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A clustering approach based on a quadripartitioned neutrosophic minimal spanning tree and its application using probabilistic membership values

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Abstract. The paper proposes an algorithm with an aim to define a preferencing order on clusters obtained by partitioning quadripartitioned neutrosophic graphs (QNGs). For this purpose, a network system is represented as a QNG and a new probabilistic approach has been proposed in deducing the membership values corresponding to the degrees of the vertices. The technique to determine the minimal spanning tree (MST) using cost and benefit score functions is introduced. The MST, thus obtained, is used to partition the graph into multiple clusters (as viable). Finally a preferencing order is defined on the clusters so isolated. An application in the form of designing a plan for running customized advertisements on social media platforms depending on an individual's online engagement data is shown using the proposed technique.

Keywords: minimal spanning tree, neutrosophic clustering, probabilistic membership

1. Introduction

Graph structures are indispensible tools for representing and studying network design of any kind. Networking structures can range from anything as simple as a family tree to something as complex as transportation systems, traffic flows, social media interaction networks, travel history, communication pattern of a contagious disease etc. Quite naturally, due to its unique capability to represent complex networks in the form of a diagram, comprising nodes or vertices (the reference points) and edges (line segments connecting any pair of nodes that are related or, are in communication)– graph theory has immense potential as a tool and has huge applications in several fields.

Of the several prominent concepts that form the foundation of the theory of graphs, the concept of a Minimal Spanning Tree (MST; in abbreviated form) is a powerful tool which is instrumental in generating a skeletal framework or the blueprint of the network under consideration, wherein all nodes of the said network are connected and are reachable with minimal cost. The Kruskal's algorithm and the Prim's algorithms are the most popular techniques used for this purpose. MSTs have wide application in several areas where optimization is essential.

Being a versatile branch of study in itself, it does not instill an element of surprise when one notices a convergence of the theory of graphs with other popular theories such as the theory of fuzzy sets and other generalized structures alike. As a proof of this, the literature, replete with prominent findings in several diverse areas, boasts of novel findings ranging from foundational theories to multifaceted, multidisciplinary and diversified application oriented studies as can be seen in the works of [1]- [9], [11], [13]- [24], [26], [27], etc. just to acknowledge a few of the contributions.

The present study is a result of the confluence of traditional graph theory results and the theory of *neutrosophics*. Of the several prominent stuctures used to model uncertainty, the theory of neutrosophic sets, established by Smarandache [12] introduced the study of neutralities for the very first time. This theory paved a groundbreaking path for researchers across the globe working on modeling uncertainty and this theory was later generalized by Chatterjee et. al [10] making way for the theory of quadripartitioned single valued neutrosophic sets (QSVNS; in abbreviated form). QSVNSs are powerful set theoretic structures which equip each member of the underlying universe of discourse with four different membership degrees denoting *truth, contradiction, ignorance* and *falsity* respectively. QSVNSs, due to their capacity to represent imprecision with higher degrees of freedom have found immense applicability. Consequently, the concepts of QSVNSs and graph theoretic concepts were merged resulting in the concept of quadripartitioned single valued neutrosophic graphs. It becomes absolutely necessary to highlight the fact that QNGs emerge as graph theoretic structures that are generalizations of fuzzy graphs, intuitionistic fuzzy graphs and neutrosophic graphs. Several prominent research studies have been conducted using these advanced graph theoretic structures and the literature is abound with their diverse applications ([1]- [3], [11], [19], [26], [4]- [5], [6], [7]- [9], [13], etc.).

Due to their versatile applicability and a more generalized framework, QNGs have been considered as the chosen framework to represent a dynamic network of an interactive model for this study. The research focuses on partitioning a QNG into well-defined clusters using a MST. The motivation behind this study stems from the fact that clusters in a QNG can be viewed as a collection of nodes with maximal inter-communications and this can help in analyzing the graph in terms of such individual components. This can further aid in designing a cluster-specific strategy which is the main focus of this study. A thorough description of the presentation of the manuscript's contents is summarized in the following.

