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Enhancing Decision Making with Interval - Valued Refined Neutrosophic Hesitant Fuzzy Sets

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Abstract: The Interval-Valued Refined Neutrosophic Hesitant Fuzzy Set (IVRNHFS) is a sophisticated mathematical set that is capable of capturing and handling complicated uncertainty, hesitancy, and vagueness in decision-making problems. In this paper, the formal definition of IVRNHFS is proposed by combining the refined aspects of truth, indeterminacy, and falsity with interval-valued hesitant information, thus providing a richer and more flexible representation of imprecise data. To facilitate practical use, new score and accuracy functions are constructed to facilitate useful comparison and ranking of alternatives. Moreover, fundamental operations like union, intersection, complement, and aggregation are redefined in the IVRNHFS context to maintain the natural interval-based refinement and hesitation. The proposed framework is demonstrated with examples and compared with existing models to show its expressiveness, computational depth, and decision correctness advantage.

Keywords: Interval Valued Refined Neutrosophic Hesitant Fuzzy Set (IVRNHFS), Hesitant Fuzzy (HF), Neutrosophic Set, Score Function

1. Introduction

As we live in a rapidly more complex and uncertain world, decision-making processes tend to be challenged by handling imprecise, vague, and conflicting information. Most mathematical models and classical decision-making methods require crisp values and full information, which are hardly available in real-world scenarios. Phenomena like medical diagnosis, financial prediction, engineering assessment, environmental monitoring, and social decision-making are often characterized by expert judgments that are hesitant, incomplete, or even conflicting. Therefore, increasingly there is a need for more expressive and robust frameworks that can capture and represent these levels of uncertainty very well.

The concept of handling uncertainty in mathematical modeling began with the seminal work of Zadeh [1], who introduced fuzzy set theory**.** This framework provided a way to mathematically model vagueness by assigning membership values ranging from 0 to 1, instead of strict binary classifications. It became a cornerstone for decision-making models under uncertainty.

Atanassov [2] extended this idea by developing intuitionistic fuzzy sets, along with a hesitation degree. This innovation allowed for more comprehensive modeling of uncertainty, especially in situations where decision-makers face hesitation about the belongingness of elements.

Torra [3] further enhanced fuzzy modeling by formulating hesitant fuzzy sets, which allow elements to have multiple possible membership values. Rodriguez, Martinez, and Herrera [4] expanded this concept into hesitant fuzzy linguistic term sets, which enabled decision-makers to express hesitation linguistically. This work bridged the gap between linguistic evaluation and fuzzy modeling.

Xia and Xu [5] contributed by exploring aggregation methods for hesitant fuzzy information, developing operators for combining hesitant fuzzy values in decision-making. Zhang and Xu [6] advanced this by introducing interval-valued hesitant fuzzy sets. which further strengthened the ability to capture uncertainty in complex multi-criteria problems. Farhadinia [7] entropy-based measures to quantify uncertainty. Liao, Xu, and Zeng [8] developed distance and similarity measures for hesitant fuzzy linguistic term sets, which significantly improved the applicability of hesitant fuzzy theory.

Moving beyond fuzzy-based models, Smarandache [9] introduced neutrosophy, a generalization that explicitly incorporates truth, indeterminacy, and falsity values as independent components. This theory broadened the capacity of uncertainty modeling to capture paradoxes and contradictions in human reasoning. Building on this, Wang, Smarandache, Zhang, and Sunderraman [10] proposed SVNS, which allowed practical application of neutrosophic theory by assigning crisp values.

Biswas, Pramanik, and Giri [11] developed a methodology under SVNS with unknown weight information, addressing real-world problems where attribute weights are not always predefined. Pramanik, Dey, and Giri [12] extended neutrosophic decision making by integrating SVNS into the TOPSIS method, enabling systematic ranking of alternatives under uncertainty. Broumi et al. [13] studied correlation coefficients for interval-valued neutrosophic sets, providing a tool for measuring relationships among neutrosophic data. Later, Pramanik, Dey, and Smarandache [14] refined this by introducing correlation coefficients for interval bipolar neutrosophic sets, extending correlation-based analysis to more complex settings.

Al-Quran, Awang, Ali, and Abdullah [15] developed the hesitant bipolar-valued neutrosophic set, which combines bipolar information with neutrosophic hesitation, thereby extending modeling capacity for complex decision-making environments. Ramaroson and Andriamanohisoa [16] introduced the IVBNHFS, exploring its properties and operations, thereby opening new avenues in hesitant and bipolar neutrosophic theory.

