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Multiparameterized Hypersoft Set and Type-2 Hypersoft Set

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Abstract

To model real-world uncertainty, various mathematical frameworks have been proposed, including Fuzzy Sets [78,79], Hyperfuzzy Sets [29,34,45], Intuitionistic Fuzzy Sets [7,8], Neutrosophic Sets [66,67], Plithogenic Sets [25, 68], and Rough Sets [57–59]. Soft sets provide a robust mathematical framework for decision-making by associating parameters with subsets of a universal set, thereby effectively addressing uncertainty and imprecision [46, 49]. Over time, various extensions of soft sets—such as Hypersoft Sets [41], SuperHypersoft Sets [62], and Treesoft Sets [5]—have been introduced to model increasingly complex decision-making environments.

This paper introduces and rigorously formalizes the concepts of the Multiparameterized Hypersoft Set and the Type-2 Hypersoft Set. These models naturally extend the classical Multiparameterized Soft Set and Type-2 Soft Set, respectively, enhancing their ability to represent multi-dimensional and hierarchical uncertainty in practical applications.

Keywords: Superhypersoft set, Soft Set, Hypersoft set, Multiparameterized Soft Set, Type-2 Soft Set

1 Introduction

Modeling many problems in the real world usually involves addressing vague, imprecise, or doubtful data, especially in a lot of complicated decision-making problems [**?**]. For many years, numerous mathematical paradigms have been advanced to depict this data, remarkably plithogenic sets [68], rough sets [57], neutrosophic sets [66], intuitionistic fuzzy sets [8], and fuzzy sets [78]. Amidst these, soft set theory, proposed by Molodtsov [49], has shown to be an intuitive framework and particularly flexible for handling uncertainty. In contrast with fuzzy sets, which depend on membership functions, soft sets connect parameters with subsets of a universal set, therefore providing a parameterization-based scheme to model indeterminacy.

Continuing soft set theory, numerous expanded models have been established to address multi-parameter and multi-criteria problems more advantageously. A notable extension is the hypersoft set, which makes a generalization to the soft sets by combining combinations of multiple parameters (attributes) and mapping these to subsets of the universe [41, 69]. This enables hypersoft sets to address further challenging scenarios where specific attributes might not be taken into account independently.

Due to the investigation into soft computing and how it is developed, it has become evident that even hypersoft sets had restrictions in addressing hierarchical uncertainty structures and higher-order interactions. In order to settle these restrictions, superhypersoft sets were proposed by Fujita and Smarandache [32]. Such sets enable mappings from power set combinations of attribute domains to subsets of the universe. Accordingly, catching abundant interdependencies and allowing deeper modeling of decision systems with composite, overlapping, or nested criteria. This reflects analogous advancements in type-2 soft set theory [16], where, at multiple levels, the uncertainty is modeled.

To enhance the modeling of such complex relationships, numerous other generalizations have been established, including plithogenic hypersoft models [2, 69] and multiparameterized soft sets [63]. These aim to offer data analysts and decision-makers better diagrammatical power in the considered areas, including information sciences, systems analysis, and artificial intelligence.

In this work, we additionally expand the existing body of work by proposing and analyzing *Multiparameterized Hypersoft Sets* and *Type-2 Hypersoft Sets*, together with their *SuperHyperSoft* counterparts. Such developments act as natural extensions of conventional soft set variants, capable of catching high-dimensional uncertainty with improved granularity. The established models also unify and generalize former structures, providing a powerful mathematical apparatus for theoretical analysis and real-world implementations.

2 Preliminaries and Definitions

In this section, we introduce the fundamental concepts of Soft Sets, Hypersoft Sets, and SuperHypersoft Sets. Throughout this paper, all sets are assumed to be finite. For the basic operations associated with each concept, the reader is referred to the respective references.

2.1 Soft Sets

A Soft Set provides a versatile approach to parameter-driven decision analysis by mapping attributes (parameters) to subsets of a universal set. This structure offers a powerful mechanism for addressing uncertainty and imprecision in complex decision-making processes [4, 18, 35, 43, 44, 46, 48, 49]. The definition is stated as follows.

Definition 2.1 (Set)**.** [42] A *set* is a well-defined collection of distinct elements or objects. If a is an element of a set A, we write $a \in A$; otherwise, we write $a \notin A$.

Definition 2.2 (Subset)**.** [42] Let A and B be sets. A is called a *subset* of B, denoted $A \subseteq B$, if every element of A is also an element of B. If $A \subseteq B$ but $A \neq B$, then A is called a *proper subset* of B, denoted $A \subset B$.

Definition 2.3 (Empty Set)**.** [42] The *empty set*, denoted by $\emptyset$, is the unique set containing no elements. Formally, for any set A, $\emptyset \subseteq A$.

Definition 2.4 (Power Set)**.** [42] Let A be a set. The *power set* of A, denoted by $\mathcal{P}(A)$, is the set of all subsets of A. That is,

$$\mathcal{P}(A) = \{X \mid X \subseteq A\}.$$

Definition 2.5 (Universal Set)**.** A *universal set*, denoted by U, is the set that contains all elements under consideration in a particular context. Every set discussed is assumed to be a subset of U.

Definition 2.6 (Cartesian Product)**.** (cf. [40]) Let A and B be sets. The *Cartesian Product* $A \times B$ is defined as

$$A \times B \;=\; \{\,(a,b) \;\mid\; a \in A,\; b \in B\},$$

i.e., the set of all ordered pairs (a, b) where a comes from A and b comes from B.