1.1. *Outine of the study and organization of the paper*

This study presented in this manuscript, intends to explore the concept and potential of a minimal spanning tree (MST) for a quadripartitioned neutrosophic graph (QNG). In this regard, it needs to be stated that the basic notion of deriving a MST for a QNG is proposed as a generalization of the approach due to the Kruskal's algorithm and is framed in the lines of the work of [6]. However, it also needs to be stated that the proposed algorithm is a more generalized one, as it introduces the concepts of separate score functions based on cost and benefit criteria. The concept of a MST is then used to remove edges which are least visited, hence leaving behind the clusters which are most frequented. This technique is then used to propose a customized marketing strategy for the placement of an advertisement on media platforms based on customer-specific *app* (abbreviated form of an application) engagement data such that the advertisement gets the maximum visibility.

The study presented in this paper has been organized thus, as mentioned in the following. Section 2 reviews some basic preliminary definitions which are indispensible for the framework of the whole study. Section 3 presents a detailed documentation of the methodologies used. This section introduces a novel technique of deriving quadripartitioned neutrosophic membership values based on a probabilistic approach. A few relevant terminologies have also been introduced in this section in order to ensure smooth reading of the manuscript. The working algorithm for deriving the MST and thereby partitioning the graph into clusters and imposing the order of preferencing on clusters concludes this section. The consecutive section viz. Section 4 provides a detailed, step-by-step illustration of the application of the working algorithm and is concluded with a comprehensive discussion of the results. It becomes necessary to mention that all relevant tables and figures have been presented at the end of the manuscript in order to ensure a well-structured presentation of the content. Care has been taken to cite the relevant data (either presented in figures or tables) in the main text to ensure that all results can be referred to without any loss of clarity and precise conclusions can be derived. The findings of the study is concluded in Section 5 with a brief discussion of the major research results, followed by a conclusive discussion regarding future research directions.

2. **Preliminaries**

Definition 2.1. [10] Let X be a non-empty set. A quadripartitioned single valued neutrosophic set A over X characterizes each element x in X by a truth-membership function t_A, a contradiction-membership function c_A, an ignorance-membership function u_A and a falsity membership function f_A such that for each $x \in X$, $t_A, c_A, u_A, f_A \in [0,1]$ and $0 \leq t_A(x) + c_A(x) + u_A(x) + f_A(x) \leq 4$.

When X is discrete, A is represented as,

$$A = \sum_{i=1}^{n} \langle t_A(x_i), c_A(x_i), u_A(x_i), f_A(x_i) \rangle / x_i, x_i \in X \tag{1}$$

However, when the universe of discourse is continuous, A is represented as,

$$A = \int_X \langle t_A(x), c_A(x), u_A(x), f_A(x) \rangle / x, x \in X \tag{2}$$

Definition 2.2. [16] Let $\mathbb{R}$ be a non-empty set and $\{\mathbb{E}_1, \mathbb{E}_2, ..., \mathbb{E}_n\}$ relations on $\mathbb{R}$. $\mathbb{G} = (\mathbb{U}, \mathbb{B}_1, \mathbb{B}_2, ..., \mathbb{B}_n)$ is said to be a quadripartitioned neutrosophic graph structure if

$$\mathbb{U} = \{n, \mathbb{T}_i(l), \mathbb{C}_i(l), \mathbb{U}_i(l), \mathbb{F}_i(l), n \in \mathbb{R}\}$$

is a quadripartitioned neutrosophic set on $\mathbb{R}$ and

$$\mathbb{B}_i = \{(k,l), \mathbb{T}(k,l), \mathbb{C}(k,l), \mathbb{U}(k,l), \mathbb{F}(k,l), n \in \mathbb{E}_i\}$$

is a quadripartitioned neutrosophic set on $\mathbb{E}_i$ such that $\mathbb{T}_i(k,l) \leq min\{\mathbb{T}(k), \mathbb{T}(l)\}$, $\mathbb{C}_i(k,l) \leq min\{\mathbb{C}(k), \mathbb{C}(l)\}$, $\mathbb{U}_i(k,l) \leq max\{\mathbb{U}(k), \mathbb{U}(l)\}$, $\mathbb{F}_i(k,l) \leq max\{\mathbb{F}(k), \mathbb{F}(l)\}$

