agent perspectives), soft parameterization, and neutrosophic logic.

Example 3.2. The universe is represented by $\mathcal{P} = \{\wp_1, \wp_2, \wp_3\}$, the set of parameters by $\mathcal{U} = \{u_1, u_2\}$, and the three polarities—positive, neutral, and negative —by $\psi = \{+, 0, -\}$.

For every value of $u_i \in \mathcal{U}$, construct an m-polar neutrosophic set over $\mathcal{P}$. For every polarity $\lambda \in \psi$, allocate a triple $(\Psi_\lambda(\wp), \mathcal{T}_\lambda(\wp), \mathcal{Y}_\lambda(\wp)) \in [0,1]^3$ to every component $\wp \in \mathcal{P}$, signifying truth-membership, indeterminacy, with falsity-membership, respectively.

Give the following definition for the MPSNTS $\widetilde{\mathcal{H}_{\mathrm{N}}} : \mathcal{U} \to \Omega_N^{\psi}(\mathcal{P})$:

- For $u_1 \in \mathcal{U}$:

$$\widetilde{\mathcal{H}_{\mathrm{N}}}(u_1) = \begin{cases} p_1 \mapsto \{(+): (0.8,0.2,0.0), (0): (0.4,0.4,0.4), (-): (0.3,0.4,0.6)\} \\ p_2 \mapsto \{(+): (0.7,0.3,0.3), (0): (0.5,0.3,0.2), (-): (0.2,0.2,0.6)\} \\ p_3 \mapsto \{(+): (0.4,0.5,0.4), (0): (0.3,0.4,0.3), (-): (0.7,0.3,0.3)\} \end{cases}$$

- For $u_2 \in \mathcal{U}$:

$$\widetilde{\mathcal{H}_{\mathrm{N}}}(u_2) = \begin{cases} p_1 \mapsto \{(+): (0.7,0.2,0.1), (0): (0.5,0.4,0.4), (-): (0.4,0.1,0.8)\} \\ p_2 \mapsto \{(+): (0.7,0.3,0.2), (0): (0.5,0.3,0.2), (-): (0.1,0.2,0.8)\} \\ p_3 \mapsto \{(+): (0.1,0.4,0.6), (0): (0.3,0.3,0.6), (-): (0.8,0.1,0.2)\} \end{cases}$$

Now, identify a collection of such sets $\widetilde{\tau_{\mathrm{N}}}$ over $(\mathcal{P}, \mathcal{U})$.that satisfy.:

- The null soft-set $\widetilde{\Phi_N}$ and the universal soft set $\widetilde{\mathcal{P}_N}$ is the owner of N,
- Any set in $\widetilde{\tau_{\mathrm{N}}}$ that is arbitrarily united is in $\widetilde{\tau_{\mathrm{N}}}$,
- Sets in $\widetilde{\tau_{\mathrm{N}}}$ have a finite intersection in $\widetilde{\tau_{\mathrm{N}}}$.

In such case, the structure $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{\mathrm{N}}})$ represents an MPSNTS.

Remark 3.3.

• Not all Neutrosophic open sets in the Neutrosophic topology are necessarily m-polar Neutrosophic open, but all m-polar sets are Neutrosophic open in $\widetilde{\tau_{\mathrm{N}}}$.

• Since $\widetilde{\Phi_N}$ may be written as an empty union with $\widetilde{\mathcal{P}_N}$ may be the union of any m-polar sets whenever they cover it, both the $\widetilde{\Phi_N}$ with $\widetilde{\mathcal{P}_N}$ are trivially m-polar open.

Definition 3.4. Assume $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{\mathrm{N}}})$ be an MPSNTS. Then a triple $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{\mathrm{N}}})$ represents m-polar supra soft neutrosophic topological structures (MPSSNTS) if satisfied the following criteria:

i. $\widetilde{\Phi_N}$, $\widetilde{\mathcal{P}_N} \in \widetilde{\tau_{\mathrm{N}}}$ where $\widetilde{\mathcal{P}_N}$ and $\widetilde{\Phi_N}$, represent the absolute with null m-polar soft neutrosophic sets, respectively,

ii. Any set of compounds in $\widetilde{\tau_{\mathrm{N}}}$ that are united belong to $\widetilde{\tau_{\mathrm{N}}}$,

Example 3.5. The universe is represented by $\mathcal{P} = \{p_1, p_2, p_3, p_4\}$, the set of parameters by $\mathcal{U} = \{u_1, u_2, u_3\}$, and the M-Polarities sets defined by $\psi = \{\lambda_1, \lambda_2, \lambda_3\}$.