Example 2.7 (Example of Cartesian Product)**.** Let

$$A = \{1, 2, 3\} \quad \text{and} \quad B = \{x, y\}.$$

Then the Cartesian product is

$$A \times B = \{\,(1,x),\ (1,y),\ (2,x),\ (2,y),\ (3,x),\ (3,y)\,\}.$$

Definition 2.8 (Soft Set)**.** [49] Let U be a finite universal set and A be a set of attributes. Let $S \subseteq A$ denote a chosen subset of parameters. A *soft set* over U is defined as a pair $(\mathcal{F}, S)$ where

$$\mathcal{F} : S \to \mathcal{P}(U)$$

is a function that assigns to each parameter $\alpha \in S$ a subset $\mathcal{F}(\alpha) \subseteq U$. Formally,

$$(\mathcal{F}, S) = \{\,(\alpha, \mathcal{F}(\alpha)) \mid \alpha \in S,\ \mathcal{F}(\alpha) \subseteq U\,\}.$$

Example 2.9 (Example of a Soft Set)**.** Let the universal set be

$$U = \{\text{House1, House2, House3}\}.$$

Let the set of attributes be

$$A = \{\text{Expensive, Cheap}\}.$$

Choose the subset $S = A$. Define the mapping

$$\mathcal{F} : S \to \mathcal{P}(U)$$

by setting:

$$\begin{aligned} \mathcal{F}(\text{Expensive}) &= \{\text{House1, House3}\}, \\ \mathcal{F}(\text{Cheap}) &= \{\text{House2}\}. \end{aligned}$$

Thus, the soft set over U is

$$(\mathcal{F}, S) = \{\,(\text{Expensive}, \{\text{House1, House3}\}),\ (\text{Cheap}, \{\text{House2}\})\,\}.$$

2.2 Hypersoft Sets

A Hypersoft Set enhances multi-attribute decision analysis by associating combinations of multiple attributes with subsets of a universal set, enabling a more nuanced and comprehensive evaluation [2, 50–55, 69]. The definition is stated as follows.

Definition 2.10 (Hypersoft Set)**.** [69] Let U be a finite universal set and let $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_m$ be m distinct attribute domains. Define the Cartesian product

$$C = \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_m,$$

so that each element $\gamma \in C$ is an m-tuple

$$\gamma = (\gamma_1, \gamma_2, \ldots, \gamma_m) \quad \text{with } \gamma_i \in \mathcal{A}_i \text{ for } i = 1, 2, \ldots, m.$$

A *hypersoft set* over U is a pair (G, C) where

$$G : C \to \mathcal{P}(U)$$

assigns to each $\gamma \in C$ a subset $G(\gamma) \subseteq U$. Formally,

$$(G, C) = \{\, (\gamma, G(\gamma)) \mid \gamma \in C,\ G(\gamma) \subseteq U \,\}.$$

Example 2.11 (Hypersoft Set with Computer Memory Configurations)**.** Computer memory refers to physical devices used to store data or instructions temporarily or permanently, enabling efficient processing, retrieval, and execution by a computer system (cf. [39, 65]). Let the finite universal set be the set of laptop models:

$$U = \{\text{Laptop 1, Laptop 2, Laptop 3, Laptop 4, Laptop 5}\}.$$

Consider two attribute domains corresponding to computer hardware specifications:

$$\mathcal{A}_1 = \{\text{8GB, 16GB}\} \quad \text{(Memory Capacity)}$$

and

$$\mathcal{A}_2 = \{\text{SSD, HDD}\} \quad \text{(Storage Type)}.$$

The Cartesian product is then

$$C = \mathcal{A}_1 \times \mathcal{A}_2 = \{(\text{8GB}, SSD),\ (\text{8GB}, HDD),\ (\text{16GB}, SSD),\ (\text{16GB}, HDD)\}.$$

Define a mapping

$$G : C \to \mathcal{P}(U)$$

by specifying:

$$\begin{aligned} G(\text{8GB}, SSD) &= \{\text{Laptop 1, Laptop 3}\}, \\ G(\text{8GB}, HDD) &= \{\text{Laptop 2}\}, \\ G(\text{16GB}, SSD) &= \{\text{Laptop 4}\}, \\ G(\text{16GB}, HDD) &= \{\text{Laptop 5}\}. \end{aligned}$$

Thus the hypersoft set is the pair

$$(G, C) = \{\, ((\gamma_1, \gamma_2),\ G(\gamma_1, \gamma_2)) \mid (\gamma_1, \gamma_2) \in C,\ G(\gamma_1, \gamma_2) \subseteq U \,\}.$$

In this example, each ordered pair—for instance, $(\text{8GB}, SSD)$—is assigned the set of laptops that feature that specific configuration.

2.3 SuperHypersoft Sets

SuperHypersoft Sets extend the concept of Hypersoft Sets by mapping power set combinations of multiple attributes to subsets of a universal set. This extension supports high-dimensional decision-making and captures intricate interdependencies among attributes, offering significant flexibility for addressing advanced decision-making challenges [21–24, 26–28, 30–33, 73]. The definition is stated as follows.

Definition 2.12 (SuperHypersoft Set)**.** [32,70] Let U be a finite universal set and let $a_1, a_2, \ldots, a_n$ be n distinct attributes with corresponding finite sets of values $A_1, A_2, \ldots, A_n$ that are pairwise disjoint:

$$A_i \cap A_j = \emptyset \quad \text{for all } i \neq j.$$

For each i, let $\mathcal{P}(A_i)$ denote the power set of A_i. Define the Cartesian product

$$C = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \cdots \times \mathcal{P}(A_n).$$

A *SuperHypersoft Set* over U is a pair (F, C) where

$$F : C \to \mathcal{P}(U)$$

assigns to each element $\gamma = (\alpha_1, \alpha_2, \ldots, \alpha_n) \in C$ (with $\alpha_i \in \mathcal{P}(A_i)$) a subset $F(\gamma) \subseteq U$. Formally,

$$(F, C) = \{\, (\gamma, F(\gamma)) \mid \gamma \in C,\ F(\gamma) \subseteq U \,\}.$$

Example 2.13 (SuperHypersoft Set with Car Model Attributes)**.** Let the finite universal set be the set of car models:

U = {Toyota Camry(cf. [11]), Honda Accord(cf. [19]), Ford Focus(cf. [76]), BMW 3 Series (cf. [12]), Audi A4(cf. [75])}.

