Possibility Interval-Valued Fermatean Neutrosophic Soft Sets in Decision making

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Abstract

Presenting and analyzing Possibility Interval-Valued Fermatean Neutrosophic Soft Sets (PIVFNSS) and their properties is the main objective of this research study. The necessity of the introduction of PIVFNSS is that the set incorporates the possibility value which increases the set's robustness. First, definitions and verification of fundamental mathematical characteristics including complement, intersection, and union are made. For easier comprehension, the operations "AND" and "OR" are defined with pertinent examples. This unique set is used to a decision-making problem, and its robustness and dependability are examined through sensitivity analysis.

Keywords: Interval-valued intuitionistic fuzzy soft sets, Possibility sets, Possibility Interval-valued intuitionistic fuzzy soft sets, Fermatean Fuzzy soft sets, Possibility Fermatean Neutrosophic soft sets.

1 Introduction

These days, uncertainty plays a crucial role in people's daily lives. Giving the proper solution is extremely difficult whenever someone solves decision-making with linguistic variables like "talent," "beautiful," etc. Zadeh [21] attempted to improve this complexity in 1965 with his invention of fuzzy sets. Even though fuzzy sets were helpful in handling these kinds of situations, making decisions is not precise when situations be like electing a person based on polling. In order to address this complexity, Atanassov [5] included non-membership values to fuzzy sets and in 1986 he named the new set as intuitionistic fuzzy sets. The more generalized version, interval-valued intuitionistic fuzzy sets, was introduced by Atanassov & Gargov [6] in 1989. In addition to these developments, Molodtsov [16] introduced soft set theory, which Maji et al. [15] developed in 2003.

Further fuzzy soft sets, intuitionistic fuzzy soft sets, interval-valued intuitionistic fuzzy soft sets are introduced and analyzed by Maji et al. [13], [17], Salleh [20] and Jiang [12] respectively. Due to the researchers' insatiable curiosity, possibility fuzzy set theory—a generalized version of Zadeh's possibility set theory [22] which was elaborately studied by Dubois [8] —was created. It was further developed to possibility fuzzy soft sets, possibility intuitionistic fuzzy soft set, possibility interval-valued intuitionistic fuzzy soft sets by Alkhazaleh [2], Bashir et al. [18], and Alkhazaleh [1]. The combination of possibility set theory and soft set theory significantly improved the ability to handle problems with imprecise data. Despite all of these advancements, researchers' desire to make the sets even better persisted, and in 2005 Smarandache [10] created Neutrosophic sets, which incorporate indeterminacy. It was further developed into Possibility Neutrosophic Soft sets by Karaaslan [9], Fermatean neutrosophic sets by Antony [4] in 2007 and possibility Fermatean neutrosophic soft sets by Alkhazaleh [3].

Regarding the uses of the sets that were previously addressed, these sets have a significant impact on decision-making issues. With linguistic variables and imprecise data, every advancement improves the solution to every decision-making challenge. Maji et al.'s work on soft sets in decision making [14] opened the door for the researchers. Fuzzy soft sets were used in decision-making by Roy and Maji [19], who made a substantial improvement. Alkhazaleh [2] showed that decision accuracy is increased by including the possibility values to Fermatean neutrosophic soft sets. In order to further refine the solution in decision making, this work presents the potential of interval-valued Fermatean neutrosophic soft sets. First, the theoretical approach is developed and the mathematical properties are analyzed. A decision-making issue with assumed data is used to test the recently suggested set, and the sensitivity analysis shows how resilient it is.

2 Prerequisites

This section provides a thorough summary of the core ideas and theoretical underpinnings necessary to comprehend the approaches and frameworks covered in this research study.

Definition 2.1. [16] Let U be the universe set and E be the set of parameters. A pair (F, E) is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U.

Definition 2.2. [11] Let U be an initial universe set, IVN(U) denotes the set of all interval-valued neutrosophic sets of U and E be a set of parameters. An interval-valued neutrosophic soft set over U is a set defined by a set-valued function Y_K representing a mapping $\nu_K : E \to \text{IVN}(U)$. It can be written as a set of ordered pairs $Y_K = \{(x, \nu_K(x)) : x \in E\}$.

Definition 2.3. [4] Let X be a non-empty set (Universe). A Fermatean Neutrosophic (FN) set on X is an object of the form

$$A = \{\langle x, \mu_m(x), \zeta_m(x), \nu_m(x)\rangle / x \in X\}$$

where $\mu_m(x), \zeta_m(x), \nu_m(x) \in [0, 1]$,

$$0 \leq (\mu_m(x))^3 + (\nu_m(x))^3 \leq 1, \quad 0 \leq (\zeta_m(x))^3 \leq 1.$$

Then

$$0 \leq (\mu_m(x))^3 + (\zeta_m(x))^3 + (\nu_m(x))^3 \leq 2, \quad \text{for all } x \in X.$$

Here, $\mu_m(x)$ is the degree of membership, $\zeta_m(x)$ is the degree of indeterminacy, and $\nu_m(x)$ is the degree of non-membership. $\mu_m(x)$ and $\nu_m(x)$ are dependent components, and $\zeta_m(x)$ is the independent component.

