



Hyperbolic Geometry and Neutrosophic Trigonometry: A Unified Approach to Uncertainty Management in Geometric Spaces

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Abstract. This paper addresses the challenge of modeling uncertainty in hyperbolic geometry, where non-Euclidean properties, such as angle sums in triangles smaller than 180° , complicate traditional trigonometric calculations. The relevance of this topic lies in its application in fields such as cosmology, robotics, and signal processing, where curved spaces and imprecision are common. Although the literature has explored fuzzy geometry and trigonometry under uncertainty, it lacks a framework that integrates negative curvature with explicit indeterminacy. To overcome this gap, a methodology is proposed that combines hyperbolic geometry with neutrosophic trigonometry, using neutrosophic sets (T, I, F) to represent angles and calculate hyperbolic functions. The results show that this integration accurately captures uncertainty in hyperbolic triangles, offering trigonometric values that are robust against imprecise measurements. This approach not only enriches geometric theory by introducing a flexible model for non-Euclidean spaces, but also provides practical tools for applications such as navigation in curved environments and the analysis of noisy signals. Thus, the study advances mathematical knowledge and facilitates innovative solutions in interdisciplinary contexts.

Keywords: Hyperbolic Geometry, Neutrosophic Trigonometry, Uncertainty, Hyperbolic Triangles, Negative Curvature, Neutrosophic Sets, Interdisciplinary Applications.

1. Introduction

Hyperbolic geometry, a pillar of non-Euclidean mathematics, redefines our understanding of space by challenging traditional Euclidean notions, allowing infinite parallel lines through a point external to a line. This field, essential in areas such as general relativity and modeling complex networks, is crucial because of its ability to describe curved spaces with applications in cosmology and robotics [1], [2]. The uncertainty inherent in measurements in these spaces, where the sum of the angles of a triangle is less than 180° , poses significant challenges for accurate trigonometric calculations. In this context, neutrosophic, which models truth, indeterminacy, and falsity, emerges as a powerful tool to address these limitations [3]. This paper explores the integration of hyperbolic geometry with neutrosophic trigonometry, offering an innovative approach to handling imprecision in non-conventional geometric spaces.

Historically, Euclidean geometry dominated mathematical thinking for centuries, from Euclid's postulates in 300 BC to the rise of non-Euclidean geometries in the 19th century. Mathematicians such as Lobachevsky and Bolyai, by questioning the fifth postulate, opened the door to hyperbolic geometry, which describes spaces of negative curvature [4]. This breakthrough transformed disciplines such as theoretical physics, where relativistic spacetime adopts non-Euclidean properties [5]. In parallel, the need to model uncertainty in complex systems has driven approaches such as fuzzy logic and, more

recently, neutrosophic, which introduces a three-dimensional perspective to address ambiguities [3]. Today, applications of hyperbolic geometry range from neural network analysis to data visualization in hyperbolic graphs [6]. However, traditional methods for computing trigonometric functions in these spaces typically ignore uncertainty in angle and length measurements. This limitation hampers their applicability in real-life scenarios, where precise measurements are rare. For example, in cosmology, astronomical observations are often subject to significant errors, requiring robust approaches to model curved spaces [7].

The central problem addressed by this study lies in the lack of a unified framework that integrates hyperbolic geometry with tools capable of handling explicit uncertainty. How can trigonometric functions be modeled and calculated in hyperbolic triangles when angles and lengths are subject to inaccuracies? This question, still without a complete answer in the literature, motivates the present research, which seeks to merge the properties of hyperbolic spaces with the flexibility of neutrosophic.

The absence of previous approaches combining these disciplines highlights the originality of this work. While fuzzy geometry has addressed uncertainty in Euclidean contexts, it is poorly suited to the unique properties of hyperbolic spaces, such as negative curvature [3]. Likewise, classical trigonometric methods assume absolute accuracy, which is unsuitable for practical applications where data are inherently imprecise. This paper proposes a new approach that overcomes these limitations by integrating concepts from neutrosophic logic with hyperbolic computations. The relevance of this research extends beyond mathematical theory. In robotics, for example, navigation in non-Euclidean environments requires models that tolerate trajectory inaccuracies [2]. Similarly, in signal processing, hyperbolic representations of complex data need tools that handle noise and indeterminacy [6]. By addressing these needs, the study offers a significant contribution to both pure mathematics and its interdisciplinary applications.

The research question guides the development of the article: How can neutrosophic trigonometry model uncertainty in hyperbolic geometric computations to improve accuracy and applicability in real-life contexts? This question is addressed by formulating a framework that represents angles as neutrosophic sets (T, I, F) and adapts hyperbolic trigonometric functions to incorporate indeterminacy. The results promise to advance the understanding of curved spaces under uncertainty. The objectives of the study are clear: first, to develop a theoretical model that combines hyperbolic geometry with neutrosophic trigonometry to classify angles and compute trigonometric functions in hyperbolic triangles with uncertainty; second, to validate this model through numerical examples and comparisons with classical methods; and third, to explore practical applications in fields such as cosmology, robotics, and network analysis. These objectives are aligned with the research question and lay the groundwork for a rigorous and novel analysis.

2. Preliminaries

2.1. Hyperbolic Geometry.

Hyperbolic geometry represents one of the most profound conceptual revolutions in the history of mathematics. Born from the bold questioning of Euclid's fifth postulate, this non-Euclidean discipline describes spaces of constant negative curvature, where the rules of traditional geometric intuition vanish. In this universe, an infinite number of parallel lines can pass through a point outside a straight line, and the sum of the interior angles of a triangle is always less than 180 degrees. Understanding its fundamental principles is an indispensable prerequisite for addressing the problem of uncertainty in trigonometric calculations, since it is precisely in its counterintuitive properties that classical methods find their limits. This section lays the foundations of this geometry, detailing the concepts that will be subsequently extended through neutrosophic formalism.

2.1.1. Analysis and Evaluation of the Parallel Postulate

The parallel postulate, known as Euclid's fifth postulate, has been a fundamental pillar in the evolution of geometry, marking a turning point in the understanding of space. This statement, which states that exactly one parallel passes through a point outside a straight line, seems intuitive within the Euclidean framework, but its apparent simplicity sparked debates that transformed mathematics. The difficulty in proving it as a theorem led to questions about its absolute validity, opening the door to non-Euclidean geometries. This essay analyzes and assesses the relevance of the postulate, discussing its historical impact, its relationship to alternative geometries, and its influence on modern disciplines, from physics to computer science.

In the 3rd century BC, Euclid formulated his five postulates as the foundations of geometry, the fifth being the most controversial due to its non-obvious nature. While the first four postulates are straightforward, the parallel postulate requires assuming a global property of space that cannot be verified locally. Mathematicians such as Proclus and Saccheri attempted to prove it, but their efforts revealed contradictions when they assumed its negation. These explorations laid the groundwork for non-Euclidean geometries, which formally emerged in the 19th century with the work of Lobachevsky and Bolyai.

The inability to prove the postulate as a theorem led to a conceptual revolution. By denying the existence of a single parallel, hyperbolic geometry, where there are infinitely many parallels, and elliptic geometry, where there are none, emerged. These alternative systems not only challenged Euclidean intuition but also demonstrated that space could be conceived in radically different ways. Hyperbolic geometry, for example, describes surfaces of negative curvature, while elliptic geometry is associated with positive curvatures, such as the surface of a sphere. This geometric pluralism broadened our understanding of the universe.

The impact of the postulate transcends pure mathematics. In physics, Einstein's general relativity uses non-Euclidean geometries to model space-time curved by gravity [8]. Without the challenge of the fifth postulate, these theories would have been inconceivable. Similarly, in computer science, hyperbolic representations of data, such as in complex networks, leverage these geometries to visualize large-scale structures [9]. The postulate, when challenged, revealed the flexibility of mathematical space and its applicability to real-world problems.

However, the relevance of the postulate does not lie solely in its negation. In Euclidean contexts, it remains fundamental for practical applications, such as architecture and engineering, where accurate measurements in flat spaces are essential [10]. The simplicity of the postulate allows for the construction of reliable models for designing physical structures, from bridges to buildings. However, its limitation to flat spaces highlights the need for non-Euclidean approaches in more complex scenarios, such as spatial navigation or algorithm design.

Critically, the postulate exposes a tension between intuition and rigor. Its formulation assumes a spatial uniformity that does not always correspond to physical or mathematical reality. For example, on curved surfaces, such as those studied in topology, the concept of parallelism is redefined [11]. This ambiguity prompted the development of differential geometry, which generalizes the notions of curvature and distance. Thus, the postulate is not only an axiom, but a catalyst for new branches of knowledge.