Assume we have two disjoint attribute sets:

$$A_1 = \{\text{Luxury, Economy}\} \quad \text{(Market Segment)}$$

and

$$A_2 = \{\text{Sedan, SUV}\} \quad \text{(Body Style)}.$$

Since SuperHypersoft Sets use power set combinations, form the power sets:

$$\mathcal{P}(A_1) = \Big\{\varnothing,\ \{\text{Luxury}\},\ \{\text{Economy}\},\ \{\text{Luxury, Economy}\}\Big\},$$

$$\mathcal{P}(A_2) = \Big\{\varnothing,\ \{\text{Sedan}\},\ \{\text{SUV}\},\ \{\text{Sedan, SUV}\}\Big\}.$$

Then, define the Cartesian product

$$C = \mathcal{P}(A_1) \times \mathcal{P}(A_2).$$

An element $\gamma \in C$ is a pair (B_1, B_2) with $B_1 \subseteq A_1$ and $B_2 \subseteq A_2$. Now define a mapping

$$F : C \to \mathcal{P}(U)$$

by assigning values to selected elements of C as follows:

$$\begin{aligned}
F\Big(\{\text{Economy}\},\ \{\text{Sedan}\}\Big) &= \{\text{Toyota Camry, Honda Accord}\},\\
F\Big(\{\text{Luxury}\},\ \{\text{Sedan}\}\Big) &= \{\text{BMW 3 Series, Audi A4}\},\\
F\Big(\{\text{Economy}\},\ \{\text{SUV}\}\Big) &= \{\text{Ford Focus}\},\\
F\Big(\{\text{Luxury}\},\ \{\text{SUV}\}\Big) &= \varnothing.
\end{aligned}$$

For any other $\gamma \in C$ not explicitly assigned above, one may define $F(\gamma) = \varnothing$ (or assign other suitable subsets of U). Consequently, the SuperHypersoft Set is the pair

$$(F, C) = \{\, (\gamma, F(\gamma)) \mid \gamma \in C,\ F(\gamma) \subseteq U \,\}.$$

This example encodes more complex attribute combinations than a hypersoft set. For instance, the combination ({Economy}, {Sedan}) is associated with the set {Toyota Camry, Honda Accord}, whereas the combination ({Luxury}, {SUV}) yields no car model from the given universal set.

2.4 Multiparameterized Soft Set

The definition is stated as follows [1, 63].

Definition 2.14 (Multiparameterized Soft Set)**.** [63] Let U be a universal set and let $\{E_i\}_{i\in I}$ be a collection of parameter sets such that

$$\bigcap_{i\in I} E_i \neq \emptyset.$$

Define

$$E = \bigcup_{i\in I} E_i \quad \text{and} \quad \mathcal{P}(E) = \{\, X : X \subseteq E \,\}.$$

If A is a nonempty subset of $\mathcal{P}(E)$ and if

$$F : A \to \mathcal{P}(U)$$

is a mapping, then the pair (F, A) is called a *multiparameterized soft set* over U. In this formulation, each $\alpha \in A$ represents a (possibly composite) set of parameters and $F(\alpha)$ is viewed as the set of all α-approximate elements of U.

Example 2.15 (Multiparameterized Soft Set in a Housing Decision Context)**.** Housing decision refers to the process of choosing where and how to live, based on factors like cost, location, and lifestyle needs [61, 77]. Consider the following setting:

$$U = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9, h_{10}\},$$

which represents 10 houses under consideration.

Define three parameter sets:

$$\begin{aligned}
E_1 &= \{e_{1,1}, e_{1,2}, e_{1,3}, e_{1,4}\}, \quad \text{with} \\
&\quad e_{1,1} = \text{“expensive”}, \quad e_{1,2} = \text{“cheap”}, \\
&\quad e_{1,3} = \text{“very expensive”}, \quad e_{1,4} = \text{“very cheap”}; \\
E_2 &= \{e_{2,1}, e_{2,2}, e_{2,3}, e_{2,4}, e_{2,5}\}, \quad \text{with} \\
&\quad e_{2,1} = \text{“Subang Jaya”}, \quad e_{2,2} = \text{“Klang”}, \\
&\quad e_{2,3} = \text{“Ampang”}, \quad e_{2,4} = \text{“Shah Alam”}, \quad e_{2,5} = \text{“Kajang”}; \\
E_3 &= \{e_{3,1}, e_{3,2}, e_{3,3}, e_{3,5}\}, \quad \text{with} \\
&\quad e_{3,1} = \text{“green”}, \quad e_{3,2} = \text{“dark green”}, \\
&\quad e_{3,3} = \text{“blue”}, \quad e_{3,5} = \text{“red”}.
\end{aligned}$$

Let

$$E = E_1 \cup E_2 \cup E_3 \quad \text{and} \quad A \subseteq \mathcal{P}(E)$$

be chosen so that each element of A represents a specific combination of parameters to be considered.

For instance, define

$$A = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7\}.$$

Now, suppose we define the mapping $F : A \to \mathcal{P}(U)$ by

$$\begin{aligned}
F(\alpha_1) &= \{h_2,\ h_3,\ h_6,\ h_7,\ h_{10}\}, \\
F(\alpha_2) &= \{h_6,\ h_7\}, \\
F(\alpha_3) &= \emptyset, \\
F(\alpha_4) &= \{h_1,\ h_8,\ h_9\}, \\
F(\alpha_5) &= \{h_8\}, \\
F(\alpha_6) &= \emptyset, \\
F(\alpha_7) &= \{h_9\}.
\end{aligned}$$

Here, each $F(\alpha_i)$ represents the subset of houses that approximately satisfy the composite parameter α_i (which might, for example, encode a combination of cost, location, and color preferences).