Definition 2.4. [3] Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, \ldots, e_m\}$ be the universal set of parameters. The pair (U, E) will be called a *soft universe*.
Let $F : E \to FN(U)$ where $FN(U)$ is the collection of all Fermatean neutrosophic subsets of U. Let $p : E \to I^U$ be a fuzzy subset of U and let $F_p : E \to FN(U) \times I^U$ be a function defined as follows:

$$F_p = \left\{ \left(e, \left[\frac{x}{F(e)(x)}, p(e)(x) \right] \right) \right\}, \quad \forall x \in U,\ e \in E.$$

where $F(e) \in FN(U)$, then F_p is called a *possibility Fermatean neutrosophic soft set.*

3 Possibility Interval-Valued Fermatean Neutrosophic Soft Set

Inadequate and imprecise data are a constant problem while making decisions. Fuzzy, intuitionistic fuzzy, and neutrosophic sets are used by the researchers to arrive at a fair answer wherever uncertainty arises. If we generalize the solution, it might be even more precise. As a new venture, possibility interval-valued Fermatean neutrosophic soft set is introduced and its application in decision making is examined.

Definition 3.1. Let $U = \{x_1, x_2, \ldots, x_n\}$ be the universal set of elements and $E = \{e_1, e_2, \ldots, e_m\}$ be the universal set of parameters. The pair (U, E) will be called a *soft universe*. Let $p : E \to I^U$ be a fuzzy subset of U and let $F_p : E \to IVFN(U) \times I^U$ be a function, where $IVFN(U)$ is the collection of all interval-valued Fermatean neutrosophic subsets of U and I^U is the collection of all fuzzy subsets of U, defined as follows:

$$F_p(e) = \left(\frac{x}{F(e)(x)},\ p(e)(x) \right)$$

where $F(e)(x) = ([\mu^-(x), \mu^+(x)],\ [\zeta^-(x), \zeta^+(x)],\ [\nu^-(x), \nu^+(x)])\,, \quad \forall x \in U.$

Then F_p is called a *possibility interval-valued Fermatean neutrosophic soft set* (PIVFNSS) over the soft universe (U, E). Here $0 \leq \mu^-(x) \leq \mu^+(x) \leq 1, \quad 0 \leq (\mu^+(x))^3 + (\nu^+(x))^3 \leq 1,$
$0 \leq (\zeta^+(x))^3 \leq 1$ and $0 \leq (\mu^+(x))^3 + (\zeta^+(x))^3 + (\nu^+(x))^3 \leq 2, \quad \forall x \in U.$

Example 3.2. Let $U = \{p_1, p_2, p_3\}$ be a set of three patients and $E = \{\alpha_1, \alpha_2\}$ be the parameter set where $\alpha_1 =$ Blood pressure and $\alpha_2 =$ Sugar level. Then the function, $F_p : E \to IVFN(U) \times I^U$

is defined as follows:

$$F_p(\alpha_1) = \left\{ \begin{array}{l} \left(\frac{p_1}{\langle[0.6,0.8],[0.5,0.5],[0.6,0.7]\rangle}, 0.5\right), \left(\frac{p_2}{\langle[0.4,0.6],[0.2,0.4],[0.2,0.3]\rangle}, 0.2\right), \\ \left(\frac{p_3}{\langle[0.5,0.6],[0.3,0.5],[0.3,0.4]\rangle}, 0.1\right) \end{array} \right\}$$

$$F_p(\alpha_2) = \left\{ \begin{array}{l} \left(\frac{p_1}{\langle[0.5,0.6],[0.3,0.4],[0.2,0.4]\rangle}, 0.4\right), \left(\frac{p_2}{\langle[0.3,0.5],[0.2,0.4],[0.3,0.6]\rangle}, 0.3\right), \\ \left(\frac{p_3}{\langle[0.6,0.7],[0.2,0.2],[0.2,0.3]\rangle}, 0.6\right) \end{array} \right\}$$

Then F_p is a PIVFNSS over (U, E).

Definition 3.3. Let F_{p_1} and G_{p_2} be any two PIVFNSSs defined as below:

$$F_{p_1}(e) = \left\{ \frac{x}{\langle[\mu_F^-(x), \mu_F^+(x)], [\zeta_F^-(x), \zeta_F^+(x)], [\nu_F^-(x), \nu_F^+(x)]\rangle}, p_1(e) \right\}$$

$$G_{p_2}(e) = \left\{ \frac{x}{\langle[\mu_G^-(x), \mu_G^+(x)], [\zeta_G^-(x), \zeta_G^+(x)], [\nu_G^-(x), \nu_G^+(x)]\rangle}, p_2(e) \right\}$$

Then $F_{p_1} \subseteq G_{p_2}$, if $[\mu_F^-(x), \mu_F^+(x)] \subseteq [\mu_G^-(x), \mu_G^+(x)], [\zeta_F^-(x), \zeta_F^+(x)] \subseteq [\zeta_G^-(x), \zeta_G^+(x)]$, $[\nu_F^-(x), \nu_F^+(x)] \supseteq [\nu_G^-(x), \nu_G^+(x)]$, and $p_1(e) \leq p_2(e)$.

