An Approach for Solving Unconstrained and Constrained Neutrosophic Geometric Integer Programming Problems

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Abstract: Neutrosophic logic, introduced by Smarandache, offers a powerful framework for modeling uncertainty and inconsistency in real-world problems. This paper presents a novel approach for solving both unconstrained and constrained Neutrosophic Geometric Programming (NGP) problems with integer decision variables under a three-level framework of truth, indeterminacy, and falsity. By representing uncertain coefficients as triangular neutrosophic numbers, the proposed method translates the NGP model into a crisp equivalent using score and accuracy functions. Standard optimization techniques, including duality and normality conditions, are applied to derive optimal integer solutions. The approach addresses limitations in fuzzy and intuitionistic fuzzy systems by incorporating indeterminacy, thus providing a more robust solution framework. This method increases the degree of truth and minimizes indeterminacy and falsity, making it a viable tool for solving uncertainty problems within a neutrosophic environment. To validate the methodology's effectiveness and demonstrate the NGP's potential, numerical examples and a real-world case application were solved, showing its use in operations research, such as the Gravel Box Design Problem, and engineering optimization, such as supply chain management and truss structure design.

Keywords: Neutrosophic logic; Geometric programming, Integer optimization; Uncertainty modeling; Triangular neutrosophic numbers.

1. Introduction

Florentine Smarandache introduced Neutrosophic logic in (1999), which is essential when dealing with incomplete, inconsistent, or generalizes classical, fuzzy, and intuitionistic fuzzy logics by introducing components using three levels: these degrees called acceptance (T), indeterminacy (I), falsity (F) unlike traditional frameworks that consider only degrees of truth or membership, neutrosophic logic models uncertainty more comprehensively by explicitly incorporating indeterminacy contradictory information [1].

Geometric programming (GP) is an effective method to solve special structure problems, which is easier than non-linear programming problems, especially in design problems fluid dynamics robotics and control systems where we minimize cost and maximize volume. GP has many advantages compared with other optimization techniques. In GP the complexity of the solution determines according to degree of difficulty. The degree of difficulty is defined as the total number of terms in the objective function and constraints, minus the total number of decision variables and one. Since the late 1960s, GP has gained prominence and has been applied across various fields, including operations research and engineering. Pioneering works by Duffin, Peterson, and Zener [2, 3] established its theoretical foundations, while later references [4] offered expanded applications. Ecker [5], in a survey paper, noted many applications and methods that handle problems with both positive and zero degrees of difficulty. However, real case problems that need to be decided in a lot of fields like multi-dimensional and multi-objective in nature, spanning economic, environmental, social, and technical domains.

Integer Programming (IP) is a subfield of mathematical optimization where some or all the variables are restricted to being integers. This distinguishes it from traditional linear programming, which allows variables to take on any real value, making IP problems computationally more complex. IP problems are used to solve real case problems requiring discrete decisions, like resource allocation, production scheduling, and network design, where units or tasks cannot be fractional. Solving these problems often involves sophisticated algorithms like the branch and bound method or cutting plane techniques for solving problems and getting the optimal solution using the feasible integer-constrained set [6].

This paper addresses the challenge of effectively handling uncertainty and imprecision in real-world optimization problems, which traditional methods often fail to manage. It directly tackles a significant gap in the field by proposing a comprehensive approach to solve NGP problems that simultaneously incorporates truth, indeterminacy, and falsity. The methodology achieves this by systematically converting these uncertain problems into a solvable crisp model using specific membership and score functions. The main objective is to provide a validated, efficient tool applicable to both unconstrained and constrained optimization problems with integer variables, demonstrating its potential for a wide range of practical applications.

This paper is consisting of 6 sections: In section 2 we present preliminaries of the Neutrosophic. NGP problems with an unconstrained NGP optimization model and constrained NGP optimization model presented in section 3. Section 4 discusses general solutions for NGP Problems with unconstrained NGP optimization and constrained NGP Optimization. Section 5 presents the proposed approach steps. Lastly, we solved several numerical examples, with an unconstrained NGP model and constrained NGP model in section 6.