We construct two MPSSNTS over $\mathcal{P}$ and $\mathcal{U}$ as below:

For $\widetilde{\mathcal{H}_{\mathrm{N}}}$:

$$\widetilde{\mathcal{H}_{\mathrm{N}}}(u_1) = \{\langle p_1, (1,0,0), \lambda_1\rangle, \langle p_2, (0.8,0.1,0.1), \lambda_2\rangle\}$$
$$\widetilde{\mathcal{H}_{\mathrm{N}}}(u_2) = \{\langle p_3, (1,0,0), \lambda_2\rangle\}$$

For $\widetilde{\mathcal{W}}_N$

$$\widetilde{\mathcal{W}}_N(u_1) = \{\langle p_2, (1,0,0), \lambda_2\rangle, \langle p_4, (0.7,0.3,0.1), \lambda_3\rangle\}$$
$$\widetilde{\mathcal{H}_{\mathrm{N}}}(u_2) = \{\langle p_3, (1,0,0), \lambda_2\rangle\}$$

Next, construct the SSNT as:

$\widetilde{\tau_{MSSN}} = \{\widetilde{\Phi_N}, \widetilde{\mathcal{P}_N}, \widetilde{\mathcal{H}_{\mathrm{N}}}, \widetilde{\mathcal{W}}_N, \tilde{\mathcal{F}}_N\}$

Where $\tilde{\mathcal{F}}_N$ represents a soft set such that:

$\tilde{\mathcal{F}}_N(u_1) = \{\langle p_1, (1,0,0), \lambda_1\rangle\}$,$\tilde{\mathcal{F}}_N(u_2) = \widetilde{\Phi_N}$

The intersection for $\widetilde{\mathcal{H}_{\mathrm{N}}} \cap \widetilde{\mathcal{W}}_N$ gives:

$$\widetilde{\mathcal{H}_{\mathrm{N}}}(u_1) \,\tilde{\cap}\, \widetilde{\mathcal{W}}_N(u_1) = \{\langle p_2, (0.8,0.1,0.1), \lambda_2\rangle\}$$
$$\widetilde{\mathcal{H}_{\mathrm{N}}}(u_2) \,\tilde{\cap}\, \widetilde{\mathcal{W}}_N(u_2) = \{\langle p_3, (1,0,0), \lambda_2\rangle\}$$

Therefore, $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{MSSN}})$ represents an MPSSNTS because the intersection is not closed**.**

Definition 3.6. Let $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ be an MPSSNTS. Then the MPSSNTS $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ considered to be supra soft connected (resp., supra soft pre− connected, supra soft β − connected (for short; $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ −con) (resp., $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ −con, $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ −con)) iff $\widetilde{X}$ and $\widetilde{\emptyset}$ represents MPSS −clopen (resp., MPSSP −clopen, MPSSβ −clopen) only.

Definition 3.7. Let $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ be an MPSSNTS. Then the supra soft closure (resp., supra soft pre closure, supra soft β closure) (for short; $\mathrm{SScl}(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$) (resp., $\mathrm{SSPcl}(\widetilde{\mathcal{H}_N})_{\mathcal{U}}, \mathrm{SS\beta\,cl}(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$). Also, supra soft interior of $(\widetilde{\mathcal{H}_N},\tilde{\mathcal{F}}_N)_{\mathcal{U}}$ (for short; $\mathrm{SSint}(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$) (resp., supra soft pre interior of $(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$, supra soft β interior of $(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$) (for short; $\mathrm{SSPint}(\widetilde{\mathcal{H}_N})_{\mathcal{U}}, \mathrm{SS\beta int}(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$) , respectively are defined by:

$\mathrm{SS\ cl}(\widetilde{\mathcal{H}_N})_{\mathcal{U}}((\text{resp.}, \mathrm{SSPcl}(\widetilde{\mathcal{H}_N})_{\mathcal{U}}, \mathrm{SS\beta\,cl}(\widetilde{\mathcal{H}_N})_{\mathcal{U}})) = \widetilde{\cap}\ \{(\widetilde{\mathcal{G}_N})_{\mathcal{U}} : \mathcal{N} \text{ is } \mathrm{MPSS}-\mathrm{CS}(X)_{\mathcal{U}}$ (resp., $\mathrm{MPSSP}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}}$, $\mathrm{MPSS\beta}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}}$) and $(\widetilde{\mathcal{H}_N})_{\mathcal{U}} \subseteq (\widetilde{\mathcal{G}_N})_{\mathcal{U}}\}$.

$\mathrm{SSint}(\widetilde{\mathcal{H}_N})_{\mathcal{U}} = \widetilde{\cup}\ \{(\widetilde{\mathcal{V}_N})_{\mathcal{U}} : (\widetilde{\mathcal{V}_N})_{\mathcal{U}}$ is $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ (resp., $\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$, $\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$) and $(\widetilde{\mathcal{V}_N})_{\mathcal{U}} \subseteq (\widetilde{\mathcal{H}_N})_{\mathcal{U}}\}$.

Theorem 3.8. Suppose $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ be an MPSSNTS. Then the below holds:

1. Every $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ −con represents $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ −con.
2. Every $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ −con represents $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ −con.
3. Every $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ −con represents $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ −con.

Proof: 1) Assume $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ be an MPSSNTS, so $\widetilde{\Phi_N}$ and $\widetilde{\mathcal{P}_N}$ are $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ and $\mathrm{MPSS}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}}$ only. Because every SSN-OS is SSNP-OS. This implies $\widetilde{\Phi_N}$ and $\widetilde{\mathcal{P}_N}$ are $\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ and $\mathrm{MPSSP}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}}$ only. Therefore, every $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ −con represents $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ −con.
The others of the proofs are the same way.

Remark 3.9. As demonstrated by the next examples, the opposite of the previous Theorem 3.8 cannot be valid.

Example 3.10. Suppose the universe is $\mathcal{P} = \{\wp_1,\wp_2,\wp_3\}$, and let the parameter set be:
$\mathcal{U} = \{u_1,u_2\}$, with $m = 2$. Assume that the next set of $\mathcal{P}$ subsets create an MPSSNT as:
$\widetilde{\tau_{MSSN}} = \{\widetilde{\Phi_N},\widetilde{\mathcal{P}_N},\widetilde{\mathcal{H}_N},\widetilde{\mathcal{W}_N}\}$ where, $\widetilde{\mathcal{H}_N} = \{\langle\wp_1,(0.6,0.4,0.1),\lambda_1\rangle,\langle\wp_2,(0.3,0.1,0.3),\lambda_2\rangle,\langle\wp_3,(0.4,0.6,0.9),\lambda_3\rangle\}$, $\widetilde{\mathcal{W}_N} = \{\langle\wp_1,(0.4,0.6,0.9),\lambda_1\rangle,\langle\wp_2,(0.3,0.1,0.3),\lambda_2\rangle,\langle\wp_3,(0.6,0.4,0.1),\lambda_3\rangle\}$. Thus, $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ represents $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$–con. But $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ not represents $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$–con, since, $\tilde{\mathcal{F}}_N = \{\langle\wp_1,(0.3,0.5,0.1),\lambda_1\rangle,\langle\wp_2,(0.4,0.5,0.4),\lambda_2\rangle,\langle\wp_3,(0.7,0.5,0.9),\lambda_3\rangle\}$ belong to $\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ but $\tilde{\mathcal{F}}_N \notin \mathrm{MPSS}-\mathrm{OS}(X)_{\mathcal{U}}$.