Deli, Şubaş, Smarandache, and Ali [17] proposed IVBNS and demonstrated their utility in decision-making problems. Saqlain, Hamza, Saeed, and Zulqarnain [18] studied aggregate, arithmetic, and geometric operators of octagonal neutrosophic numbers, providing new aggregation tools for multi-criteria decision making. Ajay, Aldring, and Nivetha [19] introduced neutrosophic cubic fuzzy Dombi Hamy mean operators, which allow for advanced aggregation of neutrosophic cubic fuzzy information.

In terms of applications, Abdel-Basset, Atef, and Smarandache [20] developed a hybrid neutrosophic group decision-making approach for project selection, demonstrating the utility of neutrosophic sets in real-world project management. Nabeeh, Abdel-Basset, El-Ghareeb, and Aboelfetouh [21] extended this approach to IoT-based enterprises, providing solutions for decision-making challenges in emerging technologies. Mondal and Pramanik [22] applied neutrosophic decision-making models to the problem of school choice, showing their relevance in the education sector. Ye [23] proposed an advancing techniques for handling hesitation in neutrosophic decision problems.

Beyond technical and decision-focused studies, neutrosophic theory has been applied in social and assessment contexts. Aslan, Kargın, and Şahin [24] demonstrating the flexibility of neutrosophic systems in social sciences. Voskoglou [25] introduced the neutrosophic theory into educational and evaluative frameworks.

The aim of this research is to establish the definition and form of IVRNHFS formally and illustrate its utility in improving decision-making under uncertainty. It seeks to define appropriate score and ranking functions, construct aggregation operators, and determine the underlying set-theoretic operations essential for real-world implementation. Integrating this model into a decision-making problem, we illustrate how it facilitates stronger and more transparent assessments, particularly when the decision space consists of fuzzy, interval-based, and hesitant information. The importance of this method is further emphasized via a case study from the real world, indicating its utility in resolving intricate decision problems where traditional models are likely to be insufficient.

This paper is organized as follows: Section 2 gives the basic definitions. Section 3 gives the definition of IVRNHFS. Section 4 introduces a new operation on these sets and explains its main properties. Section 5 gives the suggested algorithm for the score function. Section 6 shows a numerical example. Lastly, Section 7 concludes the research.

1.1 . Contribution of the Work

The primary contributions of this paper are as follows. Firstly, we propose a new score function for IVRNHFS, enhancing the ranking and comparison capacities of hesitant information in complicated decision contexts. Secondly, we introduce a new operation on IVRNHFS and explore its key mathematical characteristics, thus enriching the theoretical framework of neutrosophic set extensions. Lastly, we offer a example that illustrates the practical efficacy of the proposed method.

1.2. Significance of Neutrosophic Sets in this Research

Neutrosophic theory offers a robust mathematical framework to manage indeterminacy, inconsistency, and hesitation, which are most often encountered in real-world problems. In this case, IVRNHFS offers a more flexible and more subtle manner of representing uncertain and hesitant information than classical fuzzy and intuitionistic fuzzy methods. The application of neutrosophic sets in this research not only improves the precision of decision analysis but also enables more realistic modeling for complicated cases, so the designed method is particularly valuable for real-world applications in engineering, management, and social sciences.

2. Preliminaries

2.1 Fuzzy Set [11]

A fuzzy set, introduced by Zadeh (1965), is characterized by a membership function that assigns to each element in the universe of discourse a value between 0 and 1. Formally, a fuzzy set F in a universe V is defined as:

$$F = \{\langle x, \mu_F(v)\rangle \mid v \in V\}$$

where $\mu_F(v) \in [0,1]$ represents the degree of membership of element v in set F.

2.2 Neutrosophic sets [11]

Let V be a universal set then, the neutrosophic set F is an object having the form

$$F = \{\langle v : T'_F(v), I'_F(v), F'_F(v)\rangle, v \in V\},$$

where the functions $T', I', F' : V \to]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $v \in V$ to the element F with the condition.

$$0 \le T'_F(v) + I'_F(v) + F'_F(v) \le 3^+$$

2.3 Single-valued Neutrosophic sets [11]

Let V be a universal set, with generic element of V denoted by v. A single valued neutrosophic set F in V is characterized by a truth-membership function TA, indeterminacy function IA and falsity-membership function FA, with for each, $v \in V$, $T'_F(v)$, $I'_F(v)$, $F'_F(v) \in [0,1]$.