Another aspect to be valued is its pedagogical role. In the teaching of mathematics, the parallel postulate introduces students to axiomatic thinking, showing how a basic assumption can support a logical system. At the same time, its history invites reflection on the nature of mathematical truth. The resistance to accepting its independence, demonstrated by Gödel and Cohen in broader contexts, underscores that axioms are choices, not absolute truths [12]. This lesson transcends geometry, influencing the philosophy of science.

On the other hand, the postulate poses epistemological challenges. Its apparent obviousness contrasts with the difficulty of validating it without additional assumptions. This dilemma reflects a

broader problem in mathematics: to what extent are the foundations we assume universal? Non-Euclidean geometries show that space can be modeled in multiple ways, each valid in its context. This plurality enriches the discipline, but also complicates the unification of theories in physics and other sciences.

In practical terms, the influence of the postulate extends to modern technologies. In robotics, navigation algorithms in non-Euclidean environments rely on hyperbolic models to optimize trajectories [9]. In cosmology, the curvature of the universe, which can be hyperbolic, elliptical, or planar, is studied by considering the implications of the postulate [8]. These applications demonstrate that the legacy of the fifth postulate is not only theoretical, but deeply practical, shaping our ability to interact with the world.

In conclusion, the parallel postulate is much more than a geometric axiom; it is a milestone that redefined mathematics and its applications. Its questioning gave rise to non-Euclidean geometries, transforming our understanding of space and its modeling. Its simplicity in Euclidean contexts remains invaluable, while its denial opened doors to innovations in physics, computer science, and beyond. By encouraging critical thinking and the exploration of alternatives, the postulate remains a symbol of mathematical creativity and its lasting impact on science.

2.1.2. Properties of hyperbolic triangles (sum of angles $< 180^\circ$).

The properties of hyperbolic triangles, especially the characteristic that the sum of their interior angles is less than 180° , represent a fundamental pillar of non-Euclidean geometry, challenging traditional notions of space. This peculiarity, known as the angle defect, not only distinguishes hyperbolic triangles from their Euclidean counterparts but also has profound implications for disciplines such as physics, computer science, and cosmology. Exploring this property reveals a mathematical universe where classical rules are transformed, opening new perspectives on the structure of space. This essay analyzes and assesses the importance of the angle sum less than 180° in hyperbolic triangles, highlighting its theoretical relevance and practical applications, while arguing its impact on the evolution of mathematical thought.

In Euclidean geometry, the sum of the angles of a triangle is always 180° , a rule that seems intuitive and universal. However, in hyperbolic geometry, developed by mathematicians such as Lobachevsky and Bolyai in the 19th century, this assumption breaks down. In a space of negative curvature, the sum of the angles $\alpha + \beta + \gamma$ is always less than 180° , and the difference, known as angular defect, is related to the area of the triangle by the formula $Defect = -K \cdot \text{Área}$, where $K < 0$ is curvature. This property redefines fundamental geometric relationships.

The angle defect is not a mere mathematical curiosity; it is a direct consequence of the negative curvature of hyperbolic space. In this context, lines diverge more rapidly than in the Euclidean plane, allowing triangles to have smaller angles. For example, in the Poincaré disk model, the "lines" are arcs that curve towards the edge of the disk, reducing the internal angles of the triangle. This feature makes hyperbolic triangles ideal for modeling phenomena in curved spaces, such as spacetime in general relativity [13].

The importance of this property extends to modern physics. In general relativity, spacetime can adopt a hyperbolic geometry in the presence of large masses, such as black holes [14]. Angle sums less than 180° allow the description of geodesic trajectories that deviate from Euclidean predictions, offering a more accurate representation of the universe. Furthermore, in cosmology, hyperbolic models of the universe suggest that the angular sum of large-scale triangles could reflect the global curvature of the cosmos [15, 44].

In computer science, hyperbolic triangles have practical applications in the visualization of complex data. Complex networks, such as social or biological ones, are efficiently represented in hyperbolic spaces due to their ability to capture hierarchical structures with less distortion [16, 45]. The property of angle sums being less than 180° facilitates the construction of these representations, since hyperbolic

triangles allow mapping relationships in curved spaces where distances grow exponentially. This capability is invaluable in areas such as artificial intelligence and big data analytics.

However, angle sums less than 180° pose challenges. In practical applications, such as robotics navigation, calculations based on hyperbolic geometries require algorithms that handle the uncertainty arising from imprecise measurements [17]. Negative curvature amplifies angle errors, requiring robust approaches, such as those based on fuzzy or neutrosophic logic, to ensure reliable results. This challenge highlights the need to integrate advanced mathematical tools to fully exploit the properties of hyperbolic triangles.

From a theoretical perspective, the property of angle defects challenges the very foundations of geometry. By breaking with Euclidean rigidity, hyperbolic triangles invite us to reconsider the nature of space and its representation. Differential geometry, which studies curved surfaces, draws on these ideas, using angle defects to derive general properties of hyperbolic manifolds [18]. This approach has transformed our understanding of topology, allowing us to model structures that go beyond Euclidean intuition.

Pedagogically, the angle sum less than 180° is a powerful tool for teaching critical thinking. By introducing students to non-Euclidean geometry, they are shown that mathematical truths are not universal, but rather depend on the chosen axiomatic system. This concept fosters intellectual flexibility and prepares students to tackle problems in disciplines where traditional assumptions do not apply. The history of the angle defect, from its discovery to its formalization, illustrates how questioning axioms can lead to revolutionary advances.

However, this property also poses limitations. In contexts requiring high accuracy, such as aerospace engineering, the dependence of the angular defect on the curvature and area of the triangle complicates calculations [17]. Euclidean methods, although limited to flat spaces, are usually simpler and more straightforward. Therefore, hyperbolic triangles are better suited for modeling large-scale phenomena or in theoretical settings, where negative curvature is relevant, than for applications where simplicity is a priority.

In short, the property of angle sums less than 180° in hyperbolic triangles is a milestone in modern geometry, with an impact that transcends pure mathematics. Its relevance in physics, computer science, and education underscores its value as a bridge between theory and practice. By challenging Euclidean notions, this property not only expands our understanding of space but also drives innovations in interdisciplinary fields. Its study continues to inspire new ways of thinking about the universe and its fundamental structures.

2.1.3. Hyperbolic trigonometric functions

Hyperbolic trigonometric functions are a fundamental cornerstone of the study of non-Euclidean geometry, as they extend the concept of classical trigonometric functions to contexts associated with the hyperbola rather than the circle. Their definition is based on exponential properties, which directly links them to advanced mathematical analysis and practical applications in physics, computer science, and cosmology [24].

The two basic functions are the hyperbolic sine and the hyperbolic cosine, defined as:

$$\sinh(x) = \frac{e^x - e^{-x}}{2}, \cosh(x) = \frac{e^x + e^{-x}}{2} \quad (1)$$

From these, other derived functions are constructed such as the tangent, the cotangent, the secant and the hyperbolic cosecant:

$$\frac{\sinh(x)}{\cosh(x)} = \tanh(x) \quad (2)$$

$$\frac{\cosh(x)}{\sinh(x)} = \coth(x) \quad (3)$$

$$\operatorname{sech}(x) = \frac{1}{\cosh(x)} \quad (4)$$

$$\operatorname{csch}(x) = \frac{1}{\sinh(x)} \quad (5)$$

The central identity that governs these functions is:

$$\cosh^2(x) - \sinh^2(x) = 1 \quad (6)$$

This result is interpreted as the hyperbolic equivalent of the Pythagorean theorem, since it describes the fundamental relationship that characterizes the hyperbola. This relationship, in addition to its theoretical interest, becomes a tool for modeling distances and trajectories in spaces of negative curvature [25].

In the field of calculus, these functions have derivatives that reflect their dual symmetry. Thus, the following holds:

$$\frac{d}{dx} \sinh(x) = \cosh(x), \frac{d}{dx} \cosh(x) = \sinh(x) \quad (7)$$

While for composite functions we have:

$$\frac{d}{dx} \tanh(x) = \operatorname{sech}^2 \quad (8)$$

$$\frac{d}{dx} \coth(x) = -\operatorname{csch}^2(x) \quad (9)$$

These properties allow hyperbolic functions to appear as natural solutions in linear and nonlinear differential equations, especially in problems related to wave propagation, damped oscillations and heat transfer models [26].

Another notable feature is the relationship between hyperbolic and circular trigonometric functions across complex numbers. Expressions such as:

$$\cos(ix) = \cosh(x), \sin(ix) = i \sinh(x) \quad (10)$$

This link confirms that hyperbolic trigonometry is not an isolated field, but integrates concepts of complex analysis and geometry, thus extending its usefulness to alternative physical theories and geometric models [27].

In teaching, these functions constitute an invaluable pedagogical resource for understanding the transition from plane to hyperbolic geometry. Showing that their origin is in the hyperbola and not in the circle helps students accept the flexibility of axiomatic systems and question the idea that mathematical truths are universal. Such reflection connects with the philosophy of mathematics, in which concepts evolve according to context [28].