Thus, (F, A) constitutes a multiparameterized soft set over U that can be used, for example, to represent the attractiveness of houses based on multiple decision parameters.

2.5 Type-2 Soft Set

The definition is stated as follows [15, 16, 36–38, 47, 64].

Definition 2.16 (Type-2 Soft Set)**.** [37] Let X be a universal set and let E be a set of parameters. Denote by $\sigma(X)$ the collection of all type-1 soft sets over X, where a type-1 soft set is a pair (S, L) with $L \subseteq E$ and

$$S : L \to \mathcal{P}(X).$$

Let $A \subseteq E$ be a nonempty set of *primary parameters*. A *type-2 soft set* over (X, E) is a mapping

$$S^* : A \to \sigma(X)$$

such that for each $\alpha \in A$, there exists a type-1 soft set (S_α, L_α) with $L_\alpha \subseteq E$ satisfying

$$S^*(\alpha) = (S_\alpha, L_\alpha),$$

where

$$S_\alpha : L_\alpha \to \mathcal{P}(X).$$

We denote the type-2 soft set by $[S^*, A]$. In this setting, A is called the *primary set of parameters* and

$$\bigcup_{\alpha \in A} L_\alpha$$

is referred to as the *underlying set of parameters*.

Example 2.17 (Type-2 Soft Set in a Laptop Selection Scenario)**.** (cf. [9,71]) Let the universal set be a collection of laptops:

$$X = \{\text{Dell(cf. [56])}, \text{HP(cf. [14])}, \text{Lenovo(cf. [74])}\}.$$

Let the set of all parameters be:

$$E = \{\text{Price, Memory, OS}\}.$$

Choose the primary set of parameters as:

$$A = \{\text{Price, OS}\} \subseteq E.$$

For the primary parameter "Price": Set the underlying parameter set for "Price" as

$$L_{\text{Price}} = \{\text{High, Low}\}.$$

Define the mapping

$$S_{\text{Price}} : L_{\text{Price}} \to \mathcal{P}(X)$$

by assigning:

$$\begin{aligned} S_{\text{Price}}(\text{High}) &= \{\text{Dell, HP}\}, \\ S_{\text{Price}}(\text{Low}) &= \{\text{Lenovo}\}. \end{aligned}$$

Thus, the type-1 soft set corresponding to "Price" is $(S_{\text{Price}}, L_{\text{Price}})$.

For the primary parameter "OS": Set the underlying parameter set for "OS" as

$$L_{\text{OS}} = \{\text{Windows, Linux}\}.$$

Define the mapping

$$S_{\text{OS}} : L_{\text{OS}} \to \mathcal{P}(X)$$

by assigning:

$$\begin{aligned} S_{\text{OS}}(\text{Windows}) &= \{\text{Dell, HP, Lenovo}\}, \\ S_{\text{OS}}(\text{Linux}) &= \{\text{Lenovo}\}. \end{aligned}$$

Thus, the type-1 soft set corresponding to "OS" is $(S_{\text{OS}}, L_{\text{OS}})$.

Now, we define the type-2 soft set $[S^*, A]$ by setting:

$$S^*(\text{Price}) = (S_{\text{Price}}, L_{\text{Price}}) \quad \text{and} \quad S^*(\text{OS}) = (S_{\text{OS}}, L_{\text{OS}}).$$

Consequently, $[S^*, A]$ is a type-2 soft set over (X, E) which layers the decision information as follows:

- For the primary parameter "Price", the underlying parameters "High" and "Low" assign the laptops as:

$$S_{\text{Price}}(\text{High}) = \{\text{Dell, HP}\} \quad \text{and} \quad S_{\text{Price}}(\text{Low}) = \{\text{Lenovo}\}.$$

- For the primary parameter "OS", the underlying parameters "Windows" and "Linux" assign the laptops as:

$$S_{\text{OS}}(\text{Windows}) = \{\text{Dell, HP, Lenovo}\} \quad \text{and} \quad S_{\text{OS}}(\text{Linux}) = \{\text{Lenovo}\}.$$

This layered structure provides a nuanced decision model where each primary criterion (Price and OS) is further detailed by its own set of subcriteria.

3 Result of this Paper

This section presents the results of this paper.

3.1 Multiparameterized HyperSoft Sets

The definition is stated as follows.

Definition 3.1 (Multiparameterized HyperSoft Set)**.** Let U be a universal set and let $\{E_i\}_{i\in I}$ be a collection of parameter sets. Define

$$E = \bigcup_{i\in I} E_i \quad \text{and} \quad C = \prod_{i\in I} E_i,$$

the Cartesian product of the parameter sets. Let A be a nonempty subset of $\mathcal{P}(C)$ (that is, a collection of subsets of C). A *multiparameterized HyperSoft Set* over U is a pair (H, A) where

$$H : A \to \mathcal{P}(U)$$

assigns to each $\alpha \in A$ (with $\alpha \subseteq C$) a subset $H(\alpha) \subseteq U$.

Remarks:

- If we choose

$$A = \{\, \{\gamma\} \mid \gamma \in C\},$$

then the mapping H reduces (up to the obvious identification) to a function from C to $\mathcal{P}(U)$, and (H, A) coincides with a hypersoft set.

- If the index set I is a singleton, then $C \cong E$ and the definition of (H, A) reduces to that of a multiparameterized soft set.