Example 3.11. Suppose the universe is $\mathcal{P} = \{\wp_1,\wp_2,\wp_3\}$, and let the parameter set be:
$\mathcal{U} = \{u_1,u_2\}$, with $m = 2$. Assume that the next set of $\mathcal{P}$ subsets create an MPSSNT as: $\widetilde{\tau_{MSSN}} = \{\widetilde{\Phi_N},\widetilde{\mathcal{P}_N},\widetilde{\mathcal{H}_N},\widetilde{\mathcal{W}_N}\}$. where, $\widetilde{\mathcal{H}_N} = \{\langle\wp_1,(0.2,0.3,0.5),\lambda_1\rangle,\langle\wp_2,(0.4,0.4,0.4),\lambda_2\rangle,\langle\wp_3,(0.8,0.4,0.6),\lambda_3\rangle\}$, $\widetilde{\mathcal{W}_N} = \{\langle\wp_1,(0.5,0.4,0.5),\lambda_1\rangle,\langle\wp_2,(0.5,0.5,0.6),\lambda_2\rangle,\langle\wp_3,(0.5,0.6,0.5),\lambda_3\rangle\}$.
Hence, $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ represents $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ −con. But $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ not represents $\mathrm{MPSS}(X)_{\mathcal{U}}$ −con, since, $\tilde{\mathcal{F}}_N = \{\langle\wp_1,(0.5,0.6,0.5),\lambda_1\rangle,\langle\wp_2,(0.5,0.5,0.6),\lambda_2\rangle,\langle\wp_3,(0.5,0.4,0.5),\lambda_3\rangle\}$ belong to $\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ but $\tilde{\mathcal{F}}_N \notin \mathrm{MPSS}-\mathrm{OS}(X)_{\mathcal{U}}$. Furthermore, $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ is not $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ −con because, $\tilde{\mathcal{R}}_N = \{\langle\wp_1,(0.2,0.4,0.3),\lambda_1\rangle,\langle\wp_2,(0.4,0.3,0.5),\lambda_2\rangle,\langle\wp_3,(0.8,0.6,0.7),\lambda_3\rangle\} \notin \mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$.

Theorem 3.12. Assume $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ be an MPSSNTS. Thus $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ represents $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con (resp., $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$–con, $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$–con) iff both $\widetilde{\mathcal{P}_N}$ and $\widetilde{\Phi_N}$ are $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}})$ and $\mathrm{MPSS}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}})$ only.

Proof: Suppose $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ be $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con (resp., $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ –con, $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ –con). Let $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ be any $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}})$ and $\mathrm{MPSS}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{CS}(\mathrm{X})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{CS}(\mathrm{X})_{\mathcal{U}})$ subset of $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$. Then $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}{}^{c}$ is both $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}})$ and $\mathrm{MPSS}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}})$. Since $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ is disjoint union of the $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}})$ are $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ and $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$, the hypothesis implies that either $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}=\widetilde{\Phi_N}$ or $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}=\widetilde{\mathcal{P}_N}$.

Conversely, suppose that $\widetilde{\mathcal{P}_N}=(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\cup}\,(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$, where $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ and $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ are
disjoint non-empty $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}})$. Thus, $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ is both $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}})$ and $\mathrm{MPSS}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{CS}(\mathrm{X})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{CS}(\mathcal{P})_{\mathcal{U}})$. By assumption, $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}=\widetilde{\Phi_N}$ or $\widetilde{\mathcal{P}_N}$. Consequently, $\mathcal{P}$ is $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con (resp., $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ –con, $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ –con).

Theorem 3.13. Assume $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ be an MPSSNTS. If the $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$
$(resp.,\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}})$ $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ and $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ form a supra soft pre separation (resp., supra soft pre separation, supra soft β separation) of $\mathcal{P}$ and if $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ is a $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con (resp., $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ –con, $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ –con) subspaces of $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$, then $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\subseteq}\,(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ or $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\subseteq}\,(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}$.

Proof: Suppose $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ be an MPSSNTS. Since $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ and $(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}$ are disjoint $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$ $(resp.,\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}})$, $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\cap}\,(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ and $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\cap}\,(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}$ are $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$
$(resp.,\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}},\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}})$ in $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$. So these two supra pre soft sets $=\widetilde{\Phi_N}$ and their $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\cap}\,(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}=(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$. If there were both non-empty, they will make up the separation of $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$, so one of them is empty. Therefore, $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ must completely fall into it $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ or in $(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}$.

Theorem 3.14. Assume $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ be an MPSSNTS, and let $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ be a $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con (resp., $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ –con, $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ –con) subspace of $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$. If $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\subseteq}\,(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\subseteq}\,\mathrm{SScl}(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ then $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ is also $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con (resp., $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ –con, $\mathrm{MPSS\beta}(\mathrm{X})_{\mathcal{U}}$ –con).

