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A Novel Approach on Digital Pentapartitioned Neutrosophic Topological Structures in $\hat{\dot{B}}P$-$\mathring{A}$lgebra

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Abstract. The notion of Digital Pentapartitioned Neutrosophic Set ($\mathcal{DPNS}$) in $\hat{\dot{B}}P$-$\mathring{A}$lgebra ($\hat{\dot{B}}P$-$\mathring{A}$) is introduced in this work. A number of aspects of the concept of Digital Pentapartitioned Neutrosophic Topological Structures ($\mathcal{DPNTS}t's$) in $\hat{\dot{B}}P$-$\mathring{A}$ are presented, developed, and examined in this paper. In addition, we present the characteristics of $\hat{\dot{B}}P$-$\mathring{A}$ whose Digital Topological Structure ($\mathcal{DTS}$) is generated by a family of $\mathcal{DPNS}$. In this article, the characteristics of the $\mathcal{DPNTS}t's$ of the inverse, Homomorphic image, Hausdorff space,Compactness and $\mathfrak{C}_5$-connectedness of $\hat{\dot{B}}P$-$\mathring{A}$ are also covered.

1. Introduction

The research by Zadeh [45] is based on forming sets with the inclusion of degrees of membership which was proposed in 1965. The creation of fuzzy sets led to the formation of Atanassov's [6] intuitionistic fuzzy sets in 1986. In 2005, Florentin Smarandache [39] proposed a neutrosophic set which is an extension of intuitionistic fuzzy sets based on three components $(t, f, i) = (\text{truth}, \text{falsehood}, \text{indeterminacy})$. Surpati Pramanik and Rama Mallick [24] put forward the Pentapartitioned Neutrosophic Set in the year 2020.

Following the footsteps of Rosenfeld's [33] research, many mathematicians attempted to merge various algebras under one umbrella. $\mathcal{BCK}$ and $\mathcal{BCI}$ algebras were also studied for the first time by Imai and Iseki [15] in 1966. The concept of $\mathcal{BP}$ algebra was first put forward by

Sun Shin Ahn and Han [2] in the year 2013. Xi [44] introduced algebras of $\mathcal{BCK}$ with fuzzy sets in 1991. Akram etal [3] introduce the concept of fuzzy $\mathcal{K}$algebra in 2010. Intuitionistic fuzzy ideals of $\mathcal{BCK}$ algebras were investigated by Jun and Kim [18] in the year 2000.

Algebraic structures are important because they are applicable in many areas of life. For related study, algebraic structures offer a mathematical representation. In neutrosophic set theory, some study neutrosophic set theory. In 2015, Agboola and Davvaz [1] introduced Neutrosophic $\mathcal{BCK}/\mathcal{BCI}$-algebras and discussed their basic characteristics. Jun et al. [20]first proposed (f, y) neutrosophic subalgebras of $\mathcal{BCK}/\mathcal{BCI}$-algebras in 2017. Many scholars have developed a variety of set theories along with corresponding topological structures to resolve the problem of uncertainty. Chang [7] proposed the concept of fuzzy topology for the first time in 1968. Later, in 1976, Lowan etal. [23] put forth the concepts of fuzzy topological spaces and fuzzy compactness. Shalini etal. [36], in 2019, were the first to introduce and construct On Intuitionistic Fuzzy $\mathcal{BP}$ ideals in $\mathcal{BP}$ algebra. In 2024, Shalini and Sindhu [37] developed fuzzy and intuitionistic fuzzy topological structures in $\mathcal{BP}$ algebra. Shalini and Sindhu [38] introduced digital pentapartitioned neutrosophic sets in digital topological spaces in 2024.

The 1960's saw the rise of Computer Image Analysis. Rosenfeld [35] proposed further ideas on digital topology fuzzy sets which he claimed were first proposed in 1979. Known as the Khalimsky Line or the digital line is the set of integers $\mathbb{Z}$, equipped with the topology K having $2n + 1$, $2n$, $2n - 1$, $n \in \mathbb{Z}$ as a subset and is represented by $(\mathbb{Z}, K)$. U is a subset of K where $x - 1$ or $x + 1$ is contained in U, which is a set of integer values. let $(\mathbb{Z}^2, K^2)$ be the topological product of two digital lines $(\mathbb{Z}, U)$. There is a smallest open set containing x, let's say $U(x)$, for any point $x \in \mathbb{Z}^2$.

In the digital plane, a set is referred to as closed if its corner points are even, while it is considered open if its corner points are odd. Topological properties of rectangular shapes determine plane topology in relation to Rosenfeld [34] fuzzy digital topology ideas. Would $\mathbb{Z}^2 = \mathbb{Z} \times \mathbb{Z}$ and $K^2 = K \times K$ represent the bounded space. If so, the two sets are called the digital plane. Neutrosophic digital topology, which extends digital topology using neutrosophic sets, was introduced by R. Narmada Devi et all. [29] in 2022.

This theoretical paradigm has promising real-world applications. Among the domains in which $\mathcal{DPNTS}$ can be applied are: Image Processing: By utilizing neutrosophic metrics to improve digital image analysis, the structured partitioning provided by $\mathcal{DPNTS}$ can be used for image segmentation and noise filtering applications. Decision Support Systems: When making decisions in the face of uncertainty, where precise classifications are challenging, the uncertainty and partial membership functions offer a natural framework. The algebraic structure can be utilized to create durable logic circuits that incorporate neutrosophic measures of uncertainty and supports digital logic operations. Examples showing how the neutrosophic structure enhances performance and robustness are provided for each application area.

Certain digital pentapartitioned neutrosophic topological structures in $\mathring{B}\hat{P}$-$\mathring{A}$ are explored thoroughly in this study. We examine certain characteristics of $\mathcal{DPNTS}t's$ in $\mathring{B}\hat{P}$-$\mathring{A}$ such as super connectedness and their Hausdorff nature. Topological $\mathring{B}\hat{P}$-$\mathring{A}$ are studied under homomorphism quite thoroughly with digital pentapartitioned neutrosophic structures, specifically in image and preimage simultaneously.

2. Preliminaries

Several basic concepts and examples that are essential to our work are given here.

Definition 2.1. Take $\hat{\mathring{X}}$ be an Universe of discourse. The condition of $\mathring{A}$ as a *Neutrosophic Set* ($\mathcal{NS}$) over $\hat{\mathring{X}}$ is given by:

$$\mathring{A} = \left\{ \left(\mathfrak{a}, Tr_{\mathring{A}}(\mathfrak{a}), In_{\mathring{A}}(\mathfrak{a}), Fa_{\mathring{A}}(\mathfrak{a}) \right) : \mathfrak{a} \in \hat{\mathring{X}} \right\},$$

where $\mathrm{Tr}_{\mathring{A}}$ $(\mathfrak{a})$, $In_{\mathring{A}}$ $(\mathfrak{a})$, $Fa_{\mathring{A}}$ are the values of truth-level, indeterminacy-level and falsity-level respectively, and

$$Tr_{\mathring{A}}(\mathfrak{a}), In_{\mathring{A}}(\mathfrak{a}), Fa_{\mathring{A}} \in [0,1] \quad \text{such that} \quad 0 \leq Tr_{\mathring{A}}(\mathfrak{a}) + In_{\mathring{A}}(\mathfrak{a}) + Fa_{\mathring{A}} \leq 3, \quad \forall \mathfrak{a} \in \hat{\mathring{X}}.$$

Definition 2.2. Take $\hat{\mathring{X}}$ be an Universe of discourse. The condition of $\mathring{A}$ as a *Pentapartitioned Neutrosophic Set* ($\mathcal{PNS}$) over $\hat{\mathring{X}}$ is given by:

$$\mathring{A} = \left\{ \left(\mathfrak{a}, Tr_{\mathring{A}}(\mathfrak{a}), Co_{\mathring{A}}(\mathfrak{a}), Ig_{\mathring{A}}(\mathfrak{a}), Un_{\mathring{A}}(\mathfrak{a}), Fa_{\mathring{A}}(\mathfrak{a}) \right) : \mathfrak{a} \in \hat{\mathring{X}} \right\},$$

where $\mathrm{Tr}_{\mathring{A}}$ $(\mathfrak{a})$, $Co_{\mathring{A}}$ $(\mathfrak{a})$, $\mathrm{Ig}_{\mathring{A}}$ $(\mathfrak{a})$, $\mathrm{Un}_{\mathring{A}}$ $(\mathfrak{a})$, $Fa_{\mathring{A}}$ are the values of truth-level, Contradiction-level, ignorance - level, unknown-level and falsity-level respectively, and

$$Tr_{\mathring{A}}(\mathfrak{a}), Co_{\mathring{A}}(\mathfrak{a}), Ig_{\mathring{A}}(\mathfrak{a}), Un_{\mathring{A}}(\mathfrak{a}), Fa_{\mathring{A}} \in [0,1] \quad \text{such that} \quad 0 \leq Tr_{\mathring{A}}(\mathfrak{a}) + Co_{\mathring{A}}(\mathfrak{a}) + Ig_{\mathring{A}}(\mathfrak{a}) + Un_{\mathring{A}}(\mathfrak{a}) + Fa_{\mathring{A}}(\mathfrak{a}) \leq 3, \quad \forall \mathfrak{a}$$

Definition 2.3. The absolute $\mathcal{PNS}$ $(1_{\mathcal{P}\mathring{\mathcal{N}}\mathcal{S}})$ and the null $\mathcal{PNS}$ $(0_{\mathcal{P}\mathring{\mathcal{N}}\mathcal{S}})$ over a fixed set $\hat{\mathring{X}}$ are defined as follows: The Pentapartitioned Neutrosophic Set ($\mathcal{PNS}'s$) $(0_{\mathcal{P}\mathring{\mathcal{N}}\mathcal{S}})$ and $(1_{\mathcal{P}\mathring{\mathcal{N}}\mathcal{S}})$ are defined by

$$0_{\mathcal{P}\mathring{\mathcal{N}}\mathcal{S}} = \langle p, 0, 0, 1, 1, 1 \rangle, \quad 1_{\mathcal{P}\mathring{\mathcal{N}}\mathcal{S}} = \langle q, 1, 1, 0, 0, 0 \rangle.$$

Definition 2.4. Consider a nonmemty set $\mathring{W}$.If the following criteria are met, an $\hat{\mathring{X}}$ collection of *Pentapartitioned Neutrosophic Topological Space* ($\mathcal{PNTS}$) in $\mathring{W}$ is referred to as a pentapartitioned neutrosophic topology ($\mathcal{PNT}$) on $\mathring{W}$:

(i) $0_{\mathcal{P}\mathring{\mathcal{N}}\mathcal{S}}, 1_{\mathcal{P}\mathring{\mathcal{N}}\mathcal{S}} \in \hat{\mathring{X}}$,
(ii) If $\mathring{P}, \mathring{Q} \in \hat{\mathring{X}}$, then $\mathring{P} \cap \mathring{Q} \in \hat{\mathring{X}}$,
(iii) If $\mathring{P}_i \in \hat{\mathring{X}}$ for all $i \in I, then \bigcup_{i \in I} \mathring{P}_i \in \hat{\mathring{X}}$.

A pentapartitioned neutrosophic topological space ($\mathcal{PNTS}$) is known as the pair ($\mathring{W}$,$\hat{\dot{X}}$). A pentapartitioned neutrosophic closed set ($\mathcal{PNCS}$) is the complement of each open pentapartitioned neutrosophic set ($\mathcal{PNOS}$), and each element of $\hat{\dot{X}}$ is referred to as an $\hat{\dot{X}}$-open or $\mathcal{PNOS}$. A topology is considered discrete if it comprises every pentapartitioned neutrosophic subset of $\mathring{W}$, and indiscrete if its elements consist of only $0_{\mathcal{P}\mathring{\mathcal{N}}\mathcal{S}}$ and $1_{\mathcal{P}\mathring{\mathcal{N}}\mathcal{S}}$.

Definition 2.5. A Digital Neutrosophic Set ($\mathcal{DNS}$) $\mathring{A}$ is defined an object as $Tr_{\mathring{A}}(\mathfrak{a})$, $In_{\mathring{A}}(\mathfrak{a})$, $Fa_{\mathring{A}}(\mathfrak{a})$ where $Tr_{\mathring{A}}(\mathfrak{a})$, $In_{\mathring{A}}(\mathfrak{a})$, $Fa_{\mathring{A}}(\mathfrak{a})$ are functions mapping rectangular arrays of integer coordinate points $\mathring{\mathbb{Z}}$ to the interval $[0,1]$. These functions represent the truth-level, indeterminacy-level, and falsity-level respectively, such that

$$0 \leq Tr_{\mathring{A}}(\mathfrak{a}) + In_{\mathring{A}}(\mathfrak{a}) + Fa_{\mathring{A}}(\mathfrak{a}) \leq 3, \quad \forall \mathfrak{a} \in \mathring{\mathbb{Z}}.$$

Definition 2.6. The absolute Digital Neutrosophic Set ($\mathcal{DNS}$) ($1_{\mathcal{D}\mathring{\mathcal{N}}\mathcal{S}}$) and the null $\mathcal{DNS}$ ($0_{\mathcal{D}\mathring{\mathcal{N}}\mathcal{S}}$) over a fixed set $\mathring{\mathbb{Z}}$ are defined as follows: ($\mathcal{DNS}'s$) ($0_{\mathcal{D}\mathring{\mathcal{N}}\mathcal{S}}$) and ($1_{\mathcal{D}\mathring{\mathcal{N}}\mathcal{S}}$) are defined by

$$0_{\mathcal{D}\mathring{\mathcal{N}}\mathcal{S}} = \langle p, 0, 0, 1 \rangle, \quad 1_{\mathcal{D}\mathring{\mathcal{N}}\mathcal{S}} = \langle q, 1, 1, 0 \rangle.$$

Definition 2.7. Let $\mathring{\mathbb{Z}}$ be represent integer coordinate points arranged as rectangular arrays. A Digital Neutrosophic Topology ($\mathcal{DNT}$) on $\mathring{\mathbb{Z}}$ is defined as a collection $\hat{\dot{D}}$ of $\mathcal{NS}'s$ within $\mathring{\mathbb{Z}}$ that fulfills the following conditions:

(i) $0_{\mathcal{D}\mathring{\mathcal{N}}\mathcal{S}}, 1_{\mathcal{D}\mathring{\mathcal{N}}\mathcal{S}}$ are elements of $\hat{\dot{D}}$,
(ii) For any $\mathring{\mathbb{Z}}_1, \mathring{\mathbb{Z}}_2 \in \hat{\dot{D}} \Rightarrow \mathring{\mathbb{Z}}_1 \cap \mathring{\mathbb{Z}}_2 \in \hat{\dot{D}}$,
(iii) The union $\bigcup \mathring{\mathbb{Z}}_i$ belongs to $\hat{\dot{D}}$, for any arbitrary family $\{\mathring{\mathbb{Z}}_i : i \in I\} \subseteq \hat{\dot{D}}$.

Definition 2.8. For a set $\mathring{F}$ and digital topology $\hat{\dot{D}}$:

- $\mathcal{DN}\, int(\mathring{F}) = \bigcup\{\mathring{Z} : \mathring{F}$ is a $\mathcal{DNOS}$ and $\mathring{Z} \subseteq \mathring{F}\}$,
- $\mathcal{DN}\, cl(\mathring{F}) = \bigcap\{\mathring{Z} : \mathring{Z}$ is a $\mathcal{DNCS}$ and $\mathring{Z} \supseteq \mathring{F}\}$, respectively.

Definition 2.9. Let $(\mathring{\mathbb{Z}}^2, \hat{\dot{D}}^2)$ be a digital plane. Then the following properties are hold:

(i) Even point $\mathfrak{m} = (2\mathfrak{n}, 2\mathfrak{m}) \Rightarrow cl(2\mathfrak{n}, 2\mathfrak{m}) = \{(2\mathfrak{n}, 2\mathfrak{m})\}$,
(ii) Odd point $\mathfrak{m} = (2\mathfrak{n}+1, 2\mathfrak{m}+1) \Rightarrow cl(2\mathfrak{n}+1, 2\mathfrak{m}+1) = \{(2\mathfrak{n}, 2\mathfrak{n}+1, 2\mathfrak{n}+2, 2\mathfrak{m}, 2\mathfrak{m}+1, 2\mathfrak{m}+2)\}$,
(iii) Mixed point $\mathfrak{m} = (2\mathfrak{n}+1, 2\mathfrak{m})$ or $(2\mathfrak{n}, 2\mathfrak{m}+1) \Rightarrow cl(2\mathfrak{n}, 2\mathfrak{m}+1) = \{(2\mathfrak{n} \times 2\mathfrak{m}, 2\mathfrak{m}+1, 2\mathfrak{m}+2)\}, cl(2\mathfrak{n}+1, 2\mathfrak{m}) = \{(2\mathfrak{n}, 2\mathfrak{n}+1, 2\mathfrak{n}+2, 2\mathfrak{n}+2 \times 2\mathfrak{m})\}$.

Definition 2.10. A Digital Pentapartitioned Neutrosophic Set ($\mathcal{DPNS}$) is an entity of the form

$$\mathring{A} = \left\{ \left(\mathfrak{q}, Tr_{\mathring{A}}(\mathfrak{q}), Co_{\mathring{A}}(\mathfrak{q}), Ig_{\mathring{A}}(\mathfrak{q}), Un_{\mathring{A}}(\mathfrak{q}), Fa_{\mathring{A}}(\mathfrak{q})\right) : \mathfrak{q} \in \hat{\dot{X}} \right\},$$

where $Tr_{\mathring{A}}(\mathfrak{q})$, $Co_{\mathring{A}}(\mathfrak{q})$, $Ig_{\mathring{A}}(\mathfrak{q})$, $Un_{\mathring{A}}(\mathfrak{q})$ and $Fa_{\mathring{A}}(\mathfrak{q})$ are the mappings from rectangular arrays of integer co-ordinate points $\mathring{\mathbb{Z}}$ to $[0,1]$. These are referred to as the Truth membership, Contradiction membership, Ignorance Membership, Unknown membership, and falsity membership functions, respectively. For every $\mathfrak{q} \in \mathring{\mathbb{Z}}$ it holds that

$$0 \leq Tr_{\mathring{A}}(\mathfrak{q}) + Co_{\mathring{A}}(\mathfrak{q}) + Ig_{\mathring{A}}(\mathfrak{q}) + Un_{\mathring{A}}(\mathfrak{q}) + Fa_{\mathring{A}}(\mathfrak{q}) \leq 5,\ for\ each\ \mathfrak{q} \in \mathring{\mathbb{Z}}.$$

Definition 2.11. The Digital Pentapartitioned Neutrosophic Set ($\mathcal{DPNS}'s$) ($0_{\mathcal{DP\mathring{N}S}}$) and ($1_{\mathcal{DP\mathring{N}S}}$) are defined by

$$0_{\mathcal{DP\mathring{N}S}} = \langle p, 0, 0, 1, 1, 1\rangle\,, \quad 1_{\mathcal{DP\mathring{N}S}} = \langle q, 1, 1, 0, 0, 0\rangle\,.$$

Definition 2.12. Assume that $\mathring{L}$ and $\mathring{M}$ be the $\mathcal{DPNS}'s$. Then,

(i) $Com(\mathring{L}) = \{Fa_{\mathring{L}}, Un_{\mathring{L}}, 1 - Ig_{\mathring{L}}, Co_{\mathring{L}}, Tr_{\mathring{L}}\}$is defined as the complement of a $\mathcal{DPNS}(\mathring{L})$,

(ii) $\mathring{L} \subseteq \mathring{M}$ iff $Tr_{\mathring{L}}(q) \leq\ Tr_{\mathring{M}}(q)$, $Co_{\mathring{L}}(q) \leq\ Co_{\mathring{M}}(q)$, $Ig_{\mathring{L}}(q) \geq\ Ig_{\mathring{M}}(q)$, $Un_{\mathring{L}}(q) \geq Un_{\mathring{M}}(q)$, $Fa_{\mathring{L}}(q) \geq\ Fa_{\mathring{M}}(q)$, $\forall\ (q) \in \mathring{\mathbb{Z}}$

(iii) $\mathring{L} \cap \mathring{M} = \langle (q),\ Tr_{\mathring{L}} \wedge Tr_{\mathring{M}},\ Co_{\mathring{L}} \wedge Co_{\mathring{M}}\ ,\ Ig_{\mathring{L}} \vee Ig_{\mathring{M}},\ Un_{\mathring{L}} \vee Un_{\mathring{M}},\ Fa_{\mathring{L}} \vee Fa_{\mathring{M}}\ \rangle$,

(iv) $\mathring{L} \cup \mathring{M} = \langle (q),\ Tr_{\mathring{L}} \vee Tr_{\mathring{M}},\ Co_{\mathring{L}} \vee Co_{\mathring{M}}\ ,\ Ig_{\mathring{L}} \wedge Ig_{\mathring{M}},\ Un_{\mathring{L}} \wedge Un_{\mathring{M}},\ Fa_{\mathring{L}} \wedge Fa_{\mathring{M}}\ \rangle$,

Definition 2.13. Let $\mathcal{DPN}$ interior and $\mathcal{DPN}$ closure of $\mathring{H}(\mathring{\mathbb{Z}}, \hat{\mathring{D}})$ are given by,

(i) $\mathcal{DPN}\,int(\mathring{H}) = \bigcup\{\mathring{\mathbb{Z}} : \mathring{\mathbb{Z}}$ is a $\mathcal{DPNOS}$ and $\mathring{\mathbb{Z}} \subseteq \mathring{H}\}$,

(ii) $\mathcal{DPN}\,cl(\mathring{H}) = \bigcap\{\mathring{\mathbb{Z}} : \mathring{\mathbb{Z}}$ is a $\mathcal{DPNCS}$ and $\mathring{\mathbb{Z}} \supseteq \mathring{H}\}$.