2. Mathematical Preliminaries

2.1 Fuzzy Set [7]

Suppose an exist fixed set Z with fuzzy set X: where X set of $\boldsymbol{Z}$ is an objective can be expressed as $\widetilde{\boldsymbol{X}} = \{(\boldsymbol{z}, \boldsymbol{T_x}(\boldsymbol{x})) : \boldsymbol{z} \in \boldsymbol{Z}\}$ the function $\boldsymbol{T_x}$: Z → [0, 1] defined the acceptance membership of the element $\boldsymbol{z} \in \boldsymbol{Z}$ to the set X.

2.2 Intuitionistic Fuzzy Set [8]

Suppose an exist fixed set Z with an intuitionistic fuzzy set $\widetilde{\boldsymbol{X^\iota}}$ in Z is an object of the form

$\widetilde{\boldsymbol{X^\iota}} = \{< \boldsymbol{Z}, \boldsymbol{T_X}(\boldsymbol{z}), \boldsymbol{F_x}(\boldsymbol{z}) > | \, \boldsymbol{z} \in \boldsymbol{Z}\}$ where $\boldsymbol{T_x}$: Z → [0, 1] where $\boldsymbol{F_x}$: Z → [0, 1] define the true and false membership respectively, $\forall \boldsymbol{z} \in \boldsymbol{Z}$, $0 \le \boldsymbol{T_x}(\boldsymbol{z}) + \boldsymbol{F_x}(\boldsymbol{z}) \le 1$

2.3 Neutrosophic Set

A neutrosophic set (NS) is an advanced mathematical framework that handles uncertainty and ambiguity more flexibly than traditional and fuzzy sets. It's defined on a space of objects X by membership functions with three independent levels: T-membership (acceptance) function, I-membership function, and F-membership (reject) function F(x). These functions are unique because they map to non-standard intervals, denoted as] 0^{-},1^{+}[, which allows for values infinitesimally smaller than zero or larger than one, providing greater flexibility. A key feature of NS is no limitation on the summation of T-membership function, I-membership function, and F-membership function, which allows it to model situations where belief, disbelief, and doubt are entirely independent of one another [9].

2.4 Single Valued Neutrosophic Sets

Single valued neutrosophic sets are a mathematical concept that extends traditional and fuzzy set theories to handle uncertainty and ambiguity more comprehensively. Neutrosophic sets with single valued, denoted as A, are defined over a universe of discourse X where each element x is associated with three independent membership degrees: acceptance degree, I-membership degree, and reject degree. All these degrees fall within the range [0,1], and their sum must satisfy the condition 0 ≤ acceptance degree + indeterminacy degree + reject degree ≤ 3 [10].

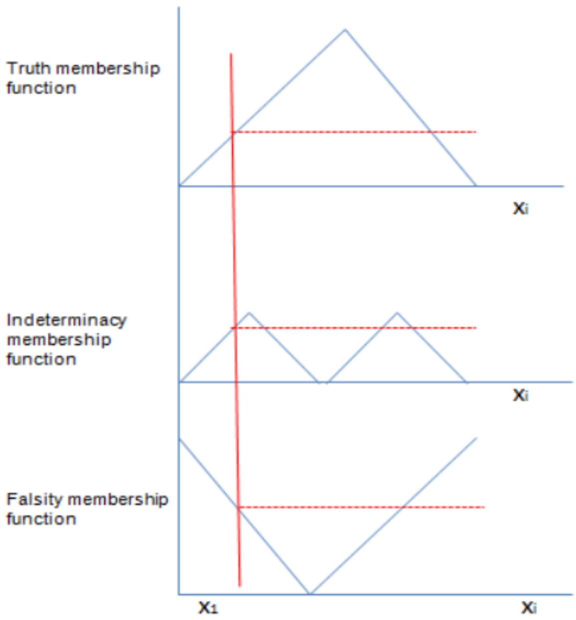

Figure 1. Neutrosophication process [9]