Proof:

Suppose $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ be $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con and let $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\subset(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}\subset\mathrm{SScl}(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$. Assume that $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}=(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\cup}\,(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ is a supra soft separation of $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ by $\mathrm{MPSS}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$. Thus, by Theorem 3.13, $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ must completely fall into it $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ or in $(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}$.

Conversely: Suppose that $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\subseteq}\,(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}$, then $SS\mathrm{cl}(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\subseteq}\,\mathrm{SScl}(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}$. Since $(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\cap}\,(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}=\widetilde{\Phi_N}$, $(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}$ cannot intersect $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$. This contradicts because $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}\neq\widetilde{\Phi_N}$ is a supra soft pre subset of $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$. So $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}=\widetilde{\Phi_N}$. Consequently, $(\widetilde{\mathcal{W}_\mathrm{N}})_{\mathcal{U}}$ is $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con.

Theorem 3.15. Assume $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$ be an MPSSNTS, and let $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ be a $\mathrm{MPSSP}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}}$, $(resp.,\mathrm{MPSS\beta}-\mathrm{OS}(\mathcal{P})_{\mathcal{U}})$ subspace of $(\mathcal{P},\mathcal{U},\widetilde{\tau_{MSSN}})$.

1. If $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\subseteq}\,(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\subseteq}\,\mathrm{SSPcl}(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ then $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ is also $\mathrm{MPSSP}(\mathcal{P})_{\mathcal{U}}$ –con.
2. If $(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\subseteq}\,(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}\,\tilde{\subseteq}\,\mathrm{SS\beta cl}(\widetilde{\mathcal{H}_\mathrm{N}})_{\mathcal{U}}$ then $(\widetilde{\mathcal{F}_\mathrm{N}})_{\mathcal{U}}$ is also $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ –con.

Proof: Obvious.

Theorem 3.16. Assume $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{MSSN}})$ be an MPSSNTS, and let $(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$ be a $\text{MPSS}\beta(\mathcal{P})_{\mathcal{U}}$ –con subspace of $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{MSSN}})$ .If $(\widetilde{\mathcal{H}_N})_{\mathcal{U}} \widetilde{\subseteq} \text{SS}\beta\text{cl}(\widetilde{\mathcal{F}_N})_{\mathcal{U}} \widetilde{\subseteq} \text{SS}\beta\text{cl}(\text{cl}(\widetilde{\mathcal{H}_N})_{\mathcal{U}})$ then $(\widetilde{\mathcal{F}_N})_{\mathcal{U}}$ is also $\text{MPSS}\beta(\mathcal{P})_{\mathcal{U}}$ –con.
Proof: Suppose $(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$ be $\text{MPSS}\beta - \text{OS}(\mathcal{P})_{\mathcal{U}}$ and let $(\widetilde{\mathcal{H}_N})_{\mathcal{U}} \subseteq (\widetilde{\mathcal{W}_N})_{\mathcal{U}} \subseteq \text{SS}\beta\text{cl}(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$.
Suppose that $(\widetilde{\mathcal{W}_N})_{\mathcal{U}} = (\widetilde{\mathcal{G}_N})_{\mathcal{U}} \widetilde{\cup} (\widetilde{\mathcal{F}_N})_{\mathcal{U}}$ is a supra soft β separation of $(\widetilde{\mathcal{W}_N})_{\mathcal{U}}$ by $\text{MPSS}\beta - \text{OS}(\mathcal{P})_{\mathcal{U}}$. So, by Theorem 3.14, $(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$ it must completely fall into $(\widetilde{\mathcal{F}_N})_{\mathcal{U}}$ or in $(\widetilde{\mathcal{F}_N})_{\mathcal{U}}$.
Conversely: suppose that $(\widetilde{\mathcal{H}_N})_{\mathcal{U}} \widetilde{\subset} (\widetilde{\mathcal{F}_N})_{\mathcal{U}}$, then $(\widetilde{\mathcal{H}_N})_{\mathcal{U}} \widetilde{\subseteq} \text{SS}\beta\text{cl}(\widetilde{\mathcal{F}_N})_{\mathcal{U}}$ implies that $\text{SS}\beta\text{cls}(\widetilde{\mathcal{H}_N})_{\mathcal{U}} \widetilde{\subset} \text{SS}\beta\text{cl}(\text{cl}(\widetilde{\mathcal{H}_N})_{\mathcal{U}})$. Since $(\widetilde{\mathcal{F}_N})_{\mathcal{U}}$ and $(\widetilde{\mathcal{G}_N})_{\mathcal{U}} = \widetilde{\Phi_N}$, $(\widetilde{\mathcal{W}_N})_{\mathcal{U}}$ cannot intersect $(\widetilde{\mathcal{F}_N})_{\mathcal{U}}$ and $(\widetilde{\mathcal{G}_N})_{\mathcal{U}}$. This contradicts the fact that $(\widetilde{\mathcal{G}_N})_{\mathcal{U}}$ is a non-empty supra β soft subset of $(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$. So, $(\widetilde{\mathcal{G}_N})_{\mathcal{U}} = \widetilde{\Phi_N}$. Therefore, $(\widetilde{\mathcal{F}_N})_{\mathcal{U}}$ is $\text{MPSS}\beta(\mathcal{P})_{\mathcal{U}}$ –con.