Definition 2.14. Let $\mathring{\mathbb{Z}}$ represent integer coordinate points arranged as rectangular arrays. A $\mathcal{DPNT}$ on $\mathring{\mathbb{Z}}$ is defined as a collection $\hat{\mathring{D}}$ of $\mathcal{DPNS}'s$ within $\mathring{\mathbb{Z}}$ that fulfills the following conditions:

(i) $0_{\mathcal{DP\mathring{N}S}}$ and $1_{\mathcal{DP\mathring{N}S}}$ are elements of $\hat{\mathring{D}}$.

(ii) For any $\mathring{\mathbb{Z}}_1, \mathring{\mathbb{Z}}_2 \in \hat{\mathring{D}}$, their intersection $\mathring{\mathbb{Z}}_1 \cap \mathring{\mathbb{Z}}_2$ is also in $\hat{\mathring{D}}$.

(iii) The union $\bigcup \mathbb{Z}_i$ belongs to $\mathbb{D}$, for any arbitrary family $\{\mathbb{Z}_i \mid i \in I\} \subseteq \mathbb{D}$.

A $\mathcal{DPNTS}$ is the pair of $(\mathring{\mathbb{Z}}^2, \hat{\mathring{D}}^2)$.

A $\mathcal{DPN}$ closed set is the complement of each $\mathcal{DPN}$ open set, and each member of $\hat{\mathring{D}}$ is referred to as a $\hat{\mathring{D}}$-open or $\mathcal{DPN}$ open set.

A topology is considered *discrete* if it comprises every $\mathcal{DPN}$ subset of $\mathring{\mathbb{Z}}$, and *indiscrete* if its elements consist of only $0_{\mathcal{DP\mathring{N}S}}$ and $1_{\mathcal{DP\mathring{N}S}}$

Definition 2.15. A nonempty set $\mathring{X}$ with a constant $\mathring{0}$ and a binary operation $*$ that satisfies the subsequent requirements is called a $\mathcal{BCK}$-$\mathring{A}gebra$ $(\mathring{X}, *, \mathring{0})$:

($\mathcal{BCK}$ 1) $[(\mathfrak{x} * \mathfrak{y}) * (\mathfrak{x} * \mathfrak{z})] * (\mathfrak{z} * \mathfrak{y}) = \mathring{0}$,

($\mathcal{BCK}$ 2) $(\mathfrak{x} * (\mathfrak{x} * \mathfrak{y})) * \mathfrak{y}) = \mathring{0}$,

($\mathcal{BCK}$ 3) $\mathfrak{x} * \mathfrak{x} = \mathring{0}$,

($\mathcal{BCK}$ 4) $\mathring{0} * \mathfrak{x} = \mathring{0}$,

($\mathcal{BCK}$ 5) $\mathfrak{x} * \mathfrak{y} = \mathring{0}$ and $\mathfrak{y} * \mathfrak{x} = \mathring{0}$ implies $\mathfrak{x} = \mathfrak{y}$, for all $, \mathfrak{y}, \mathfrak{z} \in \mathring{X}$.

Definition 2.16. A nonempty set $\mathring{X}$ with a constant $\mathring{0}$ and a binary operation $*$ that satisfies the subsequent requirements is called a *$\mathcal{BCI}$-$\mathring{A}$lgebra* $(\mathring{X}, *, \mathring{0})$:

($\mathcal{BCI}$ 1) $[(\mathfrak{x} * \mathfrak{y}) * (\mathfrak{x} * \mathfrak{z})] * (\mathfrak{z} * \mathfrak{y}) = \mathring{0}$,

($\mathcal{BCI}$ 2) $(\mathfrak{x} * (\mathfrak{x} * \mathfrak{y})) * \mathfrak{y}) = \mathring{0}$,

($\mathcal{BCI}$ 3) $\mathfrak{x} * \mathfrak{x} = \mathring{0}$,

($\mathcal{BCI}$ 4) $\mathfrak{x} * \mathfrak{y} = \mathring{0}$ and $\mathfrak{y} * \mathfrak{x} = \mathring{0}$ implies $\mathfrak{x} = \mathfrak{y}$, for all $, \mathfrak{y}, \mathfrak{z} \in \mathring{X}$.

Definition 2.17. A nonempty set $\mathring{X}$ with a constant $\mathring{0}$ and a binary operation $*$ that satisfies the subsequent requirements is called a *d-$\mathring{A}$lgebra* $(\mathring{X}, *, \mathring{0})$:

(d 1) $\mathfrak{x} * \mathfrak{x} = \mathring{0}$,

(d 2) $\mathring{0} * \mathfrak{x} = \mathring{x}$,

(d 3) $\mathfrak{x} * \mathfrak{y} = \mathring{0}$ and $\mathfrak{y} * \mathfrak{x} = \mathring{0}$ implies $\mathfrak{x} = \mathfrak{y}$, for all $, \mathfrak{y}, \mathfrak{z} \in \mathring{X}$.

Definition 2.18. A nonempty set $\mathring{X}$ with a constant $\mathring{0}$ and a binary operation $*$ that satisfies the subsequent requirements is called a *$\mathcal{B}$-$\mathring{A}$lgebra* $(\mathring{X}, *, \mathring{0})$:

($\mathcal{B}$ 1) $\mathfrak{x} * \mathfrak{x} = \mathring{0}$,

($\mathcal{B}$ 2) $\mathfrak{x} * \mathring{0} = \mathfrak{x}$,

($\mathcal{B}$ 3) $(\mathfrak{x} * \mathfrak{y}) * \mathfrak{z} = \mathfrak{x} * [\mathfrak{z} * (\mathring{0} * \mathfrak{y})]$, for all $\mathfrak{x}, \mathfrak{y} \in \mathring{X}$.

Definition 2.19. A nonempty set $\mathring{X}$ with a constant $\mathring{0}$ and a binary operation $*$ that satisfies the subsequent requirements is called a *$\mathcal{BH}$-$\mathring{A}$lgebra* $(\mathring{X}, *, \mathring{0})$:

($\mathcal{BH}$ 1) $\mathfrak{x} * \mathfrak{x} = \mathring{0}$,

($\mathcal{BH}$ 2) $\mathfrak{x} * \mathring{0} = \mathfrak{x}$,

($\mathcal{BH}$ 3) $\mathfrak{x} * \mathfrak{y} = \mathring{0}$ and $\mathfrak{y} * \mathfrak{x} = \mathring{0}$ implies $\mathfrak{x} = \mathfrak{y}$, for all $, \mathfrak{y}, \mathfrak{z} \in \mathring{X}$.

Definition 2.20. A nonempty set $\mathring{X}$ with a constant $\mathring{0}$ and a binary operation $*$ that satisfies the subsequent requirements is called a *$\mathcal{BF}$-$\mathring{A}$lgebra* $(\mathring{X}, *, \mathring{0})$:

($\mathcal{BF}$ 1) $\mathfrak{x} * \mathfrak{x} = \mathring{0}$,

($\mathcal{BF}$ 2) $\mathfrak{x} * \mathring{0} = \mathfrak{x}$,

($\mathcal{BF}$ 3) $\mathring{0} * (\mathfrak{x} * \mathfrak{y}) = \mathfrak{y} * \mathfrak{x}$, for all $\mathfrak{x}, \mathfrak{y} \in \mathring{X}$.

Definition 2.21. Let $\mathring{X}$ be a $\mathcal{BCK}$-$\mathring{A}$lgebra with a constant $\mathring{0}$ and a binary operator $*$. Then, if the following is true, $\mathring{I} \in \mathring{W}$ is called a *$\mathcal{BCK}$-ideal* of $\mathring{X}$:

(i) $\mathring{0} \in \mathring{I}$,

(ii) $\mathfrak{p} * \mathfrak{q} \in \mathring{I}$ and $\mathfrak{q} \in \mathring{I} \Rightarrow \mathfrak{p} \in \mathring{I}$, for every $\mathfrak{p}, \mathfrak{q} \in \mathring{X}$.

Definition 2.22. A binary operation $*$ and a nonempty set $\mathring{X}$ is called a $\hat{\dot{\mathcal{B}\mathcal{P}}}$-*$\mathring{A}lgebra$* $(\mathring{X}, *, \mathring{0})$ if the following hold:

($\hat{\dot{\mathcal{B}\mathcal{P}}}$ 1) $\mathfrak{x} * \mathfrak{x} = \mathring{0}$,

($\hat{\dot{\mathcal{B}\mathcal{P}}}$ 2) $\mathfrak{x} * (\mathfrak{x} * \mathfrak{y}) = \mathfrak{y}$,

($\hat{\dot{\mathcal{B}\mathcal{P}}}$ 3) $(\mathfrak{x} * \mathfrak{z}) * (\mathfrak{y} * \mathfrak{z}) = \mathfrak{x} * \mathfrak{y}$, for all $\mathfrak{x}, \mathfrak{y}, \mathfrak{z} \in \mathring{X}$.

Definition 2.23. If a nonempty subset $\mathring{I}$ of $\hat{\dot{\mathcal{B}\mathcal{P}}}$-$\mathring{A}$lgebra $(\mathring{X}, *, \mathring{0})$ meets the following criteria,it is considered a $\hat{\dot{\mathcal{B}\mathcal{P}}}$ -ideal of $\mathring{X}$.

(i) $\mathring{0} \in \mathring{I}$,

(ii) $\mathfrak{x} * \mathfrak{y} \in mathringI$ and $\mathfrak{y} \in \mathring{I} \Rightarrow \mathfrak{x} \in \mathring{I}$, for all $\mathfrak{x}, \mathfrak{y} \in \mathring{X}$.

Definition 2.24. If $\mathring{0} * \mathfrak{x} \in \mathring{I}$ for every $\mathfrak{x} \in \mathring{I}$, then a $\hat{\dot{\mathcal{B}\mathcal{P}}}$-ideal $\mathring{I}$ of a $\hat{\dot{\mathcal{B}\mathcal{P}}}$-$\mathring{A}$ $(\mathring{X}, *, \mathring{0})$ is *closed.*

Definition 2.25. A *fuzzy $\hat{\dot{\mathcal{B}\mathcal{P}}}$-Subalgebra* of $\mathring{X}$ is defined as a fuzzy set $\mathring{T}r$ of a $\hat{\dot{\mathcal{B}\mathcal{P}}}$-$\mathring{A}$ $(\mathring{X}, *, \mathring{0})$ if the following subsequent conditions holds for any $\mathfrak{x}, \mathfrak{y} \in \mathring{X}$:

$$\mathring{T}r(\mathfrak{x} * \mathfrak{y}) \geq \min\{\mathring{T}r(\mathfrak{x}), \mathring{T}r(\mathfrak{y})\}.$$

Definition 2.26. Let $\mathring{X}$ be a $\hat{\dot{\mathcal{B}\mathcal{P}}}$-$\mathring{A}$. A fuzzy set $\mathring{T}r$ of $\mathring{X}$ is called a *fuzzy $\hat{\dot{\mathcal{B}\mathcal{P}}}$-ideal* of $\mathring{X}$ if the following subsequent condition holds for any $\mathfrak{x}, \mathfrak{y} \in \mathring{X}$:

(i) $\mathring{T}r(\mathring{0}) \geq \mathring{T}r(\mathfrak{x})$,

(ii) $\mathring{T}r(\mathfrak{x}) \geq \min\{\mathring{T}r(\mathfrak{x} * \mathfrak{y}), \mathring{T}r(\mathfrak{y})\}$.

Definition 2.27. An intuitionistic fuzzy set $\mathring{A} = \langle \mathfrak{x}, \mathring{T}r(\mathfrak{x}), \mathring{F}a(\mathfrak{x}) \,|\, \mathfrak{x} \in \mathring{X} \rangle$ in a $\hat{\dot{\mathcal{B}\mathcal{P}}}$-$\mathring{A}$ $\mathring{X}$ is said to be an *intuitionistic fuzzy $\hat{\dot{\mathcal{B}\mathcal{P}}}$-Subalgebra* of $\mathring{X}$. Then following subsequent condition holds for all $\mathfrak{x}, \mathfrak{y} \in \mathring{X}$:

(i) $\mathring{T}r_{\mathring{A}}(\mathfrak{x} * \mathfrak{y}) \geq \min\{\mathring{T}r_{\mathring{A}}(x), \mathring{T}r_{\mathring{A}}(y)\}$,

(ii) $\mathring{F}a_{\mathring{A}}(\mathfrak{x} * \mathfrak{y}) \leq \max\{\mathring{F}a_{\mathring{A}}(\mathfrak{x}), \mathring{F}a_{\mathring{A}}(\mathfrak{y})\}$.

Definition 2.28. Let $\mathring{A}$ be the intuitionistic fuzzy set of a $\hat{\dot{\mathcal{B}\mathcal{P}}}$-$\mathring{A}$. An intuitionistic fuzzy $\hat{\dot{\mathcal{B}\mathcal{P}}}$-ideal of $\mathring{X}$ is defined as follows:

(i) $\mathring{T}r_{\mathring{A}}(\mathring{0}) \geq \mathring{T}r_{\mathring{A}}(\mathfrak{x}), \quad \mathring{F}a_{\mathring{A}}(\mathring{0}) \leq \mathring{F}a_{\mathring{A}}(\mathfrak{x})$,

(ii) $\mathring{T}r_{\mathring{A}}(\mathfrak{x}) \geq \min\{\mathring{T}r_{\mathring{A}}(\mathfrak{x} * \mathfrak{y}), \mathring{T}r_{\mathring{A}}(\mathfrak{y})\}$,

(iii) $\mathring{F}a_{\mathring{A}}(\mathfrak{x}) \leq \max\{\mathring{F}a_{\mathring{A}}(\mathfrak{x} * \mathfrak{y}), \mathring{F}a_{\mathring{A}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$.

Definition 2.29. Assume that $\mathring{D} = \langle \mathfrak{x}, \mathring{T}r_{\mathring{D}}(\mathfrak{x}), \mathring{I}n_{\mathring{D}}(\mathfrak{x}), \mathring{F}a_{\mathring{D}}(\mathfrak{x}); \mathfrak{x} \in \mathring{X} \rangle$ be a neutrosophic set over $\mathring{X}$. Then, the following is true, $\mathring{D}$ is referred to as a *neutrosophic* $\hat{\dot{\mathcal{B}\mathcal{P}}}$-$\mathring{A}$ ($\mathcal{N}$-$\hat{\dot{\mathcal{B}\mathcal{P}}}$-$\mathring{A}$):

(i) $\mathring{T}r_{\mathring{D}}(\mathfrak{x}) \geq \min\{\mathring{T}r_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{T}r_{\mathring{D}}(\mathfrak{y})\}$,

(ii) $\mathring{I}n_{\mathring{D}}(\mathfrak{x}) \leq \max\{\mathring{I}n_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{I}n_{\mathring{D}}(\mathfrak{y})\}$,

(iii) $\mathring{F}a_{\mathring{D}}(\mathfrak{x}) \leq \max\{\mathring{F}a_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{F}a_{\mathring{D}}(\mathfrak{y})\}$, Where $\mathfrak{x}, \mathfrak{y} \in \mathring{X}$.

Definition 2.30. A pentapartitioned neutrosophic set $\mathring{D} = \{\langle \mathfrak{x}, \mathring{T}r_{\mathring{D}}(\mathfrak{x}), \mathring{C}o_{\mathring{D}}(\mathfrak{x}), \mathring{I}g_{\mathring{D}}(\mathfrak{x}), \mathring{U}n_{\mathring{D}}(\mathfrak{x}), \mathring{F}a_{\mathring{D}}(\mathfrak{x}); \mathfrak{x} \in \mathring{X}\rangle\}$ over a $\mathcal{B}\dot{\hat{\mathcal{P}}}$-$\mathring{A}$ $\mathring{X}$ is said to be ($\mathcal{PN}$-$\mathcal{B}\dot{\hat{\mathcal{P}}}$-ideal) if and only if the following inequalities hold:

(i) $\mathring{T}r_{\mathring{D}}(\mathring{0}) \geq \mathring{T}r_{\mathring{D}}(\mathfrak{x})$ & $\mathring{T}r_{\mathring{D}}(\mathfrak{x}) \geq \min\{\mathring{T}r_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{T}r_{\mathring{D}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$,

(ii) $\mathring{C}o_{\mathring{D}}(\mathring{0}) \geq \mathring{C}o_{\mathring{D}}(x)$ & $\mathring{C}o_{\mathring{D}}(\mathfrak{x}) \leq \max\{\mathring{C}o_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{C}o_{\mathring{D}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$,

(iii) $\mathring{I}g_{\mathring{D}}(\mathring{0}) \geq \mathring{I}g_{\mathring{D}}(\mathfrak{x})$ & $\mathring{I}g_{\mathring{D}}(\mathfrak{x}) \leq \max\{\mathring{I}g_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{I}g_{\mathring{D}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$,

(iii) $\mathring{U}n_{\mathring{D}}(\mathring{0}) \geq \mathring{U}n_{\mathring{D}}(\mathfrak{x})$ & $\mathring{U}n_{\mathring{D}}(\mathfrak{x}) \leq \max\{\mathring{U}n_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{U}n_{\mathring{D}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$,

(iii) $\mathring{F}a_{\mathring{D}}(\mathring{0}) \geq \mathring{F}a_{\mathring{D}}(\mathfrak{x})$ & $\mathring{F}a_{\mathring{D}}(\mathfrak{x}) \leq \max\{\mathring{F}a_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{F}a_{\mathring{D}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$,

Definition 2.31. Assume that $\mathring{D} = \{\langle \mathfrak{x}, \mathring{T}r_{\mathring{D}}(\mathfrak{x}), \mathring{C}o_{\mathring{D}}(\mathfrak{x}), \mathring{I}g_{\mathring{D}}(\mathfrak{x}), \mathring{U}n_{\mathring{D}}(\mathfrak{x}), \mathring{F}a_{\mathring{D}}(\mathfrak{x}); \mathfrak{x} \in \mathring{X}\rangle\}$ be a $\mathcal{PNS}$ over $\mathring{X}$. Then, the follwoing is true, $\mathring{D}$ is referred to as a $\mathcal{PN}$-$\mathcal{B}\dot{\hat{\mathcal{P}}}$-$\mathring{A}$:

(i) $\mathring{T}r_{\mathring{D}}(\mathfrak{x}) \geq \min\{\mathring{T}r_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{T}r_{\mathring{D}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$,

(ii) $\mathring{C}o_{\mathring{D}}(\mathfrak{x}) \leq \max\{\mathring{C}o_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{C}o_{\mathring{D}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$,

(iii) $\mathring{I}g_{\mathring{D}}(\mathfrak{x}) \leq \max\{\mathring{I}g_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{I}g_{\mathring{D}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$,

(iii) $\mathring{U}n_{\mathring{D}}(\mathfrak{x}) \leq \max\{\mathring{U}n_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{U}n_{\mathring{D}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$,

(iii) $\mathring{F}a_{\mathring{D}}(\mathfrak{x}) \leq \max\{\mathring{F}a_{\mathring{D}}(\mathfrak{x} * \mathfrak{y}), \mathring{F}a_{\mathring{D}}(\mathfrak{y})\}, \quad \forall \mathfrak{x}, \mathfrak{y} \in \mathring{X}$,

Definition 2.32. Let $\mathbb{x}_{\mathcal{B}\hat{\mathcal{P}}}$ be a topology on $\mathcal{B}\dot{\hat{\mathcal{P}}}$ and let $\mathcal{B}\dot{\hat{\mathcal{P}}} = (\mathring{H}, *, \mathring{0})$ be a $\mathcal{B}\dot{\hat{\mathcal{P}}}$ -$\mathring{A}$lgebra. Let $\mathbb{x}_{\mathcal{B}\hat{\mathcal{P}}}$ be a topology on $\mathring{X}$ and let $\mathring{P}$ be an $\mathcal{PNS}$ in $\mathcal{B}\dot{\hat{\mathcal{P}}}$. Next, an induced $\mathcal{PNT}$ on $\mathcal{P}$ is a set or family of $\mathcal{PN}$ subsets of $\mathcal{P}$ that are defined as $\mathbb{x}_{\mathcal{P}} = \{\mathcal{P} \cap \mathcal{E}\} : \mathcal{E} \in \mathbb{x}_{\mathcal{B}\hat{\mathcal{P}}}$ and are the intersection with $\mathcal{P}$ and $\mathcal{PNOS}'s$ in $\mathcal{B}\dot{\hat{\mathcal{P}}}$. Next, the pair $(\mathcal{P}, \mathbb{x}_{\mathcal{P}})$ is referred to as an induced topological space or $\mathcal{PN}$ subspace of $(\mathcal{B}\dot{\hat{\mathcal{P}}}$, $\mathbb{x}_{\mathcal{P}})$ and $\mathbb{x}_{\mathcal{P}}$ is called $\mathcal{PN}$ induced topology on $\mathcal{P}$ or relative topology.

3. Digital Pentapartitioned Neutrosophic Topological Structures ($\mathcal{DPNTS}$)in $\mathcal{B}\dot{\hat{\mathcal{P}}}$-$\mathring{A}$

$\mathcal{BCK}/\mathcal{BCI}$-$\mathring{A}$, d-$\mathring{A}$, $\mathcal{BH}$-$\mathring{A}$, and $\mathcal{BF}$-$\mathring{A}$ are the examples of basic algebras that need the definition of $\mathcal{DPNS}$-$\mathcal{BP}$-$\mathring{A}$ before proceeding to this section. Given that $\mathcal{B}\dot{\hat{\mathcal{P}}}$-$\mathring{A}$ is an extension of the basic algebras listed above. Next, explain the concept of $\mathcal{DPN}$-$\mathcal{B}\dot{\hat{\mathcal{P}}}$-$\mathring{A}$.

Definition 3.1. Let $\mathring{\mathbb{Z}}$ be represent integer coordinate points arranged as rectangular arrays. A $\mathcal{DPNT}$ on $\mathring{\mathbb{Z}}$ is defined as a collection $\dot{\hat{\mathcal{D}}}$ of $\mathcal{PNS}'s$ within $\mathring{\mathbb{Z}}$ that fulfills the following conditions:

(i) $0_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}}$ and $1_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}}$ are elements of $\dot{\hat{\mathcal{D}}}$.