Note that for a SVNS A, the relation

$$0 \leq T'_F(v) + I'_F(v) + F'_F(v) \leq 3^+$$

holds. When the universal set V is continuous, a SVNS F can be written as

$$F = \int_v \langle T'_F(v),\ I'_F(v),\ F'_F(v) \rangle / v, v \in V$$

When the universal set V is discrete, a SVNS F can be written as

$$F = \sum_{i=1}^{n} \langle T'_F(v_i),\ I'_F(v_i),\ F'_F(v_i) \rangle v_i, v \in V$$

2.4 Hesitant sets [3]

Let V be a fixed set, a hesitant fuzzy set (HFS) on V is in terms of a function that when applied to V returns a subset of [0,1] and it is expressed as

$$F = \{\langle v, h_F(v)\rangle \mid v \in V\}$$

where $h_F(v)$ is a set of some values in [0, 1], denoting the possible membership degrees of the element $v \in V$ to the set F.

3. Interval Valued Refined Neutrosophic Hesitant Fuzzy Set (IVRNHFS)

Let V be the universal set. Then an Interval-Valued Refined Neutrosophic Hesitant Fuzzy Set (IVRNHFS) A on V is defined as :

$$A = \{\langle x : h_{T_A}(v), h_{I_A}(v), h_{F_A}(v)\rangle, v \in V\} \tag{1}$$

where:

$$h_T(v) = \left\{ \left[T_A^{L_{i,j}}(v),\ T_A^{U_{i,j}}(v)\right] : i = 1,2,\ldots,k;\ j = 1,2,\ldots,m(i)\right\}$$
$$h_I(v) = \left\{ \left[I_A^{L_{i,j}}(v),\ I_A^{U_{i,j}}(v)\right] : i = 1,2,\ldots,k;\ j = 1,2,\ldots,n(i)\right\}$$
$$h_F(v) = \left\{ \left[F_A^{L_{i,j}}(v),\ F_A^{U_{i,j}}(v)\right] : i = 1,2,\ldots,k;\ j = 1,2,\ldots,p(i)\right\}$$

Here, $h_T(v), h_I(v), h_F(v)$ are hesitant sets of intervals representing the refined truth, indeterminacy and falsity membership degrees and k is the number of refined levels. $m(i), n(i), p(i)$ are the number of hesitant intervals for the i-th refined level of truth, indeterminacy and falsity respectively.

For each interval:

$$\left[T_A^{L_{i,j}}(v),\ T_A^{U_{i,j}}(v)\right] \subseteq [0,1]$$
$$\left[I_A^{L_{i,j}}(v),\ I_A^{U_{i,j}}(v)\right] \subseteq [0,1]$$
$$\left[F_A^{L_{i,j}}(v),\ F_A^{U_{i,j}}(v)\right] \subseteq [0,1]$$

Additionally, for each $v \in V$, it must hold that:

$$0 \leq \sup\ T_A^{U_{i,j}}(v) + \sup\ I_A^{U_{i,j}}(v) + \sup\ F_A^{U_{i,j}}(v) \leq 3$$

4. Operations on IVRNHFS

Let $A = \{\langle x : h_{T_A}(v), h_{I_A}(v), h_{F_A}(v)\rangle, v \in V\}$ and $B = \{\langle x : h'_{T_A}(v), h'_{I_A}(v), h'_{F_A}(v)\rangle, v \in V\}$ b2 two IVRNHFS over the universe V where each component consists of sets of interval-valued refined hesitant degrees.

4.1 Union

The union of two IVRNHFSs A and B is a new IVRNHFS C, written as C = A ∪ B, where the refined truth-membership, indeterminacy and falsity membership functions are defined as:

$$T_C^{ij}(v) = \max\left(T_A^{ij}(v),\ T_B^{ij}(v)\right),$$
$$I_C^{ij}(v) = \min\left(I_A^{ij}(v),\ I_B^{ij}(v)\right),$$
$$F_C^{ij}(v) = \min\left(F_A^{ij}(v),\ F_B^{ij}(v)\right),\quad for\ all\ \ v \in V$$

4.2 Intersection

The intersection of two IVRNHFSs A and B is a new IVRNHFS C, written as C = A ∩ B, where the refined truth-membership, indeterminacy and falsity membership functions are defined as:

$$T_C^{ij}(v) = \min\left(T_A^{ij}(v),\ T_B^{ij}(v)\right),$$
$$I_C^{ij}(v) = \max\left(I_A^{ij}(v),\ I_B^{ij}(v)\right),$$
$$F_C^{ij}(v) = \max\left(F_A^{ij}(v),\ F_B^{ij}(v)\right),\quad for\ all\ \ v \in V$$