3. Materials and Methods

It becomes very essential to introduce the following terminologies at the very outset.

Definition 3.1. The engagement of a node or a vertex is defined as the number of times or the time duration (as applicable) for which a node (vertex) is visited.

Definition 3.2. Parallel engagement refers to engagement in multiple nodes (vertices) simulatneously at the same time.

Parallel engagement can be best understood when multiple search tabs are active simultaneously, or when multiple apps are used simultaneously before placing an order for a commodity online.

Definition 3.3. A switch refers to the process of transitioning from one node(vertex) to another.

Definition 3.4. A false impression refers to an act when a node (vertex) has been engaged unintentionally.

An act of false impression is best illustrated when a call is made to a customer by dialing a wrong number or by mistake, calling the most recent number on the previous dialed list or when an app is opened as a result of a mis-touch. This usually happens when either there was no previous intent on visiting any vertex or when the vertex is visited by mistake when the intent was to visit another vertex.

Consider a quadripartitioned neutrosophic graph characterized by n vertices. The aim of this study is to partition this graph into specific clusters with each cluster representing a group of nodes or vertices that have some common characteristics namely, maximum areas of congestion of traffic (for graphs representing traffic data), or maximum user engagement time (for graphs representing user engagement data) etc. as applicable for the underlying graph under consideration. The clustering and hence the preferencing is done as follows:

- deriving the minimal spanning tree
- removal of the edges with least engagement to generate closely packed clusters
- generating the order of preferencing by arranging the clusters from highest to least scores

A detailed description of the technique is provided in the following portions.

3.1. *A probabilistic approach in defining quadripartitioned membership degrees*

The probabilistic approach introduces a technique of generating membership values using the elementary concept of probability, where the probability of occurrence of an event is defined as the ratio of the number of outcomes by the total number of possible outcomes. In a similar line of approach, if for a vertex v, the membership degrees of v are denoted by $\langle t_v, c_v, u_v, f_v \rangle$ then the membership values can be derived as mentioned below:

$$t_v = \frac{engagement\, of\, v}{sum\, total\, of\, engagement\, of\, all\, vertices} \tag{3}$$

$$c_v = \frac{sum\, total\, of\, parallel\, engagements\, along\, with\, v}{sum\, total\, of\, parallel\, engagements\, among\, all\, vertices} \tag{4}$$

$$u_v = \frac{total\, number\, of\, switches\, from\, v}{sum\, total\, of\, switches\, among\, all\, vertices} \tag{5}$$

$$f_v = \frac{number\, of\, false\, impressions\, of\, v}{sum\, total\, of\, all\, false\, impressions\, among\, all\, vertices} \tag{6}$$

3.2. *Algorithm for defining a preference-based ordering of clusters using the MST and score functions*

At the outset, it becomes relevant to point out that a *cost criteria* refers to a parameter that involves expenditure of resources and require to be minimized, wheres a *benefit criteria* is more of an asset and should ideally be maximized. It depends on the specific study to decide which parameters are to be classified as cost criteria and which as benefit criteria.

Consider a quadripartitioned neutrosophic graph with the set of vertices being denoted by $\{v_1, v_2, ..., v_n\}$ where the vertex membership values for each vertex v_i, $i \in [1, n]$ are denoted by $\langle t_i, c_i, u_i, f_i \rangle$, $i = 1, 2, ..., n$.