(ii) For any $\mathring{\mathbb{Z}}_1$, $\mathring{\mathbb{Z}}_2 \in \dot{\hat{\mathcal{D}}}$, their intersection $\mathring{\mathbb{Z}}_1 \cap \mathring{\mathbb{Z}}_2$ is also in $\dot{\hat{\mathcal{D}}}$.

(iii) The union $\bigcup \mathring{\mathbb{Z}}_i$ belongs to $\dot{\hat{\mathcal{D}}}$, for any arbitrary family $\{\mathring{\mathbb{Z}}_i / i \in \mathring{I}\} \subseteq \dot{\hat{\mathcal{D}}}$.

Definition 3.2. $\mathcal{DPN}$-$\mathcal{BCK}/\mathcal{BCI}$-$\mathring{A}$: Let $\mathcal{DPNS}$ be an entity of the format $\mathring{A} = \{\mathfrak{a}, \mathring{T}r_{\mathring{A}}(\mathfrak{a}), \mathring{C}o_{\mathring{A}}(\mathfrak{a}), \mathring{I}g_{\mathring{A}}(\mathfrak{a}), \mathring{U}n_{\mathring{A}}(\mathfrak{a}), \mathring{F}a_{\mathring{A}}(\mathfrak{a})\}$ where $\mathring{T}r_{\mathring{A}}, \mathring{C}o_{\mathring{A}}, \mathring{I}g_{\mathring{A}}, \mathring{U}n_{\mathring{A}}, \mathring{F}a_{\mathring{A}}$ are the mappings from rectangular arrays of integer co-ordinate points $\mathring{\mathbb{Z}}$ to [0, 1]. These are referred to as the Truth level, Contradiction level, Ignorance level, Unknown level and falsity level functions, respectively. For every $\mathfrak{a} \in \mathring{\mathbb{Z}}$ it holds that $0 \leq \mathring{Tr}_{\mathring{A}} + \mathring{Co}_{\mathring{A}} + \mathring{Ig}_{\mathring{A}} + \mathring{Un}_{\mathring{A}} + \mathring{Fa}_{\mathring{A}} \leq 5$ for each $\mathfrak{a} \in \mathring{\mathbb{Z}}$. Let $\mathring{A} = \{\mathfrak{a}, \mathring{Tr}_{\mathring{A}}(\mathfrak{a}), \mathring{Co}_{\mathring{A}}(\mathfrak{a}), \mathring{Ig}_{\mathring{A}}(\mathfrak{a}), \mathring{Un}_{\mathring{A}}(\mathfrak{a}), \mathring{Fa}_{\mathring{A}}(\mathfrak{a}) : \mathfrak{a} \in \mathring{\mathbb{Z}}\}$ be a $\mathcal{DPNS}$ over $\mathring{\mathbb{Z}}$. A $\mathcal{DPN}$ $\mathcal{BCK}/\mathcal{BCI}$-$\mathring{A}$lgebra on $\mathring{\mathbb{Z}}$ is defined as a collection $\hat{\mathring{\mathcal{D}}}^2$ of $\mathcal{DPNS}'s$ within $\mathring{\mathbb{Z}}$ that fulfills the following conditions:

(i) $\mathring{Tr}_{\mathring{A}}(\mathfrak{a}) \geq \min\{Tr_{\mathring{A}}(\mathfrak{a} * \mathfrak{b} * \mathfrak{c}), \mathring{Tr}_{\mathring{A}}(\mathfrak{c})\}$
(ii) $\mathring{Co}_{\mathring{A}}(\mathfrak{a}) \geq \min\{Co_{\mathring{A}}(\mathfrak{a} * \mathfrak{b} * \mathfrak{c}), \mathring{Co}_{\mathring{A}}(\mathfrak{c})\}$
(iii) $\mathring{Ig}_{\mathring{A}}(\mathfrak{a}) \leq \max\{\mathring{Ig}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b} * \mathfrak{c}), \mathring{Ig}_{\mathring{A}}(\mathfrak{c})\}$
(iv) $\mathring{Un}_{\mathring{A}}(\mathfrak{a}) \leq \max\{\mathring{Un}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b} * \mathfrak{c}), \mathring{Un}_{\mathring{A}}(\mathfrak{c})\}$
(v) $\mathring{Fa}_{\mathring{A}}(\mathfrak{a}) \leq \max\{\mathring{Fa}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b} * \mathfrak{c}), \mathring{Fa}_{\mathring{A}}(\mathfrak{c})\}$

Example 3.3. Let $\mathbb{U} = \{\mathring{1}, \mathring{2}\} \times \{\mathring{1}, \mathring{2}\}$ as the Universal Sets in the digital plane $\hat{\mathring{\mathcal{D}}}^2$.

Consider all subsets of $\mathbb{U}$ is defined by $\mathring{B} = \{(\mathring{1}, \mathring{1}), (\mathring{1}, \mathring{2}), (\mathring{2}, \mathring{1}), (\mathring{2}, \mathring{2})\}$ as an rectangular arrays of integer co-ordinate points of $\mathring{\mathbb{Z}}^2$. Therefore $(\mathring{\mathbb{Z}}^2, \hat{\mathring{\mathcal{D}}}^2)$ is the $\mathcal{DTS}$.

Let consider the $\mathcal{PN}$ partitions of $\mathring{B}$ is defined by $\mathring{A} = \{\mathring{P}_1 < 0.8, 0.7, 0.4, 0.1, 0.2 >, \mathring{P}_2 < 0.8, 0.7, 0.3, 0.6, 0.7 >, \mathring{P}_3 < 0.5, 0.4, 0.2, 0.3, 0.3 >, \mathring{P}_4 < 0.4, 0.3, 0.2, 0.3, 0.3 >\}$.

Let consider the $(\mathfrak{x}, *, \mathfrak{o})$ be a non empty set with the following cayley table:

$*$	$\mathfrak{o}$	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{c}$
$\mathfrak{o}$	$\mathfrak{o}$	$\mathfrak{a}$	$\mathfrak{b}$	$\mathfrak{c}$
$\mathfrak{a}$	$\mathfrak{a}$	$\mathfrak{o}$	$\mathfrak{c}$	$\mathfrak{b}$
$\mathfrak{b}$	$\mathfrak{b}$	$\mathfrak{c}$	$\mathfrak{o}$	$\mathfrak{a}$
$\mathfrak{c}$	$\mathfrak{c}$	$\mathfrak{b}$	$\mathfrak{a}$	$\mathfrak{o}$

Table 1. Cayley table of $\mathcal{BCK}/\mathcal{BCI}$-$\mathring{A}$.

Where $\mathfrak{x} = \{\mathfrak{o}, \mathfrak{a}, \mathfrak{b}, \mathfrak{c}\}$, apply the conditions of $\mathcal{BCK}/\mathcal{BCI}$-$\mathring{A}$, then $(\mathfrak{x}, *, \mathfrak{o})$ is a $\mathcal{BCK}/\mathcal{BCI} - \mathring{A}$lgebra. Define $\mathring{Tr}_{\mathring{A}}(\mathfrak{x}), \mathring{Co}_{\mathring{A}}(\mathfrak{x}), \mathring{Ig}_{\mathring{A}}(\mathfrak{x}), \mathring{Un}_{\mathring{A}}(\mathfrak{x}), \mathring{Fa}_{\mathring{A}}(\mathfrak{x})$ as follows:

$$\mathring{Tr}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.8 & \text{if } \mathfrak{x} = \mathfrak{a} \\ 0.5 & \text{if } \mathfrak{x} = \mathfrak{b} \\ 0.4 & \text{if } \mathfrak{x} = \mathfrak{c} \end{cases}$$

$$\mathring{Co}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.7 & \text{if } \mathfrak{x} = \mathfrak{a} \\ 0.4 & \text{if } \mathfrak{x} = \mathfrak{b} \\ 0.3 & \text{if } \mathfrak{x} = \mathfrak{c} \end{cases}$$

$$\mathring{Ig}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.4 & \text{if } \mathfrak{x} = \mathfrak{a} \\ 0.3 & \text{if } \mathfrak{x} = \mathfrak{b} \\ 0.2 & \text{if } \mathfrak{x} = \mathfrak{c} \end{cases}$$

$$\mathring{Un}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.1 & \text{if } \mathfrak{x} = \mathfrak{a} \\ 0.6 & \text{if } \mathfrak{x} = \mathfrak{b} \\ 0.3 & \text{if } \mathfrak{x} = \mathfrak{b} \end{cases}$$

$$\mathring{Fa}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.2 & \text{if } \mathfrak{x} = \mathfrak{a} \\ 0.7 & \text{if } \mathfrak{x} = \mathfrak{b} \\ 0.3 & \text{if } \mathfrak{x} = \mathfrak{c} \end{cases}$$

Then $((\mathring{\mathbb{Z}}^2, \hat{\mathring{\mathcal{D}}}^2), *, 0)$ denotes the $\mathcal{DPN} - \mathcal{BCK}/\mathcal{BCI} - \mathring{A}$lgebra of $\mathring{\mathbb{Z}}^2$ over $\mathbb{U}$.

Definition 3.4. *$\mathcal{DPN}$-d-$\mathring{A}$*: Let $\mathcal{DPNS}$ be an entity of the format $\mathring{A} = \{\mathfrak{a}, \mathring{Tr}_{\mathring{A}}(\mathfrak{a}), \mathring{Co}_{\mathring{A}}(\mathfrak{a}), \mathring{Ig}_{\mathring{A}}(\mathfrak{a}), \mathring{Un}_{\mathring{A}}(\mathfrak{a}), \mathring{Fa}_{\mathring{A}}(\mathfrak{a})\}$ where $\mathring{Tr}_{\mathring{A}}, \mathring{Co}_{\mathring{A}}, \mathring{Ig}_{\mathring{A}}, \mathring{Un}_{\mathring{A}}, \mathring{Fa}_{\mathring{A}}$ are the mappings from rectangular arrays of integer co-ordinate points $\mathring{\mathbb{Z}}$ to [0, 1]. These are referred to as the Truth level, Contradiction level, Ignorance level, Unknown level and falsity level functions, respectively. For every $\mathfrak{a} \in \mathring{\mathbb{Z}}$ it holds that $0 \leq \mathring{Tr}_{\mathring{A}} + \mathring{Co}_{\mathring{A}} + \mathring{Ig}_{\mathring{A}} + \mathring{Un}_{\mathring{A}} + \mathring{Fa}_{\mathring{A}} \leq 5$ for each $\mathfrak{a} \in \mathring{\mathbb{Z}}$. Let $\mathring{A} = \{\mathfrak{a}, \mathring{Tr}_{\mathring{A}}(\mathfrak{a}), \mathring{Co}_{\mathring{A}}(\mathfrak{a}), \mathring{Ig}_{\mathring{A}}(\mathfrak{a}), \mathring{Un}_{\mathring{A}}(\mathfrak{a}), \mathring{Fa}_{\mathring{A}}(\mathfrak{a}) : \mathfrak{a} \in \mathring{\mathbb{Z}}\}$ be a $\mathcal{DPNS}$ over $\mathring{\mathbb{Z}}$. A $\mathcal{DPN}$ *d*-$\mathring{A}$lgebra on $\mathring{\mathbb{Z}}$ is defined as a collection $\hat{\mathring{\mathcal{D}}}^2$ of $\mathcal{DPNS}'s$ within $\mathring{\mathbb{Z}}$ that fulfills the following conditions:

(i) $\mathring{Tr}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}) \geq \min\{Tr_{\mathring{A}}(\mathfrak{a}), \mathring{Tr}_{\mathring{A}}(\mathfrak{b})\}$
(ii) $\mathring{Co}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}) \geq \min\{Co_{\mathring{A}}(\mathfrak{a}), \mathring{Co}_{\mathring{A}}(\mathfrak{b})\}$
(iii) $\mathring{Ig}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b})) \leq \max\{\mathring{Ig}_{\mathring{A}}(\mathfrak{a}), \mathring{Ig}_{\mathring{A}}(\mathfrak{b})\}$
(iv) $\mathring{Un}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}) \leq \max\{\mathring{Un}_{\mathring{A}}(\mathfrak{a}), \mathring{Un}_{\mathring{A}}(\mathfrak{b})\}$
(v) $\mathring{Fa}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}) \leq \max\{\mathring{Fa}_{\mathring{A}}(\mathfrak{a}), \mathring{Fa}_{\mathring{A}}(\mathfrak{b})\}$

Example 3.5. Let $\mathbb{U} = \{\mathring{1}, \mathring{2}\} \times \{\mathring{1}, \mathring{2}\}$ as the Universal Sets in the digital plane $\hat{\mathring{\mathcal{D}}}^2$.

Consider all subsets of $\mathbb{U}$ is defined by $\mathring{B} = \{(\mathring{1}, \mathring{1}), (\mathring{1}, \mathring{2}), (\mathring{2}, \mathring{1}), (\mathring{2}, \mathring{2})\}$ as an rectangular arrays of integer co-ordinate points of $\mathring{\mathbb{Z}}^2$. Therefore $(\mathring{\mathbb{Z}}^2, \hat{\mathring{\mathcal{D}}}^2)$ is the $\mathcal{DTS}$.

Let consider the $\mathcal{PN}$ partitions of $\mathring{B}$ is defined by $\mathring{A} = \{\mathring{P}_1 < 0.8, 0.7, 0.3, 0.1, 0.1 >, \mathring{P}_2 < 0.7, 0.6, 0.3, 0.1, 0.1 >, \mathring{P}_3 < 0.7, 0.6, 0.1, 0.2, 0.4 >, \mathring{P}_4 < 0.6, 0.4, 0.1, 0.2, 0.4 >\}$.

Let consider the $(\mathfrak{x}, *, 0)$ be a non empty set with the following cayley table:

Where $\mathfrak{x} = \{0, 1, 2, 3\}$, apply the conditions of *d*-$\mathring{A}$, then $(\mathfrak{x}, *, 0)$ is a *d*-$\mathring{A}$lgebra.

Define $\mathring{Tr}_{\mathring{A}}(\mathfrak{x}), \mathring{Co}_{\mathring{A}}(\mathfrak{x}), \mathring{Ig}_{\mathring{A}}(\mathfrak{x}), \mathring{Un}_{\mathring{A}}(\mathfrak{x}), \mathring{Fa}_{\mathring{A}}(\mathfrak{x})$ as follows:

*	0	1	2	3
0	0	0	0	0
1	1	0	0	1
2	2	2	0	0
3	3	3	2	0

TABLE 2. Cayley table of d-$\mathring{A}$algebra.

$$\mathring{Tr}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.8 & \text{if } \mathfrak{x} = 1 \\ 0.7 & \text{if } \mathfrak{x} = 2 \\ 0.6 & \text{if } \mathfrak{x} = 3 \end{cases}$$

$$\mathring{Co}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.7 & \text{if } \mathfrak{x} = 1 \\ 0.6 & \text{if } \mathfrak{x} = 2 \\ 0.4 & \text{if } \mathfrak{x} = 3 \end{cases}$$

$$\mathring{Ig}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.3 & \text{if } \mathfrak{x} = 1 \\ 0.1 & \text{if } \mathfrak{x} = 2 \\ 0.1 & \text{if } \mathfrak{x} = 3 \end{cases}$$

$$\mathring{Un}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.2 & \text{if } \mathfrak{x} = 1 \\ 0.1 & \text{if } \mathfrak{x} = 2 \\ 0.1 & \text{if } \mathfrak{x} = 3 \end{cases}$$

$$\mathring{Fa}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.4 & \text{if } \mathfrak{x} = 1 \\ 0.1 & \text{if } \mathfrak{x} = 2 \\ 0.1 & \text{if } \mathfrak{x} = 3 \end{cases}$$

Then $((\mathring{\mathbb{Z}}^2, \hat{\mathring{\mathcal{D}}}^2), *, 0))$ denotes $\mathcal{DPN} - d - \mathring{A}$lgebra of $\mathring{\mathbb{Z}}^2$ over $\mathbb{U}$.

Note: In the above similar manner we can define and prove the $\mathcal{BF} - \mathring{A}$. Since the conditions of both d and $\mathcal{BF} - \mathring{A}'s$ are same.

Definition 3.6. $\mathcal{DPN}$-$\mathcal{BH}$-$\mathring{A}$: Let $\mathcal{DPNS}$ be an entity of the format $\mathring{A} = \{\mathfrak{a}, \mathring{Tr}_{\mathring{A}}(\mathfrak{a}), \mathring{Co}_{\mathring{A}}(\mathfrak{a}), \mathring{Ig}_{\mathring{A}}(\mathfrak{a}), \mathring{Un}_{\mathring{A}}(\mathfrak{a}), \mathring{Fa}_{\mathring{A}}(\mathfrak{a})\}$ where $\mathring{Tr}_{\mathring{A}}, \mathring{Co}_{\mathring{A}}, \mathring{Ig}_{\mathring{A}}, \mathring{Un}_{\mathring{A}}, \mathring{Fa}_{\mathring{A}}$ are the mappings from rectangular arrays of integer co-ordinate points $\mathring{\mathbb{Z}}$ to [0, 1]. These are referred to as the Truth level, Contradiction level, Ignorance level, Unknown level and falsity level functions, respectively. For every $\mathfrak{a} \in \mathring{\mathbb{Z}}$ it holds that $0 \leq \mathring{Tr}_{\mathring{A}} + \mathring{Co}_{\mathring{A}} + \mathring{Ig}_{\mathring{A}} + \mathring{Un}_{\mathring{A}} + \mathring{Fa}_{\mathring{A}} \leq 5$ for each $\mathfrak{a} \in \mathring{\mathbb{Z}}$. Let $\mathring{A} = \{\mathfrak{a}, \mathring{Tr}_{\mathring{A}}(\mathfrak{a}), \mathring{Co}_{\mathring{A}}(\mathfrak{a}), \mathring{Ig}_{\mathring{A}}(\mathfrak{a}), \mathring{Un}_{\mathring{A}}(\mathfrak{a}), \mathring{Fa}_{\mathring{A}}(\mathfrak{a}) : \mathfrak{a} \in \mathring{\mathbb{Z}}\}$ be a $\mathcal{DPNS}$ over $\mathring{\mathbb{Z}}$. A $\mathcal{DPN}$ $\mathcal{BH}$-$\mathring{A}$lgebra on $\mathring{\mathbb{Z}}$ is defined as a collection $\hat{\mathring{\mathcal{D}}}^2$ of $\mathcal{DPNS}'s$ within $\mathring{\mathbb{Z}}$ that fulfills the following conditions:

(i) $\mathring{Tr}_{\mathring{A}}(\mathfrak{a}) \geq \min\{Tr_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}), \mathring{Tr}_{\mathring{A}}(\mathfrak{b})\}$
(ii) $\mathring{Co}_{\mathring{A}}(\mathfrak{a}) \geq \min\{Co_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}), \mathring{Co}_{\mathring{A}}(\mathfrak{b})\}$
(iii) $\mathring{Ig}_{\mathring{A}}(\mathfrak{a}) \leq \max\{\mathring{Ig}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}), \mathring{Ig}_{\mathring{A}}(\mathfrak{b})\}$
(iv) $\mathring{Un}_{\mathring{A}}(\mathfrak{a}) \leq \max\{\mathring{Un}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}), \mathring{Un}_{\mathring{A}}(\mathfrak{b})\}$
(v) $\mathring{Fa}_{\mathring{A}}(\mathfrak{a}) \leq \max\{\mathring{Fa}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}), \mathring{Fa}_{\mathring{A}}(\mathfrak{b})\}$

Example 3.7. Let $\mathbb{U} = \{\mathring{1}, \mathring{2}\} \times \{\mathring{1}, \mathring{2}\}$ as the Universal Sets in the digital plane $\hat{\mathring{\mathcal{D}}}^2$.

Consider all subsets of $\mathbb{U}$ is defined by $\mathring{B} = \{(\mathring{1}, \mathring{1}), (\mathring{1}, \mathring{2}), (\mathring{2}, \mathring{1}), (\mathring{2}, \mathring{2})\}$ as an rectangular arrays of integer co-ordinate points of $\mathring{\mathbb{Z}}^2$. Therefore $(\mathring{\mathbb{Z}}^2, \hat{\mathring{\mathcal{D}}}^2)$ is the $\mathcal{DTS}$.

Let consider the $\mathcal{PN}$ partitions of $\mathring{B}$ is defined by $\mathring{A} = \{\mathring{P}_1 < 0.7, 0.6, 0.2, 0.1, 0.1 >, \mathring{P}_2 < 0.6, 0.5, 0.2, 0.1, 0.1 >, \mathring{P}_3 < 0.6, 0.5, 0.1, 0.3, 0.4 >, \mathring{P}_4 < 0.5, 0.1, 0.1, 0.3, 0.4 >\}$.

Let consider $(\mathfrak{x}, *, \mathfrak{o})$ be a non empty set with the following cayley table:

$*$	$\mathfrak{o}$	$\mathfrak{u}$	$\mathfrak{v}$	$\mathfrak{w}$
$\mathfrak{o}$	$\mathfrak{o}$	$\mathfrak{w}$	$\mathfrak{o}$	$\mathfrak{v}$
$\mathfrak{u}$	$\mathfrak{u}$	$\mathfrak{o}$	$\mathfrak{o}$	$\mathfrak{o}$
$\mathfrak{v}$	$\mathfrak{v}$	$\mathfrak{v}$	$\mathfrak{o}$	$\mathfrak{w}$
$\mathfrak{w}$	$\mathfrak{w}$	$\mathfrak{w}$	$\mathfrak{u}$	$\mathfrak{o}$

TABLE 3. Cayley table of $\mathcal{BH}$-$\mathring{A}$algebra.