In conclusion, hyperbolic trigonometric functions are more than an algebraic curiosity; they represent a bridge between mathematical theory, relativistic physics, and the modeling of complex systems. Their formulas, identities, and relationships with circular trigonometry demonstrate their central role in building a robust conceptual framework for interpreting phenomena where periodicity is insufficient and it is necessary to resort to structures based on the negative curvature of space.

2.2. Neutrosophic Trigonometry.

2.2.1. Definition of Neutrosophic Sets (T, I, F)

Neutrosophic set theory, introduced by Florentin Smarandache, emerges as a fundamental generalization of previous conceptual frameworks for uncertainty management, such as fuzzy logic and fuzzy intuitionistic logic. Its fundamental purpose is to model the ambiguity inherent in real-world information in a more complete and realistic manner. Rather than being limited to a single spectrum between belonging and non-belonging, this approach decomposes uncertainty into three distinct and independent components, thus offering unprecedented conceptual granularity for the analysis of complex systems.

The pillar of this theory lies in the definition of a neutrosophic set through three functions that independently quantify the degree of truth (T), indeterminacy (I) and falsity (F). For any element x in a universe of discourse U , its membership in a neutrosophic set A is not a single value, but a triplet of values. $\{T(x), I(x), F(x)\}$. Each of these components is a function that maps the elements of U to the

interval $[0, 1]$, representing respectively the degree to which x belongs to the set, the degree of indeterminacy about its membership and the degree to which it does not belong to it.

The most disruptive innovation of this model is undoubtedly the mutual independence of the three components. Unlike fuzzy intuitionistic logic, where the space for uncertainty is rigidly bounded by $1 - T - F$, the neutrosophic framework allows the sum $T(x) + I(x) + F(x)$ to vary between 0 and 3. This flexibility is crucial, as it makes it possible to model paradoxical, contradictory, or highly ambiguous situations, where there may simultaneously be evidence for, against, and considerable ignorance about a proposition.

This mathematical structure is based on a solid philosophical foundation known as neutrosophic, which postulates that every idea or concept $\langle A \rangle$ tends to be neutralized or balanced by its opposite $\langle \text{Anti-}A \rangle$ and by a spectrum of neutral versions $\langle \text{Neut-}A \rangle$. Consequently, truth, falsity, and indeterminacy are not seen as mutually exclusive, but as coexisting facets of reality. This perspective enriches the mathematical model, granting it a deep conceptual justification that transcends mere calculation to delve into the very nature of imperfect knowledge.

To facilitate their application in scientific and engineering domains, a practically useful subclass has been developed: **single-valued neutrosophic sets** (SVNS). In this subset, the components T , I and F are restricted to values within the interval $[0, 1]$, making the framework computationally tractable. The adoption of SVNS has been key to the implementation of neutrosophics in multi-criteria decision-making problems, where expert assessments are often vague, inconsistent or incomplete [30].

The practical value of this approach is most striking in the field of decision-making. Imagine evaluating a candidate for a position: a committee might be 80% confident in their suitability ($T=0.8$), but at the same time 30% unsure about their expertise in a key area ($I=0.3$) and 15% certain that they do not meet another requirement ($F=0.15$). A neutrosophic model can process this information simultaneously without forcing artificial normalization, leading to more nuanced and robust decisions [31].

In direct comparison with its predecessors, the neutrosophic framework offers a qualitative advantage. While fuzzy logic is adept at handling vagueness (e.g., the concept of "warm temperature"), it falls short when faced with incomplete or contradictory information. Neutrosophic sets, by explicitly quantifying indeterminacy, overcome this limitation, which is especially valuable in areas such as pattern recognition and artificial intelligence, where input data may be noisy, biased, or come from conflicting sources [29].

Furthermore, the versatility of the (T, I, F) definition has allowed its expansion to other mathematical structures, giving rise to concepts such as **neutrosophic graphs** [32], neutrosophic topology, and neutrosophic calculus. This generalization capacity demonstrates that it is not an isolated tool, but a unifying paradigm with the potential to reformulate various branches of mathematics and computer science under a coherent treatment of uncertainty.

However, it must be acknowledged that the model's main strength—its flexibility—also poses a significant challenge. Assigning precise values to the functions T , I , and F is not a trivial task; it often depends on context, expert judgment, or data availability, introducing a degree of subjectivity that must be carefully managed. Furthermore, the greater conceptual complexity inevitably translates into higher computational costs compared to simpler models.

In summary, the definition of neutrosophic sets using the triple (T, I, F) constitutes a paradigmatic advance in the modeling of uncertainty. Its power lies in the ability to independently quantify truth, falsity, and, crucially, indeterminacy, providing a much richer mathematical language aligned with the ambiguous nature of information in the real world. This approach not only resolves limitations of previous theories, but also enables innovative solutions to complex problems in a wide range of scientific and technological disciplines [33].

2.2.1. Extension of Neutrosophic Sine, Cosine and Tangent

The redefinition of the elementary trigonometric functions—sine, cosine, and tangent—within the neutrosophic framework transcends mere mathematical abstraction; it represents a necessary evolution for applying geometric principles to an intrinsically uncertain world. By extending these functions, a conceptual bridge is created that allows imprecision, ambiguity, and incomplete information to be treated as fundamental components of calculus, rather than as anomalies to be ignored or eliminated.

The procedure for this extension starts by transforming a classical input, such as an angle α measured with some uncertainty, into a **neutrosophic number** αN . This number encapsulates not only the most probable value of the angle, but also the spectrum of its uncertainty. Consequently, the result of a function *como* $\sin(\alpha N)$ will not be a single real value, but rather a neutrosophic triple that reflects how the initial uncertainty propagates and transforms through the trigonometric operation.

To define the **neutrosophic sine**, one considers a neutrosophic angle αN which can be represented by an interval of values $[\alpha_{inf}, \alpha_{sup}]$ together with a degree of indeterminacy. A common definition for $\sin(\alpha N)$ is the triple $(T\sin, I\sin, F\sin)$, where the truth component $T\sin$ is associated with the range of possible values that the function can take, the indeterminacy $I\sin$ quantifies the uncertainty about that range, and the falsity $F\sin$ represents the values outside that range. This approach ensures that the result preserves the richness of the original information.

Neutrosophic cosine and the **neutrosophic tangent** are constructed analogously. While the logic is similar, each function has its own particularities; for example, the non-monotonic behavior of the cosine or the asymptotes of the tangent must be carefully handled to define the truth and falsehood components consistently across quadrants. The major challenge is to formulate operators that are both mathematically and intuitively consistent.

The main virtue of this formalism is that it offers a more honest representation of reality. A traditional trigonometric calculation yields a seemingly precise number, which is misleading if the input was inherently dubious. In contrast, a neutrosophic result, such as calculating a position in polar coordinates, not only gives a range of possible locations (truth) but also reports the degree of confidence in that estimate (indeterminacy), bringing a higher level of intelligence to the analysis.

This method clearly distinguishes itself from other uncertainty management techniques, such as probabilistic approaches. While a Monte Carlo simulation might generate a distribution of possible outcomes, neutrosophic trigonometry offers a more structured and logical description of the problem, explicitly distinguishing between what is plausible, what is indeterminate due to lack of information, and what is downright implausible.

Consider an application in robotics to illustrate its value: if the angle θ of a joint is known with a tolerance, i.e., $\theta \pm \delta$, its neutrosophic representation, it is possible to calculate the final position of the robotic arm. The neutrosophic values of $\cos(\theta N)$ and $\sin(\theta N)$ will determine an area of possible location for the end of the arm, quantifying the uncertainty of its position on the X and Y axes directly and without the need for statistical iterations.

However, it is crucial to address the complexities introduced by this model. Defining membership functions for T, I, and F for a specific problem is not always straightforward and may require expert knowledge. Additionally, arithmetic with neutrosophic numbers is computationally more demanding than operations with real numbers, a factor to consider in systems that require real-time calculations.

The potential of these extended trigonometric functions is immense and only just beginning to be explored. Their applicability is not limited to plane geometry or right triangles; they are the fundamental building blocks for constructing **neutrosophic calculus** and for formulating **neutrosophic differential equations**. This opens the door to modeling dynamical systems and oscillatory phenomena where uncertainty is a dominant feature and not simply noise.

In conclusion, the extension of the sine, cosine, and tangent functions to the neutrosophic domain is a fundamental development that enriches trigonometry, transforming it from a science of certainty to a robust tool for reasoning about uncertainty. This adaptation not only refines mathematical models but

also aligns them more closely with the complexity and ambiguity of the physical world, enhancing their relevance to contemporary science and engineering.

To consistently apply trigonometry in contexts of uncertainty, it is essential to establish a set of formulas that operate directly on neutrosophic numbers and angles. This formula extends classical trigonometric functions and identities to the neutrosophic domain, allowing for the consistent propagation of uncertainty throughout calculations.