4.3 Complement

The complement of a Interval valued Refined Neutrosophic Hesitant Fuzzy Set (IVRNHFS) A is denoted by A^c, and is defined as:

$$T_{A^c}^{ij}(v) = F_A^{ij}(v),$$
$$I_{A^c}^{ij}(v) = I_A^{ij}(v),$$
$$F_{A^c}^{ij}(v) = T_A^{ij}(v), \text{for all } \ v \in V$$

4.4 Containment

An Interval Valued Refined Neutrosophic Hesitant Fuzzy Set (IVRNHFS) A is said to be contained in another IVRNHFS B, denoted by $A \subseteq B$, if and only if:

$$T_A^{ij}(v) \le T_B^{ij}(v),$$
$$I_A^{ij}(v) \ge I_B^{ij}(v),$$
$$F_A^{ij}(v) \ge F_B^{ij}(v), \quad \text{for all } \ v \in V$$

Example

Let the two Interval-Valued Refined Neutrosophic Hesitant Fuzzy Sets (IVRNHFS) over the universe V be

$$A = \left\{ \begin{array}{l} \left(\begin{array}{l} v_1, \{[0.6, 0.8], [0.7, 0.9], [0.5, 0.7], [0.6, 0.75]\},\ \{[0.2, 0.4], [0.1, 0.3], [0.3, 0.5], [0.2, 0.4]\}, \\ \{[0.1, 0.2], [0.15, 0.25], [0.05, 0.15], [0.1, 0.2]\} \end{array} \right), \\ \left(\begin{array}{l} v_2, \{[0.4, 0.6], [0.5, 0.65], [0.45, 0.6], [0.5, 0.7]\},\ \{[0.3, 0.5], [0.35, 0.6], [0.3, 0.4], [0.2, 0.4]\}, \\ \{[0.1, 0.2], [0.15, 0.25], [0.1, 0.15], [0.1, 0.2]\} \end{array} \right) \\ \left(\begin{array}{l} v_3, \{[0.7, 0.9], [0.75, 0.95], [0.8, 0.9], [0.7, 0.85]\}, \{[0.1, 0.2], [0.15, 0.25], [0.05, 0.15], [0.1, 0.2]\}, \\ \{[0.05, 0.1], [0.1, 0.15], [0.05, 0.1], [0.05, 0.1]\} \end{array} \right) \end{array} \right\}$$

$$B=\left\{\begin{array}{l}\left(\begin{array}{l}v_1,\{[0.7,0.9],[0.8,0.9],[0.6,0.8],[0.7,0.75]\},\ \{[0.3,0.4],[0.2,0.3],[0.4,0.45],[0.3,0.35]\},\\ \{[0.01,0.02],[0.15,0.25],[0.05,0.1],[0.01,0.02]\}\end{array}\right),\\ \left(\begin{array}{l}v_2,\{[0.5,0.7],[0.6,0.65],[0.5,0.6],[0.6,0.8]\},\ \{[0.3,0.4],[0.35,0.4],[0.3,0.35],[0.2,0.3]\},\\ \{[0.1,0.15],[0.1,0.2],[0.1,0.15],[0.01,0.02]\}\end{array}\right)\\ \left(\begin{array}{l}v_3,\{[0.8,0.9],[0.8,0.95],[0.8,0.9],[0.8,0.9]\},\{[0.3,0.4],[0.25,0.35],[0.25,0.35],[0.2,0.3]\},\\ \{[0.05,0.1],[0.2,0.25],[0.05,0.1],[0.05,0.1]\}\end{array}\right)\end{array}\right\}$$

Then, the union of two IVRNHFS is

$$C=\left\{\begin{array}{l}\left(\begin{array}{l}v_1,\{[0.7,0.9],[0.8,0.9],[0.6,0.8],[0.7,0.75]\},\ \{[0.2,0.4],[0.1,0.3],[0.3,0.45],[0.2,0.35]\},\\ \{[0.01,0.02],[0.15,0.25],[0.05,0.1],[0.01,0.02]\}\end{array}\right),\\ \left(\begin{array}{l}v_2,\{[0.5,0.7],[0.6,0.65],[0.5,0.6],[0.6,0.8]\},\ \{[0.3,0.4],[0.35,0.4],[0.3,0.35],[0.2,0.3]\},\\ \{[0.1,0.15],[0.1,0.2],[0.1,0.15],[0.01,0.02]\}\end{array}\right)\\ \left(\begin{array}{l}v_3,\{[0.8,0.9],[0.8,0.95],[0.8,0.9],[0.8,0.9]\},\{[0.1,0.2],[0.15,0.25],[0.05,0.15],[0.1,0.2]\},\\ \{[0.05,0.1],[0.1,0.15],[0.05,0.1],[0.05,0.1]\}\end{array}\right)\end{array}\right\}$$