Definition 3.4. A null PIVFNSS, denoted by F_{N_0}, is defined as follows:

$$F_{N_0}(e) = \left(\frac{x}{\langle[0,0],[0,0],[1,1]\rangle}, 0 \right)$$

Definition 3.5. An absolute PIVFNSS, denoted by F_{N_1}, is defined as follows:

$$F_{N_1}(e) = \left(\frac{x}{\langle[1,1],[0,0],[0,0]\rangle}, 1 \right)$$

Definition 3.6. The complement of a PIVFNSS F_p, denoted as F_p^c, is defined by,

$$F_p^c(e) = \left(\frac{x}{\langle[\nu_F^-(x), \nu_F^+(x)],\ [1-\zeta_F^-(x), 1-\zeta_F^+(x)],\ [\mu_F^-(x), \mu_F^+(x)]\rangle}, 1 - p(e) \right).$$

Definition 3.7. The union of two PIVFNSSs F_p and G_q denoted as $H_r = F_p \cup G_q$ is a PIVFNSS defined by

$$H_r(e) = (H(e)(x), r(e)(x)), \quad \forall \quad e \in E$$

where

$$H(e) = \left\{ \frac{x}{\left\langle \left[\max(\mu_F^-, \mu_G^-), \max(\mu_F^+, \mu_G^+)\right], \left[\min(\zeta_F^-, \zeta_G^-), \min(\zeta_F^+, \zeta_G^+)\right], \left[\min(\nu_F^-, \nu_G^-), \min(\nu_F^+, \nu_G^+)\right] \right\rangle} \right\}$$

and

$$r(e) = \max\left(p(e), q(e)\right), \quad \forall e \in E$$

Definition 3.8. The intersection of two PIVFNSSs F_p and G_q denoted as $W_s = F_p \cap G_q$ is a PIVFNSS defined by

$$W_s(e) = \left(W(e)(x), s(e)(x)\right), \quad \forall \quad e \in E$$

where

$$W(e) = \left\{ \frac{x}{\left\langle \left[\min(\mu_F^-, \mu_G^-), \min(\mu_F^+, \mu_G^+)\right], \left[\max(\zeta_F^-, \zeta_G^-), \max(\zeta_F^+, \zeta_G^+)\right], \left[\max(\nu_F^-, \nu_G^-), \max(\nu_F^+, \nu_G^+)\right] \right\rangle} \right\}$$

and

$$s(e) = \min\left(p(e), q(e)\right), \quad \forall e \in E$$

Example 3.9. Let $U = \{x_1, x_2\}$ be the universe set, $E = \{e_1, e_2\}$ be the parameter set. Let F_p and G_q be defined as follows:

$$F_p(e_1) = \left\{ \left(\frac{x_1}{\langle [0.6, 0.7], [0.5, 0.6], [0.3, 0.4] \rangle}, 0.7 \right), \left(\frac{x_2}{\langle [0.5, 0.8], [0.6, 0.8], [0.2, 0.4] \rangle}, 0.2 \right) \right\},$$

$$F_p(e_2) = \left\{ \left(\frac{x_1}{\langle [0.2, 0.6], [0.5, 0.5], [0.3, 0.5] \rangle}, 0.3 \right), \left(\frac{x_2}{\langle [0.4, 0.8], [0.3, 0.6], [0.1, 0.3] \rangle}, 0.4 \right) \right\}$$

and

$$G_q(e_1) = \left\{ \left(\frac{x_1}{\langle [0.5, 0.6], [0.3, 0.4], [0.4, 0.5] \rangle}, 0.6 \right), \left(\frac{x_2}{\langle [0.7, 0.8], [0.6, 0.7], [0.3, 0.4] \rangle}, 0.7 \right) \right\},$$

$$G_q(e_2) = \left\{ \left(\frac{x_1}{\langle [0.4, 0.5], [0.2, 0.3], [0.6, 0.7] \rangle}, 0.4 \right), \left(\frac{x_2}{\langle [0.4, 0.6], [0.3, 0.4], [0.3, 0.3] \rangle}, 0.6 \right) \right\}.$$

Then their union is obtained as,

$$H_r(e_1) = \left\{ \left(\frac{x_1}{\langle [0.6, 0.7], [0.3, 0.4], [0.3, 0.4] \rangle}, 0.7 \right), \left(\frac{x_2}{\langle [0.7, 0.8], [0.6, 0.7], [0.2, 0.4] \rangle}, 0.7 \right) \right\},$$

$$H_r(e_2) = \left\{ \left(\frac{x_1}{\langle [0.4, 0.6], [0.2, 0.3], [0.3, 0.5] \rangle}, 0.4 \right), \left(\frac{x_2}{\langle [0.4, 0.8], [0.3, 0.4], [0.1, 0.3] \rangle}, 0.6 \right) \right\}$$

And their intersection is obtained as,

$$W_r(e_1) = \left\{ \left(\frac{x_1}{\langle [0.5, 0.6], [0.5, 0.6], [0.4, 0.5] \rangle}, 0.6 \right), \left(\frac{x_2}{\langle [0.5, 0.8], [0.6, 0.8], [0.3, 0.4] \rangle}, 0.2 \right) \right\},$$

$$W_r(e_2) = \left\{ \left(\frac{x_1}{\langle [0.2, 0.5], [0.5, 0.5], [0.6, 0.7] \rangle}, 0.3 \right), \left(\frac{x_2}{\langle [0.4, 0.6], [0.3, 0.6], [0.3, 0.3] \rangle}, 0.4 \right) \right\}$$

The complement of $F_p(e_1)$ is,

$$(F_p(e_1))^c = \left\{ \left(\frac{x_1}{\langle [0.3, 0.4], [0.5, 0.4], [0.6, 0.7] \rangle}, 0.3 \right), \left(\frac{x_2}{\langle [0.2, 0.4], [0.4, 0.2], [0.5, 0.8] \rangle}, 0.8 \right) \right\}$$