Example 3.2 (Multiparameterized HyperSoft Set)**.** Let the universal set be

$$U = \{1, 2, 3, 4, 5, 6\}.$$

Consider two parameter sets:

$$E_1 = \{\text{red, blue}\} \quad \text{and} \quad E_2 = \{\text{small, large}\}.$$

The Cartesian product of E_1 and E_2 is

$$C = E_1 \times E_2 = \{(\text{red, small}),\ (\text{red, large}),\ (\text{blue, small}),\ (\text{blue, large})\}.$$

Now, choose a specific collection of subsets of C (these subsets represent composite parameter combinations):

$$A = \Big\{\{(\text{red, small}), (\text{blue, large})\},\ \{(\text{red, large})\},\ \{(\text{blue, small})\}\Big\} \subseteq \mathcal{P}(C).$$

Define a mapping

$$H : A \to \mathcal{P}(U)$$

by setting

$$\begin{aligned} H\big(\{(\text{red}, \text{small}), (\text{blue}, \text{large})\}\big) &= \{1, 3, 5\}, \\ H\big(\{(\text{red}, \text{large})\}\big) &= \{2, 4\}, \\ H\big(\{(\text{blue}, \text{small})\}\big) &= \{3, 6\}. \end{aligned}$$

Thus, the pair (H, A) is a *multiparameterized HyperSoft Set* over U. Notice that this structure generalizes both a hypersoft set (if we chose singletons from C) and a multiparameterized soft set (if the parameter set were not a Cartesian product but a single set).

Theorem 3.3. *Let U be a universal set, and let $\{E_i\}_{i\in I}$ be a collection of parameter sets with $C = \prod_{i\in I} E_i$. Consider a multiparameterized HyperSoft Set (H, A) over U with $A \subseteq \mathcal{P}(C)$. Then:*

1. *If $A = \{\{\gamma\} : \gamma \in C\}$, then (H, A) is equivalent to a hypersoft set.*
2. *If I is a singleton (so that $C \cong E$), then (H, A) reduces to a multiparameterized soft set.*

Proof. **(1)** Suppose $A = \{\{\gamma\} : \gamma \in C\}$. For each singleton $\{\gamma\} \in A$, define the mapping $G : C \to \mathcal{P}(U)$ by

$$G(\gamma) = H(\{\gamma\}).$$

Then the pair (G, C) satisfies the definition of a hypersoft set. Hence, in this case, the multiparameterized HyperSoft Set (H, A) generalizes the hypersoft set.

(2) Now assume that I is a singleton, i.e., $I = \{i_0\}$. Then the Cartesian product $C = \prod_{i\in I} E_i$ reduces to E_{i_0}, and we may denote it by E. Also, $A \subseteq \mathcal{P}(E)$. In this case, the pair (H, A) fits exactly the definition of a multiparameterized soft set.

Thus, in both cases, (H, A) generalizes the intended earlier concepts. □

3.2 Multiparameterized SuperHyperSoft Sets

The definition is stated as follows.

Definition 3.4 (Multiparameterized SuperHyperSoft Set)**.** Let U be a universal set and let $\{A_i\}_{i\in I}$ be a collection of pairwise disjoint attribute sets. For each i, denote by $\mathcal{P}(A_i)$ the power set of A_i. Define

$$\mathcal{D} = \prod_{i\in I} \mathcal{P}(A_i).$$

Let B be a nonempty subset of $\mathcal{P}(\mathcal{D})$ (that is, a collection of subsets of $\mathcal{D}$). A *multiparameterized SuperHyperSoft Set* over U is a pair (S, B) where

$$S : B \to \mathcal{P}(U)$$

assigns to each $\beta \in B$ (with $\beta \subseteq \mathcal{D}$) a subset $S(\beta) \subseteq U$.

Remarks:

- If we set
$$B = \{\{\delta\} \mid \delta \in \mathcal{D}\},$$
then the structure (S, B) reduces to a SuperHypersoft set.
- If in addition each A_i is replaced by an ordinary parameter set E_i (so that $\mathcal{P}(A_i)$ is (isomorphic to) E_i) then $\mathcal{D} \cong \prod_{i\in I} E_i$ and (S, B) becomes a multiparameterized HyperSoft Set.

Example 3.5 (Multiparameterized SuperHyperSoft Set)**.** Let the universal set be

$$U = \{a, b, c, d, e\}.$$

Assume we have two disjoint attribute sets:

$$A_1 = \{\text{low, high}\} \quad \text{and} \quad A_2 = \{\text{old, new}\}.$$

Their power sets are given by

$$\mathcal{P}(A_1) = \{\varnothing, \{\text{low}\}, \{\text{high}\}, \{\text{low, high}\}\}$$

and

$$\mathcal{P}(A_2) = \{\varnothing, \{\text{old}\}, \{\text{new}\}, \{\text{old, new}\}\}.$$

Form the Cartesian product of these power sets:

$$\mathcal{D} = \mathcal{P}(A_1) \times \mathcal{P}(A_2).$$

Now, choose a specific collection B of subsets of $\mathcal{D}$. For example, let

$$B_1 = \Big\{\big(\{\text{low}\}, \{\text{old}\}\big), \big(\{\text{low}\}, \{\text{new}\}\big)\Big\}$$

and

$$B_2 = \Big\{\big(\{\text{high}\}, \{\text{new}\}\big)\Big\}.$$

Thus, set

$$B = \{B_1, B_2\} \subseteq \mathcal{P}(\mathcal{D}).$$

Define a mapping

$$S : B \to \mathcal{P}(U)$$

by

$$\begin{aligned} S(B_1) &= \{a, b, d\}, \\ S(B_2) &= \{c, e\}. \end{aligned}$$

Then the pair (S, B) is a *multiparameterized SuperHyperSoft Set* over U. Notice that if we had chosen

$$B = \Big\{\{(\alpha_1, \alpha_2)\} : (\alpha_1, \alpha_2) \in \mathcal{D}\Big\},$$

the structure would collapse to a standard SuperHypersoft set. Moreover, by an appropriate identification of $\mathcal{P}(A_i)$ with a conventional parameter set, (S, B) reduces to a multiparameterized HyperSoft Set.