Where $\mathfrak{x} = \{\mathfrak{o}, \mathfrak{u}, \mathfrak{v}, \mathfrak{w}\}$, apply the conditions of $\mathcal{BH}$-$\mathring{A}$, then $(\mathfrak{x}, *, \mathfrak{o})$ is a $\mathcal{BH}$-$\mathring{A}$lgebra.
Define $\mathring{Tr}_{\mathring{A}}(\mathfrak{x}), \mathring{Co}_{\mathring{A}}(\mathfrak{x}), \mathring{Ig}_{\mathring{A}}(\mathfrak{x}), \mathring{Un}_{\mathring{A}}(\mathfrak{x}), \mathring{Fa}_{\mathring{A}}(\mathfrak{x})$ as follows:

$$\mathring{Tr}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.7 & \text{if } \mathfrak{x} = \mathfrak{u} \\ 0.6 & \text{if } \mathfrak{x} = \mathfrak{v} \\ 0.5 & \text{if } \mathfrak{x} = \mathfrak{w} \end{cases}$$

$$\mathring{Co}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.6 & \text{if } \mathfrak{x} = \mathfrak{u} \\ 0.5 & \text{if } \mathfrak{x} = \mathfrak{v} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{w} \end{cases}$$

$$\mathring{Ig}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.2 & \text{if } \mathfrak{x} = \mathfrak{u} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{v} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{w} \end{cases}$$

$$\mathring{Un}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.3 & \text{if } \mathfrak{x} = \mathfrak{u} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{v} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{w} \end{cases}$$

$$\mathring{Fa}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.4 & \text{if } \mathfrak{x} = \mathfrak{u} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{v} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{w} \end{cases}$$

Then $((\mathring{\mathbb{Z}}^2, \hat{\dot{\mathcal{D}}}^2), *, \mathfrak{o}))$ denotes $\mathcal{DPN} - \mathcal{BH} - \mathring{A}$lgebra of $\mathring{\mathbb{Z}}^2$ over $\mathbb{U}$.

Definition 3.8. $\mathcal{DPN}$-$\mathcal{B}\hat{\dot{\mathcal{P}}}$-$\mathring{A}$: Let $\mathcal{DPNS}$ be an entity of the format $\mathring{A} = \{\mathfrak{a}, \mathring{Tr}_{\mathring{A}}(\mathfrak{a}), \mathring{Co}_{\mathring{A}}(\mathfrak{a}), \mathring{Ig}_{\mathring{A}}(\mathfrak{a}), \mathring{Un}_{\mathring{A}}(\mathfrak{a}), \mathring{Fa}_{\mathring{A}}(\mathfrak{a})\}$ where $\mathring{Tr}_{\mathring{A}}, \mathring{Co}_{\mathring{A}}, \mathring{Ig}_{\mathring{A}}, \mathring{Un}_{\mathring{A}}, \mathring{Fa}_{\mathring{A}}$ are the mappings from rectangular arrays of integer co-ordinate points $\mathring{\mathbb{Z}}$ to [0, 1]. These are referred to as the Truth level, Contradiction level, Ignorance level, Unknown level and falsity level functions, respectively. For every $\mathfrak{a} \in \mathring{\mathbb{Z}}$ it holds that $0 \leq \mathring{Tr}_{\mathring{A}} + \mathring{Co}_{\mathring{A}} + \mathring{Ig}_{\mathring{A}} + \mathring{Un}_{\mathring{A}} + \mathring{Fa}_{\mathring{A}} \leq 5$ for each $\mathfrak{a} \in \mathring{\mathbb{Z}}$. Let $\mathring{A} = \{\mathfrak{a}, \mathring{Tr}_{\mathring{A}}(\mathfrak{a}), \mathring{Co}_{\mathring{A}}(\mathfrak{a}), \mathring{Ig}_{\mathring{A}}(\mathfrak{a}), \mathring{Un}_{\mathring{A}}(\mathfrak{a}), \mathring{Fa}_{\mathring{A}}(\mathfrak{a}) : \mathfrak{a} \in \mathring{\mathbb{Z}}\}$ be a $\mathcal{DPNS}$ over $\mathring{\mathbb{Z}}$. A $\mathcal{DPN}$ $\mathcal{B}\hat{\dot{\mathcal{P}}}$-$\mathring{A}$lgebra on $\mathring{\mathbb{Z}}$ is defined as a collection $\hat{\dot{\mathcal{D}}}^2$ of $\mathcal{DPNS}'s$ within $\mathring{\mathbb{Z}}$ that fulfills the following conditions:

(i) $\mathring{Tr}_{\mathring{A}}(\mathfrak{a}) \geq \min\{Tr_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}), \mathring{Tr}_{\mathring{A}}(\mathfrak{b})\}$
(ii) $\mathring{Co}_{\mathring{A}}(\mathfrak{a}) \geq \min\{Co_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}), \mathring{Co}_{\mathring{A}}(\mathfrak{b})\}$
(iii) $\mathring{Ig}_{\mathring{A}}(\mathfrak{a}) \leq \max\{\mathring{Ig}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}), \mathring{Ig}_{\mathring{A}}(\mathfrak{b})\}$
(iv) $\mathring{Un}_{\mathring{A}}(\mathfrak{a}) \leq \max\{\mathring{Un}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}), \mathring{Un}_{\mathring{A}}(\mathfrak{b})\}$
(v) $\mathring{Fa}_{\mathring{A}}(\mathfrak{a}) \leq \max\{\mathring{Fa}_{\mathring{A}}(\mathfrak{a} * \mathfrak{b}), \mathring{Fa}_{\mathring{A}}(\mathfrak{b})\}$

Example 3.9. Let $\mathbb{U} = \{\mathring{1}, \mathring{2}\} \times \{\mathring{1}, \mathring{2}\}$ as the Universal Sets in the digital plane $\hat{\dot{\mathcal{D}}}^2$.

Consider all subsets of $\mathbb{U}$ is defined by $\mathring{B} = \{(\mathring{1}, \mathring{1}), (\mathring{1}, \mathring{2}), (\mathring{2}, \mathring{1}), (\mathring{2}, \mathring{2})\}$ as an rectangular arrays of integer co-ordinate points of $\mathring{\mathbb{Z}}^2$. Therefore $(\mathring{\mathbb{Z}}^2, \hat{\dot{\mathcal{D}}}^2)$ is the $\mathcal{DTS}$.

Let consider the $\mathcal{PN}$ partitions of $\mathring{B}$ is defined by $\mathring{A} = \{\mathring{P}_1 < 0.8, 0.7, 0.1, 0.1, 0.1 >, \mathring{P}_2 < 0.6, 0.5, 0.1, 0.1, 0.1 >, \mathring{P}_3 < 0.6, 0.5, 0.2, 0.2, 0.3 >, \mathring{P}_4 < 0.4, 0.2, 0.2, 0.2, 0.3 >\}$.

Let consider $(\mathfrak{x}, *, \mathfrak{o})$ be a non empty set with the following cayley table:

$*$	$\mathfrak{o}$	$\mathfrak{p}$	$\mathfrak{q}$	$\mathfrak{r}$
$\mathfrak{o}$	$\mathfrak{o}$	$\mathfrak{p}$	$\mathfrak{q}$	$\mathfrak{r}$
$\mathfrak{p}$	$\mathfrak{p}$	$\mathfrak{o}$	$\mathfrak{r}$	$\mathfrak{q}$
$\mathfrak{q}$	$\mathfrak{q}$	$\mathfrak{r}$	$\mathfrak{o}$	$\mathfrak{p}$
$\mathfrak{r}$	$\mathfrak{r}$	$\mathfrak{q}$	$\mathfrak{p}$	$\mathfrak{o}$

TABLE 4. Cayley table of $\mathcal{B}\hat{\dot{\mathcal{P}}}$-$\mathring{A}$.

Where $\mathfrak{x} = \{\mathfrak{o}, \mathfrak{p}, \mathfrak{q}, \mathfrak{r}\}$, apply the conditions of $\mathcal{B}\hat{\dot{\mathcal{P}}}$-$\mathring{A}$, then $(\mathfrak{x}, *, \mathfrak{o})$ is a $\mathcal{B}\hat{\dot{\mathcal{P}}}$-$\mathring{A}$.
Define $\mathring{Tr}_{\mathring{A}}(\mathfrak{x}), \mathring{Co}_{\mathring{A}}(\mathfrak{x}), \mathring{Ig}_{\mathring{A}}(\mathfrak{x}), \mathring{Un}_{\mathring{A}}(\mathfrak{x}), \mathring{Fa}_{\mathring{A}}(\mathfrak{x})$ as follows:

$$\mathring{Tr}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.8 & \text{if } \mathfrak{x} = \mathfrak{p} \\ 0.6 & \text{if } \mathfrak{x} = \mathfrak{q} \\ 0.4 & \text{if } \mathfrak{x} = \mathfrak{r} \end{cases}$$

$$\mathring{Co}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.7 & \text{if } \mathfrak{x} = \mathfrak{p} \\ 0.5 & \text{if } \mathfrak{x} = \mathfrak{q} \\ 0.2 & \text{if } \mathfrak{x} = \mathfrak{r} \end{cases}$$

$$\mathring{Ig}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.2 & \text{if } \mathfrak{x} = \mathfrak{p} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{q} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{r} \end{cases}$$

$$\mathring{Un}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.2 & \text{if } \mathfrak{x} = \mathfrak{p} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{q} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{r} \end{cases}$$

$$\mathring{Fa}_{\mathring{A}}(\mathfrak{x}) = \begin{cases} 0.3 & \text{if } \mathfrak{x} = \mathfrak{p} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{q} \\ 0.1 & \text{if } \mathfrak{x} = \mathfrak{r} \end{cases}$$

Then $((\mathring{\mathbb{Z}}^2, \dot{\hat{\mathcal{D}}}^2), *, 0))$ denotes $\mathcal{B}\dot{\hat{\mathcal{P}}}$-$\mathring{A}$ of $\mathring{\mathbb{Z}}^2$ over $\mathbb{U}$.

Definition 3.10. Let $\varkappa_{\mathcal{B}\hat{\mathcal{P}}}$ be a digital pentapartitioned neutrosophic topology ($\mathcal{DPNT}$) on $\mathcal{B}\dot{\hat{\mathcal{P}}}$ and let $\mathcal{B}\dot{\hat{\mathcal{P}}} = ((\mathring{\mathbb{Z}}^2, \dot{\hat{\mathcal{D}}}^2), *, 0))$ be a $\mathcal{B}\dot{\hat{\mathcal{P}}}$-$\mathring{A}$. Let $\varkappa_{\mathcal{B}\hat{\mathcal{P}}}$ be a $\mathcal{DPNT}$ on $\varkappa$ and let $\mathfrak{P}$ be a $\mathcal{DPNS}$ in $\mathcal{B}\dot{\hat{\mathcal{P}}}$. Next, an induced digital pentapartitioned neutrosophic topology on $\mathfrak{P}$ is a set or family of $\mathcal{DPNS}$ subsets of $\mathfrak{P}$ that are defined as $\varkappa_{\mathcal{B}\hat{\mathcal{P}}} = \{\mathfrak{P} \cap \varepsilon : \varepsilon \in \varkappa_{\mathcal{B}\hat{\mathcal{P}}}\}$ and are the intersection with $\mathfrak{P}$ and $\mathcal{DPNOS}'s$ in $\mathcal{B}\dot{\hat{\mathcal{P}}}$. Next, the pair $(\mathfrak{P}, \varkappa_{\mathcal{B}\hat{\mathcal{P}}})$ is referred to as an induced digital topological space or digital pentapartitioned neutrosophic subspace of $(\mathcal{B}\dot{\hat{\mathcal{P}}}, \varkappa_{\mathcal{B}\hat{\mathcal{P}}})$ and $\varkappa_{\mathcal{B}\hat{\mathcal{P}}}$ is called digital pentapartitioned neutrosophic induced topology on $\mathfrak{P}$ or relative digital topology.

Definition 3.11. Let $(\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1)$ and $(\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$ be two $\mathcal{DPNTS}'s$ and let f : $(\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1) \to (\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$. Then, f is called digital pentapartitioned neutrosophic continuous if following conditions hold:

(i) For each $\mathcal{DPNS}$, $\mathfrak{P} \in \varkappa_2$, $\mathrm{f}^{-1}(\mathfrak{P}) \in \varkappa_1$.
(ii) For each $\mathcal{DPN}$ $\mathcal{B}\dot{\hat{\mathcal{P}}}$-subalgebra $\mathfrak{P} \in \varkappa_2$, $\mathrm{f}^{-1}(\mathfrak{P})$ is a $\mathcal{DPNBP}$- subalgebra $\in \varkappa_1$.

Definition 3.12. Let $(\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1)$ and $(\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$ be two $\mathcal{DPNTS}'\int$ and let $(\mathfrak{P}, \varkappa_{\mathfrak{P}})$ and $(\mathfrak{Q}, \varkappa_{\mathfrak{Q}})$ be two digital pentapartitioned neutrosophic subspaces over $(\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1)$ and $(\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$. Let f be a mapping from $(\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1)$ into $(\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$, then f is a mapping from $(\mathfrak{P}, \varkappa_{\mathfrak{P}})$ to $(\mathfrak{Q}, \varkappa_{\mathfrak{Q}})$ if $\mathrm{f}(\mathfrak{P}) \subset \mathfrak{Q}$.

Definition 3.13. Let f be a mapping from $(\mathfrak{P},\varkappa_{\mathfrak{P}})$ to $(\mathfrak{Q}, \varkappa_{\mathfrak{Q}})$. Then, f is relatively digital pentapartitioned neutrosophic continuous if for every $\mathcal{DPNOS}$, $\Upsilon_{\mathfrak{Q}}$ in $\varkappa_{\mathfrak{Q}}$, $\mathrm{f}^{-1}(\Upsilon_{\mathfrak{Q}}) \cap \mathfrak{P} \in \varkappa_{\mathfrak{P}}$.

Definition 3.14. Let f be a mapping from $(\mathfrak{P}, \varkappa_{\mathfrak{P}})$ to $(\mathfrak{Q}, \varkappa_{\mathfrak{Q}})$. Then, f is relatively digital pentapartitioned neutrosophic open if for every $\mathcal{DPNOS}$, $\chi_{\mathfrak{P}}$ in $\varkappa_{\mathfrak{P}}$, the image $\mathrm{f}(\chi_p) \in \chi_{\mathfrak{P}}$.

Theorem 3.15. *Let $(\mathfrak{P}, \varkappa_{\mathfrak{P}})$ to $(\mathfrak{Q}, \varkappa_{\mathfrak{Q}})$ be a digital pentapartitioned neutrosophic subspaces of $(\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1)$ and $(\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$, where $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$ and $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$ are $\mathcal{B}\dot{\hat{\mathcal{P}}} - \mathring{A}$. If* f *is a digital pentapartitioned neutrosophic continuous function from $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$ to $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$ and $\mathrm{f}(\mathfrak{P}) \subset \mathfrak{Q}$. Then,* f *is relatively digital pentapartitioned neutrosophic continuous function from $\mathfrak{P}$ into $\mathfrak{Q}$.*

Definition 3.16. Let $(\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1)$ and $(\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$ be two $\mathcal{DPNTS}'s$. A mapping $\mathrm{f} : (\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1) \to (\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$ is called a digital pentapartitioned neutrosophic homomorphism if following conditions hold:

(i) f is a one-one and onto function.
(ii) f is a $\mathcal{DPN}$ continuous function from $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$ to $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$.
(iii) f^{-1} is a $\mathcal{DPN}$ continuous function from $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$ to $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$.

Proposition 3.17. *Let $(\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1)$ be a $\mathcal{DPNTS}$ and $(\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$ be an indiscrete $\mathcal{DPNTS}$ on $\mathcal{B}\dot{\hat{\mathcal{P}}} - \mathring{A}'s$ $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$ and $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$ respectively. Then, each function* f *defined as $\mathrm{f} : (\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1) \to (\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$ is a digital pentapartitioned neutrosophic continuous function from $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$ to $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$. If $(\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1)$ and $(\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$ be two discrete $\mathcal{DPNTS}'s$ $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$ and $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$, respectively, then each homomorphism* $\mathrm{f} : (\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1) \to (\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$ *is a digital pentapartitioned neutrosophic continuous function from $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$ to $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$.*

Proof. Let f be a mapping defined as $\mathrm{f} : \mathcal{B}\dot{\hat{\mathcal{P}}}_1 \to \mathcal{B}\dot{\hat{\mathcal{P}}}_2$. Let $\varkappa_1$ be $\mathcal{DPNT}$ on $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$ and $\varkappa_2$ be $\mathcal{DPNT}$ on $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$, where $\varkappa_2 = \{0_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}}, 1_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}}\}$.
We show that $\mathrm{f}^{-1}(\mathfrak{P})$ is a $\mathcal{DPN}$ $\mathcal{B}\dot{\hat{\mathcal{P}}}$-subalgebra of $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$, i.e., for each $\mathfrak{P} \in \varkappa_2$, $\mathrm{f}^{-1}(\mathfrak{P}) \in \varkappa_2$.
Since $\varkappa_2 = \{0_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}}, 1_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}}\}$, then for any $\nu \in \varkappa_1$,
consider $0_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}} \in \varkappa_2$ such that $\mathrm{f}^{-1}(0_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}})$ (ν)= $0_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}}$ $(\mathrm{f}(\nu))$= $0_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}}(\nu)$.
Therefore, $\mathrm{f}^{-1}(0_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}}) = 0_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}} \in \varkappa_1$. Similarly, $\mathrm{f}^{-1}(1_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}}) = 1_{\mathcal{DP}\mathring{\mathcal{N}}\mathcal{S}} \in \varkappa_1$. Hence, f is a $\mathcal{DPNS}$ continuous function from $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$ to $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$.
Both $\varkappa_1$ and $\varkappa_2$ are $\mathcal{DPNTS}'s$ on $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$ and $\mathcal{B}\dot{\hat{\mathcal{P}}}_2$, respectively, and $\mathrm{f} : (\mathcal{B}\dot{\hat{\mathcal{P}}}_1, \varkappa_1) \to (\mathcal{B}\dot{\hat{\mathcal{P}}}_2, \varkappa_2)$ is a homomorphism. Therefore, for all $\mathfrak{P} \in \varkappa_2$, and $\mathrm{f}^{-1}(\mathfrak{P}) \in \varkappa_1$, where fis not a usual inverse homomorphism. To prove that $\mathrm{f}^{-1}(\mathfrak{P})$ is a $\mathcal{DPN}$ $\mathcal{B}\dot{\hat{\mathcal{P}}}$-subalgebra of $\mathcal{B}\dot{\hat{\mathcal{P}}}_1$. Let for $\nu, \upsilon \in \mathcal{B}\dot{\hat{\mathcal{P}}}_1$,

$$
\begin{aligned}
f^{-1}(\mathring{Tr}_{\mathfrak{P}})(\mathfrak{s}*\mathfrak{t}) &= \mathring{Tr}_{\mathfrak{P}}(f(\mathfrak{s}*\mathfrak{t}))\\
&= \mathring{Tr}_{\mathfrak{P}}(f(\mathfrak{s}*\mathfrak{t}))\\
&\geq min\{\mathring{Tr}_{\mathfrak{P}}(f(\mathfrak{s}))*\mathring{Tr}_{\mathfrak{P}}(f(\mathfrak{t}))\}\\
f^{-1}(\mathring{Tr}_{\mathfrak{P}})(\mathfrak{s}*\mathfrak{t}) &= min\{f^{-1}(\mathring{Tr}_{\mathfrak{P}})(\mathfrak{s}), f^{-1}(\mathring{Tr}_{\mathfrak{P}})(\mathfrak{t})\},\\
f^{-1}(\mathring{Co}_{\mathfrak{P}})(\mathfrak{s}*\mathfrak{t}) &= \mathring{Co}_{\mathfrak{P}}(f(\mathfrak{s}*\mathfrak{t}))\\
&= \mathring{Co}_{\mathfrak{P}}(f(\mathfrak{s}*\mathfrak{t}))\\
&\geq min\{\mathring{Co}_{\mathfrak{P}}(f(\mathfrak{s}))*\mathring{Co}_{\mathfrak{P}}(f(\mathfrak{t}))\}\\
f^{-1}(\mathring{Co}_{\mathfrak{P}})(\mathfrak{s}*\mathfrak{t}) &= min\{f^{-1}(\mathring{Co}_{\mathfrak{P}})(\mathfrak{s}), f^{-1}(\mathring{Co}_{\mathfrak{P}})(\mathfrak{t})\},\\
f^{-1}(\mathring{Ig}_{\mathfrak{P}})(\mathfrak{s}*\mathfrak{t}) &= \mathring{Ig}_{\mathfrak{P}}(f(\mathfrak{s}*\mathfrak{t}))\\
&= \mathring{Ig}_{\mathfrak{P}}(f(\mathfrak{s}*\mathfrak{t}))\\
&\leq max\{\mathring{Ig}_{\mathfrak{P}}(f(\mathfrak{s}))*\mathring{Ig}_{\mathfrak{P}}(f(\mathfrak{t}))\}\\
f^{-1}(\mathring{Ig}_{\mathfrak{P}})(\mathfrak{s}*\mathfrak{t}) &= max\{f^{-1}(\mathring{Ig}_{\mathfrak{P}})(\mathfrak{s}), f^{-1}(\mathring{Ig}_{\mathfrak{P}})(\mathfrak{t})\},\\
f^{-1}(\mathring{Un}_{\mathfrak{P}})(\mathfrak{s}*\mathfrak{t}) &= \mathring{Un}_{\mathfrak{P}}(f(\mathfrak{s}*\mathfrak{t}))\\
&= \mathring{Un}_{\mathfrak{P}}(f(\mathfrak{s}*\mathfrak{t}))\\
&\leq max\{\mathring{Un}_{\mathfrak{P}}(f(\mathfrak{s}))*\mathring{Un}_{\mathfrak{P}}(f(\mathfrak{t}))\}\\
f^{-1}(\mathring{Un}_{\mathfrak{P}})(\mathfrak{s}*\mathfrak{t}) &= max\{f^{-1}(\mathring{Un}_{\mathfrak{P}})(\mathfrak{s}), f^{-1}(\mathring{Un}_{\mathfrak{P}})(\mathfrak{t})\},\\
f^{-1}(\mathring{Fa}_{\mathfrak{P}})(\mathfrak{s}*\mathfrak{t}) &= \mathring{Fa}_{\mathfrak{P}}(f(\mathfrak{s}*\mathfrak{t}))\\
&= \mathring{Fa}_{\mathfrak{P}}(f(\mathfrak{s}*\mathfrak{t}))\\
&\leq max\{\mathring{Fa}_{\mathfrak{P}}(f(\mathfrak{s}))*\mathring{Fa}_{\mathfrak{P}}(f(\mathfrak{t}))\}\\
f^{-1}(\mathring{Fa}_{\mathfrak{P}})(\mathfrak{s}*\mathfrak{t}) &= max\{f^{-1}(\mathring{Fa}_{\mathfrak{P}})(\mathfrak{s}), f^{-1}(\mathring{Fa}_{\mathfrak{P}})(\mathfrak{t})\},
\end{aligned}
$$