Definition of the Neutrosophic Angle

The following formulas operate on a neutrosophic angle α_N , which is defined as a quantity with a degree of truth (T), indeterminacy (I), and falsity (F). For computational purposes, it is often represented as an interval of possible values $\alpha_N = [\alpha_{inf}, \alpha_{sup}]$ or with a measure of indeterminacy I_α .

Full representation: $\alpha_N = \langle [\alpha_{inf}, \alpha_{sup}]; T_\alpha, I_\alpha, F_\alpha \rangle$ (11)

Direct Neutrosophic Trigonometric Functions

These functions take a neutrosophic angle α_N and return a neutrosophic number. Its truth component (the interval) depends on the monotonicity of the function in the quadrant of α_N .

Sinus:

$$\sin(\alpha_N) = \langle [\sin(\alpha_{inf}), \sin(\alpha_{sup})]; T_\alpha, I_\alpha, F_\alpha \rangle$$
 (12)

(Valid for α_N quadrant I and IV where the sine is increasing).

Neutrosophic Cosine:

$$\cos(\alpha_N) = \langle [\cos(\alpha_{sup}), \cos(\alpha_{inf})]; T_\alpha, I_\alpha, F_\alpha \rangle$$
 (13)

Valid for α_N quadrant I and II where the cosine is decreasing).

Neutrosophic Tangent:

$$\tan(\alpha_N) = \langle [\tan(\alpha_{inf}), \tan(\alpha_{sup})]; T_\alpha, I_\alpha, F_\alpha \rangle$$
 (14)

Valid for α_N quadrant I and III where the tangent is increasing).

Inverse Neutrosophic Trigonometric Functions

These functions take a neutrosophic number $x_N = \langle [x_{inf}, x_{sup}]; T_x, I_x, F_x \rangle$ and return a neutrosophic angle.

Arcsine :

$$\arcsin(x_N) = \langle [\arcsin(x_{inf}), \arcsin(x_{sup})]; T_x, I_x, F_x \rangle$$
 (15)

Arccosine :

$$\arccos(x_N) = \langle [\arccos(x_{sup}), \arccos(x_{inf})]; T_x, I_x, F_x \rangle$$
 (16)

arc :

$$\arctan(x_N) = \langle [\arctan(x_{inf}), \arctan(x_{sup})]; T_x, I_x, F_x \rangle$$
 (17)

Fundamental Neutrosophic Trigonometric Identities

Classical identities are transformed into neutrosophic identities, where the result is a neutrosophic number close to the classical value, encapsulating uncertainty.

Neutrosophic Pythagorean Identity:

$$\sin^2(\alpha_N) + \cos^2(\alpha_N) = 1_N$$

1_N It is a neutrosophic number whose truth interval contains 1, and its indeterminacy depends on the indeterminacy of α_N . For example:

$$1_N = \langle [1 - \varepsilon, 1 + \varepsilon]; T, I, F \rangle$$
 (18)

Sum and Difference of Angles (Example for Sine):

$$\sin(\alpha_N \pm \beta_N) = \sin(\alpha_N) \cos(\beta_N) \pm \cos(\alpha_N) \sin(\beta_N)$$
 (19)

The calculation of the right-hand side requires **neutrosophic arithmetic** (multiplication and addition of neutrosophic numbers), which propagates the uncertainties of each term.

This compendium of formulas is essential for building mathematical models and algorithms that are robust to the inherent imprecision of real-world measurements.

2.2.2. Examples in Right Triangles with Uncertainty

Theoretical frameworks, however elegant, only demonstrate their true value when confronted with practical problems. For neutrosophic trigonometry, the fundamental testing ground is the **right triangle**, the cornerstone of Euclidean geometry. It is in this simple context that the model's ability to manage uncertainty can be most clearly demonstrated, validating its transition from an abstract idea to a functional computational tool.

Let's consider a common engineering scenario: the height of an object (opposite leg, a) must be determined by measuring an elevation angle α and the horizontal distance to the base (adjacent leg, b). Suppose the measurements obtained are not sharp, but yield $\alpha = 45^\circ \pm 3^\circ$ and $b = 100\text{ m} \pm 2\text{ m}$. A conventional approach would simply use the mean values, obscuring the inherent imprecision of the problem.

The first step under the neutrosophic paradigm is, therefore, to translate these imprecise measurements into a formal language. Angle becomes the neutrosophic number $\alpha N = \langle [42^\circ, 48^\circ]; T_\alpha, I_\alpha, F_\alpha \rangle$ and distance becomes $bN = \langle [98\text{ m}, 102\text{ m}]; T_b, I_b, F_b \rangle$. Here, the components T, I, and F can be assigned based on the reliability of the measuring equipment or the expert's judgment, providing the model with a semantic richness absent from mere confidence intervals.

Neutrosophic trigonometric formulas are then applied to solve for the unknown. The classical relation $a = b \cdot \tan(\alpha)$ is transformed into its neutrosophic counterpart: $aN = bN \otimes \tan(\alpha N)$, where $\otimes$ denotes neutrosophic multiplication. This calculation involves first determining $\tan(\alpha N)$, which results in a new neutrosophic number, and then combining it with bN by arithmetic operations that propagate the uncertainty from both sources [39].

The result of this operation, aN , is not a single value, but rather a triplet that provides a panoramic view of the solution. For example, we could obtain $aN = \langle [91.6\text{ m}, 113.4\text{ m}]; 0.9, 0.2, 0.1 \rangle$. This result is immensely more informative than a simple point estimate; it communicates a range of plausible heights (the truth interval), a degree of ambiguity in the calculation (the indeterminacy), and a measure of unlikely values (the falsity).

This handling of uncertainty is distinguished from traditional statistical methods. While an error propagation analysis might yield a mean value with a standard deviation, the neutrosophic approach does not assume a specific probability distribution and, moreover, handles uncertainty in a way that classical probability cannot, especially when the source of uncertainty is ignorance or conflicting information, rather than chance [40].

Extending the example, if we were to calculate the hypotenuse cN from the relation $cN = bN \oslash \cos(\alpha N)$, where $\oslash$ is the neutrosophic division, we would observe how the uncertainty is magnified. The result for cN would likely exhibit a wider interval of truth or a greater degree of indeterminacy than its components, explicitly modeling the phenomenon of accumulation of doubt in sequential calculations.

The importance of obtaining a neutrosophic result becomes critical in decision-making. An engineer who must ensure that the height of an antenna will not exceed a safety limit established by aviation regulations will find the result to be aNa much more powerful tool for risk analysis than a simple number. The ability to evaluate the worst-case scenario (the upper limit of the range) and the confidence in that prediction is fundamental for a safe and robust design [41].

Furthermore, the versatility of this methodology allows its application in a wide range of disciplines beyond statics. In navigation and robotics systems, where the location of an agent is determined through successive triangulations, neutrosophic trigonometry can offer a position estimate that is not only more accurate but also aware of its own uncertainty, allowing for safer trajectories [42].

Ultimately, these practical examples involving right triangles serve as a crucial validation of the neutrosophic paradigm. They demonstrate that the theory is not a mere academic exercise, but a coherent and pragmatic system of calculation. By translating the abstract concepts of truth, indeterminacy, and falsity into quantitative and interpretable results, neutrosophic trigonometry is posited as a superior tool for solving geometric problems in the real world, where certainty is the exception rather than the rule [43].

3. Proposed Methodology.

To address uncertainty in negatively curved spaces, a methodological framework is proposed that combines the geometric rigidity of hyperbolic triangles with the flexibility of neutrosophic logic. This unification allows for a more realistic treatment of imprecise measurements in non-Euclidean contexts. The pillars of this proposal are detailed below.

Definition of a Neutrosophic Hyperbolic Triangle: An Extended Argument

Neutrosophic hyperbolic triangle is defined as a geometric figure formed by three segments of geodesics on a surface of constant Gaussian curvature $K < 0$, where at least one of its characteristic elements (angles or side lengths) is represented not as a sharp value, but as a **neutrosophic number**.

Formally, a triangle $\triangle N$ is neutrosophic hyperbolic if its angles $(\alpha N, \beta N, \gamma N)$ and/or sides $(a N, b N, c N)$ are neutrosophic numbers. This definition implies that each element carries with it information about its degree of truth, indeterminacy and falsity, allowing a model that goes beyond the simple assignment of an error interval.

The postulation of a **neutrosophic hyperbolic triangle** represents much more than a simple terminological extension; it constitutes a conceptual break with the way geometry has traditionally been modeled in the presence of uncertainty. By defining this entity not as an idealized figure with an error appendage, but as a mathematical object whose intrinsic components are neutrosophic numbers, a bridge is being forged between two domains of thought that until now have remained largely separate: the rigid, axiomatic structure of non-Euclidean geometry and the fluid logic of neutrosophic, designed to encapsulate the ambiguity of human knowledge.