Example 3.6 (Multiparameterized SuperHyperSoft Set for House Construction)**.** Let the universal set be the set of houses under construction:

$$U = \{\text{House A, House B, House C, House D, House E}\}.$$

Consider three disjoint attribute sets relevant to house construction:

- **Architectural Style** A_1:
$$A_1 = \{\text{Modern, Traditional}\}.$$
- **Structural Material** A_2:
$$A_2 = \{\text{Wood, Concrete}\}.$$
- **Building Height** A_3:
$$A_3 = \{\text{Low-rise, High-rise}\}.$$

For each attribute set A_i, form its power set:

$$\mathcal{P}(A_1) = \Big\{\varnothing,\ \{\text{Modern}\},\ \{\text{Traditional}\},\ \{\text{Modern, Traditional}\}\Big\},$$
$$\mathcal{P}(A_2) = \Big\{\varnothing,\ \{\text{Wood}\},\ \{\text{Concrete}\},\ \{\text{Wood, Concrete}\}\Big\},$$
$$\mathcal{P}(A_3) = \Big\{\varnothing,\ \{\text{Low-rise}\},\ \{\text{High-rise}\},\ \{\text{Low-rise, High-rise}\}\Big\}.$$

Then, form the Cartesian product of these power sets:

$$\mathcal{D} = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \mathcal{P}(A_3).$$

An element of $\mathcal{D}$ is a triple (B_1, B_2, B_3) with $B_1 \subseteq A_1$, $B_2 \subseteq A_2$, and $B_3 \subseteq A_3$.

Now, consider the following specific attribute combinations:

$$\gamma_1 = \Big(\{\text{Modern}\},\ \{\text{Wood}\},\ \{\text{Low-rise}\}\Big),$$
$$\gamma_2 = \Big(\{\text{Traditional}\},\ \{\text{Concrete}\},\ \{\text{High-rise}\}\Big),$$
$$\gamma_3 = \Big(\{\text{Modern}\},\ \{\text{Concrete}\},\ \{\text{Low-rise}\}\Big),$$
$$\gamma_4 = \Big(\{\text{Traditional}\},\ \{\text{Wood}\},\ \{\text{Low-rise}\}\Big).$$

Let the collection of attribute combinations be

$$B = \Big\{\{\gamma_1\},\ \{\gamma_2\},\ \{\gamma_3\},\ \{\gamma_4\}\Big\} \subseteq \mathcal{P}(\mathcal{D}).$$

Define the mapping

$$S : B \to \mathcal{P}(U)$$

by assigning:

$$\begin{aligned} S\big(\{\gamma_1\}\big) &= \{\text{House A, House B}\},\\ S\big(\{\gamma_2\}\big) &= \{\text{House C, House D}\},\\ S\big(\{\gamma_3\}\big) &= \{\text{House B, House E}\},\\ S\big(\{\gamma_4\}\big) &= \{\text{House A, House E}\}. \end{aligned}$$

Thus, the pair (S, B) constitutes a *Multiparameterized SuperHyperSoft Set* over U.

In this model:

- The combination $\gamma_1 = (\{\text{Modern}\}, \{\text{Wood}\}, \{\text{Low-rise}\})$ represents houses built in a Modern style using Wood as the structural material with a Low-rise design, and it corresponds to {House A, House B}.
- The combination $\gamma_2 = (\{\text{Traditional}\}, \{\text{Concrete}\}, \{\text{High-rise}\})$ represents houses built in a Traditional style with Concrete structures and a High-rise design, and is associated with {House C, House D}.
- The combination $\gamma_3 = (\{\text{Modern}\}, \{\text{Concrete}\}, \{\text{Low-rise}\})$ describes houses with a Modern style, Concrete construction, and Low-rise design, mapping to {House B, House E}.
- The combination $\gamma_4 = (\{\text{Traditional}\}, \{\text{Wood}\}, \{\text{Low-rise}\})$ represents houses in a Traditional style built with Wood and having a Low-rise design, corresponding to {House A, House E}.

Theorem 3.7. *Let U be a universal set, and let $\{A_i\}_{i\in I}$ be pairwise disjoint attribute sets. Define*

$$\mathcal{D} = \prod_{i\in I} \mathcal{P}(A_i).$$

Consider a multiparameterized SuperHyperSoft Set (S, B) over U with $B \subseteq \mathcal{P}(\mathcal{D})$. Then:

1. *If $B = \{\{\delta\} : \delta \in \mathcal{D}\}$, then (S, B) is equivalent to a SuperHypersoft set.*
2. *If each A_i is replaced by a conventional parameter set E_i (so that $\mathcal{P}(A_i)$ is (isomorphic to) E_i), then $\mathcal{D} \cong \prod_{i \in I} E_i$ and (S, B) reduces to a multiparameterized HyperSoft Set.*

Proof. **(1)** Assume $B = \{\{\delta\} : \delta \in \mathcal{D}\}$. For each $\{\delta\} \in B$, define the mapping $F : \mathcal{D} \to \mathcal{P}(U)$ by

$$F(\delta) = S(\{\delta\}).$$

Then the pair $(F, \mathcal{D})$ satisfies the definition of a SuperHypersoft set. Hence, under this singleton restriction on B, the multiparameterized SuperHyperSoft Set (S, B) coincides with a SuperHypersoft set.

(2) Now suppose that for every $i \in I$ the attribute set A_i is replaced by a standard parameter set E_i (with the identification $\mathcal{P}(A_i) \cong E_i$). Then the Cartesian product

$$\mathcal{D} = \prod_{i \in I} \mathcal{P}(A_i)$$

becomes isomorphic to

$$\prod_{i \in I} E_i,$$

which is exactly the domain used in the definition of a multiparameterized HyperSoft Set. With the appropriate identification on the family B, the structure (S, B) becomes equivalent to a multiparameterized HyperSoft Set.