Hence, f is a $\mathcal{DPN}$ continuous function from $\hat{\dot{\mathcal{BP}}}_1$ to $\hat{\dot{\mathcal{BP}}}_2$. □

Proposition 3.18. *Let $\varkappa_1$ and $\varkappa_2$ be two $\mathcal{DPNTS}'\int$ on $\hat{\dot{\mathcal{BP}}}$. Then, each homomorphism f : ($\hat{\dot{\mathcal{BP}}}$, $\varkappa_1$) →($\hat{\dot{\mathcal{BP}}}$, $\varkappa_2$) is a $\mathcal{DPN}$ continuous function.*

Proof: Let ($\hat{\dot{\mathcal{BP}}}$, $\varkappa_1$) and ($\hat{\dot{\mathcal{BP}}}$, $\varkappa_2$) be two $\mathcal{DPNTS}'\int$, where $\hat{\dot{\mathcal{BP}}}$ is a $\hat{\dot{\mathcal{BP}}} - \mathring{A}$. To prove the above result, it is enough to show that result is false for a particular topology. Let $\mathfrak{P}$ = $(\mathring{Tr}_{\mathfrak{P}}, \mathring{Co}_{\mathfrak{P}}, \mathring{Ig}_{\mathfrak{P}}, \mathring{Un}_{\mathfrak{P}}, \mathring{Fa}_{\mathfrak{P}})$ and $\mathfrak{Q}$ =$(\mathring{Tr}_{\mathfrak{Q}}, \mathring{Co}_{\mathfrak{Q}}, \mathring{Ig}_{\mathfrak{Q}}, \mathring{Un}_{\mathfrak{Q}}, \mathring{Fa}_{\mathfrak{Q}})$ be two $\mathcal{DPNS}'s$ in $\hat{\dot{\mathcal{BP}}}$. Take $\varkappa_1 = 0_{\mathcal{DPNS}}, 1_{\mathcal{DPNS}}, \mathfrak{P}$ and $\varkappa_2 = 0_{\mathcal{DPNS}}, 1_{\mathcal{DPNS}}, \mathfrak{Q}$. If f : ($\hat{\dot{\mathcal{BP}}}$, $\varkappa_1$) →($\hat{\dot{\mathcal{BP}}}$, $\varkappa_2$), defined by f($\mathfrak{s}$) $= 0*\mathfrak{s}$, for all $\mathfrak{s} \in \hat{\dot{\mathcal{BP}}}$, then f is a homomorphism. Now, for $\mathfrak{s} \in \mathfrak{P}$, $\mathfrak{t} \in \varkappa_2$, (f$^{-1}(\mathfrak{Q})$)($\mathfrak{s}$) = $\mathfrak{Q}$(f($\mathfrak{s}$)), for all $\mathfrak{s} \in \hat{\dot{\mathcal{BP}}}$, f$^{-1}(\mathfrak{Q}) = \mathfrak{Q}$. Therefore, f$^{-1}(\mathfrak{Q}) \notin \varkappa_2$. Hence, f is not a $\mathcal{DPN}$ continuous mapping.

Definition 3.19. Let $\hat{\dot{\mathcal{BP}}} = \{\mathcal{H}, *, \mathring{0}\}$ be a $\hat{\dot{\mathcal{BP}}}$-$\mathring{A}$ and χ be a $\mathcal{DPNT}$ on $\hat{\dot{\mathcal{BP}}}$. Let $\mathfrak{P}$ be a $\mathcal{DPN}$ $\hat{\dot{\mathcal{BP}}}$-$\mathring{A}$ of $\hat{\dot{\mathcal{BP}}}$ and $\varkappa_{\mathfrak{P}}$ be a $\mathcal{DPNT}$ on $\mathfrak{P}$. Then, $\mathfrak{P}$ is said to be a $\mathcal{DPN}$ $\hat{\dot{\mathcal{BP}}}$-$\mathring{A}$ on $\hat{\dot{\mathcal{BP}}}$ if the self mapping $\varrho_{\wp}$: $(\mathfrak{P}, \varkappa_{\mathfrak{P}}) \to (\mathfrak{P}, \varkappa_{\mathfrak{P}})$ defined as $\varrho_{\mathfrak{P}}(\mathfrak{s}) = \mathfrak{s}*\mathfrak{P}$, for all $\mathfrak{P} \in \hat{\dot{\mathcal{BP}}}$, is a relatively digital pentapartitioned neutrosophic continuous mapping.

Theorem 3.20. *Let $\varkappa_1$ and $\varkappa_2$ be two $\mathcal{DPNTS}'\int$ on $\hat{\dot{\mathcal{BP}}}_1$ and $\hat{\dot{\mathcal{BP}}}_2$ respectively and* $\mathrm{f} : \hat{\dot{\mathcal{BP}}}_1 \to \hat{\dot{\mathcal{BP}}}_2$ *be a homomorphism such that* $\mathrm{f}^{-1}(\varkappa_2) = \varkappa_1$. *If* $\mathfrak{P} = (\mathring{Tr}_{\mathfrak{P}}, \mathring{Co}_{\mathfrak{P}}, \mathring{Ig}_{\mathfrak{P}}, \mathring{Un}_{\mathfrak{P}}, \mathring{Fa}_{\mathfrak{P}})$ *is a* $\mathcal{DPN}$ $\hat{\mathcal{BP}}$-$\mathring{A}$ *of* $\hat{\dot{\mathcal{BP}}}_2$, *then* $\mathrm{f}^{-1}(\mathfrak{P})$ *is a* $\mathcal{DPNT}$ $\hat{\mathcal{BP}}$-$\mathring{A}$ *of* $\hat{\dot{\mathcal{BP}}}_1$.

Proof. Let $\mathfrak{P} = (\mathring{Tr}_{\mathfrak{P}}, \mathring{Co}_{\mathfrak{P}}, \mathring{Ig}_{\mathfrak{P}}, \mathring{Un}_{\mathfrak{P}}, \mathring{Fa}_{\mathfrak{P}})$ be $\mathcal{DPNT}$ $\hat{\dot{\mathcal{BP}}}$-$\mathring{A}$ of $\hat{\dot{\mathcal{BP}}}_2$. To prove that $\mathrm{f}^{-1}(\mathfrak{P})$ be a $\mathcal{DPNT}$ $\hat{\dot{\mathcal{BP}}}$-$\mathring{A}$ of $\hat{\dot{\mathcal{BP}}}_1$. Let for any $\mathfrak{s}, \mathfrak{t} \in \hat{\dot{\mathcal{BP}}}_1$,

$$\begin{aligned}
\mathring{Tr}_{f^{-1}(\mathfrak{P})}(\mathfrak{s} * \mathfrak{t}) &= \mathring{Tr}_{\mathfrak{P}}(f(\mathfrak{s} * \mathfrak{t})) \\
&\geq min\{\mathring{Tr}_{\mathfrak{P}}(f(\mathfrak{s})) * \mathring{Tr}_{\mathfrak{P}}(f(\mathfrak{t}))\} \\
&= min\{\mathring{Tr}_{f^{-1}(\mathfrak{P})}(\mathfrak{s}), \mathring{Tr}_{f^{-1}(\mathfrak{P})}(\mathfrak{t})\}, \\
\mathring{Co}_{f^{-1}(\mathfrak{P})}(\mathfrak{s} * \mathfrak{t}) &= \mathring{Co}_{\mathfrak{P}}(f(\mathfrak{s} * \mathfrak{t})) \\
&\geq min\{\mathring{Co}_{\mathfrak{P}}(f(\mathfrak{s})) * \mathring{Co}_{\mathfrak{P}}(f(\mathfrak{t}))\} \\
&= min\{\mathring{Co}_{f^{-1}(\mathfrak{P})}(\mathfrak{s}), \mathring{Co}_{f^{-1}(\mathfrak{P})}(\mathfrak{t})\}, \\
\mathring{Ig}_{f^{-1}(\mathfrak{P})}(\mathfrak{s} * \mathfrak{t}) &= \mathring{Ig}_{\mathfrak{P}}(f(\mathfrak{s} * \mathfrak{t})) \\
&\leq max\{\mathring{Ig}_{\mathfrak{P}}(f(\mathfrak{s})) * \mathring{Ig}_{\mathfrak{P}}(f(\mathfrak{t}))\} \\
&= max\{\mathring{Ig}_{f^{-1}(\mathfrak{P})}(\mathfrak{s}), \mathring{Ig}_{f^{-1}(\mathfrak{P})}(\mathfrak{t})\}, \\
\mathring{Un}_{f^{-1}(\mathfrak{P})}(\mathfrak{s} * \mathfrak{t}) &= \mathring{Un}_{\mathfrak{P}}(f(\mathfrak{s} * \mathfrak{t})) \\
&\leq max\{\mathring{Un}_{\mathfrak{P}}(f(\mathfrak{s})) * \mathring{Un}_{\mathfrak{P}}(f(\mathfrak{t}))\} \\
&= max\{\mathring{Un}_{f^{-1}(\mathfrak{P})}(\mathfrak{s}), \mathring{Un}_{f^{-1}(\mathfrak{P})}(\mathfrak{t})\}, \\
\mathring{Fa}_{f^{-1}(\mathfrak{P})}(\mathfrak{s} * \mathfrak{t}) &= \mathring{Fa}_{\mathfrak{P}}(f(\mathfrak{s} * \mathfrak{t})) \\
&\leq max\{\mathring{Fa}_{\mathfrak{P}}(f(\mathfrak{s})) * \mathring{Fa}_{\mathfrak{P}}(f(\mathfrak{t}))\} \\
&= max\{\mathring{Fa}_{f^{-1}(\mathfrak{P})}(\mathfrak{s}), \mathring{Fa}_{f^{-1}(\mathfrak{P})}(\mathfrak{t})\},
\end{aligned}$$

Hence, $\mathrm{f}^{-1}(\mathfrak{P})$ is a $\mathcal{DPN}$ $\hat{\dot{\mathcal{BP}}}$-$\mathring{A}$ of $\hat{\dot{\mathcal{BP}}}_\infty$. Now, we prove that $\mathrm{f}^{-1}(\wp)$ is $\mathcal{DPNT}$ $\hat{\dot{\mathcal{BP}}}$-$\mathring{A}$ of $\hat{\dot{\mathcal{BP}}}_\infty$. Since f is a $\mathcal{DPN}$ continuous function, then by proposition 3.15, f is also a relatively $\mathcal{DPN}$ continuous function which maps $(\mathrm{f}^{-1}(\mathfrak{P}).\chi_{\mathrm{f}^{-1}(\mathfrak{P})})$ to $(\mathfrak{P}, \varkappa_{\mathfrak{P}})$

Assume $\mathfrak{P} \in \hat{\dot{\mathcal{BP}}}_1$ and χ be a $\mathcal{DPNS}$ in $\varkappa_P$ and let $\mathfrak{W}$ be a $\mathcal{DPNS}$ in $\chi_{\mathrm{f}^{-1}(\mathfrak{P})}$ such that $\mathrm{f}^{-1}(\chi) = \mathfrak{W}$ To prove that $\varrho\mathfrak{P} : (\mathrm{f}^{-1}(\mathfrak{P}).\chi_{\mathrm{f}^{-1}(\mathfrak{P})}) \to (\mathrm{f}^{-1}(\mathfrak{P}).\chi_{\mathrm{f}^{-1}(\mathfrak{P})})$ is relatively $\mathcal{DPN}$ continuous mapping, then for $\mathfrak{P} \in \hat{\dot{\mathcal{BP}}}_1$ we have

$$\begin{aligned}
\mathring{Tr}_{\varrho\mathfrak{P}^{-1}}(\mathfrak{W})(\mathfrak{s}) &= \mathring{Tr}_{(\mathfrak{W})}(\varrho_{\mathfrak{p}}(s)) = \mathring{Tr}_{(\mathfrak{W})}(\mathfrak{s} * \mathfrak{P}) \\
&= \mathring{Tr}_{\mathrm{f}^{-1}(\chi)}(\mathfrak{s} * \mathfrak{P}) = \mathring{Tr}_{(\chi)}(\mathrm{f}(\mathfrak{s} * \mathfrak{P})) \\
&= \mathring{Tr}_{(\chi)}(\mathrm{f}(\mathfrak{s}) * \mathrm{f}(\mathfrak{P})) = \mathring{Tr}_{(\chi)}(\varrho_{\mathrm{f}(\mathfrak{P})}(\mathrm{f}(\mathfrak{s}))) \\
&= \mathring{Tr}_{\mathfrak{P}^{-1}\mathrm{f}(\mathfrak{P})\chi}(\mathrm{f}(\mathfrak{s})) = \mathring{Tr}_{\mathrm{f}^{-1}}(\varrho^{-1}_{\mathrm{f}(\mathfrak{P})}(\chi)(\mathfrak{s})),
\end{aligned}$$

$$\begin{aligned}
\mathring{Co}_{\varrho\mathfrak{P}^{-1}}(\mathfrak{W})(\mathfrak{s}) &= \mathring{Co}_{(\mathfrak{W})}(\varrho_{\mathfrak{p}}(s)) = \mathring{Co}_{(\mathfrak{W})}(\mathfrak{s} * \mathfrak{P}) \\
&= \mathring{Co}_{\mathrm{f}^{-1}(\chi)}(\mathfrak{s} * \mathfrak{P}) = \mathring{Co}_{(\chi)}(\mathrm{f}(\mathfrak{s} * \mathfrak{P})) \\
&= \mathring{Co}_{(\chi)}(\mathrm{f}(\mathfrak{s}) * \mathrm{f}(\mathfrak{P})) = \mathring{Co}_{(\chi)}(\varrho_{\mathrm{f}(\mathfrak{P})}(\mathrm{f}(\mathfrak{s}))) \\
&= \mathring{Co}_{\mathfrak{P}^{-1}\mathrm{f}(\mathfrak{P})\chi}(\mathrm{f}(\mathfrak{s})) = \mathring{Co}_{\mathrm{f}^{-1}}(\varrho^{-1}_{\mathrm{f}(\mathfrak{P})}(\chi)(\mathfrak{s})),
\end{aligned}$$

$$\begin{aligned}\mathring{I}g_{\varrho\mathfrak{P}^{-1}(\mathfrak{W})}(\mathfrak{s}) &= \mathring{I}g_{(\mathfrak{W})}(\varrho\mathfrak{P}(\mathfrak{s})) = \mathring{I}g_{(\mathfrak{W})}(\mathfrak{s}*\mathfrak{P})\\ &= \mathring{I}g_{\mathrm{f}^{-1}(\chi)}(\mathfrak{s}*\mathfrak{P}) = \mathring{I}g_{(\chi)}(\mathrm{f}(\mathfrak{s}*\mathfrak{P}))\\ &= \mathring{I}g_{(\chi)}(\mathrm{f}(\mathfrak{s})*\mathrm{f}(\mathfrak{P})) = \mathring{I}g_{(\chi)}(\varrho\mathrm{f}\mathfrak{P}(\mathrm{f}(\mathfrak{s}))\\ &= \mathring{I}g_{\mathfrak{P}^{-1}\mathrm{f}(\mathfrak{P})\chi}(\mathrm{f}(\mathfrak{s})) = \mathring{I}g_{\mathrm{f}^{-1}}(\varrho^{-1}_{\mathrm{f}(\mathfrak{P})}(\chi)(\mathfrak{s})),\end{aligned}$$

$$\begin{aligned}\mathring{U}n_{\varrho\mathfrak{P}^{-1}(\mathfrak{W})}(\mathfrak{s}) &= \mathring{U}n_{(\mathfrak{W})}(\varrho\mathfrak{P}(\mathfrak{s})) = \mathring{U}n_{(\mathfrak{W})}(\mathfrak{s}*\mathfrak{P})\\ &= \mathring{U}n_{\mathrm{f}^{-1}(\chi)}(\mathfrak{s}*\mathfrak{P}) = \mathring{U}n_{(\chi)}(\mathrm{f}(\mathfrak{s}*\mathfrak{P}))\\ &= \mathring{U}n_{(\chi)}(\mathrm{f}(\mathfrak{s})*\mathrm{f}(\mathfrak{P})) = \mathring{U}n_{(\chi)}(\varrho\mathrm{f}\mathfrak{P}(\mathrm{f}(\mathfrak{s}))\\ &= \mathring{U}n_{\mathfrak{P}^{-1}\mathrm{f}(\mathfrak{P})\chi}(\mathrm{f}(\mathfrak{s})) = \mathring{U}n_{\mathrm{f}^{-1}}(\varrho^{-1}_{\mathrm{f}(\mathfrak{P})}(\chi)(\mathfrak{s})),\end{aligned}$$

$$\begin{aligned}\mathring{F}a_{\varrho\mathfrak{P}^{-1}(\mathfrak{W})}(\mathfrak{s}) &= \mathring{F}a_{(\mathfrak{W})}(\varrho\mathfrak{P}(\mathfrak{s})) = \mathring{F}a_{(\mathfrak{W})}(\mathfrak{s}*\mathfrak{P})\\ &= \mathring{F}a_{\mathrm{f}^{-1}(\chi)}(\mathfrak{s}*\mathfrak{P}) = \mathring{F}a_{(\chi)}(\mathrm{f}(\mathfrak{s}*\mathfrak{P}))\\ &= \mathring{F}a_{(\chi)}(\mathrm{f}(\mathfrak{s})*\mathrm{f}(\mathfrak{P})) = \mathring{F}a_{(\chi)}(\varrho\mathrm{f}\mathfrak{P}(\mathrm{f}(\mathfrak{s}))\\ &= \mathring{F}a_{\mathfrak{P}^{-1}\mathrm{f}(\mathfrak{P})\chi}(\mathrm{f}(\mathfrak{s})) = \mathring{F}a_{\mathrm{f}^{-1}}(\varrho^{-1}_{\mathrm{f}(\mathfrak{P})}(\chi)(\mathfrak{s})),\end{aligned}$$

It concludes that $\varrho_{\mathfrak{P}^{-1}}(\mathfrak{W}) = \mathrm{f}^{-1}(\varrho^{-1}_{\mathrm{f}(\mathfrak{P})}(\chi))$. Thus, $\varrho^{-1}_{\mathfrak{P}}(\mathfrak{W})\cap\mathrm{f}^{-1}(\mathfrak{P}) = \mathrm{f}^{-1}(\varrho^{-1}_{\mathrm{f}(\mathfrak{P})}(\chi))\cap\mathrm{f}^{-1}(\mathfrak{P})$ is a $\mathcal{DPNS}$ in $\mathrm{f}^{-1}(\mathfrak{P})$ and a $\mathcal{DPNS}$ in $\chi_{\mathrm{f}^{-1}(\mathfrak{P}))}$ Hence, $\mathrm{f}^{-1}(\mathfrak{P})$ and a $\mathcal{DPNT}$ $\mathcal{B}\hat{\dot{\mathcal{P}}}$-$\mathring{A}$ of $\mathcal{B}\hat{\dot{\mathcal{P}}}$. Hence,the proof. □

Theorem 3.21. *Let $(\mathcal{B}\hat{\dot{\mathcal{P}}}_1, \varkappa_1)$ and $(\mathcal{B}\hat{\dot{\mathcal{P}}}_2, \varkappa_2)$ be two $\mathcal{DPNTS}'s$ on $\mathcal{B}\hat{\dot{\mathcal{P}}}_1$ and $\mathcal{B}\hat{\dot{\mathcal{P}}}_2$, respectively, and let f be a bijective homomorphism of $\mathcal{B}\hat{\dot{\mathcal{P}}}_1$ into $\mathcal{B}\hat{\dot{\mathcal{P}}}_2$ such that $\mathrm{f}(c_1) = \varkappa_2$. If $\mathfrak{P}$ is a $\mathcal{DPNT}$ $\mathcal{B}\hat{\dot{\mathcal{P}}}$-$\mathring{A}_1$, then $\mathrm{f}(\mathfrak{P})$ is a $\mathcal{DPNT}$ $\mathcal{B}\hat{\dot{\mathcal{P}}}$-$\mathring{A}_2$.*

Proof. Suppose that $\mathfrak{P} = \{\mathring{T}r_{\mathfrak{P}}, \mathring{C}o_{\mathfrak{P}}, \mathring{I}g_{\mathfrak{P}}, \mathring{U}n_{\mathfrak{P}}, \mathring{F}a_{\mathfrak{P}}$ is a $\mathcal{DPN}$ topological $\mathcal{B}\hat{\dot{\mathcal{P}}}$ of $\mathcal{B}\hat{\dot{\mathcal{P}}}_1$. To prove that $\mathrm{f}(\mathfrak{P})$ is a $\mathcal{DPNT}$ $\mathcal{B}\hat{\dot{\mathcal{P}}}$-$\mathring{A}_2$ of $\mathcal{B}\hat{\dot{\mathcal{P}}}_2$, let, for $\mathfrak{s},\mathfrak{t} \in \mathcal{B}\hat{\dot{\mathcal{P}}}_2, \mathrm{f}(\mathfrak{P}) = (\mathrm{f}_{sup}(\mathring{T}r_{\mathfrak{P}})(\mathfrak{t}), \mathrm{f}_{sup}(\mathring{C}o_{\mathfrak{P}})(\mathfrak{t}), \mathrm{f}_{inf}(\mathring{I}g_{\mathfrak{P}})(\mathfrak{t}), \mathrm{f}_{inf}(\mathring{U}n_{\mathfrak{P}})(\mathfrak{t}), \mathrm{f}_{inf}(\mathring{F}a_{\mathfrak{P}})(\mathfrak{t}))$.