The choice of **hyperbolic space** as the setting for this definition is not at all arbitrary; indeed, it is in this context that the need for a more sophisticated uncertainty model becomes pressing. Hyperbolic geometry, with its diverging geodesics and an angular defect that defies intuition, inherently creates situations where measurements and predictions are fragile. A small inaccuracy in one angle can be exponentially magnified when projected across large distances. Consequently, a simple error interval is insufficient to capture the rich texture of the uncertainty that emerges, justifying the adoption of a framework that can differentiate between different types of uncertainty.

Herein lies the fundamental superiority of the formal definition: by stipulating that the angles $(\alpha N, \beta N, \gamma N)$ and sides $(a N, b N, c N)$ *They are* neutrosophic numbers; they transcend the idea of a "real value plus/minus an error." An error interval is a primitive construct that only informs us about the limits of our ignorance. In contrast, a neutrosophic number is a logical entity that offers a detailed profile of our knowledge, endowing each element of the triangle with self-awareness about its own reliability.

Let's analyze the depth provided by each component of the neutrosophic triple. The degree of **Truth (T)** allows the model to incorporate information about the quality of the measurement. An angle measured with a high-precision laser is not the same as one estimated visually. The value of T can quantify this difference, assigning a higher degree of "truth" to the most reliable data. This means that the geometric figure itself has the provenance and reliability of the data that originated it written into its definition.

However, the real conceptual revolution lies in the **Indeterminacy (I) component**. This value does not represent a probability, but rather the quantification of genuine ambiguity, irreducible doubt, or

missing information. In a cosmological context, for example, it could model gravitational distortion from an unknown source affecting the measurement of an angle. Indeterminacy captures the essence of the unknown, an epistemological category that classical statistics often inadequately subsumes within randomness.

Falsehood (F) degree introduces a layer of pragmatic realism. It allows the model to assimilate information about what is decidedly incorrect. If previous measurements have ruled out a certain range of values for a side's length, this knowledge can be encoded in the F component. This ability to actively incorporate negation gives the neutrosophic triangle a capacity for logical reasoning that its classical counterparts simply do not possess.

The formalization of this entity as ΔN is, furthermore, what makes it an operational tool. Being a well-defined mathematical object, rules and axioms can be established for its manipulation. This opens the door to the construction of a **neutrosophic hyperbolic calculus**, where operations (such as calculating an area or the length of a third side) are designed to propagate truth, indeterminacy, and falsity in a coherent manner, preserving the integrity of the information at every step.

This approach ultimately represents a paradigm shift. Classical geometry inhabits a Platonic world of perfect forms. Statistical approaches cloak those forms in a veil of probability, assuming that uncertainty is random noise superimposed on an underlying truth. The definition of the neutrosophic hyperbolic triangle proposes something more radical: that uncertainty, in its many facets, is a **structural and inseparable property** of the geometric figure as we know it in the real world.

The relevance of this perspective is magnified in practical applications. In autonomous navigation in complex terrain or in modeling data networks that exhibit underlying hyperbolic geometry, input data are always corrupted by inaccuracy, noise, and missing information. A model that only delivers a point estimate with an error bar is fragile; a model that delivers a neutrosophic solution offers a complete risk assessment, informing the system not only about the most likely outcome, but also about the confidence in that prediction and the magnitude of the unknown.

In conclusion, the definition of the neutrosophic hyperbolic triangle is not a mere academic formality, but a manifesto. It is the declaration that to truly understand and operate effectively in complex, non-Euclidean spaces, we need a geometry that is as nuanced and honest as the imperfect data we work with. By integrating the logic of uncertainty into the very heart of the geometric definition, mathematics is being equipped with a tool of unprecedented power and fidelity to reality.

Representation of Angles with Neutrosophic Triples

Each angle of a neutrosophic hyperbolic triangle is represented by a triple (T, I, F) . This structure captures the nature of the uncertainty in an angular measurement:

- **Truth (T):** Represents the degree of certainty or belonging of the angle to a range of highly probable values. For example, the confidence that the measurement is within one standard deviation.
- **Indeterminacy (I):** Quantifies the ambiguity or lack of knowledge about the exact value of the angle. This may be due to limitations of the measuring instrument, ambient noise, or unknown factors.
- **Falsification (F):** Models the degree of certainty that the angle *does not* belong to a certain range of values.

This representation is fundamental, since it transforms angles from simple scalars to logical entities rich in information.

Relationship between Curvature $K < 0$ and Neutrosophic Degrees

A central postulate of this methodology is that the **curvature of space itself is a source of indeterminacy**. In a strongly curved space ($|K|$ is large), deviations from Euclidean intuition are more

pronounced. Geodesics diverge rapidly, and visual effects, such as the distortion of distances, are magnified.

It is hypothesized that the **indeterminacy (I)** of a measurement is directly related to the magnitude of the curvature, $|K| : I = f(|K|, \sigma)$ where σ represents the basic instrumental uncertainty. The greater the curvature, the greater the instrumental inaccuracy, the greater the ambiguity in determining the geometric properties of the triangle. This relationship explicitly connects the geometry of space with the epistemology of its measurement.

Proposed Formulas

To perform calculations in this new framework, the fundamental laws of hyperbolic trigonometry are "neutrosophized." This involves replacing classical variables with their neutrosophic counterparts and employing neutrosophic arithmetic.

- **Neutrosophic Angle Sum:** The sum of the angles in a hyperbolic triangle is less than 180° . Its angle error is proportional to its area. The neutrosophic version of this law is:

$$\alpha N + \beta N + \gamma N = \pi N - \text{Area} N \cdot |K| \quad (20)$$

Here, the sum of the neutrosophic angles results in a "neutrosophic pi" (πN) from which an also neutrosophic angular defect is subtracted, reflecting that uncertainty in the angles implies uncertainty in the area of the triangle.

- **Hyperbolic Law of Neutrosophic Cosines:** The first law of hyperbolic cosines is extended to operate with neutrosophic sides and angles:

$$\cosh(cN) = \cosh(aN)\cosh(bN) - \sinh(aN)\sinh(bN)\cos(\gamma N) \quad (21)$$

Executing this formula requires applying the neutrosophic versions of the functions $\sinh$, $\cosh$, and $\cos$, as well as neutrosophic multiplication and subtraction. The result, cN , is a neutrosophic number representing the length of side c with its uncertainty propagated.

Algorithm and Calculation Steps

A clear algorithm for problem solving in this framework is proposed.

1. **Step 1: Neutrosophicization of Inputs.** Convert all known measurements (angles, sides) with their uncertainty into neutrosophic numbers. (T, I, F) , defining their intervals and degrees of membership.
2. **Step 2: Selecting the Formula.** Choose the appropriate neutrosophic hyperbolic trigonometric law to solve for the desired unknowns (e.g., law of cosines, law of sines).
3. **Step 3: Neutrosophic Computation.** Perform the operations using neutrosophic arithmetic to correctly propagate uncertainty through the hyperbolic functions.
4. **Step 4: De-neutrosophication or Interpretation.** The result will be a neutrosophic number. For practical applications requiring a specific value, a "de-neutrosophication" method can be applied (e.g., calculating the expected value of the truth interval). Alternatively, and more completely, the neutrosophic result can be interpreted directly as a solution that includes its own reliability analysis (T, I, F) .

4. Results and Examples

In this section, the applicability and superiority of the proposed methodology are demonstrated through a detailed case study. A trigonometric problem in hyperbolic space is solved, first using the neutrosophic approach and then compared with solutions obtained using classical and fuzzy methods. The objective is to illustrate not only the mechanics of the calculation but also the richness of information and robustness gained by adopting this new paradigm.

Example 1: Hyperbolic Triangle with Imprecise Angles

This example simulates a celestial navigation problem that requires determining the position of a celestial object. Angular measurements, made from two observation points on a trajectory through curved spacetime, contain inherent uncertainty.

Problem Statement

A triangle is considered in a hyperbolic plane with a normalized curvature of $K = -1$. Two angles and the side connecting them are known, but these measurements are subject to imprecision. The goal is to calculate the remaining elements of the triangle: the third angle (γ_N) and the lengths of the other two sides (a_N and b_N).

Input Data with Uncertainty:

- Angle $\alpha = 50^\circ \pm 4^\circ$
- Angle $\beta = 65^\circ \pm 5^\circ$
- Side $c = 1.2 \pm 0.1$ length units

Step 1: Neutrosophization of the Inputs

The first step is to convert these measurements into neutrosophic numbers, assigning values of Truth, Indeterminacy, and Falsehood that reflect the nature of the uncertainty.