- Step 1: Deduce the membership degrees $\langle t_i, c_i, u_i, f_i \rangle$ of every vertex v_i, $i = 1, 2, ..., n$ using equations 3- 6.
- Step 2: Deduce the membership degree of an edge $e_{i,j}$ as

$$\langle t_{ij}, c_{ij}, u_{ij}, f_{ij} \rangle = \langle max\{t_i, t_j\}, max\{c_i, c_j\}, min\{u_i, u_j\}, min\{f_i, f_j\} \rangle \tag{7}$$

- Step 3: Construct the adjacency matrix, A such that

$$A_{ij} = \langle t_{ij}, c_{ij}, u_{ij}, f_{ij} \rangle \tag{8}$$

- Step 4: Generate the score matrix S where,
  for a benefit parameter,

$$S_{i,j} = \frac{1}{4}[t_{ij} + c_{ij} + (1 - u_{ij}) + (1 - f_{ij})], i \geq j \tag{9}$$

  and for a cost parameter,

$$S_{i,j} = \frac{1}{4}[(1 - t_{ij}) + (1 - c_{ij}) + u_{ij} + f_{ij}], i \geq j \tag{10}$$

- Step 5: Select $min_{i,j}\{S_{i,j}\}$.
- Step 6: Find the next minimal score such that it does not form a loop.
- Step 7: Repeat Step 5 until $(n - 1)$ edges are selected.
- Step 8: The $(n - 1)$ edges selected thus, in totality, generates the required minimal spanning tree.
- Step 9: Remove the edges for which the cost scores arethe highest such that the edges are so removed leave the resultant graph with at least $(\frac{n-1}{2})$ edges.
- Step 10: The scores of each cluster is computed as the sum total of the benefit scores of all the edges that belong to that cluster.
- Step 11: Rank the clusters in descending order of benefit scores.

3.2.1. *Remarks*

The following are valid with respect to the proposed algorithm:

(1) It is sufficient to consider the score matrix as an upper triangular matrix as the matrix is symmetric.

(2) The removal of the edges with high cost scores helps in isolating clusters which have vertices with highest engagement. This helps to get the minimal representation of the graph in the form of a forest and hence it becomes easier to study individual characteristics specific to each cluster.

4. Application: Preferencing of media platforms for designing customer specific advertisement strategy for a product based on the user's media engagement history

Consider a *Direct to Consumer* (D2C) brand who are about to introduce a range of personal hygine products for working women who have to use public restrooms throughtout their work hours. The target group for this product are adult, employed women within the age bracket of 25 to 50 years. In order to market this new range of products, a personalised advertisement strategy is to be designed to ensure maximum visibility of the product. In view of this, the media engagement of adult females, in the form of time spent on different apps including various social media platforms are studied.

As a part of an illustrative case, the media engagement data for a 38 year old, employed female is considered. It needs to be stated that the data used for the purpose of illustration is abstract. However, the proposed technique would work seamlessly in case of availability of real-time data. The workability of the algorithm is ensured by labeling every app used as a vertex. The app engagement and app transition data are respectively represented in Tables 1 & 2.

4.1. *Results*

- The membership degrees for each vertex representing an app, is defined using the probabilistic approach with the help of equations (3)- (6) and is mentioned in Table 3.
- The quadripartitioned neutrosophic graph for the user data is represented in Figure 1.
- The membership degrees of the different edges as obtained from the data are mentioned in Table 4.
- The adjacency matrix is obtained as:

$$A = \begin{bmatrix} 0 & e_{1,2} & 0 & e_{1,4} & e_{1,5} & e_{1,6} & e_{1,7} & 0 & 0 & 0 \\ e_{1,2} & 0 & e_{2,3} & e_{2,4} & e_{2,5} & 0 & 0 & 0 & 0 & e_{2,10} \\ 0 & e_{2,3} & 0 & e_{3,4} & e_{3,5} & 0 & 0 & 0 & 0 & 0 \\ e_{1,4} & e_{2,4} & e_{3,4} & 0 & e_{4,5} & e_{4,6} & 0 & 0 & 0 & 0 \\ e_{1,5} & e_{2,5} & e_{3,5} & e_{4,5} & 0 & e_{5,6} & e_{5,7} & 0 & 0 & 0 \\ e_{1,6} & 0 & 0 & e_{4,6} & e_{5,6} & 0 & e_{6,7} & 0 & 0 & 0 \\ e_{1,7} & 0 & 0 & 0 & e_{5,7} & e_{6,7} & 0 & 0 & 0 & e_{7,10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_{8,9} & e_{8,10} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e_{8,9} & 0 & e_{9,10} \\ 0 & e_{2,10} & 0 & 0 & 0 & 0 & e_{7,10} & e_{8,10} & e_{9,10} & 0 \end{bmatrix} \quad (11)$$

- The score matrix (in the upper traingular form) is obtained as:

$$S = \begin{bmatrix} 0 & 0.557775 & 0 & 0.55420 & 0.554625 & 0.55235 & 0.554875 & 0 & 0 & 0 \\ 0 & 0 & 0.52180 & 0.51345 & 0.53765 & 0 & 0 & 0 & 0 & 0.51705 \\ 0 & 0 & 0 & 0.510525 & 0.53115 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.51385 & 0.51555 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.532225 & 0.55060 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.538575 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.520525 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.527875 & 0.526525 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.504775 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

- The minimal spanning tree is represented in Figure 2.
- The cost score for the edges that form a part of the minimal spanning tree are calculated as follows: $S_{9,10} = 0.4952 < S_{3,4} = 0.4895 < S_{2,4} = 0.4866 < S_{4,5} = 0.4862 < S_{4,6} = 0.4845 < S_{2,10} = 0.4830 < S_{7,10} = 0.4795 < S_{8,10} = 0.4735 < S_{1,6} = 0.4477$
- The resultant clusters containing 5 edges, representing maximal user engagement is shown in Figure 3.

4.2. *Discussions*

For the ease of idedntification, let the cluster of nodes connected by edges of green color be labeled as Cluster 1 and those denoted by the color purple be denoted as Cluster 2. The total benefit score of Cluster 1 is 1.5641 whereas that of Cluster 2 is 1.0679. Since the aggregate score of Cluster 1 is more, preference should be given to the nodes of that cluster while placing advertisements of the product, since these nodes have the maximum user engagement. The preference of Cluster 2 becomes secondary.

It is interesting to note that two different approaches have been used while calculating the scores for this particular study. For generating the minimal spanning tree, the score function employed uses the benefit criteria to access information regarding maximal engagement for a node. However, while partitioning the minimal spanning tree into clusters the scores of the edges were re-computed using the score function for cost criteria. This enabled removal of edges that represented minimal user engagement thus resulting in clusters that have the maximal user engagement, ensuring maximal visibility.

5. **Conclusions**

In this paper, a working rule has been proposed to identify clusters based on the minimal spanning tree derived from a quadripartitioned neutrosophic graph. The novel technique of deriving membership values based on the probabilistic approach has been discussed. This not only makes it easy to understand the distinct role that each membership component has to contribute to a quadripartitioned neutrosophic component but also provides a relatable and realistic approach in deriving these membership values especially when real-life data is considered, which, otherwise seems challenging.

Future works include further studies on applications of quadripartitioned neutrosophic graphs using the concepts of minimal spanning tree, shortest paths and graph colorings based on probabilistic membership values.