Definition 3.10. If (F_{p_1}, E_1) and (F_{p_2}, E_2) are any two PIVFNSSs, then the "$\wedge$" (called as AND) operation between these two is defined by,

$$(F_{p_1}, E_1) \wedge (G_{p_2}, E_2) = (H_{p_3}, E_1 \times E_2),$$

where

$$H_{p_3}(a, b) = (H(a, b)(x), \wedge(a, b)(x)), \quad \forall (a, b) \in E_1 \times E_2.$$

Here,

$$H(a, b) = F(a) \cap G(b) \quad \text{and} \quad \wedge (a, b) = p_1(a) \wedge p_2(b).$$

Definition 3.11. If (F_{p_1}, E_1) and (F_{p_2}, E_2) are any two PIVFNSSs, then the "$\vee$" (called as OR) operation between these two is defined by,

$$(F_{p_1}, E_1) \vee (G_{p_2}, E_2) = (H_{p_3}, E_1 \times E_2),$$

where

$$H_{p_3}(a, b) = (H(a, b)(x), \vee(a, b)(x)), \quad \forall (a, b) \in E_1 \times E_2.$$

Here,

$$H(a, b) = F(a) \cup G(b) \quad \text{and} \quad \vee (a, b) = p_1(a) \vee p_2(b).$$

Now we construct a model using PIVFNSS and apply it in decision making as given below:

Algorithm:

Step 1: Input the PIVFNSS

Step 2: Construct the "AND" matrix

Step 3: Construct the truth, indeterminacy and falsity matrices of the "AND" matrix

Step 4: Convert each matrices from PIVFNSS to PIFSS.

Step 5: Construct the weighted matrices using the formula $\mu + p - \mu p$, $\zeta + p - \zeta p$, and $\nu + p - \nu p$, where μ, ζ, and ν are membership, indeterminacy, and falsity values, and p is their respective possibility value. Find the maximum value in each row of the matrices.

Step 6: Compute the score for each weighted matrix.

Step 7: Compute the decision score

Step 8: The maximum score is the optimal decision

Example 3.12. Suppose a family consists of a husband and wife wants to buy a gated community house and so they approach three c_1, c_2, c_3 construction companies. The selection of house is based on the following parameters: Affordability (a), locality (l), amenities (m).

Let $C = \{c_1, c_2, c_3\}$ be the universe set and $P = \{a, l, m\}$ be the parameter set. Let H_{p_1} and W_{p_2} be the observations made by the husband and wife respectively. Then,

Step 1: Input PIVFNSS values.

$$H_{p_1}(a) = \left\{ \begin{array}{l} \left(\frac{c_1}{\langle [0.1, 0.3], [0.3, 0.4], [0.5, 0.6] \rangle}, 0.1 \right), \left(\frac{c_2}{\langle [0.6, 0.7], [0.0, 0.2], [0.2, 0.3] \rangle}, 0.7 \right), \\ \left(\frac{c_3}{\langle [0.2, 0.4], [0.0, 0.1], [0.0, 0.3] \rangle}, 0.2 \right) \end{array} \right\}$$

$$H_{p_1}(l) = \left\{ \begin{array}{l} \left(\frac{c_1}{\langle [0.0, 0.2], [0.0, 0.2], [0.2, 0.7] \rangle}, 0.2 \right), \left(\frac{c_2}{\langle [0.8, 0.9], [0.3, 0.4], [0.2, 0.3] \rangle}, 0.6 \right), \\ \left(\frac{c_3}{\langle [0.2, 0.4], [0.1, 0.5], [0.0, 0.1] \rangle}, 0.3 \right) \end{array} \right\}$$

$$H_{p_1}(m) = \left\{ \begin{array}{l} \left(\frac{c_1}{\langle [0.2, 0.5], [0.1, 0.3], [0.0, 0.2] \rangle}, 0.2 \right), \left(\frac{c_2}{\langle [0.6, 0.7], [0.2, 0.3], [0.3, 0.4] \rangle}, 0.8 \right), \\ \left(\frac{c_3}{\langle [0.4, 0.5], [0.6, 0.7], [0.5, 0.6] \rangle}, 0.3 \right) \end{array} \right\}$$

$$W_{p_2}(a) = \left\{ \begin{array}{l} \left(\frac{c_1}{\langle [0.2, 0.4], [0.0, 0.3], [0.3, 0.5] \rangle}, 0.3 \right), \left(\frac{c_2}{\langle [0.7, 0.8], [0.0, 0.2], [0.2, 0.3] \rangle}, 0.5 \right), \\ \left(\frac{c_3}{\langle [0.0, 0.1], [0.4, 0.5], [0.6, 0.7] \rangle}, 0.1 \right) \end{array} \right\}$$

$$W_{p_2}(l) = \left\{ \begin{array}{l} \left(\frac{c_1}{\langle [0.1, 0.2], [0.6, 0.7], [0.4, 0.5] \rangle}, 0.1 \right), \left(\frac{c_2}{\langle [0.5, 0.7], [0.2, 0.3], [0.1, 0.2] \rangle}, 0.4 \right), \\ \left(\frac{c_3}{\langle [0.3, 0.5], [0.2, 0.3], [0.1, 0.2] \rangle}, 0.3 \right) \end{array} \right\}$$

$$W_{p_2}(m) = \left\{ \begin{array}{l} \left(\frac{c_1}{\langle [0.3, 0.4], [0.6, 0.7], [0.0, 0.2] \rangle}, 0.3 \right), \left(\frac{c_2}{\langle [0.4, 0.5], [0.5, 0.5], [0.4, 0.4] \rangle}, 0.4 \right), \\ \left(\frac{c_3}{\langle [0.5, 0.6], [0.3, 0.4], [0.0, 0.1] \rangle}, 0.6 \right) \end{array} \right\}$$