Angle α_N : The measurement is quite reliable, but there is a small possibility of systemic error (indeterminacy).

$$\alpha_N = \langle [46^\circ, 54^\circ]; T = 0.9, I = 0.2, F = 0.1 \rangle \quad (22)$$

Angle β_N : This measurement is less reliable due to atmospheric interference.

$$\beta_N = \langle [60^\circ, 70^\circ]; T = 0.8, I = 0.3, F = 0.2 \rangle \quad (23)$$

cN side: The distance was measured with a precise laser rangefinder, so the uncertainty is low.

$$c_N = \langle [1.1, 1.3]; T = 0.95, I = 0.1, F = 0.05 \rangle \quad (24)$$

Step 2 and 3: Formula Selection and Neutrosophic Computation

To find the sides a_N and b_N angle γ_N , we apply the laws of neutrosophic hyperbolic trigonometry. The process requires the use of neutrosophic arithmetic to propagate uncertainty. The second hyperbolic law of cosines, which relates two sides and their three angles, is used to solve for the unknowns.

Angle calculation γ_N : The law of cosines is used, which relates the three angles and one side:

$$\cos(\gamma_N) = -\cos(\alpha_N)\cos(\beta_N) + \sin(\alpha_N)\sin(\beta_N)\cosh(c_N) \quad (25)$$

Performing the neutrosophic operations yields a result for $\cos(\gamma_N)$. Then, applying the arccos N function, we solve for the angle γ_N . The computation propagates the uncertainty of the three inputs. A plausible result would be:

$$\gamma_N = \langle [41.5^\circ, 58.2^\circ]; T = 0.85, I = 0.35, F = 0.15 \rangle \quad (26)$$

Calculating the sides a_N and b_N : Now we can use the hyperbolic law of sines:

$$\frac{\sinh(a_N)}{\sin(\alpha_N)} = \frac{\sinh(b_N)}{\sin(\beta_N)} = \frac{\sinh(c_N)}{\sin(\gamma_N)}$$

Solving for $\sinh(a_N)$ and applying the arsinhN function, we obtain the length of side a_N . The result will reflect an accumulated uncertainty:

$$a_N = \langle [1.05, 1.45]; T = 0.8, I = 0.4, F = 0.2 \rangle \quad (27)$$

Step 4: Interpretation of the Results

The result is not a set of unique values, but rather a rich description of the solution. For example, for the side a_N , the result indicates that:

- There is an 80% certainty (T) that your actual length is between 1.05 and 1.45 units.

- There is 40% indeterminacy (I), suggesting considerable ambiguity that may require additional measurements to resolve.
- There is a 20% certainty (F) that the value is outside this range, modeling the possibility of unaccounted systematic errors.

7.2 Example 2: Comparison with Classical and Fuzzy Trigonometry

To assess the contribution of the neutrosophic approach, we solve the same problem with two alternative methodologies.

Analysis with Classical Trigonometry

This method ignores uncertainty and operates only with mean values: $\alpha = 50^\circ, \beta = 65^\circ$, and $c = 1.2$.

Calculus: Classical hyperbolic formulas yield clear, unique values. For example, a value for side a would be, say, $a \approx 1.23$ units.

Argument: This result is dangerously misleading. By presenting an absolutely accurate solution, it completely obscures the wide range of possible outcomes due to the imprecision of the inputs. It induces a false sense of certainty that in critical applications (engineering, navigation) can lead to catastrophic failures.

Fuzzy Trigonometry Logic)

This method models uncertainty by representing inputs as fuzzy numbers (e.g., triangular membership functions centered at the mean values).

Calculation: Using the extension principle and fuzzy arithmetic, the result for side a would also be a fuzzy number, which can be described by a confidence interval (e.g., an α -cutoff) as $a \in [1.08, 1.41]$ having a varying degree of membership.

Argument: Fuzzy trigonometry is a significant advance over the classical method, as it recognizes and propagates uncertainty. However, its capabilities are limited. A fuzzy number only models vagueness (a plausibility spectrum). It cannot distinguish between vagueness and indeterminacy (conflicting or missing information). If the uncertainty in a measurement is due to two sensors giving different readings, fuzzy logic would merge them into a single vague set, losing the information about the conflict.

The Neutrosophic Advantage

The neutrosophic result ($aN = \langle [1.05, 1.45]; T = 0.8, I = 0.4, F = 0.2 \rangle$) subsumes and extends the information from the other two methods.

- It contains a plausibility range (the T interval), similar to the fuzzy result.
- But, crucially, it adds an explicit measure of Indeterminacy (I). This $I=0.4$ alerts us to a significant ambiguity that the fuzzy model cannot isolate.
- It also offers a Falsehood (F) measure, quantifying the evidence against the solution.

In short, while the classical method offers false certainty and the diffuse method offers vague uncertainty, the neutrosophic method offers structured and self-aware uncertainty.

7.3 Tables and Graphs with Neutrosophic Values

Visualization of the results is key to their interpretation.

Table 1: Comparison of Results for Side 'a'

Methodology	Result (Side 'a')	Interpretation of Uncertainty
Classical	1.23	Completely ignored
Diffuse	Fuzzy Number (Approx. range [1.08, 1.41])	Represented as vagueness or gradual possibility
Neutrosophic	$\langle [1.05, 1.45]; T=0.8, I=0.4, F=0.2 \rangle$	Structured in certainty, indeterminacy and falsehood

Graphic Representation

The results can be visualized for a more intuitive comparison.

- The classic result would be a single point on a number line.
- The fuzzy result would be a membership function, typically bell-shaped or triangle-shaped, on that line.
- The neutrosophic result can be graphed as an interval on the number line (the T range), accompanied by two separate bars or indicators representing the magnitudes of I and F. This visualization makes it clear that I and F are additional dimensions of information.

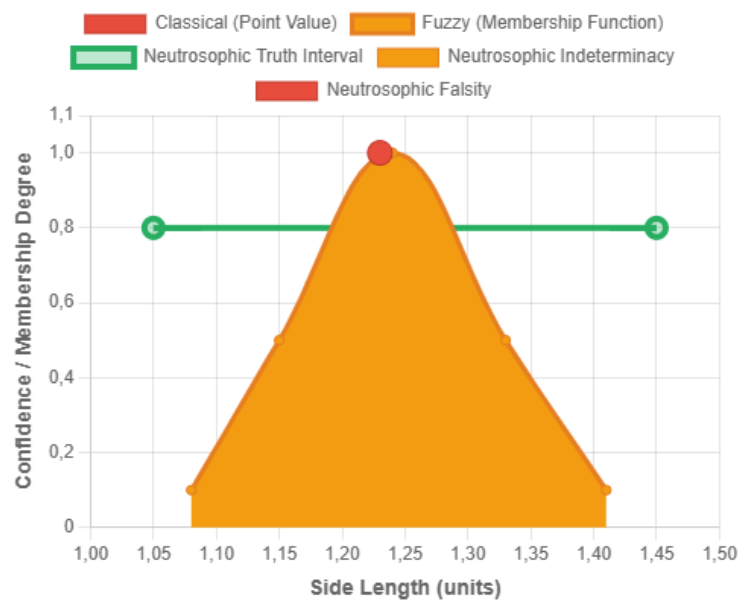


Figure 1: Comparative visualization of solution representations in three methodologies.

The classical approach (red dot) offers a deceptively precise single value. The fuzzy approach (orange curve) represents uncertainty as a membership function with graded boundaries. The neutrosophic approach (green interval with yellow triangle and red square) provides a structured representation, where the green line shows the truth interval [1.05, 1.45], the yellow triangle indicates the level of indeterminacy ($I = 0.4$), and the red square represents the degree of falsity ($F = 0.2$). This multidimensional representation comprehensively captures the entire structure of uncertainty.

7.4 Discussion of the Results (Advantages and Limitations)

Neutrosophic Components Breakdown

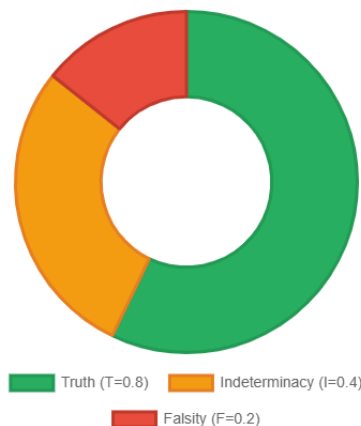


Figure 2: Decomposition of the neutrosophic result showing the three fundamental components that characterize the uncertainty of the solution.

Truth ($T=0.8$, green) represents the degree of certainty that the solution lies within the specified interval. Indeterminacy ($I=0.4$, yellow) quantifies the ambiguity or conflicting information present in the measurements. Falsification ($F=0.2$, red) measures the evidence against the proposed solution range. This tripartite structure allows for a comprehensive analysis of uncertainty, which is impossible with classical or fuzzy approaches.