Let $\mathfrak{p}_0 \in \mathrm{f}^{-1}(\mathfrak{s}), \mathfrak{q}_0 \in \mathrm{f}^{-1}(\mathfrak{t})$ such that

$\sup_{\varkappa\,\mathrm{inf}^{-1}(\mathfrak{s})} \mathring{T}r_{\mathfrak{P}}(\varkappa) = \mathring{T}r_{\mathfrak{P}}(\mathfrak{p}_0), \sup_{\varkappa\,\mathrm{inf}^{-1}(\mathring{t})} \mathring{T}r_{\mathfrak{P}}(\varkappa) = \mathring{T}r_{\mathfrak{P}}(\mathfrak{q}_0)$,
$\sup_{\varkappa\,\mathrm{inf}^{-1}(\mathfrak{s})} \mathring{C}o_{\mathfrak{P}}(\varkappa) = \mathring{C}o_{\mathfrak{P}}(\mathfrak{p}_0), \sup_{\varkappa\,\mathrm{inf}^{-1}(\mathring{t})} \mathring{C}o_{\mathfrak{P}}(\varkappa) = \mathring{C}o_{\mathfrak{P}}(\mathfrak{q}_0)$,
$\inf_{\varkappa\,\mathrm{inf}^{-1}(\mathfrak{s})} \mathring{I}g_{\mathfrak{P}}(\varkappa) = \mathring{I}g_{\mathfrak{P}}(\mathfrak{p}_0), \inf_{\varkappa\,\mathrm{inf}^{-1}(\mathfrak{t})} \mathring{I}g_{\mathfrak{P}}(\varkappa) = \mathring{I}g_{\mathfrak{P}}(\mathfrak{q}_0)$,
$\inf_{\varkappa\,\mathrm{inf}^{-1}(\mathfrak{s})} \mathring{U}n_{\mathfrak{P}}(\varkappa) = \mathring{U}n_{\mathfrak{P}}(\mathfrak{p}_0), \inf_{\varkappa\,\mathrm{inf}^{-1}(\mathfrak{t})} \mathring{U}n_{\mathfrak{P}}(\varkappa) = \mathring{U}n_{\mathfrak{P}}(\mathfrak{q}_0)$,
$\inf_{\varkappa\,\mathrm{inf}^{-1}(\mathfrak{s})} \mathring{F}a_{\mathfrak{P}}(\varkappa) = \mathring{F}a_{\mathfrak{P}}(\mathfrak{p}_0), \inf_{\varkappa\,\mathrm{inf}^{-1}(\mathfrak{t})} \mathring{F}a_{\mathfrak{P}}(\varkappa) = \mathring{F}a_{\mathfrak{P}}(\mathfrak{q}_0)$,
Now,

$$\begin{aligned}\mathring{T}r_{\mathrm{f}(\mathfrak{P})}(\mathfrak{s}*\ t) &= {}_{\varkappa\in\mathrm{f}^{-1}(\mathfrak{s}*\mathfrak{t})}{}^{\sup}\mathring{T}r_{\mathfrak{P}}(\varkappa)\\ &\geq \mathring{T}r_{\mathfrak{P}}(\mathfrak{p}_0, \mathfrak{q}_0)\\ &\geq \min\{\mathring{T}r_{\mathfrak{P}}(\mathfrak{p}_0), \mathring{T}r_{\mathfrak{P}}(\mathfrak{q}_0)\}\end{aligned}$$

$$= \min\{{}_{\varkappa\in \mathfrak{f}^{-1}(\mathfrak{s})}{}^{\sup}\mathring{T}r_{\mathfrak{P}}(\varkappa), {}_{\varkappa\in \mathrm{f}^{-1}(t)}{}^{\sup}\mathring{T}r_{\mathfrak{P}}(\varkappa)\}$$
$$= \min\{\mathring{T}r_{\mathrm{f}(\mathfrak{P})}(\mathfrak{s}) * \mathring{T}r_{\mathrm{f}(\mathfrak{P})}(\mathfrak{t})\},$$

$$\begin{aligned}\mathring{C}o_{\mathrm{f}(\mathfrak{P})}(\mathfrak{s} *\ t) &= {}_{\varkappa\in \mathrm{f}^{-1}(\mathfrak{s}*\mathfrak{t})}{}^{\sup}\mathring{C}o_{\mathfrak{P}}(\varkappa)\\ &\geq \mathring{C}o_{\mathfrak{P}}(\mathfrak{p}_0, \mathfrak{q}_0)\\ &\geq \min\{\mathring{C}o_{\mathfrak{P}}(\mathfrak{p}_0), \mathring{C}o_{\mathfrak{P}}(\mathfrak{q}_0)\}\\ &= \min\{{}_{\varkappa\in \mathfrak{f}^{-1}(\mathfrak{s})}{}^{\sup}\mathring{C}o_{\mathfrak{P}}(\varkappa), {}_{\varkappa\in \mathrm{f}^{-1}(t)}{}^{\sup}\mathring{C}o_{\mathfrak{P}}(\varkappa)\}\\ &= \min\{\mathring{C}o_{\mathrm{f}(\mathfrak{P})}(\mathfrak{s}) * \mathring{C}o_{\mathrm{f}(\mathfrak{P})}(\mathfrak{t})\},\end{aligned}$$

$$\begin{aligned}\mathring{I}g_{\mathrm{f}(\mathfrak{P})}(\mathfrak{s} *\ t) &= {}_{\varkappa\in \mathrm{f}^{-1}(\mathfrak{s}*\mathfrak{t})}{}^{\inf}\mathring{I}g_{\mathfrak{P}}(\varkappa)\\ &\leq \mathring{I}g_{\mathfrak{P}}(\mathfrak{p}_0, \mathfrak{q}_0)\\ &\leq \max\{\mathring{I}g_{\mathfrak{P}}(\mathfrak{p}_0), \mathring{I}g_{\mathfrak{P}}(\mathfrak{q}_0)\}\\ &= \max\{{}_{\varkappa\in \mathfrak{f}^{-1}(\mathfrak{s})}{}^{\inf}\mathring{I}g_{\mathfrak{P}}(\varkappa), {}_{\varkappa\in \mathrm{f}^{-1}(t)}{}^{\inf}\mathring{I}g_{\mathfrak{P}}(\varkappa)\}\\ &= \max\{\mathring{I}g_{\mathrm{f}(\mathfrak{P})}(\mathfrak{s}) * \mathring{I}g_{\mathrm{f}(\mathfrak{P})}(\mathfrak{t})\},\end{aligned}$$

$$\begin{aligned}\mathring{U}n_{\mathrm{f}(\mathfrak{P})}(\mathfrak{s} *\ t) &= {}_{\varkappa\in \mathrm{f}^{-1}(\mathfrak{s}*\mathfrak{t})}{}^{\inf}\mathring{U}n_{\mathfrak{P}}(\varkappa)\\ &\leq \mathring{U}n_{\mathfrak{P}}(\mathfrak{p}_0, \mathfrak{q}_0)\\ &\leq \max\{\mathring{U}n_{\mathfrak{P}}(\mathfrak{p}_0), \mathring{U}n_{\mathfrak{P}}(\mathfrak{q}_0)\}\\ &= \max\{{}_{\varkappa\in \mathfrak{f}^{-1}(\mathfrak{s})}{}^{\inf}\mathring{U}n_{\mathfrak{P}}(\varkappa), {}_{\varkappa\in \mathrm{f}^{-1}(t)}{}^{\inf}\mathring{U}n_{\mathfrak{P}}(\varkappa)\}\\ &= \max\{\mathring{U}n_{\mathrm{f}(\mathfrak{P})}(\mathfrak{s}) * \mathring{U}n_{\mathrm{f}(\mathfrak{P})}(\mathfrak{t})\},\end{aligned}$$

$$\begin{aligned}\mathring{F}a_{\mathrm{f}(\mathfrak{P})}(\mathfrak{s} *\ t) &= {}_{\varkappa\in \mathrm{f}^{-1}(\mathfrak{s}*\mathfrak{t})}{}^{\inf}\mathring{F}a_{\mathfrak{P}}(\varkappa)\\ &\leq \mathring{F}a_{\mathfrak{P}}(\mathfrak{p}_0, \mathfrak{q}_0)\\ &\leq \max\{\mathring{F}a_{\mathfrak{P}}(\mathfrak{p}_0), \mathring{F}a_{\mathfrak{P}}(\mathfrak{q}_0)\}\\ &= \max\{{}_{\varkappa\in \mathfrak{f}^{-1}(\mathfrak{s})}{}^{\inf}\mathring{F}a_{\mathfrak{P}}(\varkappa), {}_{\varkappa\in \mathrm{f}^{-1}(t)}{}^{\inf}\mathring{F}a_{\mathfrak{P}}(\varkappa)\}\\ &= \max\{\mathring{F}a_{\mathrm{f}(\mathfrak{P})}(\mathfrak{s}) * \mathring{F}a_{\mathrm{f}(\mathfrak{P})}(\mathfrak{t})\},\end{aligned}$$

Hence, $\mathrm{f}(\mathfrak{P})$ is a $\mathcal{DPN}$ $\hat{\dot{\mathcal{BP}}}$-subalgebra of $\hat{\dot{\mathcal{BP}}}_2$. Now, we prove that the self mapping $\varrho_q : (\mathrm{f}(\mathfrak{P}), \varkappa_{\mathrm{f}}(\mathfrak{P})) \to (\mathrm{f}(\mathfrak{P}), \varkappa_{\mathrm{f}}(\mathfrak{P}))$, defined by $\varrho_q(\mathfrak{t}) = t * \mathfrak{q}$ for all $\mathfrak{q} \in \hat{\dot{\mathcal{BP}}}_2$ is a relatively digital pentapartitioned neutrosophic continuous mapping. Let $\chi_{\mathfrak{q}}$ be a $\mathcal{DPNS}$ in $\varkappa_P$, there exists a $\mathcal{DPNS}$ "χ" in $\varkappa_1$ such that $\chi_{\mathfrak{p}} = \chi \cap \mathfrak{P}$. We show that for a $\mathcal{DPNS}$ in $\varkappa_{\mathrm{f}(\mathfrak{P})}$, $\varrho_q^{-1}(\chi_{\mathrm{f}(\mathfrak{P})}) \cap \mathrm{f}(\mathfrak{P}) \in \varkappa_{\mathrm{f}(\mathfrak{P})}$.

Since f is an injective mapping, then $\mathrm{f}(\chi_{\mathfrak{P}}) = \mathrm{f}(\chi \cap \mathfrak{P}) = \mathrm{f}(\chi) \cap\ \mathrm{f}(\mathfrak{P})$ is a $\mathcal{DPNS}$ in $\varkappa_{\mathrm{f}(\mathfrak{P})}$ which shows that f is relatively digital pentapartitioned neutrosophic open. In addition, f is surjective, then for all $\mathfrak{q} \in \hat{\dot{\mathcal{BP}}}_2, \mathfrak{p} = \mathrm{f}(\mathfrak{q})$, where $\mathfrak{p} \in \hat{\dot{\mathcal{BP}}}_1$. Now,

$$\begin{aligned}\mathring{T}r_{\mathrm{f}^{-1}(\varrho_q^{-1}(\varkappa_{\mathrm{f}(\mathfrak{P})}))}(\mathfrak{s}) &= \mathring{T}r_{\mathrm{f}^{-1}(\varrho^{-1}\mathrm{f}(\mathfrak{p})(\varkappa(\mathrm{f}(\mathfrak{P}))))}(\mathfrak{s})\\ &= \mathring{T}r_{\varrho^{-1}(\mathrm{f}(\mathfrak{p})(\varkappa\mathrm{f}(\mathfrak{P})))}(\mathfrak{f}(\mathfrak{s}))\\ &= \mathring{T}r_{(\chi_{\mathrm{f}(\mathfrak{P})})}(\varrho_{\mathrm{f}(\mathfrak{p})}(\mathrm{f}(\mathfrak{s}))\\ &= \mathring{T}r_{(\chi_{\mathrm{f}(\mathfrak{P})})}(f(\mathfrak{s}) * \mathrm{f}(\mathfrak{p}))\\ &= \mathring{T}r_{\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})})}(\mathfrak{s} * \mathfrak{p})\\ &= \mathring{T}r_{\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})})}(\varrho_{\mathfrak{p}}(\mathfrak{s}))\\ &= \mathring{T}r_{\varrho^{-1}(\mathfrak{p})(\mathrm{f}^{-1}(\chi\mathrm{f}(\mathfrak{P}))}(\mathfrak{s}),\end{aligned}$$

$$\begin{aligned}\mathring{C}o_{\mathrm{f}^{-1}(\varrho_q^{-1}(\varkappa_{\mathrm{f}(\mathfrak{P})}))}(\mathfrak{s}) &= \mathring{C}o_{\mathrm{f}^{-1}(\varrho^{-1}\mathrm{f}(\mathfrak{p})(\varkappa(\mathrm{f}(\mathfrak{P}))))}(\mathfrak{s})\\ &= \mathring{C}o_{\varrho^{-1}(\mathrm{f}(\mathfrak{p})(\varkappa\mathrm{f}(\mathfrak{P})))}(\mathfrak{f}(\mathfrak{s}))\\ &= \mathring{C}o_{(\chi_{\mathrm{f}(\mathfrak{P})})}(\varrho_{\mathrm{f}(\mathfrak{p})}(\mathrm{f}(\mathfrak{s}))\\ &= \mathring{C}o_{(\chi_{\mathrm{f}(\mathfrak{P})})}(f(\mathfrak{s}) * \mathrm{f}(\mathfrak{p}))\\ &= \mathring{C}o_{\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})})}(\mathfrak{s} * \mathfrak{p})\\ &= \mathring{C}o_{\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})})}(\varrho_{\mathfrak{p}}(\mathfrak{s}))\\ &= \mathring{C}o_{\varrho^{-1}(\mathfrak{p})(\mathrm{f}^{-1}(\chi\mathrm{f}(\mathfrak{P}))}(\mathfrak{s}),\end{aligned}$$

$$\begin{aligned}\mathring{I}g_{\mathrm{f}^{-1}(\varrho_q^{-1}(\varkappa_{\mathrm{f}(\mathfrak{P})}))}(\mathfrak{s}) &= \mathring{I}g_{\mathrm{f}^{-1}(\varrho^{-1}\mathrm{f}(\mathfrak{p})(\varkappa(\mathrm{f}(\mathfrak{P}))))}(\mathfrak{s})\\ &= \mathring{I}g_{\varrho^{-1}(\mathrm{f}(\mathfrak{p})(\varkappa\mathrm{f}(\mathfrak{P})))}(\mathfrak{f}(\mathfrak{s}))\\ &= \mathring{I}g_{(\chi_{\mathrm{f}(\mathfrak{P})})}(\varrho_{\mathrm{f}(\mathfrak{p})}(\mathrm{f}(\mathfrak{s}))\\ &= \mathring{I}g_{(\chi_{\mathrm{f}(\mathfrak{P})})}(f(\mathfrak{s}) * \mathrm{f}(\mathfrak{p}))\\ &= \mathring{I}g_{\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})})}(\mathfrak{s} * \mathfrak{p})\\ &= \mathring{I}g_{\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})})}(\varrho_{\mathfrak{p}}(\mathfrak{s}))\\ &= \mathring{I}g_{\varrho^{-1}(\mathfrak{p})(\mathrm{f}^{-1}(\chi\mathrm{f}(\mathfrak{P}))}(\mathfrak{s}),\end{aligned}$$

$$\begin{aligned}\mathring{U}n_{\mathrm{f}^{-1}(\varrho_q^{-1}(\varkappa_{\mathrm{f}(\mathfrak{P})}))}(\mathfrak{s}) &= \mathring{U}n_{\mathrm{f}^{-1}(\varrho^{-1}\mathrm{f}(\mathfrak{p})(\varkappa(\mathrm{f}(\mathfrak{P}))))}(\mathfrak{s})\\ &= \mathring{U}n_{\varrho^{-1}(\mathrm{f}(\mathfrak{p})(\varkappa\mathrm{f}(\mathfrak{P})))}(\mathfrak{f}(\mathfrak{s}))\\ &= \mathring{U}n_{(\chi_{\mathrm{f}(\mathfrak{P})})}(\varrho_{\mathrm{f}(\mathfrak{p})}(\mathrm{f}(\mathfrak{s}))\\ &= \mathring{U}n_{(\chi_{\mathrm{f}(\mathfrak{P})})}(f(\mathfrak{s}) * \mathrm{f}(\mathfrak{p}))\\ &= \mathring{U}n_{\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})})}(\mathfrak{s} * \mathfrak{p})\\ &= \mathring{U}n_{\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})})}(\varrho_{\mathfrak{p}}(\mathfrak{s}))\\ &= \mathring{U}n_{\varrho^{-1}(\mathfrak{p})(\mathrm{f}^{-1}(\chi\mathrm{f}(\mathfrak{P}))}(\mathfrak{s}),\end{aligned}$$

$$\begin{aligned}\mathring{F}a_{\mathrm{f}^{-1}(\varrho_q^{-1}(\varkappa_{\mathrm{f}(\mathfrak{P})}))}(\mathfrak{s}) &= \mathring{F}a_{\mathrm{f}^{-1}(\varrho^{-1}\mathrm{f}(\mathfrak{p})(\varkappa(\mathrm{f}(\mathfrak{P}))))}(\mathfrak{s})\\ &= \mathring{F}a_{\varrho^{-1}(\mathrm{f}(\mathfrak{p})(\varkappa\mathrm{f}(\mathfrak{P})))}(\mathfrak{f}(\mathfrak{s}))\\ &= \mathring{F}a_{(\chi_{\mathrm{f}(\mathfrak{P})})}(\varrho_{\mathrm{f}(\mathfrak{p})}(\mathrm{f}(\mathfrak{s}))\\ &= \mathring{F}a_{(\chi_{\mathrm{f}(\mathfrak{P})})}(f(\mathfrak{s}) * \mathrm{f}(\mathfrak{p}))\end{aligned}$$

$$= \mathring{F}a_{\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})})}(\mathfrak{s} * \mathfrak{p})$$
$$= \mathring{F}a_{\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})})}(\varrho_{\mathfrak{p}}(\mathfrak{s}))$$
$$= \mathring{F}a_{\varrho^{-1}(\mathfrak{p})(\mathrm{f}^{-1}(\chi \mathrm{f}(\mathfrak{P}))}(\mathfrak{s}),$$

This implies that $\mathrm{f}^{-1}(\varrho_{\mathfrak{q}}^{-1}((\chi_{\mathrm{f}(\mathfrak{P})})) = \varrho_{\mathfrak{p}}^{-1}(\mathrm{f}^{-1}(\chi_{\mathrm{f}(\mathfrak{P})}))$. Since is relatively $\varrho_{\mathfrak{p}} : (\mathfrak{P}, \varkappa_{\mathfrak{P}}) \to (\mathfrak{P}, \varkappa_{\mathfrak{P}})$ is relatively digital pentapartitioned neutrosophic continuous mapping and f is relatively digital pentapartitioned neutrosophic continues mapping from $(\mathfrak{P}, \varkappa_{\mathfrak{P}})$ into $(\mathrm{f}(\mathfrak{P}))(\varkappa_{\mathrm{f}(\mathfrak{P})}), \mathrm{f}^{-1}(\varrho_q^{-1}((\mathfrak{W}_{\mathrm{f}(\mathfrak{P})}))) \cap \mathfrak{P} = \varrho_p^{-1}(\mathrm{f}^{-1}((\chi_{\mathrm{f}(\mathfrak{P})}(\mathfrak{p}))) \cap$
$mathfrakP$ a $\mathcal{DPNS}$ in $\varkappa_{\mathfrak{P}}$. Hence, $\mathrm{f}(\mathrm{f}^{-1}(\varrho_{(q)}((\varkappa_{\mathrm{f}(\mathfrak{P})}))\cap\mathfrak{P}) = \varrho_{(}\mathfrak{p})^{-1}(\varkappa_{\mathrm{f}(\mathfrak{p})})\cap \mathrm{f}(\mathfrak{P})$ is a $\mathcal{DPNS}$ in $\varkappa_{\mathfrak{p}}$), hence the proof. □