From the Definition 3.10, we get,

$$I_{p_3}(a,a)=\left\{\begin{array}{l}\left(\frac{c_1}{\langle[0.1,0.3],[0.3,0.4],[0.5,0.6]\rangle},0.1\right),\left(\frac{c_2}{\langle[0.6,0.7],[0.0,0.2],[0.2,0.3]\rangle},0.5\right),\\\left(\frac{c_3}{\langle[0.0,0.1],[0.4,0.5],[0.6,0.7]\rangle},0.1\right)\end{array}\right\}$$

$$I_{p_3}(a,l)=\left\{\begin{array}{l}\left(\frac{c_1}{\langle[0.1,0.2],[0.6,0.7],[0.5,0.6]\rangle},0.1\right),\left(\frac{c_2}{\langle[0.5,0.7],[0.2,0.3],[0.2,0.3]\rangle},0.4\right),\\\left(\frac{c_3}{\langle[0.2,0.4],[0.2,0.3],[0.1,0.3]\rangle},0.2\right)\end{array}\right\}$$

$$I_{p_3}(a,m)=\left\{\begin{array}{l}\left(\frac{c_1}{\langle[0.1,0.3],[0.6,0.7],[0.5,0.6]\rangle},0.1\right),\left(\frac{c_2}{\langle[0.4,0.5],[0.5,0.5],[0.4,0.4]\rangle},0.4\right),\\\left(\frac{c_3}{\langle[0.2,0.4],[0.3,0.4],[0.0,0.3]\rangle},0.2\right)\end{array}\right\}$$

$$I_{p_3}(l,a)=\left\{\begin{array}{l}\left(\frac{c_1}{\langle[0.0,0.2],[0.0,0.3],[0.3,0.7]\rangle},0.2\right),\left(\frac{c_2}{\langle[0.7,0.8],[0.3,0.4],[0.2,0.3]\rangle},0.5\right),\\\left(\frac{c_3}{\langle[0.0,0.1],[0.4,0.5],[0.6,0.7]\rangle},0.1\right)\end{array}\right\}$$

$$I_{p_3}(l,l)=\left\{\begin{array}{l}\left(\frac{c_1}{\langle[0.0,0.2],[0.6,0.7],[0.4,0.5]\rangle},0.1\right),\left(\frac{c_2}{\langle[0.5,0.7],[0.3,0.4],[0.2,0.3]\rangle},0.4\right),\\\left(\frac{c_3}{\langle[0.2,0.4],[0.2,0.5],[0.1,0.2]\rangle},0.3\right)\end{array}\right\}$$

$$I_{p_3}(l,m)=\left\{\begin{array}{l}\left(\frac{c_1}{\langle[0.0,0.2],[0.2,0.7],[0.4,0.5]\rangle},0.2\right),\left(\frac{c_2}{\langle[0.4,0.5],[0.5,0.5],[0.4,0.4]\rangle},0.4\right),\\\left(\frac{c_3}{\langle[0.2,0.4],[0.3,0.5],[0.0,0.1]\rangle},0.3\right)\end{array}\right\}$$

$$I_{p_3}(m,a)=\left\{\begin{array}{l}\left(\frac{c_1}{\langle[0.2,0.4],[0.1,0.3],[0.3,0.5]\rangle},0.2\right),\left(\frac{c_2}{\langle[0.6,0.7],[0.2,0.3],[0.3,0.4]\rangle},0.5\right),\\\left(\frac{c_3}{\langle[0.0,0.1],[0.6,0.7],[0.6,0.7]\rangle},0.1\right)\end{array}\right\}$$

$$I_{p_3}(m,l)=\left\{\begin{array}{l}\left(\frac{c_1}{\langle[0.1,0.2],[0.6,0.7],[0.4,0.5]\rangle},0.1\right),\left(\frac{c_2}{\langle[0.5,0.7],[0.2,0.3],[0.3,0.4]\rangle},0.4\right),\\ \left(\frac{c_3}{\langle[0.3,0.5],[0.6,0.7],[0.5,0.6]\rangle},0.3\right)\end{array}\right\}$$

$$I_{p_3}(m,m)=\left\{\begin{array}{l}\left(\frac{c_1}{\langle[0.2,0.4],[0.6,0.7],[0.0,0.2]\rangle},0.2\right),\left(\frac{c_2}{\langle[0.4,0.5],[0.5,0.5],[0.4,0.4]\rangle},0.4\right),\\ \left(\frac{c_3}{\langle[0.4,0.5],[0.6,0.7],[0.5,0.6]\rangle},0.3\right)\end{array}\right\}$$