Theorem 3.17. Assume $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{MSSN}})$ be an MPSSNTS and let $(\widetilde{\mathcal{H}_N})_{\mathcal{U}}$ be a $\text{MPSSP}(\mathcal{P})_{\mathcal{U}}$ –con subspace of $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{MSSN}})$ .If $(\widetilde{\mathcal{H}_N})_{\mathcal{U}} \widetilde{\subseteq} \text{SSPcl}(\widetilde{\mathcal{F}_N})_{\mathcal{U}} \widetilde{\subseteq} \text{SSPcl}(\text{cl}(\widetilde{\mathcal{H}_N})_{\mathcal{U}})$ then $(\widetilde{\mathcal{F}_N})_{\mathcal{U}}$ is also $\text{MPSSP}(\mathcal{P})_{\mathcal{U}}$ –con.
Proof: Its clear.

Definition 3.18. Let $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{MSSN}})$ be an MPSSNTS and assume $\widetilde{x\omega} \widetilde{\in} \mathcal{P}$. The $\text{MPSS}(\text{P})_{\mathcal{U}}$ (resp., $\text{MPSSP}(\mathcal{P})_{\mathcal{U}}$, $\text{MPSS}\beta(\mathcal{P})_{\mathcal{U}}$) component of $\widetilde{x\omega}$ (for short, $\text{MPSS} - \text{C}(\widetilde{x\omega})$ (resp., $\text{MPSSP} - \text{C}(\widetilde{x\omega}), \text{MPSS}\beta - \text{C}(\widetilde{x\omega})$ is defined via:
$\text{MPSS} - \text{C}(\widetilde{x\omega})$ (resp., $\text{MPSSP} - \text{C}(\widetilde{x\omega}), \text{MPSS}\beta - \text{C}(\widetilde{x\omega}) = \widetilde{\cup}\,\{\text{MPSS}(\mathcal{P})_{\mathcal{U}}\ (\text{resp.}, \text{MPSSP}(\mathcal{P})_{\mathcal{U}}, \text{MPSS}\beta(\mathcal{P})_{\mathcal{U}}): \text{MPSS}(\mathcal{P})_{\mathcal{U}} - \text{con}\ (\text{res.}, \text{MPSSP}(\mathcal{P})_{\mathcal{U}}$ –con , $\text{MPSS}\beta(\mathcal{P})_{\mathcal{U}}$ –con $\widetilde{\subset} \widetilde{\text{X}}$ and $\text{xw} \widetilde{\in} \text{SSM} - \text{P}(\mathcal{P})_{\mathcal{B}} - \text{con}\ (\text{res.}, \text{SSPM} - \text{P}(\mathcal{P})_{\mathcal{B}} - \text{con}, \text{SS}\beta\text{M} - \text{P}(\mathcal{P})_{\mathcal{B}} - \text{con}\}$.