Example 3.22. Let $\hat{\mathcal{BP}} = (\mathring{\mathcal{H}},^*, \mathring{0})$ be a $\hat{\mathcal{BP}} - \mathring{\mathcal{A}}$, where $(\mathring{\mathcal{H}}) = \{0, \mathfrak{a}, \mathfrak{a}^2, \mathfrak{a}^3, \mathfrak{a}^4, \mathfrak{a}^5, \mathfrak{a}^6, \mathfrak{a}^7, \mathfrak{a}^8\}$ is the cyclic group of order 9 and Caley's table. We define a $\mathcal{DPNS}$ as:
Let Universal Set $\mathbb{U} = \{\mathring{1}, \mathring{2}\} \times \{\mathring{1}, \mathring{2}\}$ in the digital plane $\hat{\mathring{\mathcal{D}}}^2$
Consider all subsets $\mathring{B} = \{(\mathring{1}, \mathring{1}), (\mathring{1}, \mathring{2}), (\mathring{2}, \mathring{1}), (\mathring{2}, \mathring{2})\}$ as an rectangular arrays of integer coordinate points of $\mathring{\mathbb{Z}}^2$
Therefore $(\mathring{\mathbb{Z}}^2, \hat{\mathring{\mathcal{D}}}^2)$ is the $\mathcal{DTS}$. Let $\mathring{A} = \{\mathfrak{P}_1 < 0.1, 0.2, 0.7, 0.6, 0.5 >, \mathfrak{P}_2 < 0.7, 0.6, 0.5, 0.1, 0.2 >, \mathfrak{P}_3 < 0.2, 0.1, 0.6, 0.5, 0.4 >, \mathfrak{P}_4 < 0.8, 0.7, 0.5, 0.3, 0.1 >\}$
$\mathcal{P} = \{(0, 0.1, 0.2, 0.7, 0.6, 0.5), (\mathring{u}, 0.7, 0.6, 0.5, 0.1, 0.2)\}$,
$\mathcal{Q} = \{(0, 0.2, 0.1, 0.6, 0.5, 0.4), (\mathring{u}, 0.8, 0.7, 0.5, 0.3, 0.1)\}$,
For all $\mathring{u} \neq \mathring{0} \in \mathring{\mathcal{H}}$, where $\mathcal{P}, \mathcal{Q} \in [0,\ 1]$. The collection $\varkappa_{\hat{\mathcal{BP}}} = \{0_{D\mathring{P}NS}, 1_{D\mathring{P}NS}, \mathcal{P}, \mathcal{Q}\}$ of $\mathcal{DPNS}'\int$ of $\hat{\mathcal{BP}}$ is a $\mathcal{DPNT}$ on $\hat{\mathcal{BP}}$ and $(\hat{\mathcal{BP}},\ \varkappa_{\hat{\mathcal{BP}}})$ is a $\mathcal{DPNTS}$. Let $\mathcal{R}$ be a $\mathcal{DPNS}$ in $\hat{\mathcal{BP}}$, defined as:
$\mathcal{R} = \{(0, 0.1, 0.1, 0.7, 0.6, 0.5), (\mathring{u}, 0.8, 0.7, 0.5, 0.1, 0.1)\}$, for all $\mathring{u} \neq \mathring{0} \in \mathring{\mathcal{H}}$.
Clearly, $\mathcal{R}$ is a $\mathcal{DPN}$ $\hat{\mathcal{BP}}$-$\mathring{A}$ of $\hat{BP}$. By direct calculations relative topology $\varkappa_{\mathfrak{R}}$ is obtained as $\varkappa_{\mathcal{R}} = \{\mathring{0}_{\mathcal{P}}, \mathring{1}_{\mathcal{P}}, \mathcal{P}\}$. Then, the pair $(\mathcal{R}, \varkappa_{\mathcal{R}})$ is a digital pentapartitioned neutrosophic subspace of $(\hat{\mathcal{BP}}, \varkappa_{\hat{\mathcal{BP}}})$. We show that $\mathcal{R}$ is a $\mathcal{DPNT}$ $\hat{\mathcal{BP}}$-subalgebra of $\hat{\mathcal{BP}}$, i.e., the self mapping $\varrho_{\mathbb{p}} : (\mathcal{R}, \varkappa_{\mathcal{R}}) \to (\mathcal{R}, \varkappa_{\mathcal{R}})$ defined by $\varrho_{\mathbb{p}}(\mathfrak{s}) = \mathfrak{s} * \mathbb{p}, \in \hat{\mathcal{BP}}$ for all $\mathbb{p}$ is relatively digital pentapartitioned neutrosophic continuous mapping, i.e., for a $\mathcal{DPNOS}$, $\mathcal{P}$ in $(\mathcal{R}, \varkappa_{\mathcal{R}}), \varrho_{(\mathbb{p})}^{-1}(\mathcal{P}) \cap \mathcal{R} \in \varkappa_{\mathcal{R}}$. Since $\varrho_{\mathbb{p}}$ is homomorphism, then $\varrho_{(\mathbb{p})}^{-1}(\mathcal{P}) \cap \mathcal{R} = \mathcal{P} \in \varkappa_{\mathcal{R}}$. Therefore, $\varrho_{\shortmid} : (\mathcal{R}, \varkappa_{\mathcal{R}}) \to (\mathcal{R}, \varkappa_{\mathcal{R}})$ is relatively digital pentapartitioned neutrosophic continuous mapping. Hence, $\mathcal{R}$ is a $\mathcal{DPNT}$ $\hat{\mathcal{BP}}$-$\mathring{A}$ of $\hat{\mathcal{BP}}$-$\mathring{A}$.

Definition 3.23. Let $\varkappa$ be a $\mathcal{DPNT}$ on $\hat{\mathcal{BP}}$ and $(\hat{\mathcal{BP}}, \varkappa)$ be a $\mathcal{DPNT}$. Then, $(\hat{\mathcal{BP}}, \varkappa)$ is called digital pentapartitioned neutrosophic $\mathfrak{C}_5$-disconnected topological space if there exist a $\mathcal{DPNOS}$ and $\mathcal{DPNCS}$ $\mathcal{J}$ such that $\mathcal{J} = (\mathring{T}r_{\mathcal{J}}, \mathring{C}o_{\mathcal{J}}, \mathring{I}g_{\mathcal{J}}, \mathring{U}n_{\mathcal{J}}, \mathring{F}a_{\mathcal{J}}) \neq 1_{D\mathring{P}NS}$ and $\mathcal{J} =$

$(\mathring{Tr}_{\mathcal{J}}, \mathring{Co}_{\mathcal{J}}, \mathring{Ig}_{\mathcal{J}}, \mathring{Un}_{\mathcal{J}}, \mathring{Fa}_{\mathcal{J}}) \neq 0_{D\mathring{P}NS}$, otherwise $(\hat{\dot{\mathcal{B}\mathcal{P}}}, \varkappa)$ is called digital pentapartitioned neutrosophic $\mathfrak{C}_5$-connected.

Example 3.24. Every indiscrete $\mathcal{DPNT}$ space on $\hat{\dot{\mathcal{B}\mathcal{P}}}$ is $\mathfrak{C}_5$-connected.

Proposition 3.25. *Let $(\hat{\dot{\mathcal{B}\mathcal{P}}}_1, \varkappa_1)$ and $(\hat{\dot{\mathcal{B}\mathcal{P}}}_2, \varkappa_2)$ be two $\mathcal{DPNTS}'\int$ and* f : $(\hat{\dot{\mathcal{B}\mathcal{P}}}_1, \varkappa_1)$ *and $(\hat{\dot{\mathcal{B}\mathcal{P}}}_2, \varkappa_2)$ be a surjective digital pentapartitioned neutrosophic continuous mapping. If $(\hat{\dot{\mathcal{B}\mathcal{P}}}_1, \varkappa_1)$ is a digital pentapartitioned neutrosophic $\mathfrak{C}_5$-connected space, then $(\hat{\dot{\mathcal{B}\mathcal{P}}}_2, \varkappa_2)$ is also a digital pentapartitioned neutrosophic $\mathfrak{C}_5$-connected space.*

Proof. Suppose on contrary that $(\hat{\dot{\mathcal{B}\mathcal{P}}}_2, \varkappa_2)$ is a digital pentapartitioned neutrosophic $\mathfrak{C}_5$-disconnected space. Then, by Definition 3.23, there exist both $\mathcal{DPNOS}$ and $\mathcal{DPNCS}$ $\mathcal{J}$ be such that $\mathcal{J} \neq 1_{D\mathring{P}SN}$ and $J \neq 0_{D\mathring{P}SN}$. Since f is a digital pentapartitioned continuous and onto function, so $\mathrm{f}^{-1}(\mathcal{J}) = 1_{D\mathring{P}NS}$ or $\mathrm{f}^{-1}(\mathcal{J}) = 0_{D\mathring{P}SN}$ where $\mathrm{f}^{-1}(\mathcal{J})$ is both $\mathcal{DPNOS}$ and $\mathcal{DPNCS}$. Therefore, $\mathcal{J} = \mathrm{f}(\mathrm{f}^{-1}(\mathcal{J})) = \mathrm{f}(1_{D\mathring{P}NS}) = 1_{D\mathring{P}NS}$ and $\mathcal{J} = \mathrm{f}(\mathrm{f}^{-1}(\mathcal{J})) = \mathrm{f}(0_{D\mathring{P}SN}) = 0_{D\mathring{P}SN}$ contradiction. Hence, $(\hat{\dot{\mathcal{B}\mathcal{P}}}_2, \varkappa_2)$ is a digital pentapartitioned neutrosophic $\mathfrak{C}_5$-connected space. □

Corollary 3.26. *Let $\varkappa$ be a $\mathcal{DPNT}$ on $\hat{\dot{\mathcal{B}\mathcal{P}}}$. Then, $(\hat{\dot{\mathcal{B}\mathcal{P}}}, \varkappa)$ is called a digital pentapartitioned neutrosophic $\mathfrak{C}_5$-connected space if and only if there does not exist a digital pentapartitioned neutrosophic continuous map* $\mathrm{f} : (\hat{\dot{\mathcal{B}\mathcal{P}}}, \varkappa) \to (\mathcal{S}_T, \varkappa_T)$ *such that* $\mathrm{f} \neq 0_{D\mathring{P}SN}$ *and* $\mathrm{f} \neq 1_{D\mathring{P}SN}$.

Definition 3.27. Let $\mathcal{P} = (\mathring{Tr}_{\mathcal{J}},\ \mathring{Co}_{\mathcal{J}}, \mathring{Ig}_{\mathcal{J}}, \mathring{Un}_{\mathcal{J}},\ \mathring{Fa}_{\mathcal{J}})$ be a $\mathcal{DPNS}$ in $\hat{\dot{\mathcal{B}\mathcal{P}}}$. Let $\varkappa$ be a $\mathcal{DPNT}$ on $\hat{\dot{\mathcal{B}\mathcal{P}}}$. The interior and closure of $\mathcal{P}$ in $\hat{\dot{\mathcal{B}\mathcal{P}}}$ is defined as:
$\mathcal{P}^{Int}$: The union of $\mathcal{DPNOS}'s$ which contained in $\mathcal{P}$.
$\mathcal{P}^{Clo}$: The intersection of $\mathcal{DPNCS}'s$ for which $\mathcal{P}$ is a subset of these $\mathcal{DPNCS}'s$.

Remark 3.28. Being union of $\mathcal{DPNOS}$ $\mathcal{P}^{Int}$ is a $\mathcal{DPNO}$ and $\mathcal{P}^{Clo}$ being intersection of $\mathcal{DPNCS}$ is $\mathcal{DPNC}$.

Theorem 3.29. *Let $\mathcal{P}$ be a $\mathcal{DPNS}$ in a $\mathcal{DPNTS}$ $(\hat{\dot{\mathcal{B}\mathcal{P}}}, \varkappa)$. Then, P^{Int} is such an open set which is the largest open set of $\hat{\dot{\mathcal{B}\mathcal{P}}}$ contained in P.*

Corollary 3.30. *$\mathcal{P} = \{\mathring{Tr}_{\mathcal{J}},\ \mathring{Co}_{\mathcal{J}}, \mathring{Ig}_{\mathcal{J}}, \mathring{Un}_{\mathcal{J}}, \mathring{Fa}_{\mathcal{J}}\}$ is a $\mathcal{DPNOS}$ in $\hat{\dot{\mathcal{B}\mathcal{P}}}$ if and only if $\mathcal{P}^{Int} = \mathcal{P}$ and $\mathcal{P} = \{\mathring{Tr}_{\mathcal{J}},\ \mathring{Co}_{\mathcal{J}}, \mathring{Ig}_{\mathcal{J}}, \mathring{Un}_{\mathcal{J}}, \mathring{Fa}_{\mathcal{J}}\}$ is a $\mathcal{DPNCS}$in $\hat{\dot{\mathcal{B}\mathcal{P}}}$ if and only if $\hat{\dot{\mathcal{B}\mathcal{P}}}^{Clo} = \mathcal{P}$.*

Proposition 3.31. *Let $\mathcal{P}$ be a $\mathcal{DPNS}$ in $\hat{\dot{\mathcal{B}\mathcal{P}}}$. Then, following results hold for $\mathcal{P}$:*

- $(1_{D\mathring{P}NS})^{Int} = 1_{D\mathring{P}NS}$.
- $(0_{D\mathring{P}NS})^{Clo} = 0_{D\mathring{P}NS}$.
- $\overline{(\mathcal{P})}^{Int} = \overline{(\mathcal{P})^{Clo}}$

- $\overline{(\mathcal{P})}^{Clo} = \overline{(\mathcal{P})^{Int}}$

Definition 3.32. Let $\mathcal{B}\hat{\mathring{\mathcal{P}}}$ be a $\mathcal{B}\hat{\mathring{\mathcal{P}}} - \mathring{A}$ and $\varkappa$ be a $\mathcal{DPNT}$ on $\mathcal{B}\hat{\mathring{\mathcal{P}}}$. A $\mathcal{DPNOSP}$ in $\mathcal{B}\hat{\mathring{\mathcal{P}}}$ is said to be digital pentapartitioned neutrosophic regular open if $\mathcal{P} = (\mathcal{P}^{Clo})^{Int}$

Remark 3.33. Every $\mathcal{DPNOS}$ which is regular is $\mathcal{DPNO}$ and every $\mathcal{DPN}$ closed and open set is a digital pentapartitioned neutrosophic regular open.

Definition 3.34. A $\mathcal{DPN}$ super connected $\mathcal{B}\hat{\mathring{\mathcal{P}}}-\mathring{A}$ algebra is such a $\mathcal{B}\hat{\mathring{\mathcal{P}}}-\mathring{A}$ in which there does not exist a $\mathcal{DPN}$ regular open set $\mathcal{P} = \{\mathring{T}r_{\mathcal{J}},\ \mathring{C}o_{\mathcal{J}}, \mathring{I}g_{\mathcal{J}}, \mathring{U}n_{\mathcal{J}}, \mathring{F}a_{\mathcal{J}}\}$ such that $\mathcal{P} \neq 0_{D\mathring{P}NS}$ and $\mathcal{P} \neq 1_{D\mathring{P}NS}$. If there exists such a digital pentapartitioned neutrosophic regular open set $\mathcal{P} = \{\mathring{T}r_{\mathcal{J}},\ \mathring{C}o_{\mathcal{J}}, \mathring{I}g_{\mathcal{J}}, \mathring{U}n_{\mathcal{J}}, \mathring{F}a_{\mathcal{J}}\}$ such that $\mathcal{P} \neq 0_{D\mathring{P}NS}$ and $\mathcal{P} \neq 1_{D\mathring{P}NS}$, then $\mathcal{B}\hat{\mathring{\mathcal{P}}} - \mathring{A}$ is said to be a digital pentapartitioned neutrosophic super disconnected.

Example 3.35. Let $\mathcal{B}\hat{\mathring{\mathcal{P}}} = \{\mathring{\mathcal{H}},^*, \mathring{0}\}$ be a $\mathcal{B}\hat{\mathring{\mathcal{P}}} - \mathring{A}$, where $\mathring{\mathcal{H}} = \{0, \mathfrak{a}, \mathfrak{a}^2, \mathfrak{a}^3, \mathfrak{a}^4, \mathfrak{a}^5, \mathfrak{a}^6, \mathfrak{a}^7, \mathfrak{a}^8\}$ is the cyclic group of order 9 and Caley's table for $*$ is define a $\mathcal{DPNS}$ as:
Let Universal Set $\mathbb{U} = \{1,2\} \times \{1,2\}$ in the digital plane $\hat{\mathring{\mathcal{D}}}^2$
Consider all subsets $\mathring{B} = \{(1,1),(1,2),(2,1),(2,2)\}$ as an rectangular arrays of integer coordinate points of $\mathring{\mathbb{Z}}^2$
Therefore $(\mathring{\mathbb{Z}}^2, \hat{\mathring{\mathcal{D}}}^2)$ is the digital topological space.
Let $\mathring{A} = \{\mathfrak{P}_1 < 0.2, 0.2, 0.6, 0.7, 0.8 >, 2 < 0.8, 0.6, 0.3, 0.2, 0.1 >, 3 < 0.1, 0.1, 0.5, 0.7, 0.8 >, 4 < 0.7, 0.7, 0.4, 0.2, 0.1 >\}$
$\mathcal{P} = \{(0, 0.2, 0.2, 0.6, 0.7, 0.8), (\mathring{u}, 0.8, 0.6, 0.3, 0.2, 0.1)\}$,
Let $\varkappa_{\mathcal{B}\hat{\mathring{\mathcal{P}}}} = \{0_{D\mathring{P}NS}, 1_{D\mathring{P}NS}, \mathcal{P}\}$ be a $\mathcal{DPNT}$ on $\mathcal{B}\hat{\mathring{\mathcal{P}}}$
Here
$\mathcal{DPNOS}'s$: $0_{D\mathring{P}NS} = \{0,0,1,1,1\}$, $1_{D\mathring{P}NS} = \{1,1,0,0,0\}$, $\mathcal{P} = (0, 0.2, 0.2, 0.6, 0.7, 0.8),\ (\mathring{u}, 0.8, 0.6, 0.3, 0.2, 0.1)$
$\mathcal{DPNCS}'s$:$(0_{D\mathring{P}NS})^C = (0,0,1,1,1)^C = (\{1,1,0,0,0\}) = 1_{D\mathring{P}NS}$, $(1_{\mathcal{DPNS}})^C = (1,1,0,0,0)^C = (\{0,0,1,1,1\}) = 0_{\mathcal{DPNS}}$,
$(\mathcal{P})^C = ((0, 0.2, 0.2, 0.6, 0.7, 0.8),\ (\mathring{u}, 0.8, 0.6, 0.3, 0.2, 0.1))^C = \{(0, 0.8, 0.7, 0.4, 0.2, 0.2),\ (\mathring{u}, 0.1, 0.2, 0.7, 0.6, 0.8)\} = (\mathcal{P})'$ (Say)
Then, closure of $\mathcal{Q}$ is the intersection of closed sets which contain $\mathcal{Q}$. Therefore,
$(\mathcal{P})' = ((\mathcal{Q})^{Clo}$
Now, interior of $\mathcal{Q}$ is the union of open sets which contain in $\mathcal{Q}$. Therefore,

$$0_{D\mathring{P}NS} \cup \mathcal{P} = \mathcal{P}$$

$\mathcal{P} = (\mathcal{Q})^{Int}$

Note that $((\mathcal{Q})^{Clo})^{Clo} = (\mathcal{Q})^{Clo}$. Now, if we consider a $\mathcal{DPNS}$ in a $\mathcal{P} = (0, 0.2, 0.2, 0.6, 0.7, 0.8), (\mathring{u}, 0.8, 0.6, 0.3, 0.2, 0.1)$ in a BP-algebra $\hat{\dot{\mathcal{BP}}}$ and if $\varkappa_{\hat{\dot{\mathcal{BP}}}} = \{0_{DP\mathring{N}S}, 1_{DP\mathring{N}S}, \mathcal{P}\}$ is a $\mathcal{DPNT}$ on $\hat{\dot{\mathcal{BP}}}$.
Then,$(\mathcal{P})^{Clo} = \mathcal{P}$ and $(\mathcal{P})^{Int} = \mathcal{P}$.
Consequently, $\mathcal{P} = (\mathcal{P}^{Clo})^{Int}$,
which shows that $\mathcal{P}$ is a $\mathcal{DPN}$ regular open set in BP-algebra $\hat{\dot{\mathcal{BP}}}$. Since $\mathcal{P}$ is a $\mathcal{DPN}$ regular open set in $\hat{\dot{\mathcal{BP}}}$ and $\mathcal{P} \neq 0_{\mathcal{DP\mathring{N}S}}$,
$\mathcal{P} \neq 1_{\mathcal{DP\mathring{N}S}}$ then, by Definition, $\hat{\dot{\mathcal{BP}}}$algebra $\hat{\dot{\mathcal{BP}}}$ is a digital pentapartitioned neutrosophic supper disconnected BP-algebra.

Proposition 3.36. *Let $\hat{\dot{\mathcal{BP}}}$ be a $\hat{\dot{\mathcal{BP}}} - \mathring{A}$ and let $\mathcal{P}$ be a $\mathcal{DPNOS}$. Then, the following statements are equivalent:*

(i) A $\hat{\dot{\mathcal{BP}}} - \mathring{A}$ is digital pentapartitioned neutrosophic super connected.
(ii) $(\mathcal{P})^{Clo} = 1_{DP\mathring{N}S}$, for each $\mathcal{DPNOS}$ $\mathcal{P} \neq 0_{DP\mathring{N}S}$.
(iii) $(\mathcal{P})^{Int} = 0_{DP\mathring{N}S}$, for each $\mathcal{DPNOS}$ $\mathcal{P} \neq 1_{DP\mathring{N}S}$.
(iv) There do not exist $\mathcal{DPNOS}'\int$ $\mathcal{P}, \mathcal{S}$ such that $\mathcal{P} \subseteq \mathcal{S}$ and $\mathcal{P} \neq 0_{DP\mathring{N}S} \neq \mathcal{S}$ in $\hat{\dot{\mathcal{BP}}} - \mathring{A}$-algebra $\hat{\dot{\mathcal{BP}}} - \mathring{A}$.

Definition 3.37. Let $(\hat{\dot{\mathcal{BP}}} - \mathring{A}, \varkappa)$ be a $\mathcal{DPNTS}$, where $\hat{\dot{\mathcal{BP}}} - \mathring{A}$ is a $\hat{\dot{\mathcal{BP}}}$-algebra. Let $\mathfrak{U}$ be a collection of $\mathcal{DPNOS}$'s in $\hat{\dot{\mathcal{BP}}} - \mathring{A}$ denoted by $\mathcal{S} = \{(\mathring{T}r_{\mathcal{P}_{\mathcal{J}}}, \mathring{C}o_{\mathcal{P}_{\mathcal{J}}}, \mathring{I}g_{\mathcal{P}_{\mathcal{J}}}, \mathring{U}n_{\mathcal{P}_{\mathcal{J}}}, \mathring{F}a_{\mathcal{P}_{\mathcal{J}}}) : j \in \mathcal{J}\}$. Let $\mathcal{P}$ be a $\mathcal{DPNOS}$ in $\hat{\dot{\mathcal{BP}}} - \mathring{A}$. Then, $\mathfrak{U}$ is called a digital pentapartitioned neutrosophic open covering of $\mathcal{P}$ if $\mathcal{P} \subseteq \cup\mathfrak{U}$.

Definition 3.38. Let $\hat{\dot{\mathcal{BP}}}$ be a BP-algebra and $(\hat{\dot{\mathcal{BP}}}, \varkappa)$ be a $\mathcal{DPNTS}$. Let $\mathfrak{D}$ be a finite sub-collection of $\mathfrak{U}$. If $\mathfrak{D}$ is also a digital pentapartitioned neutrosophic open covering of $\mathcal{P}$, then it is called a finite sub-covering of $\mathfrak{U}$ and $\mathcal{P}$ is called digital pentapartitioned neutrosophic compact if each digital pentapartitioned neutrosophic open covering $\mathfrak{U}$ of $\mathcal{P}$ has a finite sub-cover. Then, $(\hat{\dot{\mathcal{BP}}}, \varkappa)$ is called compact BP-algebra.