2.5 Complement

Single valued neutrosophic sets complement is defined by symbol Ac and it defined given by swapping the truth $T_A(x)$, and falsity membership degrees $F_A(x)$, while the indeterminacy degree is calculated as one minus its original value [9]. For an element x in the universe of discourse, if A = {⟨x, $T_A(x)$, $I_A(x)$, $F_A(x)$⟩}, as equation (1) Ac is given by:

$$A^c=\{\langle x,T_A{}^c(x),I_A{}^c(x),F_A{}^c(x)\rangle\} \quad (1)$$

2.6 Union

Single valued neutrosophic sets union [9] denoted as K=M ∪ N, membership functions defined as in equation (2):

$$\text{T-membership: } T_K(x) = \max(T_M(x), T_N(x))$$
$$\text{I-membership: } I_K(x) = \min(I_M(x), I_N(x)) \quad (2)$$
$$\text{F-membership: } F_K(x) = \min(F_M(x), F_N(x))$$

2.7 Intersection

The intersection of two SVN set M and N [9], denoted as K=M ∩ N, is a NSSV with membership functions defined as in equation (3):

$$\text{T-membership: } T_K(x) = \min(T_M(x), T_N(x))$$
$$\text{I-membership: } I_K(x) = \max(I_M(x), I_N(x)) \quad (3)$$
$$\text{F-membership: } F_K(x) = \max(F_M(x), F_N(x))$$

3. Mathematical model for NGP Problems

NGP problem is essentially an extension of a traditional geometric programming problem, sharing the same fundamental structure of an objective function, general constraints, and decision variables. The key distinction, however, is that an NGP problem incorporates at least one neutrosophic geometric function which can serve as neutrosophic objective function, neutrosophic constraints, or even all of them. Consequently, solving these types of problems requires the development of various specialized NGP optimization models to effectively handle the unique neutrosophic components [11].

3.1 Unconstrained NGP Optimization Model

Based on the principles of NGP, as equation (4) an unconstrained NGP problem in n decision variables is an optimization problem that seeks to minimize or maximize a neutrosophic objective function without any constraints [12].

$$\boldsymbol{f}(\mathbf{x}) = \sum_{j=1}^{N} \boldsymbol{U_j}(\boldsymbol{x}) = \sum_{j=1}^{N} (\widetilde{\boldsymbol{C}_j}) \prod_{i=1}^{n} \boldsymbol{x}_i^{a_{ij}} \quad (4)$$

Where:

$x_i \geq 0, \quad i = 1, \dots n,$

x_i: integer for $i \in \{0,1, \dots n\}$.

$(\widetilde{\boldsymbol{C}_j})$: is neutrosophic numbers.

3.2 Constrained NGP Optimization Model

NGP problem with constraints is a specific type of optimization problem. As equations (5) Both the objective and constraints are defined using neutrosophic geometric functions.

Maximize

$$\boldsymbol{f}(\mathbf{x}) = \boldsymbol{g_o}(\mathbf{x}) = \sum_{j=1}^{N} (\widetilde{\boldsymbol{C}_j}) \prod_{i=1}^{n} \boldsymbol{x}_i^{a_{ij}} \quad (5)$$

Subject to

$$g_k(\mathbf{x}) = \sum_{j=1}^{N}\left(\widetilde{a_{ij}}^{n}\prod_{i=1}^{n} x_i^{a_{ij}}\right) \le b_i$$

$x_j \ge 0$, i = 1, 2,….,m; j = 1, 2, ……,n

x_j Integer for $j \in \{0,1, \dots n\}$.

Where $\widetilde{c_j}$, $\tilde{a}_{ij}^{n}$ are neutrosophic numbers.

The single valued neutrosophic sets ($a_{ij}\tilde{n}$) is expressed by A=(l,m,n) where l,m,n ∈ [0,1] and l,m,n ≤ 3

$\mathbf{T}\tilde{a}_{ij}^{n}(\mathbf{x})$ function of neutrosophic number $\tilde{a}_{ij}^{n}$ is expressed as equation (6):

$$\mathrm{T}\tilde{a}_{ij}^{n}(\mathrm{x}) = \begin{cases} \dfrac{x - l_1}{l_2 - l_1} & l_1 \le x \le l_2 \\ \dfrac{l_2 - x}{l_3 - l_2} & l_2 \le x \le l_3 \\ 0 & otherwise \end{cases} \tag{6}$$

$\mathbf{I}\tilde{a}_{ij}^{n}(\mathbf{x})$ function of neutrosophic number $\tilde{a}_{ij}^{n}$is expressed as equation (7):

$$\mathrm{I}\tilde{a}_{ij}^{n}(\mathrm{x}) = \begin{cases} \dfrac{x - m_1}{m_2 - m_1} & m_1 \le x \le m_2 \\ \dfrac{m_2 - x}{m_3 - m_2} & m_2 \le x \le m_3 \\ 0 & otherwise \end{cases} \tag{7}$$