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TABLE 1. Table representing total time spent on different apps by the user

App	**Vertex label**	**Engagement (in hours/ week)**	**Number of False impressions**	**Number of Parallel executions**
YouTube	v_1	20	5	1
Amazon	v_2	6	3	2
Flipkart	v_3	3	1	2
Myntra	v_4	2	2	1
Instagram	v_5	8	0	0
Tiktok	v_6	8	3	1
Netflix	v_7	10	4	3
Twitter	v_8	2	2	4
BBC	v_9	3.5	5	4
HT	v_{10}	4	5	3

TABLE 2. Table representing transition details between apps

Initial App	**Transitioned App**	**No. of transitions**
YouTube	Amazon	7
	Netflix	3
	Flipkart	2
	Tiktok	5
	Instagram	4
Amazon	Myntra	2
	Instagram	3
Flipkart	Myntra	2
	Amazon	3
	Instagram	2
Instagram	Myntra	6
	Tiktok	4
Tiktok	Myntra	3
Netflix	Instagram	6
	Tiktok	1
Twitter	BBC	4
	HT	4
BBC	Amazon	4
	Netflix	1
HT	BBC	6

TABLE 3. Table representing the membership degrees of vertices representing the apps used

Vertex label	**Membership degrees**	**Vertex label**	**Membership degrees**
v_1	$\langle 0.3008, 0.0476, 0.2727, 0.1667\rangle$	v_6	$\langle 0.1203, 0.0476, 0.039, 0.1\rangle$
v_2	$\langle 0.0902, 0.0952, 0.0649, 0.1\rangle$	v_7	$\langle 0.1504, 0.1429, 0.0909, 0.1333\rangle$
v_3	$\langle 0.0451, 0.0952, 0.0909, 0.0333\rangle$	v_8	$\langle 0.0301, 0.1905, 0.1039, 0.0667\rangle$
v_4	$\langle 0.0301, 0, 0.0649, 0.0667\rangle$	v_9	$\langle 0.0526, 0.1905, 0.0649, 0.1667\rangle$
v_5	$\langle 0.1203, 0, 0.1299, 0\rangle$	v_{10}	$\langle 0.0602, 0.1429, 0.0779, 0.1667\rangle$

TABLE 4. Table representing the membership degrees of the edges

Edge label	**Membership degrees**	**Edge label**	**Membership degrees**
$e_{1,2}$	$\langle 0.3008, 0.0965, 0.0649, 0.1000\rangle$	$e_{3,5}$	$\langle 0.1203, 0.0952, 0.0909, 0.0000\rangle$
$e_{1,4}$	$\langle 0.3008, 0.0476, 0.0649, 0.0667\rangle$	$e_{4,5}$	$\langle 0.1203, 0.0000, 0.0647, 0.0000\rangle$
$e_{1,5}$	$\langle 0.3008, 0.0476, 0.1299, 0.0000\rangle$	$e_{4,6}$	$\langle 0.1203, 0.0476, 0.0390, 0.0667\rangle$
$e_{1,6}$	$\langle 0.3008, 0.0476, 0.0390, 0.1000\rangle$	$e_{5,6}$	$\langle 0.1203, 0.0476, 0.0390, 0.0000\rangle$
$e_{1,7}$	$\langle 0.3008, 0.1429, 0.0909, 0.1333\rangle$	$e_{5,7}$	$\langle 0.1504, 0.1429, 0.0909, 0.0000\rangle$
$e_{2,3}$	$\langle 0.0902, 0.0952, 0.0649, 0.0333\rangle$	$e_{6,7}$	$\langle 0.1504, 0.1429, 0.0390, 0.1000\rangle$
$e_{2,4}$	$\langle 0.0902, 0.0952, 0.0649, 0.0667\rangle$	$e_{7,10}$	$\langle 0.1504, 0.1429, 0.0779, 0.1333\rangle$
$e_{2,5}$	$\langle 0.1203, 0.0952, 0.0649, 0.0000\rangle$	$e_{8,9}$	$\langle 0.0526, 0.1905, 0.0649, 0.0667\rangle$
$e_{2,10}$	$\langle 0.0902, 0.1429, 0.0649, 0.1000\rangle$	$e_{8,10}$	$\langle 0.0602, 0.1905, 0.0779, 0.0667\rangle$
$e_{3,4}$	$\langle 0.0451, 0.0952, 0.0649, 0.0333\rangle$	$e_{9,10}$	$\langle 0.0602, 0.1905, 0.0649, 0.1667\rangle$