Remark 3.39. If either $\hat{\dot{\mathcal{BP}}}$ is a finite BP-algebra or $\varkappa$ is a finite topology on $\hat{\dot{\mathcal{BP}}}$, i.e., consists of finite number of digital pentapartitioned neutrosophic subsets of $\hat{\dot{\mathcal{BP}}}$, then the $\mathcal{DPNT}$ $(\hat{\dot{\mathcal{BP}}}, \varkappa)$ is a digital pentapartitioned neutrosophic compact topological space.

Proposition 3.40. *Let $(\hat{\dot{\mathcal{BP}}}_1, \varkappa_1)$ and $(\hat{\dot{\mathcal{BP}}}_2, \varkappa_2)$ be two $\mathcal{DPNTS}$'s and f be a digital pentapartitioned neutrosophic continuous mapping from $(\hat{\dot{\mathcal{BP}}}_1)$ into $(\hat{\dot{\mathcal{BP}}}_2)$. Let $\mathcal{P}$ be a $\mathcal{DPNS}$ in $(\hat{\dot{\mathcal{BP}}}_1, \varkappa_1)$. If $\mathcal{P}$ is a digital pentapartitioned neutrosophic compact in $(\hat{\dot{\mathcal{BP}}}_1, \varkappa_1)$, then f$(\mathcal{P})$ is digital pentapartitioned neutrosophic compact in $(\hat{\dot{\mathcal{BP}}}_2, \varkappa_2)$.*

Proof. Let f : $(\hat{\dot{\mathcal{BP}}}_1, \varkappa_1) \to (\hat{\dot{\mathcal{BP}}}_2, \varkappa_2)$ be a digital pentapartitioned neutrosophic continuous function. Let $\mathfrak{U} = (\mathrm{f}^{-1}(\mathcal{P}_j : j \in \mathcal{J})$ be a digital pentapartitioned neutrosophic open covering of $\mathcal{P}$ since $\mathcal{P}$ be a $\mathcal{DPNS}$ in $(\hat{\dot{\mathcal{BP}}}_1, \varkappa_1)$. Let $\mathfrak{D} = (\mathcal{P}_j : j \in \mathcal{J})$ be a digital pentapartitioned neutrosophic open covering of $\mathrm{f}(\mathcal{P})$. Since $\mathcal{P}$ is compact, then there exists a digital pentapartitioned neutrosophic finite sub-cover $\bigcup_{j=1}^{n} \mathrm{f}^{-1}(\mathcal{P}_j)$ such that

$\mathcal{P} = \bigcup_{j=1}^{n} \mathrm{f}^{-1}(\mathcal{P}_j)$

We have to prove that there also exists a finite sub-cover of $\mathfrak{D}$ for $\mathrm{f}(\mathcal{P})$ such that

$\mathrm{f}(\mathcal{P}) \subseteq \bigcup_{j=1}^{n}(\mathcal{P}_j)$

Now,

$\mathcal{P} \subseteq \bigcup_{j=1}^{n} \mathrm{f}^{-1}(\mathcal{P}_j)$

$\mathrm{f}(\mathcal{P}) \subseteq \mathrm{f}(\bigcup_{j=1}^{n} \mathrm{f}^{-1}(\mathcal{P}_j))$

$\mathrm{f}(\mathcal{P}) \subseteq \bigcup_{j=1}^{n}(\mathrm{f}(\mathrm{f}^{-1}(\mathcal{P}_j)))$

$\mathrm{f}(\mathcal{P}) \subseteq \bigcup_{j=1}^{n}(\mathcal{P}_j)$

Hence, $\mathrm{f}(\mathcal{P})$ is a digital pentapartitioned neutrosophic compact in $(\hat{\dot{\mathcal{BP}}}_2, \varkappa_2)$. $\square$

Definition 3.41. A digital pentapartitioned neutrosophic set $\mathcal{P}$ in a -algebra $\hat{\dot{\mathcal{BP}}}$ is called a digital pentapartitioned neutrosophic point if

$$\mathring{Tr}_{\mathcal{P}}(t) = \begin{cases} \lambda \in (0,1], & if\ \mathfrak{t} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{Co}_{\mathcal{P}}(t) = \begin{cases} \mu \in (0,1], & if\ \mathfrak{t} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{Ig}_{\mathcal{P}}(t) = \begin{cases} \nu \in (0,1], & if\ \mathfrak{t} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{Un}_{\mathcal{P}}(t) = \begin{cases} \xi \in (0,1], & if\ \mathfrak{t} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{Fa}_{\mathcal{P}}(t) = \begin{cases} o \in (0,1], & if\ \mathfrak{t} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

with support $\mathring{u}$ and value $(\lambda, \mu, \nu, \xi, o)$, denoted by $\mathfrak{s}(\lambda, \mu, \nu, \xi, o)$. This digital pentapartitioned point is said to "belong to" a $\mathcal{DPNS}$ $\mathcal{P}$, written as $\mathfrak{s}(\lambda, \mu, \nu, \xi, o) \in \mathcal{P}$ if $\mathring{Tr}_{\mathcal{P}}(s) \geq \lambda, \mathring{Co}_{\mathcal{P}}(s) \geq \mu, \mathring{Ig}_{\mathcal{P}}(s) \leq \nu, \mathring{Un}_{\mathcal{P}}(s) \leq \xi, \mathring{Tr}_{\mathcal{P}}(s) \leq o$, and said to be "quasi-coincident with" a $\mathcal{DPNS}$ $\mathcal{P}$, written as $\mathfrak{s}(\lambda, \mu, \nu, \xi, o)\mathcal{P}\ if\ \mathring{Tr}_{\mathcal{P}}\mathfrak{s} + \lambda > 1, \mathring{Co}_{\mathcal{P}}\mathfrak{s} + \mu > 1, \mathring{Ig}_{\mathcal{P}}\mathfrak{s} + \nu < 1, \mathring{Un}_{\mathcal{P}}\mathfrak{s} + \xi < 1, \mathring{Fa}_{\mathcal{P}}\mathfrak{s} + o < 1$.

Definition 3.42. Let $\hat{\dot{\mathcal{BP}}}$ be a $\hat{\dot{\mathcal{BP}}}$-algebra and let $(\hat{\dot{\mathcal{BP}}}, \varkappa)$ be a $\mathcal{DPNTS}$. Then, $(\hat{\dot{\mathcal{BP}}}, \varkappa)$ is called a digital pentapartitioned neutrosophic Hausdorff space if and only if, for any two distinct digital pentapartitioned neutrosophic points $\mathfrak{s}_1, \mathfrak{s}_1 \in \hat{\dot{\mathcal{BP}}}$, there exist $\mathcal{DPNOS}'s$ $\mathfrak{Q}_1 = (\mathring{Tr}_{\mathfrak{Q}_1}, \mathring{Co}_{\mathfrak{Q}_1}, \mathring{Ig}_{\mathfrak{Q}_1}, \mathring{Un}_{\mathfrak{Q}_1}, \mathring{Fa}_{\mathfrak{Q}_1}), \mathfrak{Q}_2 = (\mathring{Tr}_{\mathfrak{Q}_2}, \mathring{Co}_{\mathfrak{Q}_2}, \mathring{Ig}_{\mathfrak{Q}_2}, \mathring{Un}_{\mathfrak{Q}_2}, \mathring{Fa}_{\mathfrak{Q}_2})$ such that $\mathfrak{s}_1 \in \mathfrak{Q}_1, \int_{\in} \in \mathfrak{Q}_2$ i.e. $\mathring{Tr}_{\mathfrak{Q}_1}(\mathfrak{s}_1) = 1, \mathring{Co}_{\mathfrak{Q}_1}(\mathfrak{s}_1) = 1, \mathring{Ig}_{\mathfrak{Q}_1}(\mathfrak{s}_1) = (\mathfrak{Un})_{\mathfrak{Q}_1}(\mathfrak{s}_1) = \mathring{Fa}_{\mathfrak{Q}_1}(\mathfrak{s}_1) = 0,$

$\mathring{T}r_{\mathfrak{Q}_2}(\mathfrak{s}_1) = 1, \mathring{C}o_{\mathfrak{Q}_2}(\mathfrak{s}_1) = 1, \mathring{I}g_{\mathfrak{Q}_2}(\mathfrak{s}_1) = \mathring{U}n_{\mathfrak{Q}_2}(\mathfrak{s}_1) = \mathring{F}a_{\mathfrak{Q}_2}(\mathfrak{s}_1) = 0$, and satisfy the condition that $\mathfrak{Q}_1 \cap \mathfrak{Q}_2 = 0_{D\mathring{P}NS}$. Then, $(\hat{\mathring{\mathcal{BP}}}, \varkappa)$ is called digital pentapartitioned neutrosophic Hausdorff space and $\hat{\mathring{\mathcal{BP}}}$-algebra is said to ba a Hausdorff BP-algebra. In fact, $(\hat{\mathring{\mathcal{BP}}}, \varkappa)$ is a Hausdorff $\hat{\mathring{\mathcal{BP}}}$-algebra.

Example 3.43. Let $\hat{\mathring{\mathcal{BP}}} = (\mathring{\mathcal{H}}, ^*, 0)$ be a $\hat{\mathring{\mathcal{BP}}}$-algebra and let $(\hat{\mathring{\mathcal{BP}}}, \varkappa_{\hat{\mathring{\mathcal{BP}}}})$ be a $\mathcal{DPNTS}$ on $\hat{\mathring{\mathcal{BP}}}$, where $\mathring{\mathcal{H}} = \{0, \mathfrak{a}, \mathfrak{a}^2, \mathfrak{a}^3, \mathfrak{a}^4, \mathfrak{a}^5, \mathfrak{a}^6, \mathfrak{a}^7, \mathfrak{a}^8\}$ is the cyclic group of order 9 and Caley's table for * is given in example.

Let Universal Set $\mathbb{U} = \{\mathring{1}, \mathring{2}\} \times \{\mathring{1}, \mathring{2}\}$ in the digital plane $\hat{\mathring{\mathcal{D}}}^2$.

Consider all subsets $\mathring{B} = \{(\mathring{1}, \mathring{1}), (\mathring{1}, \mathring{2}), (\mathring{2}, \mathring{1}), (\mathring{2}, \mathring{2})\}$ as an rectangular arrays of integer coordinate points of $\hat{\mathring{\mathcal{D}}}^2$.

Therefore $(\mathring{\mathbb{Z}}^2, \hat{\mathring{\mathcal{D}}}^2)$ is the digital topological space.

Let $\mathring{A} = \{\mathfrak{P}_1 < 0.3, 0.2, 0.5, 0.7, 0.8 >, \mathfrak{P}_2 < 0.7, 0.5, 0.3, 0.2, 0.1 >, \mathfrak{P}_3 < 0.1, 0.2, 0.5, 0.6, 0.7 >, \mathfrak{P}_4 < 0.7, 0.6, 0.5, 0.1, 0.1 >\}$

We define two $\mathcal{DPNS}'\int$ as $\mathcal{P} = \{(\mathfrak{e}, 1, 1, 0), (\mathfrak{u}, 0, 0, 1)\}.\mathcal{Q} = \{(\mathfrak{e}, 0, 0, 1), (\mathfrak{u}, 1, 1, 0)\}$. Consider a digital pentapartitioned neutrosophic point for $\mathfrak{e} \in (\hat{\mathring{\mathcal{BP}}})$ such that,

$$\mathring{T}r_{\mathcal{P}}(\mathfrak{e}) = \begin{cases} 0.3, & if\ \mathfrak{e} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{C}o_{\mathcal{P}}(\mathfrak{e}) = \begin{cases} 0.2, & if\ \mathfrak{e} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{I}g_{\mathcal{P}}(\mathfrak{e}) = \begin{cases} 0.5, & if\ \mathfrak{e} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{U}n_{\mathcal{P}}(\mathfrak{e}) = \begin{cases} 0.7, & if\ \mathfrak{e} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{F}a_{\mathcal{P}}(\mathfrak{e}) = \begin{cases} 0.8, & if\ \mathfrak{e} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

Then, $\mathfrak{e}(0.3, 0.2, 0.5, 0.7, 0.8)$ is a digital pentapartitioned neutrosophic point with support $\mathfrak{e}$ and value $(0.3, 0.2, 0.5, 0.7, 0.8)$. This digital pentapartitioned neutrosophic point belongs to $\mathcal{DPNS}\ \mathcal{P}$ but not $\mathcal{DPNS}\ \mathcal{Q}$. Now, for all $\mathring{u} \neq \mathfrak{e} \in (\hat{\mathring{\mathcal{BP}}})$

$$\mathring{T}r_{\mathcal{Q}}(\mathring{u}) = \begin{cases} 0.7, & if\ \mathring{u} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{C}o_{\mathcal{Q}}(\mathring{u}) = \begin{cases} 0.5, & if\ \mathring{u} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{I}g_{\mathcal{Q}}(\mathring{u}) = \begin{cases} 0.3, & if\ \mathring{u} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{Un}_{\mathcal{Q}}(\mathring{u}) = \begin{cases} 0.2, & if\ \mathring{u} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

$$\mathring{Fa}_{\mathcal{Q}}(\mathring{u}) = \begin{cases} 0.1, & if\ \mathring{u} = \mathfrak{s} \\ 0, & otherwise \end{cases}$$

Then,$\mathring{u}(0.7, 0.5, 0.3, 0.2, 0.1)$ is a digital pentapartitioned neutrosophic point with support $\mathring{u}$ and the value $(0.7, 0.5, 0.3, 0.2, 0.1)$.

This digital pentapartitioned neutrosophic point belongs to DPNS $\mathcal{Q}$ but not $\mathcal{DPNS}\ \mathcal{P}$. Thus, $\mathfrak{e}(0.3, 0.2, 0.5, 0.7, 0.8) \in \mathcal{P}$ and $\mathfrak{e}(0.3, 0.2, 0.5, 0.7, 0.8) \notin \mathcal{Q}$, $\mathring{u}(0.7, 0.5, 0.3, 0.2, 0.1) \in \mathcal{Q}$ and $\mathring{u}(0.7, 0.5, 0.3, 0.2, 0.1) \notin \mathcal{P}$ and $\mathcal{P} \cap \mathcal{Q} = 0_{D\mathring{P}NS}$. Thus, $\hat{\dot{\mathcal{BP}}}$-algebra is a Hausdorff $\hat{\dot{\mathcal{BP}}}$-algebra and $(\hat{\dot{\mathcal{BP}}}, \varkappa_{\hat{\dot{\mathcal{BP}}}})$ is a Hausdorff topological space.

Theorem 3.44. *Let $(\hat{\dot{\mathcal{BP}}}_1, \varkappa_1)$ and $(\hat{\dot{\mathcal{BP}}}_2, \varkappa_2)$ be two $\mathcal{DPNTS}$'s. Let* f *be a digital pentapartitioned neutrosophic homomorphism from $(\hat{\dot{\mathcal{BP}}}_1, \varkappa_1)$ into $(\hat{\dot{\mathcal{BP}}}_2, \varkappa_2)$. Then, $(\hat{\dot{\mathcal{BP}}}_1, \varkappa_1)$ is a digital pentapartitioned neutrosophic Hausdorff space if and only if $(\hat{\dot{\mathcal{BP}}}_2, \varkappa_2)$ is a digital pentapartitioned neutrosophic Hausdorff $\hat{\dot{\mathcal{BP}}}$-algebra.*

Proof. Let $(\hat{\dot{\mathcal{BP}}}_1, \varkappa_1), (\hat{\dot{\mathcal{BP}}}_2, \varkappa_2)$ be two DPNTSs. Let $(\hat{\dot{\mathcal{BP}}}_1)$ be a digital pentapartitioned neutrosophic Hausdorff space, then, according to the Definition, there exist two DPNOS's W and X for two distinct digital pentapartitioned neutrosophic points $\mathfrak{s}_1, \mathfrak{s}_1 \in \varkappa_2$ also $\mathfrak{p}, \mathfrak{q} \in \hat{\dot{\mathcal{BP}}}_1 (\mathfrak{p} \neq \mathfrak{q})$ such that $\mathring{W} \cap \mathring{X} = 0_{D\mathring{P}NS}$.
Now, for $\mathfrak{y} \in \hat{\dot{\mathcal{BP}}}_1$, consider $(\mathrm{f}^{-1}(\mathfrak{s}_1)(\mathfrak{y}) = \mathfrak{s}_1(f^{-1}(\mathfrak{y}))$, where $\mathfrak{s}_1(\mathrm{f}^{-1}(\mathfrak{y}) = \mathfrak{s} \in (0, 1]$ if $\mathfrak{y} = \mathrm{f}^{-1}(\mathfrak{p})$, otherwise 0. That is, $\mathrm{f}^{-1}(\mathfrak{s}_1)(\mathfrak{y}) = ((\mathrm{f}^{-1}(\mathfrak{s}))_1(\mathfrak{y}))$. Therefore, we have $\mathrm{f}^{-1}(\mathfrak{s}_1) = ((\mathrm{f}^{-1}(\mathfrak{s}))_1$. Similarly, $\mathrm{f}^{-1}(\mathfrak{s}_2) = ((\mathrm{f}^{-1}(\mathfrak{s}))_2$. Now, since f^{-1} is a digital pentapartitioned neutrosophic continuous mapping from $\hat{\dot{\mathcal{BP}}}_2$ into $\hat{\dot{\mathcal{BP}}}_1$, there exist two $\mathcal{DPNOS}$'s $\mathrm{f}(\mathring{W})$ and $\mathrm{f}(\mathring{X})$ of $\mathfrak{s}_1$ and $\mathfrak{s}_1$ respectively, such that $\mathrm{f}(\mathring{W}) \cap \mathrm{f}(\mathring{X}) = \mathrm{f}(0_{D\mathring{P}NS})$.This implies that $\hat{\dot{\mathcal{BP}}}_2$ is a digital pentapartitioned neutrosophic Hausdorff BP-algebra. The converse part can be proved similarly. □

Theorem 3.45. *Let* f *be a digital pentapartitioned neutrosophic continuous function which is both one-one and onto, where* f *is a mapping from a digital pentapartitioned neutrosophic compact $\hat{\dot{\mathcal{BP}}}$-algebra $\hat{\dot{\mathcal{BP}}}_1$ into a digital pentapartitioned neutrosophic Hausdorff BP-algebra $\hat{\dot{\mathcal{BP}}}_2$. Then,* f *is a homomorphism.*

Proof. Let $\mathrm{f} : \hat{\dot{\mathcal{BP}}}_1 \to \hat{\dot{\mathcal{BP}}}_2$ be a neutrosophic continuous bijective function from digital pentapartitioned neutrosophic compact $\hat{\dot{\mathcal{BP}}}$-algebra $\hat{\dot{\mathcal{BP}}}_1$ into a digital pentapartitioned neutrosophic Hausdorff $\hat{\dot{\mathcal{BP}}}$-algebra $\hat{\dot{\mathcal{BP}}}_2$. Since f is a digital pentapartitioned neutrosophic continuous mapping from $\hat{\dot{\mathcal{BP}}}_1$ into $\hat{\dot{\mathcal{BP}}}_2$, f is a homomorphism. Since f is bijective, we only prove that f is digital pentapartitioned neutrosophic closed. Let $\mathfrak{E} = \{\mathring{Tr}_{\mathfrak{E}}, \mathring{Co}_{\mathfrak{E}}, \mathring{Ig}_{\mathfrak{E}}, \mathring{Un}_{\mathfrak{E}}, \mathring{Fa}_{\mathfrak{E}}\}$ be a digital pentapartitioned neutrosophic closed in $\hat{\dot{\mathcal{B}}}\mathcal{P}_1$. If $\mathfrak{E} = 0_{D\mathring{P}NS}$is a digital pentapartitioned neutrosophic closed in $\hat{\dot{\mathcal{B}}}\mathcal{P}_1$, then f($\mathfrak{E}$) $= 0_{D\mathring{P}NS}$ is digital pentapartitioned neutrosophic closed in $\hat{\dot{\mathcal{B}}}\mathcal{P}_2$. However, if $\mathfrak{E} \neq 0_{D\mathring{P}NS}$, then $\mathfrak{E}$ will be a digital pentapartitioned neutrosophic compact, being subset of a digital pentapartitioned neutrosophic compact BP-algebra. Then, f(E), being digital pentapartitioned neutrosophic continuous image of a digital pentapartitioned neutrosophic compact $\hat{\dot{\mathcal{B}}}\mathcal{P}$-algebra, is also digital pentapartitioned neutrosophic compact. Therefore, $\hat{\dot{\mathcal{B}}}\mathcal{P}_2$ is closed, which implies that mapping f is closed. Thus, f is a homomorphism. □

4. Conclusion

The concept of $\mathcal{DPN} - \hat{\dot{\mathcal{B}}}\mathcal{P} - \mathring{A}$ has been defined in this article. Since $\hat{\dot{\mathcal{B}}}\mathcal{P} - \mathring{A}$ a novel branch of logical algebra, has just emerged and is used in fuzzy sets,intuitionistic fuzzy sets, neutrosophic sets and pentapartitioned neutrosophic sets, we may now expand the concept to $\mathcal{DPN}\hat{\dot{\mathcal{B}}}\mathcal{P} - \mathring{A}$ on $\mathcal{DPN}$'s. We investigated a few of the findings and established $\mathcal{DPN} - \hat{\dot{\mathcal{B}}}\mathcal{P} - \mathring{A}$ on $\mathcal{DPN}'s$ in this study. Characteristics of the $\mathcal{DPNTS}$ of the inverse, Homomorphic image, Hausdorff Space ,Compactness and $\mathfrak{C}_5-$ Connectedness in $\mathcal{DPN} - \hat{\dot{\mathcal{B}}}\mathcal{P} - \mathring{A}$ were discussed in detail.

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