$\mathbf{F}\tilde{a}_{ij}^{n}(\mathbf{x})$function of neutrosophic number $\tilde{a}_{ij}^{n}$ is expressed as equation (8):

$$\mathrm{F}\tilde{a}_{ij}^{n}(\mathrm{x}) = \begin{cases} \dfrac{x - n_1}{n_2 - n_1} & n_1 \le x \le n_2 \\ \dfrac{n_2 - x}{n_3 - n_2} & n_2 \le x \le n_3 \\ 1 & otherwise \end{cases} \tag{8}$$

As equations (9, 10), minimum and maximum for objective function can be expressed according to truth, indeterminacy and falsity membership

$$z^{max} = \max\{z(xi^*)\} \text{ and } z^{min} = \min\{z(xi^*)\}, \quad 1 \le i \le k \tag{9}$$

$$z^{F}_{min} = z^{T}_{min} \text{ and } z^{F}_{max} = z^{T}_{max} - \mathrm{P}\,(z^{T}_{max} - z^{T}_{min})$$

$$z^{I}_{max} = z^{I}_{max} \text{ and } z^{I}_{min} = z^{I}_{min} - \mathrm{Q}\,(z^{T}_{max} - z^{T}_{min}) \tag{10}$$

where P, Q ∈ (0, 1)

The acceptance, indeterminacy and falsity membership of objective function can bd defined as in equations 11, 12 and 13:

$$T^{z}(\mathrm{x}) = \begin{cases} 1 & if\, z \leq z^{min} \\ \dfrac{z^{max} - z(x)}{z^{max} - z^{min}} & z^{min} \leq z(x) \leq z^{max} \\ 0 & if z(x) > z^{max} \end{cases} \quad (11)$$

$$T^{z}(\mathrm{x}) = \begin{cases} 0 & if\, z \leq z^{min} \\ \dfrac{z(x) - z^{max}}{z^{max} - z^{min}} & z^{min} < z(x) \leq z^{max} \\ 0 & if\ z(x) > z^{max} \end{cases} \quad (12)$$

$$T^{z}(\mathrm{x}) = \begin{cases} 0 & if\, z \leq z^{min} \\ \dfrac{z(x) - z^{min}}{z^{max} - z^{min}} & z^{min} < z(x) \leq z^{max} \\ 1 & if\ z(x) > z^{max} \end{cases} \quad (13)$$

The neutrosophic set of the j^{th} decision variable a_i is expressed as equations 14, 15 and 16:

$$T_{x_j}^{(x)} = \begin{cases} 1 & if\ a_i \leq 0 \\ \dfrac{k_j - a_i}{k_j} & if\ 0 < a_i \leq k_j \\ 0 & if\ a_j > k_j \end{cases} \quad (14)$$

$$F_{x_j}^{(x)} = \begin{cases} 0 & if\ a_i \leq 0 \\ \dfrac{a_i}{k_j - l_j} & if\ 0 < a_i \leq k_j \\ 1 & if\ a_i > k_j \end{cases} \quad (15)$$

$$I_{j}^{(x)} = \begin{cases} 0 & if\ a_i \leq 0 \\ \dfrac{a_2 - a}{k_j - l_j} & if\ 0 < a_i \leq k_j \\ 0 & if\ a_i > k_j \end{cases} \quad (16)$$

where k_j, l_j are integer numbers.

4. NGP Problems Solution

The exploration of NGP problems aims to provide a robust framework for optimization under indeterminacy. Unlike classical GP, NGP accounts for incomplete, indeterminate, and inconsistent information inherent in real case scenarios by employing neutrosophic numbers. The general approach typically involves transforming the NGP problem into an equivalent crisp (classical) GP problem through various defuzzification or score function methods. This transformation allows for the application of established optimization techniques, such as duality theory and condensation methods, ultimately providing a more comprehensive and adaptable solution in an uncertain environment [13-14].