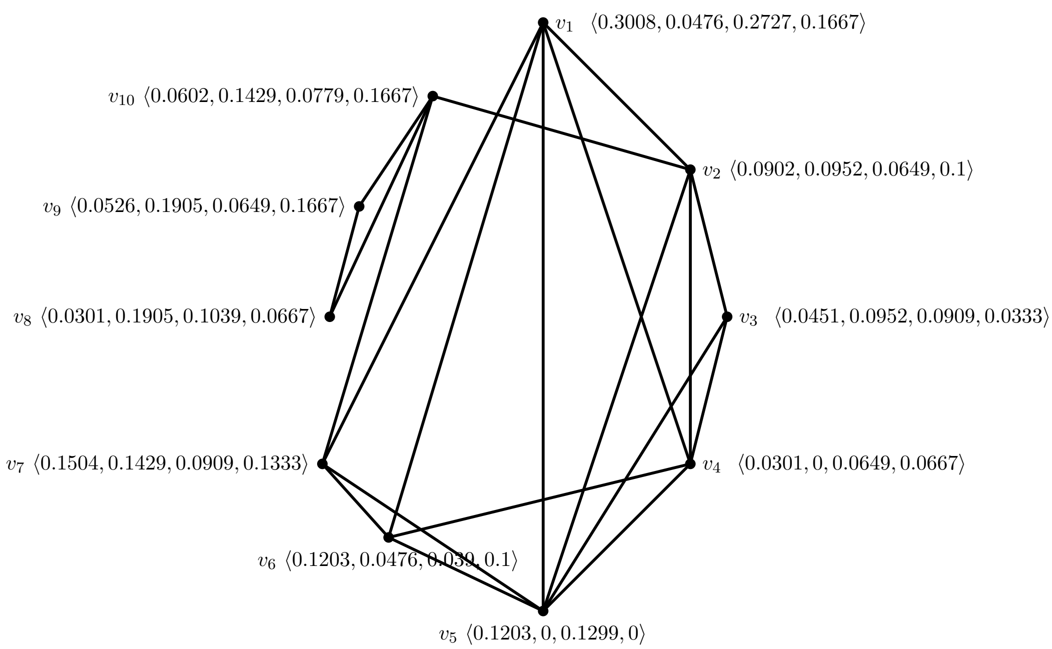

FIGURE 1. Quadripartitioned neutrosophic graph representing user app engagement data