Step 2: Construct the "AND" matrix:

$$I_{p_3}=\begin{pmatrix}
(\langle[0.1,0.3],[0.3,0.4],[0.5,0.6]\rangle,0.1) & (\langle[0.6,0.7],[0.0,0.2],[0.2,0.3]\rangle,0.5) & (\langle[0.0,0.1],[0.4,0.5],[0.6,0.7]\rangle,0.1)\\
(\langle[0.1,0.2],[0.6,0.7],[0.5,0.6]\rangle,0.1) & (\langle[0.5,0.7],[0.2,0.3],[0.2,0.3]\rangle,0.4) & (\langle[0.2,0.4],[0.2,0.3],[0.1,0.3]\rangle,0.2)\\
(\langle[0.1,0.3],[0.6,0.7],[0.5,0.6]\rangle,0.1) & (\langle[0.4,0.5],[0.5,0.5],[0.4,0.4]\rangle,0.4) & (\langle[0.2,0.4],[0.3,0.4],[0.0,0.3]\rangle,0.2)\\
(\langle[0.0,0.2],[0.0,0.3],[0.3,0.7]\rangle,0.2) & (\langle[0.7,0.8],[0.3,0.4],[0.2,0.3]\rangle,0.5) & (\langle[0.0,0.1],[0.4,0.5],[0.6,0.7]\rangle,0.1)\\
(\langle[0.0,0.2],[0.6,0.7],[0.4,0.5]\rangle,0.1) & (\langle[0.5,0.7],[0.3,0.4],[0.2,0.3]\rangle,0.4) & (\langle[0.2,0.4],[0.2,0.5],[0.1,0.2]\rangle,0.3)\\
(\langle[0.0,0.2],[0.2,0.7],[0.4,0.5]\rangle,0.2) & (\langle[0.4,0.5],[0.5,0.5],[0.4,0.4]\rangle,0.4) & (\langle[0.2,0.4],[0.3,0.5],[0.0,0.1]\rangle,0.3)\\
(\langle[0.2,0.4],[0.1,0.3],[0.3,0.5]\rangle,0.2) & (\langle[0.6,0.7],[0.2,0.3],[0.3,0.4]\rangle,0.5) & (\langle[0.0,0.1],[0.6,0.7],[0.6,0.7]\rangle,0.1)\\
(\langle[0.1,0.2],[0.6,0.7],[0.4,0.5]\rangle,0.1) & (\langle[0.5,0.7],[0.2,0.3],[0.3,0.4]\rangle,0.4) & (\langle[0.3,0.5],[0.6,0.7],[0.5,0.6]\rangle,0.3)\\
(\langle[0.2,0.4],[0.6,0.7],[0.0,0.2]\rangle,0.2) & (\langle[0.4,0.5],[0.5,0.5],[0.4,0.4]\rangle,0.4) & (\langle[0.4,0.5],[0.6,0.7],[0.5,0.6]\rangle,0.3)
\end{pmatrix}$$

Step 3: Construct the Truth, Indeterminacy and Falsity matrices from the above matrix:
Truth PIVFNSS Matrix

$$\begin{pmatrix}
([0.1,0.3],0.1) & ([0.6,0.7],0.5) & ([0.0,0.1],0.1)\\
([0.1,0.2],0.1) & ([0.5,0.7],0.4) & ([0.2,0.4],0.2)\\
([0.1,0.3],0.1) & ([0.4,0.5],0.4) & ([0.2,0.4],0.2)\\
([0.0,0.2],0.2) & ([0.7,0.8],0.5) & ([0.0,0.1],0.1)\\
([0.0,0.2],0.1) & ([0.5,0.7],0.4) & ([0.2,0.4],0.3)\\
([0.0,0.2],0.2) & ([0.4,0.5],0.4) & ([0.2,0.4],0.3)\\
([0.2,0.4],0.2) & ([0.6,0.7],0.5) & ([0.0,0.1],0.1)\\
([0.1,0.2],0.1) & ([0.5,0.7],0.4) & ([0.3,0.5],0.3)\\
([0.2,0.4],0.2) & ([0.4,0.5],0.4) & ([0.4,0.5],0.3)
\end{pmatrix}$$

Indeterminacy PIVFNSS Matrix

$$\begin{pmatrix} ([0.3,0.4],0.1) & ([0.0,0.2],0.5) & ([0.4,0.5],0.1) \\ ([0.6,0.7],0.1) & ([0.2,0.3],0.4) & ([0.2,0.3],0.2) \\ ([0.6,0.7],0.1) & ([0.5,0.5],0.4) & ([0.3,0.4],0.2) \\ ([0.0,0.3],0.2) & ([0.3,0.4],0.5) & ([0.4,0.5],0.1) \\ ([0.6,0.7],0.1) & ([0.3,0.4],0.4) & ([0.2,0.5],0.3) \\ ([0.2,0.7],0.2) & ([0.5,0.5],0.4) & ([0.3,0.5],0.3) \\ ([0.1,0.3],0.2) & ([0.2,0.3],0.5) & ([0.6,0.7],0.1) \\ ([0.6,0.7],0.1) & ([0.2,0.3],0.4) & ([0.6,0.7],0.3) \\ ([0.6,0.7],0.2) & ([0.5,0.5],0.4) & ([0.6,0.7],0.3) \end{pmatrix}$$