4.1 Unconstrained NGP Solution

Using equation (17)

$$\boldsymbol{z}(\mathbf{x}) = \sum_{j=1}^{N} \boldsymbol{U}_j(\boldsymbol{x}) = \sum_{j=1}^{N} (\widetilde{\boldsymbol{C}}_j) \prod_{i=1}^{n} \boldsymbol{x}_i^{a_{ij}} \tag{17}$$

Where x_{ij}, $c_j > 0$. The minima or maxima of a function can be expressed as equation (18), which states that the differentiation of function = 0 occurs.

$$\frac{\partial z}{\partial x_i} = 0 \tag{18}$$

The orthogonality condition gets the solution as defined in equation (19).

$$\sum_{j=1}^{N} w_j^* a_{ij} = 0 \tag{19}$$

And the normality condition as in equation (20)

$$\sum_{j=1}^{N} w_j^* = 1 \tag{20}$$

Where:

$$w_j^* = \frac{U_j(x^*)}{z^*}$$

The optimal objective function can be write as equation (21):

$$z^* = \left(\frac{U_1^*}{w_1^*}\right)^{w_1^*} \left(\frac{U_2^*}{w_2^*}\right)^{w_2^*} \dots \dots \dots . \left(\frac{U_n^*}{w_n^*}\right)^{w_n^*} \tag{21}$$

the orthogonality and normality equations can be solved using w_j^* values.

We can calculate the degree of difficulty in GP using equation DD = k – (m+1)

DD: refer Difficulty Degree.

m: refer to Decision Variables.

k: refer to Objective Function terms as Posynomial.

If the problem has one solution this is called zero difficulty. While the problem has a positive number of solutions this can express variables in many terms of variables to represent solution. The degree of difficulty negative is not allowed in GP.

Using z^* and $U_J{}^*$, we can get optimal solution for decision variables using equation (22)

$$\boldsymbol{U_j^*} = \boldsymbol{w_j^*}\, \boldsymbol{z^*} = \boldsymbol{c_j} \prod_{i=1}^{n} \boldsymbol{x_i^{a_{ij}}} \tag{22}$$

For a problem with zero DD, the equation can be reduced as equation (23)

$$\ln \frac{\boldsymbol{w_1^*\, z^*}}{\boldsymbol{c_j}} = \boldsymbol{a_{1j}} \ln \boldsymbol{x_1^*} + \boldsymbol{a_{2j}} \ln \boldsymbol{x_2^*} + \ldots\ldots\ldots\ldots\ldots\ldots + \boldsymbol{a_{nj}} \ln \boldsymbol{x_n^*} \tag{23}$$

The design variables can be obtained as equation (24)

$$\begin{gathered} \boldsymbol{k_i} = \ln \boldsymbol{x_i^*} \\ x_i^* = e^{k_i} \end{gathered} \tag{24}$$

4.2 Constrained NGP Solution

Using equation (25)

$$\boldsymbol{f}(\mathbf{x}) = \boldsymbol{g_o}(\mathbf{x}) = \sum_{j=1}^{N} (\widetilde{\boldsymbol{C}_J}) \prod_{i=1}^{n} \boldsymbol{x_i^{a_{ij}}} \tag{25}$$

S.t

$$\boldsymbol{g_k}(\mathbf{x}) = \sum_{j=1}^{N} \left(\widetilde{\boldsymbol{a_{ij}}}^{n} \prod_{i=1}^{n} \boldsymbol{x_i^{a_{ij}}} \right) \leq \mathbf{1}$$

The standard form of primal problems and dual problems are expressed as equation (26):

$$\prod_{k=0}^{m} \prod_{j=1}^{N} \left(\left(\frac{c_{kj}}{w_{kj}} \right) \sum_{J=1}^{N} w_{kI} \right)^{w_{kI}} \tag{26}$$