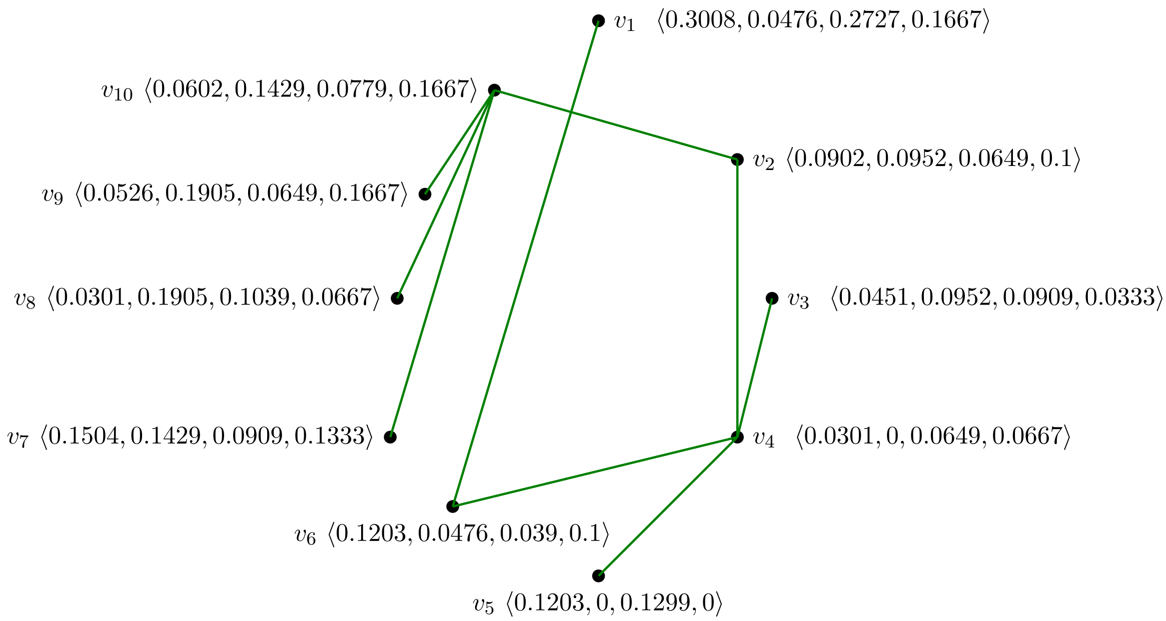

FIGURE 2. Minimal spanning tree deduced from the quadripartitioned neutrosophic graph shown in Fig. 1

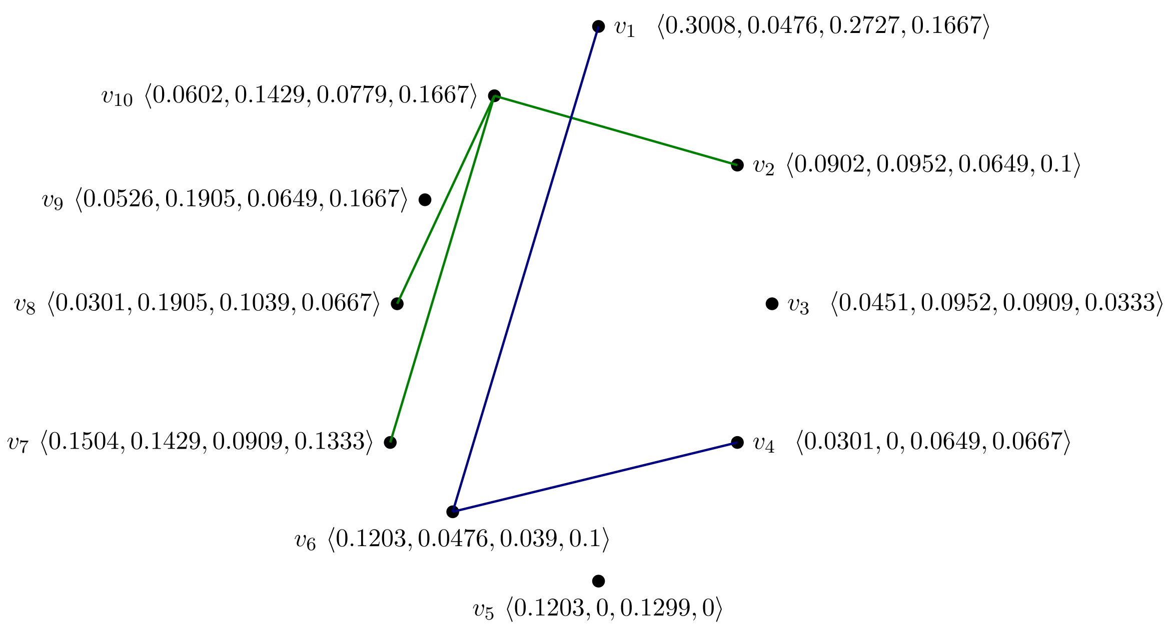

FIGURE 3. Clusters showing maximal user engagement

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