Falsity PIVFNSS Matrix

$$\begin{pmatrix} ([0.5,0.6],0.1) & ([0.2,0.3],0.5) & ([0.6,0.7],0.1) \\ ([0.5,0.6],0.1) & ([0.2,0.3],0.4) & ([0.1,0.3],0.2) \\ ([0.5,0.6],0.1) & ([0.4,0.4],0.4) & ([0.0,0.3],0.2) \\ ([0.3,0.7],0.2) & ([0.2,0.3],0.5) & ([0.6,0.7],0.1) \\ ([0.4,0.5],0.1) & ([0.2,0.3],0.4) & ([0.1,0.2],0.3) \\ ([0.4,0.5],0.2) & ([0.4,0.4],0.4) & ([0.0,0.1],0.3) \\ ([0.3,0.5],0.2) & ([0.3,0.4],0.5) & ([0.6,0.7],0.1) \\ ([0.4,0.5],0.1) & ([0.3,0.4],0.4) & ([0.5,0.6],0.3) \\ ([0.0,0.2],0.2) & ([0.4,0.4],0.4) & ([0.5,0.6],0.3) \end{pmatrix}$$

Step 4: Convert each PIVFNSS matrix into PIFSS matrix:

Truth PIVFNSS Matrix

$$\begin{pmatrix} (0.2,0.1) & (0.65,0.5) & (0.05,0.1) \\ (0.15,0.1) & (0.6,0.4) & (0.4,0.2) \\ (0.2,0.1) & (0.45,0.4) & (0.3,0.2) \\ (0.1,0.2) & (0.75,0.5) & (0.05,0.1) \\ (0.1,0.1) & (0.6,0.4) & (0.3,0.3) \\ (0.1,0.2) & (0.45,0.4) & (0.3,0.3) \\ (0.3,0.2) & (0.65,0.5) & (0.05,0.1) \\ (0.15,0.1) & (0.6,0.4) & (0.4,0.3) \\ (0.3,0.2) & (0.45,0.4) & (0.45,0.3) \end{pmatrix}$$

Indeterminacy PIVFNSS Matrix

$$\begin{pmatrix} (0.35,0.1) & (0.1,0.5) & (0.45,0.1) \\ (0.65,0.1) & (0.25,0.4) & (0.25,0.2) \\ (0.65,0.1) & (0.5,0.4) & (0.35,0.2) \\ (0.15,0.2) & (0.35,0.5) & (0.45,0.1) \\ (0.65,0.1) & (0.35,0.4) & (0.35,0.3) \\ (0.45,0.2) & (0.5,0.4) & (0.4,0.3) \\ (0.2,0.2) & (0.25,0.5) & (0.65,0.1) \\ (0.65,0.1) & (0.25,0.4) & (0.65,0.3) \\ (0.65,0.2) & (0.5,0.4) & (0.65,0.3) \end{pmatrix}$$

Falsity PIFSS Matrix

$$\begin{pmatrix} (0.55,0.1) & (0.25,0.5) & (0.65,0.1) \\ (0.55,0.1) & (0.25,0.4) & (0.2,0.2) \\ (0.55,0.1) & (0.4,0.4) & (0.15,0.2) \\ (0.5,0.2) & (0.25,0.5) & (0.65,0.1) \\ (0.45,0.1) & (0.25,0.4) & (0.15,0.3) \\ (0.45,0.2) & (0.4,0.4) & (0.05,0.3) \\ (0.4,0.2) & (0.35,0.5) & (0.65,0.1) \\ (0.45,0.1) & (0.35,0.4) & (0.55,0.3) \\ (0.1,0.2) & (0.4,0.4) & (0.55,0.3) \end{pmatrix}$$

Step 5: Construct weighted matrices

$$\begin{pmatrix} 0.28 & 0.83 & 0.15 \\ 0.24 & 0.76 & 0.52 \\ 0.28 & 0.67 & 0.44 \\ 0.28 & 0.88 & 0.15 \\ 0.19 & 0.76 & 0.51 \\ 0.28 & 0.67 & 0.51 \\ 0.44 & 0.83 & 0.15 \\ 0.24 & 0.76 & 0.58 \\ 0.44 & 0.67 & 0.62 \end{pmatrix} \quad \begin{pmatrix} 0.42 & 0.55 & 0.51 \\ 0.69 & 0.55 & 0.40 \\ 0.69 & 0.70 & 0.48 \\ 0.32 & 0.68 & 0.51 \\ 0.69 & 0.61 & 0.55 \\ 0.56 & 0.70 & 0.58 \\ 0.36 & 0.63 & 0.69 \\ 0.69 & 0.55 & 0.76 \\ 0.72 & 0.70 & 0.76 \end{pmatrix} \quad \begin{pmatrix} 0.60 & 0.63 & 0.69 \\ 0.60 & 0.55 & 0.36 \\ 0.60 & 0.64 & 0.32 \\ 0.60 & 0.63 & 0.69 \\ 0.51 & 0.55 & 0.41 \\ 0.56 & 0.64 & 0.34 \\ 0.52 & 0.68 & 0.69 \\ 0.51 & 0.61 & 0.69 \\ 0.28 & 0.64 & 0.69 \end{pmatrix}$$

Step 6: Find the score:

$$s^t(c_1) = 0,\ s^t(c_2) = 6.83,\ s^t(c_3) = 0$$

$$s^i(c_1) = 1.38,\ s^i(c_2) = 2.63,\ s^i(c_3) = 2.21$$

$$s^f(c_1) = 0.60,\ s^f(c_2) = 1.83,\ s^f(c_3) = 3.45$$

Step 7: Compute the decision score

$$\begin{aligned} d_s(c_1) &= 0 - 1.38 - 0.60 = -1.98, \\ d_s(c_2) &= 6.83 - 2.63 - 1.83 = 2.37, \\ d_s(c_3) &= 0 - 2.21 - 3.45 = -5.66. \end{aligned}$$

Step 8: c_2 has the maximum score.