S.t

$$\sum_{k=0}^{m}\sum_{j=1}^{N} a_{kij} w_{kl} = 0$$

$$\sum_{j=1}^{N} w_{kj} = 0 , \qquad k = 0 \tag{27}$$

The standard form can be defined as equation (28)

$$g_k(x) \le v(x)$$
$$\frac{g_k(x)}{v(x)} \le 1 \tag{28}$$

5. The proposed approach steps

Step 1: Use score and accuracy functions as equations (29,30) to transform NGP problem into a crisp model using triangular neutrosophic numbers. According to definition $\tilde{x}$ = ⟨(k_1, m_1, n_1), $w\tilde{x}$, $u\tilde{x}$, $y\tilde{x}$ ⟩ triangular neutrosophic number can be expressed as:

$$\text{score } (\tilde{x}) = \frac{1}{16}[\text{k + m + n}] \times (2 + \mu\tilde{x} - v\tilde{x} - \lambda\tilde{x}) \tag{29}$$

$$\text{accuracy } (\tilde{x}) = \frac{1}{16}[\text{k + m + n}] \times (2 + \mu\tilde{x} - v\tilde{x} + \lambda\tilde{x}) \tag{30}$$

Using score & accuracy degrees of $\tilde{x}$. NGP converted into crisp model using equations (29, 30).
Step 2: Construct a decision by selecting the greatest T-membership degree, I- membership and the lowest F-membership degree.
Step 3: Use equations from 25-28 to solve GP problems.
Step 4: Use roundoff technique to get optimal solution as integer decision variables. If it is, the solution is valid, else use Lingo 18 to get optimal solution.
Step 5: Otherwise return to Step 1 and repeat the process.

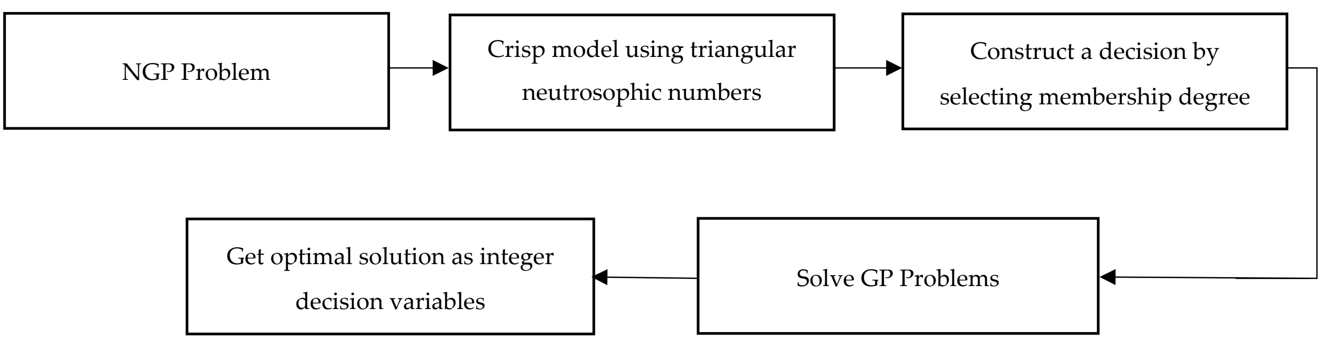

Figure 2. The proposed approach steps

6. Numerical Examples

6.1 Unconstrained problem

Illustrative Example (1):

$$\text{Minimize} \qquad z = \frac{\tilde{5}}{x_1\, x_2^3} + \frac{\tilde{3}\, x_1^2\, x_2}{x_3^2} + \frac{\tilde{4}\, x_1\, x_2^4}{x_3} + \tilde{1}\, x_3$$

Where:

$\tilde{5} = \langle (4,5,6\ ), 0.80, 0.60, 0.40\ \rangle$

$\tilde{3} = \langle (2.5,3,3.5\ ), 0.75, 0.50, 0.30 \rangle$

$\tilde{4} = \langle (3.5,4,4.1\ ), 1, 0.50, 0.0\ \rangle$

$\tilde{1} = \langle (0,1,2\ ), 1, 0.50, 0\ \rangle$

The neutrosophic model is then converted into a crisp model using the method outlined in Equation (30), as follows.