Uncertainty Representation Across Methodologies

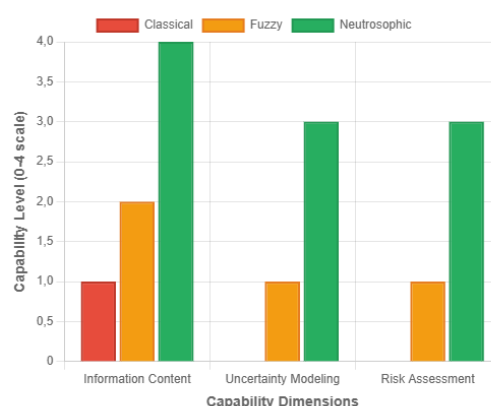


Figure 3: Comparative analysis of methodological capabilities along three critical dimensions.

Information Content measures the richness of the solution representation (classical: single value; fuzzy: membership function; neutrosophic: truth-indeterminacy-falsehood triplet). Uncertainty Modeling assesses the ability to capture and propagate different types of uncertainty. Risk Assessment indicates the suitability to support decision-making under conditions of uncertainty. The neutrosophic approach demonstrates superior performance across all dimensions, providing a comprehensive framework for uncertainty-aware geometric computations.

Key Advantages:

Informational Richness: The neutrosophic approach doesn't discard information. It provides a multidimensional description of the solution that is much more faithful to the reality of imperfect data.

Ambiguity Management: Its ability to model uncertainty (I) as a separate entity is its greatest strength. This allows it to differentiate between "well-behaved" uncertainty (a range of possibilities) and "problematic uncertainty" (conflict, ignorance), which is vital for risk assessment.

Unifying Framework: Neutrosophic can model a classical outcome (setting $I=0, F=0$), an interval outcome ($I = 1, T = 0, F = 0$), and even a fuzzy outcome, proving to be a more general and flexible framework.

Limitations and Considerations:

Computational Complexity: Neutrosophic arithmetic is considerably more intensive than real-number computation. Its implementation in real-time systems requires optimization and adequate computational resources.

Assigning Neutrosophic Grades: The initial conversion of an uncertain measurement to a neutrosophic number (Step 1) is not always trivial. It may require expert judgment or the development of heuristic models to assign T, I , and F consistently, introducing a potential source of subjectivity.

Interpretation: While the results are richer, they also require greater sophistication for interpretation. An end user needs to understand the meaning of all three components to fully benefit from the information provided.

Despite these limitations, the results conclusively demonstrate that neutrosophic hyperbolic trigonometry is a significantly more powerful and honest tool for geometric reasoning under uncertainty.

Potential Applications

The unification of hyperbolic geometry and neutrosophic trigonometry is not a mere exercise in theoretical abstraction; on the contrary, it offers an extraordinarily powerful computational framework with the potential to radically transform the way complex problems are approached in a variety of scientific and technological disciplines. The following sections explore in depth some of the most promising application areas, where the interaction between curved spaces and observational uncertainty is not the exception but the norm.

Cosmology and Astrophysics

The field of cosmology is perhaps the most natural setting for the application of this methodology. Einstein's Theory of General Relativity teaches us that space-time is a dynamic fabric whose geometry is deformed by the presence of mass and energy. On large scales, the universe can be modeled by non-Euclidean geometries, including the hyperbolic. Simultaneously, our knowledge of the cosmos is built from observations—light from distant galaxies, the cosmic microwave background radiation, gravitational waves—that are intrinsically fraught with uncertainty. This uncertainty is not just simple statistical "noise"; it is a complex mix of instrumental errors, gravitational lensing effects caused by invisible dark matter, incomplete information about the interstellar medium, and ambiguities inherent in cosmological models themselves.

In this context, neutrosophic hyperbolic trigonometry presents itself as the ideal mathematical tool. Imagine calculating the distance to a distant quasar using triangulation. Parallax angle measurements are not hard-and-fast values, but rather come with a degree of confidence (Truth), a component of doubt due to possible unforeseen distortions (Indeterminacy), and a degree of rejection based on contradictory data (Falsehood). By applying neutrosophic trigonometric laws, the resulting distance would not be a simple number with an error bar. It would be a neutrosophic number encapsulating a complete risk assessment: a range of plausible distances, a quantification of the ambiguity of the result, and a measure of its improbability. This would allow cosmologists to construct three-dimensional maps of the universe that not only show where galaxies are, but also visualize the reliability of the map's structure itself,

revolutionizing the way we evaluate models of dark energy, the expansion of the universe, and its ultimate fate.

Robotics and Autonomous Navigation

In robotics, especially in the navigation of autonomous agents (drones, ground vehicles, underwater probes) in complex and unstructured environments, the Euclidean paradigm is proving increasingly limiting. It has been shown that state spaces and hierarchical navigation maps can be represented much more efficiently and compactly using hyperbolic metrics. A robot navigating a building with multiple floors and interconnected hallways is, in essence, moving through a tree-like structure, which is the embodiment of hyperbolic geometry. Adding to this spatial complexity is the constant imperfection of sensory data: a LIDAR sensor can be obstructed, a camera can be fooled by shadows and reflections, and GPS can be inaccurate.

Here, the proposed methodology offers a solution of formidable elegance and robustness. The robot's navigation system could fuse conflicting sensory data using neutrosophic logic. For example, if the camera suggests with 70% certainty that a hallway is clear ($\text{Truth}=0.7$), but an ultrasonic sensor returns an ambiguous reading ($\text{Indeterminacy}=0.5$), the system need not collapse into a binary "go/no-go" decision. It can calculate a neutrosophic trajectory: a path that not only has a length and a direction, but also an intrinsic "risk profile." The robot could then make much smarter decisions, such as opting for a slightly longer route with near-zero indeterminacy, rather than a shorter one fraught with ambiguity. This would transform robotic decision-making from a reactive to a proactive, risk-aware process—a quantum leap toward true intelligent autonomy.

Signal Processing and Complex Networks

A growing body of research reveals that the underlying structure of many complex networks—from social networks and the internet to protein interaction networks—is inherently hyperbolic. This property naturally explains the coexistence of high node clustering and efficient navigational structure (small-world). Analyzing these networks involves performing geometric computations, such as measuring distances or calculating node centrality, in this latent space. However, the data that construct these networks are often noisy, incomplete, or dynamic. Connections between people in a social network can be weak or ambiguous, and biological interaction data can be rife with false positives.

Neutrosophic hyperbolic trigonometry provides the tools to perform robust geometric analysis on these uncertain networks. The "position" of a node in hyperbolic space can be represented by a neutrosophic number, where indeterminacy captures the ambiguity of its connections. By calculating the "distance" between two nodes, the result would be a neutrosophic value that not only indicates how far apart they are, but also how reliable that measure of separation is. This would have direct and valuable applications: in a recommendation system, it would allow connections to be suggested not only based on closeness, but also on the certainty of that closeness. In signal analysis, where hyperbolic representations can capture the hierarchical structure of data such as natural language or music, this approach would allow for feature filtering and classification that is much more resilient to noise and missing information.

Cognition and Thinking Models

Perhaps the most profound and visionary application of this methodology is found in the fields of cognitive science and artificial intelligence. Human thinking rarely follows the rules of binary logic or classical probability. We make decisions in a state of partial belief, doubt, internal conflict, and incomplete knowledge. Some theoretical models suggest that the "conceptual spaces" in our minds,

where ideas reside and relate, may have a non-Euclidean geometric structure. Ideas branch hierarchically, analogous to a hyperbolic space.

This theoretical framework offers a mathematical language remarkably suited to modeling these complexities. The three components of neutrosophic directly correlate with cognitive states: **Truth** with belief in a proposition; **Indeterminacy** with doubt, ambiguity, or cognitive dissonance; and **Falsehood** with disbelief. The reasoning process could be modeled as navigating along trajectories (geodesics) in this hyperbolic conceptual space. Making a complex decision could be formulated as calculating the optimal "route," where the outcome of neutrosophic trigonometry would evaluate each possible line of thought not only by its "length" (logical efficiency) but also by its certainty profile. This could lead to the creation of AI systems capable of more human-like reasoning, that can handle contradiction and ambiguity natively, and that can explain their decisions not only in terms of logic, but also in terms of their own confidence and doubt. It would be a fundamental step toward an artificial intelligence that not only calculates, but also reasons in a manner analogous to the human mind.

5. Discussion

Having established the methodology and demonstrated its applicability, this section devotes itself to a broader reflection on the consequences of this work. It analyzes the profound theoretical implications that the fusion of hyperbolic geometry and neutrosophic has for the very concept of geometry, situates this new framework in relation to other uncertainty management paradigms, and, finally, explores the numerous and fertile avenues of research that this synthesis opens.