Thus, in either case, the multiparameterized SuperHyperSoft Set generalizes the earlier structures. □

3.3 Type-2 HyperSoft Sets

The definition is stated as follows.

Definition 3.8 (Type-2 HyperSoft Set)**.** Let U be a universal set and let E be a set of parameters. Let $\Sigma_{HS}(U)$ denote the collection of all hypersoft sets over U (that is, each element of $\Sigma_{HS}(U)$ is a pair (G, C), with

$$G : C \to \mathcal{P}(U)$$

and $C = \mathcal{A}_1 \times \cdots \times \mathcal{A}_m$ for some attribute domains $\mathcal{A}_i$, $i = 1, \ldots, m$).

Let $A \subseteq E$ be a nonempty set of *primary parameters*. A *Type-2 HyperSoft Set* over (U, E) is defined as a mapping

$$H^* : A \to \Sigma_{HS}(U)$$

such that for each $\alpha \in A$ there exists a hypersoft set (G_α, C) satisfying

$$H^*(\alpha) = (G_\alpha, C), \quad \text{with } G_\alpha : C \to \mathcal{P}(U).$$

We denote the Type-2 HyperSoft Set by $[H^*, A]$.

This structure generalizes the concept of a hypersoft set (since if A is a singleton then $[H^*, A]$ is equivalent to a hypersoft set) and also a type-2 soft set (if the attribute domains are taken so that $C \cong S$, a simple parameter set).

Theorem 3.9 (Generalization of Hypersoft Set and Type-2 Soft Set)**.** *A Type-2 HyperSoft Set $[H^*, A]$ over (U, E) generalizes both:*

1. *A hypersoft set, and*
2. *A type-2 soft set.*

Proof. (1) **Generalization of a Hypersoft Set:** Assume that $A = \{\alpha_0\}$ is a singleton. Then $[H^*, A]$ reduces to the single hypersoft set (G_{α_0}, C), which is exactly the definition of a hypersoft set.

(2) **Generalization of a Type-2 Soft Set:**
Suppose the attribute domains $\mathcal{A}_1, \ldots, \mathcal{A}_m$ are chosen so that C is isomorphic to an ordinary parameter set (that is, each $\mathcal{A}_i$ is a singleton or C may be identified with a set $S \subseteq E$). In this case, each hypersoft set (G_α, C) becomes equivalent to a type-1 soft set and the mapping

$$H^* : A \to \sigma(U)$$

coincides with the definition of a type-2 soft set.

Thus, in both scenarios, the Type-2 HyperSoft Set $[H^*, A]$ encompasses the structures of both hypersoft sets and type-2 soft sets. □

Example 3.10 (Type-2 HyperSoft Set in a Smartphone Selection Scenario)**.** (cf. [13, 17]) Let the universal set be a set of smartphone models:

$$U = \{\text{iPhone 14(cf. [72])}, \text{Samsung Galaxy S23(cf. [20])}, \text{Google Pixel 7(cf. [60])}\}.$$

Assume there are two attribute domains for specifications:

- $\mathcal{A}_1$: Battery Capacity, where $\mathcal{A}_1 = \{3000\text{mAh}, 4000\text{mAh}\}$.
- $\mathcal{A}_2$: Camera Quality, where $\mathcal{A}_2 = \{12\text{MP}, 48\text{MP}\}$.

Then the Cartesian product is

$$C = \mathcal{A}_1 \times \mathcal{A}_2 = \{(3000\text{mAh}, 12\text{MP}), (3000\text{mAh}, 48\text{MP}), (4000\text{mAh}, 12\text{MP}), (4000\text{mAh}, 48\text{MP})\}.$$

For the primary parameter set, let

$$A = \{\text{Price}, \text{Operating System}\} \subseteq E,$$

where E is the overall parameter set (which might also include Memory and other features). We now define H^* as follows:

$$H^*(\text{Price}) = (G_{\text{Price}}, C), \quad H^*(\text{Operating System}) = (G_{\text{OS}}, C).$$

Let the mappings be defined by:

$$\begin{aligned} G_{\text{Price}}(3000\text{mAh}, 12\text{MP}) &= \{\text{iPhone 14}\}, \\ G_{\text{Price}}(3000\text{mAh}, 48\text{MP}) &= \{\text{Samsung Galaxy S23}\}, \\ G_{\text{Price}}(4000\text{mAh}, 12\text{MP}) &= \{\text{Google Pixel 7}\}, \\ G_{\text{Price}}(4000\text{mAh}, 48\text{MP}) &= \varnothing. \end{aligned}$$

Similarly, define

$$\begin{aligned} G_{\text{OS}}(3000\text{mAh}, 12\text{MP}) &= \{\text{Google Pixel 7}\}, \\ G_{\text{OS}}(3000\text{mAh}, 48\text{MP}) &= \{\text{Samsung Galaxy S23}\}, \\ G_{\text{OS}}(4000\text{mAh}, 12\text{MP}) &= \{\text{iPhone 14}\}, \\ G_{\text{OS}}(4000\text{mAh}, 48\text{MP}) &= \{\text{Samsung Galaxy S23}\}. \end{aligned}$$

Thus, the mapping H^* yields a Type-2 HyperSoft Set

$$[H^*, A] = \{H^*(\text{Price}), H^*(\text{Operating System})\},$$

which integrates decision information by using composite specification parameters.

3.4 Type-2 SuperHyperSoft Sets

The definition is stated as follows.