Minimize

$$z = \frac{5.6875}{x_1\, x_2^3} + \frac{3.5968\, x_1^2\, x_2}{x_3^2} + \frac{4.3125\, x_1\, x_2^4}{x_3} + 0.2815\, x_3$$

DD = 4 - (3+1) = 0 then a unique solution exists

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 & 0 \\ -3 & 1 & 4 & 0 \\ 0 & -2 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 5.6875 \\ 3.5968 \\ 4.3125 \\ 0.2815 \end{bmatrix}$$

Matrix form using normality and orthogonality conditions

$$\begin{bmatrix} -1 & 2 & 1 & 0 \\ -3 & 1 & 4 & 0 \\ 0 & -2 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 & 0 \\ -3 & 1 & 4 & 0 \\ 0 & -2 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

After we solve the above system, we get values of W:

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} \frac{7}{20} \\ \frac{1}{20} \\ \frac{1}{4} \\ \frac{7}{20} \end{bmatrix}$$

Substitute with the optimal values of w in equation (21) to get value of z*

$$z^* = \left(\frac{5.6875}{w_1}\right)^{w_1} \left(\frac{3.5968}{w_2}\right)^{w_2} \left(\frac{4.3125}{w_3}\right)^{w_3} \left(\frac{0.2815}{w_4}\right)^{w_4} = 6.2$$

$$\begin{bmatrix} -1 & -3 & 0 \\ 2 & 1 & -2 \\ 1 & 4 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{bmatrix} = \begin{bmatrix} \ln \frac{6.2049 \times \frac{7}{20}}{5.6875} \\ \ln \frac{6.2049 \times \frac{1}{20}}{3.5968} \\ \ln \frac{6.2049 \times \frac{1}{4}}{4.3125} \\ \ln \frac{6.2049 \times \frac{7}{20}}{0.2815} \end{bmatrix}$$

As equation (24) values of k_i is used for finding values of design variables x_i^*

$$\begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \begin{bmatrix} 9.39 \\ -3.26 \\ 7.71 \end{bmatrix}$$

Table 1. *optimal solution for Illustrative Example (1)*

	Continuous optimum	**Round off**	**Integer optimum**
x_1	9.39	9	3
x_2	−3.26	1 (0 not allowed)	1
x_3	7.71	8	8
z^*	6.2	11.8	6.4

The optimal integer solutions as Table 1 are:

$$\begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 8 \end{bmatrix}$$

6.2 Constrained problem

Application Example (2):

Application of Neutrosophic Optimization in Gravel box Design Problem [15].

Transport cost: $\frac{\widetilde{25}}{x_1 x_2 x_3}$, Material cost: $\widetilde{48}\, x_2\, x_3$. So, the problem

$$\text{Min} \quad f(x) = g_0(x) = \frac{\widetilde{25}}{x_1 x_2 x_3} + \widetilde{48}\, x_2\, x_3$$

subject to

$$g_1(x) = \frac{1}{2}\, x_1\, x_3 + \frac{1}{4}\, x_1\, x_2 \le 1$$

Where:

$\widetilde{25} = \langle (19,25,33), 0.80, 0.50, 0.0 \rangle$;

$\widetilde{48} = \langle (44,48,54), 0.90, 0.50, 0.0 \rangle$

The neutrosophic coefficient converted to a crisp using the method specified in equation (30). This conversion process is typically achieved by using a defined ranking method, such as a score function or an accuracy function, to transform the neutrosophic numbers into a single, definite value.

Minimize

$$f(x) = g_0(x) = \frac{27.9}{x_1 x_2 x_3} + 55.3\, x_2\, x_3$$

subject to

$$g_1(x) = 0.5\, x_1\, x_3 + 0.25\, x_1\, x_2 \le 1$$

DD = 3 - (2+1) = 0.

$$f^* = \left(\frac{27.9}{w_1}\right)^{w_1} \left(\frac{55.3}{w_2}\right)^{w_2} \left(\frac{0.5}{w_3}\right)^{w_3} \left(\frac{0.25}{w_4}\right)^{w_4} (w_3 + w_4)^{w_3 + w_4}$$

Matrix form of normality and orthogonality can be expressed as

$$\begin{bmatrix} -1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

After we solve the above system, we get values of W

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

Substitute as equation (21) to get the optimal value *f**

$$f^* = \left(\frac{27.9}{w_1}\right)^{w_1} \left(\frac{55.3}{w_2}\right)^{w_2} \left(\frac{0.5}{w_3}\right)^{w_3} \left(\frac{0.25}{w_4}\right)^{w_4} (w_3 + w_4)^{w_3+w_4} = 52.35$$