Theorem 3.19. Assume$(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{MSSN}})$ be an MPSSNTS. Thus
(i) each $\text{MPSS} - \text{C}(\widetilde{x\omega})$ (resp., $\text{MPSSP} - \text{C}(\widetilde{x\omega})$) is a maximal $\text{SSP}(\mathcal{P})_{\mathcal{U}}$ –connected set in $\widetilde{\text{X}}$,
(ii) the set of all distinct $\text{MPSS} - \text{C}(\widetilde{x\omega})$ (resp., $\text{MPSSP} - \text{C}(\widetilde{x\omega})$) of soft points (resp., soft pre points) of $\tilde{\mathcal{P}}$ form a partition of $\tilde{\mathcal{P}}$ and $\widetilde{x\omega}\ \widetilde{\cup}_{\widetilde{x\omega} \widetilde{\in} \tilde{\mathcal{P}}} \text{MPSS} - \text{C}(\widetilde{x\omega_1}) \widetilde{\subset} \tilde{\mathcal{P}}$ (resp., $\widetilde{x\omega}\ \widetilde{\cup}_{\widetilde{x\omega} \widetilde{\in} \tilde{\mathcal{P}}} \text{MPSSP} - \text{C}(\widetilde{x\omega_1}) \widetilde{\subset} \tilde{\mathcal{P}}$.
Proof: (i) The proof follows from definition 3.18.
(ii) Assume $\text{MPSS} - \text{C}(\widetilde{x\omega_1})$ (resp., $\text{MPSSP} - \text{C}(\widetilde{x\omega_1})$) and $\text{MPSS} - \text{C}(\widetilde{x\omega_2})$ (resp., $\text{MPSSP} - \text{C}(\widetilde{x\omega_2})$) be two $\text{MPSS} - \text{C}(\widetilde{x\omega})$ (resp., $\text{MPSSP} - \text{C}(\widetilde{x\omega})$) of distinct soft points $\widetilde{x\omega_1}$ and $\widetilde{x\omega_2}$ in $\tilde{\mathcal{P}}$. If $\text{MPSS} - \text{C}(\widetilde{x\omega_1})\ \widetilde{\cap}\ \text{MPSS} - \text{C}(\widetilde{x\omega_2}) \neq \widetilde{\emptyset}\ (resp., \text{MPSSP} - \text{C}(\widetilde{x\omega_1})\ \widetilde{\cap}\ \text{MPSSP} - \text{C}(\widetilde{x\omega_2}) \neq \widetilde{\emptyset})$, thus by Theorem 3.12, $\text{MPSS} - \text{C}(\widetilde{x\omega_1})\ \widetilde{\cup}\ \text{MPSS} - \text{C}(\widetilde{x\omega_2})\ (resp., \text{MPSSP} - \text{C}(\widetilde{x\omega_1})\ \widetilde{\cup}\ \text{MPSS} - \text{C}(\widetilde{x\omega_2}) \neq \widetilde{\emptyset})$ is $\text{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con (res., $\text{MPSSP}(\mathcal{P})_{\mathcal{U}}$ –con). But $\text{MPSS} - \text{C}(\widetilde{x\omega_1}) \widetilde{\subset} \text{MPSS} - \text{C}(\widetilde{x\omega_1})\ \widetilde{\cup}\ \text{MPSS} - \text{C}(\widetilde{x\omega_1})$ (resp., $\text{MPSSP} - \text{C}(\widetilde{x\omega_1}) \widetilde{\subset} \text{MPSSP} - \text{C}(\widetilde{x\omega_1})\ \widetilde{\cup}\ \text{MPSSP} - \text{C}(\widetilde{x\omega_1})$) which contradicts the maximality
of $\text{MPSS} - \text{C}(\widetilde{x\omega_1})$ (resp., $\text{MPSSP} - \text{C}(\widetilde{x\omega_1})$) . Next, for any supra soft point (res., supra soft pre point) $\widetilde{x\omega}\ \widetilde{\in} \tilde{\mathcal{P}}$, $\widetilde{x\omega} \widetilde{\in} \text{MPSS} - \text{C}(\widetilde{x\omega_1})$ (resp., $\text{MPSSP} - \text{C}(\widetilde{x\omega_1})$)
and $\widetilde{\cup}_{\widetilde{x\omega} \widetilde{\in} \tilde{\mathcal{P}}} (\widetilde{x\omega}) \subset \widetilde{\cup}_{\widetilde{x\omega} \widetilde{\in} \tilde{\mathcal{P}}} \text{MPSS} - \text{C}(\widetilde{x\omega_1})\ (\text{res.}, \widetilde{\cup}_{\widetilde{x\omega} \widetilde{\in} \tilde{\mathcal{P}}} (\widetilde{x\omega}) \subset \widetilde{\cup}_{\widetilde{x\omega} \widetilde{\in} \tilde{\mathcal{P}}} \text{MPSSP} - \text{C}(\widetilde{x\omega_1}))$.
Therefore, $\widetilde{x\omega}\ \widetilde{\cup}_{\widetilde{x\omega} \widetilde{\in} \tilde{\mathcal{P}}} \text{MPSS} - \text{C}(\widetilde{x\omega_1}) \widetilde{\subset} \tilde{\mathcal{P}}$ (resp., $\widetilde{x\omega}\ \widetilde{\cup}_{\widetilde{x\omega} \widetilde{\in} \tilde{\mathcal{P}}} \text{MPSSP} - \text{C}(\widetilde{x\omega_1}) \widetilde{\subset} \tilde{\mathcal{P}}$.