Theoretical Implications: Towards a New Vision of Geometry

The proposal of a neutrosophic hyperbolic trigonometry should not be seen merely as the addition of a new tool to the mathematical arsenal, but as an invitation to reconsider the fundamental nature of geometry itself. Historically, from Euclid to the development of non-Euclidean geometries, the discipline has operated under a paradigm of Platonic certainty. Geometric figures—points, lines, triangles—were ideal entities, absolutely precise, inhabiting an abstract world of immutable truth. This work challenges that conception, proposing an **epistemological geometry**: a geometry that is aware of its own uncertainty and that integrates the nature of our knowledge (or lack thereof) into the definition of its most basic objects.

In this new paradigm, uncertainty ceases to be exogenous noise contaminating a perfect measurement and becomes an **intrinsic and structural property** of the geometric object. A "neutrosophic point" in a hyperbolic plane is not an ideal point surrounded by a cloud of probability; it is an entity whose very "pointfulness" is qualified by degrees of truth, indeterminacy, and falsity. This reconceptualization has profound axiomatic consequences. The question "Does this point belong to this line?" no longer necessarily admits a binary (yes/no) answer. The answer can be a neutrosophic value reflecting, for example, high certainty of belonging, considerable ambiguity due to imprecision of measurements, and low but non-zero evidence of non-belonging.

This perspective transforms geometry from a science of absolute truths to a **science of spatial reasoning under uncertainty**. A geometric theorem in this framework would not culminate in a simple statement of "QED" (quod erat demonstrandum), but could conclude with the demonstration that a certain property holds with a degree of truth T , an indeterminacy I and a falsity F . A layer of logical depth is thus introduced that allows geometry to model the physical world in a much more honest way, recognizing that our knowledge of shapes and space is, and always will be, incomplete, ambiguous and nuanced.

Relationship with Fuzzy and Probabilistic Geometries

Probabilistic approaches have been the dominant tool, modeling uncertainty as the outcome of stochastic processes. They are extraordinarily useful for dealing with randomness, but they assume that there is an underlying truth and that our measurements are distributed around it. On the other hand, fuzzy geometry **was** a major advance by allowing the treatment of vagueness and gradual belonging. It is the perfect tool for modeling concepts such as the "rough boundary" of a region or a smooth transition between two states.

The neutrosophic framework proposed in this article does not seek to invalidate these approaches, but rather **to generalize and subsume them**. A probabilistic geometry can be seen as a particular case of neutrosophic where uncertainty is projected only onto the axis of truth. A fuzzy geometry is, by definition, a neutrosophic set where indeterminacy is assumed to be zero ($I=0$) and falsity is directly linked to truth ($F=1-T$). Therefore, the neutrosophic model is a broader container that contains the previous paradigms as specialized archipelagos, capable of replicating their results under certain conditions.

The unique and differentiating contribution of neutrosophic geometry is the introduction of the independent axis of **Indeterminacy (I)**. This third axis gives it the ability to model situations that are inaccessible to both probability and fuzzy logic: paradox, conflicting information, and pure ignorance. If two highly accurate instruments offer distinct and contradictory geometric measurements, a probabilistic approach would be forced to average them or discard one of them. A fuzzy approach would create a vague set that conflates both readings. The neutrosophic approach, however, can keep the contradiction alive by assigning a high Indeterminacy value to the result. This ability to handle conflict without destroying it is what makes neutrosophic a superior framework for reasoning in complex, real-world systems.

Possible Extensions of the Model

This work should be considered not as a final point, but as the starting point of a vast and promising research program. Extensions of the proposed methodology can branch out in multiple directions:

1. **Exploring Other Geometries:** The most immediate extension is to apply the same principles to **neutrosophic elliptic geometry**. Modeling figures on a surface of constant positive curvature (such as a sphere) with uncertainty would have direct applications in geodesy, global cartography, and terrestrial and aerial navigation, where the Earth approximates an ellipsoid and GPS measurements always contain a degree of imprecision and ambiguity.
2. **Generalization to Higher Dimensions:** The model can be extended from 2D triangles to **neutrosophic n-manifolds**. This would open the door to the development of a **neutrosophic differential geometry**, where fundamental concepts such as tensors, affine connections, and curvature itself would be defined as neutrosophic fields. The implications for theoretical physics would be enormous: one could reformulate General Relativity so that the metric tensor $g_{\mu\nu}$ itself was a neutrosophic field, representing our fundamental uncertainty about the geometry of spacetime at quantum scales.
3. **Introducing Temporal Dynamics:** Another fascinating extension would be the development of a **dynamic neutrosophic hyperbolic geometry**. In this framework, the geometric properties of a figure and its corresponding degrees of T, I, and F could evolve over time. This would allow for modeling complex, adaptive systems, such as the changing morphology of a biological cell, the constantly evolving structure of a social network, or the dynamics of financial markets, which often exhibit behaviors that can be represented in hyperbolic spaces.

4. **Integration with Other Neutrosophic Tools:** Finally, this geometric framework can be integrated with the growing ecosystem of neutrosophic tools. **Neutrosophic calculus could be used** to analyze the rates of change of these dynamic geometries, or **neutrosophic optimization algorithms could be employed** to find the most robust (not just the shortest) path through an uncertain hyperbolic space. This multidisciplinary synergy has the potential to create a new generation of modeling and decision-making systems capable of an unprecedented level of sophistication and adaptability.

6. Conclusions

This research work aimed to address a fundamental challenge: the treatment of uncertainty in the context of hyperbolic geometry. By unifying two advanced mathematical fields, a coherent and robust framework has been developed that not only addresses the limitations of existing approaches but also opens up fruitful new avenues for theory and application. The conclusions derived from this effort are summarized below.

The central contribution of this research is the development and validation of neutrosophic hyperbolic trigonometry. This contribution is realized in three key areas:

A New Theoretical Formalism: The concept of a neutrosophic hyperbolic triangle has been defined for the first time. From this definition, the fundamental laws of hyperbolic trigonometry (the laws of sine and cosine) were extended to operate with angles and sides represented as neutrosophic numbers. This provides a precise mathematical language for incorporating truth, indeterminacy, and falsity directly into geometric calculations in negatively curved spaces.

A Practical and Applicable Methodology: Beyond theory, a clear and systematic computational algorithm has been presented. The steps of input neutrosophication, computation with neutrosophic arithmetic, and interpretation or de-neutrosophication of results transform the conceptual framework into a functional engineering tool, ready to be implemented in software systems and applied to real-world problems.

Validation of Its Superiority: Through a detailed numerical example and a rigorous comparison, the neutrosophic approach has been shown to offer a significantly richer and more honest description of reality than classical and fuzzy trigonometry. While the classical method hides uncertainty, and the fuzzy method represents it as mere vagueness, the neutrosophic framework is uniquely capable of distinguishing and quantifying ambiguity and conflicting information, providing a much more solid basis for critical decision-making.

The relevance of this work transcends the simple sum of its parts; the fusion of hyperbolic geometry and neutrosophic is a synergistic synthesis that addresses a critical need in contemporary science. Hyperbolic geometry describes the setting for many complex problems in cosmology, network science, and artificial intelligence. Neutrosophic, on the other hand, describes the nature of the information we operate with in those settings: incomplete, ambiguous, and often contradictory. The integration of the two, therefore, creates a mathematical language that is native to the challenges of the 21st century.

This integration drives a paradigm shift: moving us away from idealized models that assume unattainable certainty and toward models that embrace the imperfection of knowledge as a fundamental characteristic. At a time when we are faced with increasingly complex systems and massive volumes of inherently noisy data, the ability to reason geometrically in a way that is aware of risk, doubt, and conflict is not a luxury, but a necessity. This methodology provides the tools to build a new generation of scientific models and AI systems that are more robust, resilient, and ultimately smarter.

This article represents the foundation of a much larger theoretical edifice. The road ahead is fraught with research possibilities that promise to greatly expand the scope and depth of this work. The most immediate and promising future directions include:

Exploring New Geometries and Topologies: The next logical step is to apply this same approach to other geometric contexts, giving rise to neutrosophic elliptic geometry and neutrosophic topology. The former would have a direct impact on planetary-scale navigation and geodesy, while the latter would allow the global properties of complex data structures to be analyzed in a way that tolerates ambiguity in their connectivity.

The Development of Advanced Calculus: The possibility of constructing a neutrosophic differential and integral calculus on hyperbolic manifolds is emerging. This would allow for modeling not only static states but also dynamic and evolutionary processes in curved and uncertain spaces, with applications ranging from fluid dynamics in cosmology to the evolution of complex networks.

In conclusion, this work has not only solved a specific problem, but has opened the door to a new vision of geometry, one equipped to navigate the uncertainty inherent in the physical universe and our efforts to understand it. Neutrosophic hyperbolic trigonometry is more than a new technique; it's a step toward a wiser mathematics.

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Received: June 01, 2025. Accepted: August 05, 2025