Design variables x_1 and x_2 can be calculated from the equations:

$$U_1^* = W_1 f^* = 34.9 = 27.9\, x_1^{-1}\, x_2^{-1} x_3^{-1}$$

$$U_2^* = W_2 f^* = 17.45 = 55.3\, x_2\, x_3$$

This gives:

$$\begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \begin{bmatrix} 2.534 \\ 1 \\ 0.3155 \end{bmatrix}$$

Table2. *optimal solution for Illustrative Example (2)*

	Continuous optimum	**Round off**	**Integer optimum**
x_1	2.534	3	**1**
x_2	1	1	**1**
x_3	0.3155	1	**1**

f^*	52.35	Infeasible	**83.2**

The optimal integer solutions as table 2 are:

$$\begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Illustrative Example (3):

Minimize

$$f(x) = g_0(x) = \frac{30}{x_1 x_2 x_3} + 30\, x_2\, x_3$$

$s.t$

$$g_1(x) = \tilde{4}\, x_1\, x_3 + \tilde{3}\, x_1\, x_2 \leq 1$$

Where:

$\tilde{4} = \langle (3.5,4,4.1\,), 1, 0.50, 0.0\,\rangle;$

$\tilde{3} = \langle (2.5,3,3.5\,), 0.75, 0.50, 0.25\,\rangle$

The neutrosophic expressed to a crisp as in Equation (30). This conversion process is typically achieved by using a defined ranking method.

Minimize

$$f(x) = g_0(x) = \frac{30}{x_1 x_2 x_3} + 30\, x_2\, x_3$$

$subject\ to$

$$g_1(x) = 4.31\, x_1\, x_3 + 3.63\, x_1\, x_2 \leq 1$$

DD = 3 - (2+1) = 0, so the minimization problem can be written in dual form as equation (21)

$$f^* = \left(\frac{30}{w_1}\right)^{w_1} \left(\frac{30}{w_2}\right)^{w_2} \left(\frac{4.31}{w_3}\right)^{w_3} \left(\frac{3.63}{w_4}\right)^{w_4} (w_3 + w_4)^{w_3 + w_4}$$

Matrix form using normality and orthogonality conditions

$$\begin{bmatrix} -1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$$

Substitute to get optimal value f^*

$$f^* = \left(\frac{30}{w_1}\right)^{w_1} \left(\frac{30}{w_2}\right)^{w_2} \left(\frac{4.31}{w_3}\right)^{w_3} \left(\frac{3.63}{w_4}\right)^{w_4} (w_3 + w_4)^{w_3+w_4} = 225$$

Design variables x_1 and x_2 can be calculated from the equations:

$$U_1^* = W_1 f^* = 150 = \frac{30}{x_1 x_2 x_3}$$

$$U_2^* = W_2 f^* = 75 = 30\, x_2\, x_3$$

This gives

$$\begin{bmatrix} x_1^* \\ x_2^* \\ x_3^* \end{bmatrix} = \begin{bmatrix} 0.08 \\ 1.581 \\ 1.581 \end{bmatrix}$$

Table 3. *optimal solution for Illustrative Example (3)*

	Continuous optimum	**Round off**	**Integer optimum**
x_1	0.08	1 (0 not allowed)	No integer feasible solution
x_2	1.581	2	
x_3	1.581	2	
f^*	225	Infeasible	

The optimal integer solutions are No feasible solution

7. Conclusion

Nowadays, neutrosophic environments are becoming very important research topics because they are a good tool for solving uncertainty problems. This paper proposes an approach for solving (NGP) problems under the three levels of truth, indeterminacy, and falsity. The approach was applied to two models; the first is an unconstrained optimization problem, and the second is a constrained optimization problem with integer variables in both cases. We increase the degree of truth and decrease indeterminacy and falsity as much as possible. Our approach depends on converting NGP using a crisp model with acceptance, indeterminacy and falsity membership and score functions. Finally, the efficiency of this new approach was validated through several numerical examples. In the future studies, the proposed approach can be solved by metaheuristic algorithms. Also, it can be developed to solve a lot of real-world problems such as healthcare and medical diagnosis (like breast cancer diagnosis), industrial and engineering design (like supply chain management and truss structure design), and environmental and sustainability (like water resource allocation). The paper validates its method with numerical examples, but it does not explicitly discuss the computational complexity or scalability of the approach when applied to large-scale, highly complex real-world problems.

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