The family will select the second construction company to buy the house.

4 Sensitivity Analysis

1. The parameter a is removed:
Construct weighted matrices:

$$\begin{bmatrix} 0.19 & 0.76 & 0.51 \\ 0.28 & 0.67 & 0.51 \\ 0.24 & 0.76 & 0.58 \\ 0.44 & 0.67 & 0.62 \end{bmatrix} \quad \begin{bmatrix} 0.69 & 0.61 & 0.55 \\ 0.56 & 0.70 & 0.58 \\ 0.69 & 0.55 & 0.76 \\ 0.72 & 0.70 & 0.76 \end{bmatrix} \quad \begin{bmatrix} 0.51 & 0.55 & 0.41 \\ 0.56 & 0.64 & 0.34 \\ 0.51 & 0.61 & 0.69 \\ 0.28 & 0.64 & 0.69 \end{bmatrix}$$

Step 6: Find the score:

$$s^t(c_1) = 0,\ s^t(c_2) = 2.86,\ s^t(c_3) = 0$$

$$s^i(c_1) = 0.69,\ s^i(c_2) = 0.70,\ s^i(c_3) = 1.52$$

$$s^f(c_1) = 0,\ s^f(c_2) = 1.19,\ s^f(c_3) = 1.38$$

Step 7: Compute the decision score

$$d_s(c_1) = 0 - 0.69 - 0 = -0.69$$

$$d_s(c_2) = 2.86 - 0.70 - 1.19 = 0.97$$

$$d_s(c_3) = 0 - 1.52 - 1.38 = -2.9$$

Conclusion: c_2 has the maximum score.

2. The parameter l is removed:

Step 5: Construct weighted matrices

$$\begin{bmatrix} 0.28 & 0.83 & 0.15 \\ 0.28 & 0.67 & 0.44 \\ 0.44 & 0.83 & 0.15 \\ 0.44 & 0.67 & 0.62 \end{bmatrix} \quad \begin{bmatrix} 0.42 & 0.55 & 0.51 \\ 0.69 & 0.70 & 0.48 \\ 0.36 & 0.63 & 0.69 \\ 0.72 & 0.70 & 0.76 \end{bmatrix} \quad \begin{bmatrix} 0.60 & 0.63 & 0.69 \\ 0.60 & 0.64 & 0.32 \\ 0.52 & 0.68 & 0.69 \\ 0.28 & 0.64 & 0.69 \end{bmatrix}$$

Step 6: Find the score:

$$s^t(c_1) = 0,\ s^t(c_2) = 3,\ s^t(c_3) = 0$$

$$s^i(c_1) = 0,\ s^i(c_2) = 1.25,\ s^i(c_3) = 1.45$$

$$s^f(c_1) = 0,\ s^f(c_2) = 0.64,\ s^f(c_3) = 2.07$$

Step 7: Compute the decision score

$$d_s(c_1) = 0 - 0 - 0 = 0$$

$$d_s(c_2) = 3 - 1.25 - 0.64 = 1.11$$

$$d_s(c_3) = 0 - 1.45 - 2.07 = -3.52$$

Conclusion: c_2 has the maximum score.

3. The parameter m is removed:
Step 5: Construct weighted matrices

$$\begin{bmatrix} 0.28 & 0.83 & 0.15 \\ 0.24 & 0.76 & 0.52 \\ 0.28 & 0.88 & 0.15 \\ 0.19 & 0.76 & 0.51 \end{bmatrix} \quad \begin{bmatrix} 0.42 & 0.55 & 0.51 \\ 0.69 & 0.55 & 0.40 \\ 0.32 & 0.68 & 0.51 \\ 0.69 & 0.61 & 0.55 \end{bmatrix} \quad \begin{bmatrix} 0.60 & 0.63 & 0.69 \\ 0.60 & 0.55 & 0.36 \\ 0.60 & 0.63 & 0.69 \\ 0.51 & 0.55 & 0.41 \end{bmatrix}$$

Step 6: Find the score:

$$s^t(c_1) = 0,\ s^t(c_2) = 3.23,\ s^t(c_3) = 0$$

$$s^i(c_1) = 1.38,\ s^i(c_2) = 1.23,\ s^i(c_3) = 0$$

$$s^f(c_1) = 0.60,\ s^f(c_2) = 0.55,\ s^f(c_3) = 1.38$$

Step 7: Compute the decision score

$$d_s(c_1) = 0 - 1.38 - 0.60 = -1.98$$

$$d_s(c_2) = 3.23 - 1.23 - 0.55 = 1.45$$

$$d_s(c_3) = 0 - 0 - 1.38 = -1.38$$

Conclusion: c_2 has the maximum score.

5 Conclusion

The idea of Possibility Interval-Valued Fermatean Neutrosophic Soft Sets (PIVFNSS) has been presented and investigated in this work as a strong mathematical framework for dealing with ambiguity, uncertainty, and indeterminacy in challenging decision-making situations. In comparison to current models, the suggested model greatly improves the flexibility and accuracy of capturing uncertain information by combining the soft set and possibility theory with the interval-valued Fermatean neutrosophic environment.

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