Definition 3.11 (Type-2 SuperHyperSoft Set)**.** Let U be a universal set and let E be a set of parameters. Let $\Sigma_{SHS}(U)$ denote the collection of all super–hypersoft sets over U, that is, each element of $\Sigma_{SHS}(U)$ is a pair (F, C) with

$$F : C \to \mathcal{P}(U)$$

and

$$C = \mathcal{P}(A_1) \times \mathcal{P}(A_2) \times \cdots \times \mathcal{P}(A_n),$$

where $A_1, A_2, \ldots, A_n$ are pairwise disjoint attribute sets.

Let $A \subseteq E$ be a nonempty set of primary parameters. A *Type-2 SuperHyperSoft Set* over (U, E) is a mapping

$$H^{\#} : A \to \Sigma_{SHS}(U)$$

such that for each $\alpha \in A$, there exists a super–hypersoft set (F_α, C) satisfying

$$H^{\#}(\alpha) = (F_\alpha, C), \quad \text{with } F_\alpha : C \to \mathcal{P}(U).$$

We denote the Type-2 SuperHyperSoft Set by $[H^{\#}, A]$.

This construction generalizes a super–hypersoft set (if A is chosen as a singleton) and also generalizes a Type-2 HyperSoft Set (when the underlying attribute sets A_i are such that $\mathcal{P}(A_i)$ may be identified with A_i, reducing a super–hypersoft set to a hypersoft set).

Theorem 3.12 (Generalization of SuperHypersoft Set and Type-2 HyperSoft Set)**.** *A Type-2 SuperHyperSoft Set $[H^{\#}, A]$ over (U, E) generalizes both:*

1. *A super–hypersoft set, and*
2. *A Type-2 HyperSoft Set.*

Proof. (1) **Generalization of a Super–Hypersoft Set:**
Assume that $A = \{\alpha_0\}$ is a singleton. Then $[H^{\#}, A]$ reduces to the single super–hypersoft set (F_{α_0}, C), which is exactly the definition of a super–hypersoft set.

(2) **Generalization of a Type-2 HyperSoft Set:**
If the disjoint attribute sets $A_1, \ldots, A_n$ are chosen so that each power set $\mathcal{P}(A_i)$ is isomorphic to A_i (for example, when each A_i contains only one element), then the Cartesian product

$$C = \mathcal{P}(A_1) \times \cdots \times \mathcal{P}(A_n)$$

becomes isomorphic to $A_1 \times \cdots \times A_n$, and a super–hypersoft set reduces to a hypersoft set. Hence $[H^{\#}, A]$ then coincides with a Type-2 HyperSoft Set as defined previously.

Thus, in either case, a Type-2 SuperHyperSoft Set $[H^{\#}, A]$ generalizes both super–hypersoft sets and Type-2 HyperSoft Sets. □

Example 3.13 (Type-2 SuperHyperSoft Set in a Car Selection Scenario)**.** (cf. [3, 6]) Let the universal set be the collection of car models:

$$U = \{\text{Toyota Camry, Honda Accord, Ford Fusion(cf. [10]), BMW 3 Series}\}.$$

Suppose there are two disjoint attribute sets:

- A_1 (Fuel Type): $A_1 = \{\text{Gasoline, Diesel}\}$.
- A_2 (Transmission): $A_2 = \{\text{Automatic, Manual}\}$.

Then the power sets are

$$\mathcal{P}(A_1) = \{\varnothing, \{\text{Gasoline}\},\ \{\text{Diesel}\},\ \{\text{Gasoline, Diesel}\}\},$$

$$\mathcal{P}(A_2) = \{\varnothing, \{\text{Automatic}\},\ \{\text{Manual}\},\ \{\text{Automatic, Manual}\}\}.$$

Their Cartesian product is

$$C = \mathcal{P}(A_1) \times \mathcal{P}(A_2).$$

For the primary parameter set, choose

$$A = \{\text{Price}\} \subseteq E.$$

Define the mapping

$$H^{\#} :\ A \to \Sigma_{SHS}(U)$$

by setting

$$H^{\#}(\text{Price}) = (F_{\text{Price}}, C),$$

where $F_{\text{Price}} : C \to \mathcal{P}(U)$ is defined on selected elements of C as follows:

$$\begin{aligned} F_{\text{Price}}(\{\text{Gasoline}\},\ \{\text{Automatic}\}) &= \{\text{Toyota Camry, Honda Accord}\}, \\ F_{\text{Price}}(\{\text{Diesel}\},\ \{\text{Manual}\}) &= \{\text{Ford Fusion}\}, \\ F_{\text{Price}}(\{\text{Gasoline}\},\ \{\text{Manual}\}) &= \{\text{BMW 3 Series}\}, \\ F_{\text{Price}}(\gamma) &= \varnothing, \quad \text{for all other } \gamma \in C. \end{aligned}$$

Then the pair

$$[H^{\#}, A] = \{H^{\#}(\text{Price})\}$$

forms a Type-2 SuperHyperSoft Set over (U, E) that—via its mapping—captures detailed car selection information based on the power-set combinations of Fuel Type and Transmission.

For instance, the composite attribute

$$(\{\text{Gasoline}\},\ \{\text{Automatic}\})$$

is associated with {Toyota Camry, Honda Accord}, while

$$(\{\text{Diesel}\},\ \{\text{Manual}\})$$

is mapped to {Ford Fusion}.

4 Conclusion

As natural extensions of existing soft set frameworks, this work has proposed and meticulously developed the notions of Multiparameterized Hypersoft Sets and Type-2 Hypersoft Sets. Such novel developments allow a more meaningful depiction of multi-dimensional and hierarchical uncertainty in complex decision-making systems. With the help of making generalizations about conventional soft, hypersoft, and type-2 soft models, the established frameworks provide improved flexibility and granularity in modeling real-world problems. Future forthcoming examinations may examine practical implementations of such sets in several fields, including intelligent decision support systems, medical diagnostics, data classification, and others.

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