Theorem 3.20. Assume $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{MSSN}})$ be an MPSSNTS. Then

(i) Each $\mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw})$ is a maximal $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$–con set in $\tilde{\mathcal{P}}$,

(ii) The set of all distinct $\mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw})$ of soft β points of $\tilde{\mathcal{P}}$ form a partition of $\tilde{\mathcal{P}}$ and $\widetilde{xw}\ \tilde{\cup}_{\widetilde{xw}\,\tilde{\in}\,\tilde{\mathcal{P}}}\ \mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw_1}) \tilde{\subseteq} \tilde{\mathcal{P}}$.

(iii) Each $\mathrm{SS\beta cl}(\mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw}))$ is $\mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw})$.

Proof: (i) The definition 3.18 leads to the proof.

(ii) It is clear.

(iii) Assume $\widetilde{xw}$ be any supra soft β point in $\tilde{\mathcal{P}}$. So $\mathrm{SS\beta cl}(\mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw}))$ is a $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ –con set containing $\widetilde{xw}$ {by Theorem 3.14}. But $\mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw})$ is the maximal $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$–con set containing $\widetilde{xw}$. Thus, $,\mathrm{SS\beta cl}(\mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw})) \tilde{\subseteq} \mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw})$. Therefore, $\mathrm{SS\beta}cl(\mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw}))$ is $\mathrm{MPSS\beta} - \mathrm{C}(\widetilde{xw})$.

Remark 3.21. Let $(\mathcal{P}, \mathcal{U}, \widetilde{\tau_{MSSN}})$ be an MPSSNTS . Then the following impactions are satisfied:

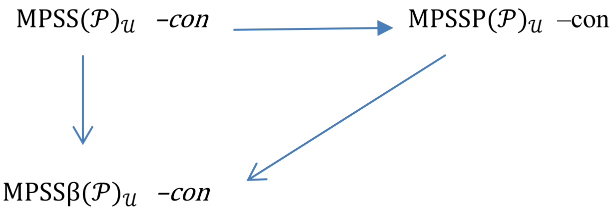

Fig. 1. The relationships among various Types of MPSSNTS.

4. Conclusions

As a result of introducing the new ideas like: supra soft (resp., supra soft Pre and supra soft β) via m-polar Neutrosophic topological spaces Also, we established generalizations of classical connectedness within supra soft Neutrosophic topological spaces. We studied their essential properties and examined their relationships with other connectedness types such as $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con (resp., $\mathrm{MPSS}(\mathcal{P})_{\mathcal{U}}$ –con, $\mathrm{MPSS\beta}(\mathcal{P})_{\mathcal{U}}$ –con) spaces. Our findings show that these new forms offer a more flexible framework for analyzing connectivity under soft structures. Furthermore, we explored how these concepts behave under soft continuous maps and how they contribute to the broader development of soft neutrosophic topological theory.

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Received: May 15, 2025. Accepted: Aug 31, 2025