The intersection of two IVRNHFS is

$$C=\left\{\begin{array}{l}\left(\begin{array}{l}v_1,\{[0.6,0.8],[0.7,0.9],[0.5,0.7],[0.6,0.75]\},\ \{[0.3,0.4],[0.2,0.3],[0.4,0.5],[0.3,0.4]\},\\ \{[0.1,0.2],[0.15,0.25],[0.05,0.15],[0.1,0.2]\}\end{array}\right),\\ \left(\begin{array}{l}v_2,\{[0.4,0.6],[0.5,0.65],[0.45,0.6],[0.5,0.7]\},\ \{[0.3,0.5],[0.35,0.6],[0.3,0.4],[0.2,0.4]\},\\ \{[0.1,0.2],[0.15,0.25],[0.1,0.15],[0.1,0.2]\}\end{array}\right)\\ \left(\begin{array}{l}v_3,\{[0.7,0.9],[0.75,0.95],[0.8,0.9],[0.7,0.85]\},\{[0.3,0.4],[0.25,0.35],[0.25,0.35],[0.2,0.3]\},\\ \{[0.05,0.1],[0.2,0.25],[0.05,0.1],[0.05,0.1]\}\end{array}\right)\end{array}\right\}$$

The complement of the set A is

$$A^c=\left\{\begin{array}{l}\left(\begin{array}{l}v_1,\{[0.1,0.2],[0.15,0.25],[0.05,0.15],[0.1,0.2]\},\ \{[0.2,0.4],[0.1,0.3],[0.3,0.5],[0.2,0.4]\},\\ \{[0.6,0.8],[0.7,0.9],[0.5,0.7],[0.6,0.75]\}\end{array}\right),\\ \left(\begin{array}{l}v_2,\{[0.1,0.2],[0.15,0.25],[0.1,0.15],[0.1,0.2]\},\ \{[0.3,0.5],[0.35,0.6],[0.3,0.4],[0.2,0.4]\},\\ \{[0.4,0.6],[0.5,0.65],[0.45,0.6],[0.5,0.7]\}\end{array}\right)\\ \left(\begin{array}{l}v_3,\{[0.05,0.1],[0.1,0.15],[0.05,0.1],[0.05,0.1]\},\{[0.1,0.2],[0.15,0.25],[0.05,0.15],[0.1,0.2]\},\\ \{[0.7,0.9],[0.75,0.95],[0.8,0.9],[0.7,0.85]\}\end{array}\right)\end{array}\right\}$$

5. Proposed Score Function

Let $A=\left\{\left\langle v:h_{T_A}(v),h_{I_A}(v),h_{F_A}(v)\right\rangle,v\in V\right\}$ be IVRNHFS

(a) Weighted Arithmetic Average Operator (WAAM) for a IVRNHFS

The WAAM for a IVRNHFS is defined as a weighted aggregation of the hesitant intervals, with individual weights assigned to each membership component:

$$AO_{WA}(v)=w_T h_T(v)+w_I h_I(v)+w_F h_F(v) \tag{2}$$

where $w_T, w_I, w_F \in [0,1]$ are the weights, with $w_T+w_I+w_F=1$. The resulting aggregated IVRNHFS is:

$$AO_{WA}(v)=\left\langle h_{T_{WA}}(v),\ h_{I_{WA}}(v),\ h_{F_{WA}}(v)\right\rangle$$

where,

$$h_{T_{WA}}(v)=\left[1-\prod_{j=1}^{mT}\left(1-T_j^L\right)^{wT},\ 1-\prod_{j=1}^{mT}\left(1-T_j^U\right)^{wT}\right]$$

$$h_{I_{WA}}(v)=\left[\prod_{j=1}^{mI}\left(I_j^L\right)^{wI},\prod_{j=1}^{mI}\left(I_j^U\right)^{wI}\right]$$

$$h_{F_{WA}}(v)=\left[\prod_{j=1}^{mF}\left(F_j^L\right)^{wF},\prod_{j=1}^{mF}\left(F_j^U\right)^{wF}\right]$$

(b) Weighted Geometric Average Operator (WGAM) for a IVRNHFS

The WGAM for a INRNHFS is defined as a weighted geometric aggregation of the hesitant intervals, with individual weights for each membership component:

$$AO_{WG}(x)=(h_T(x))^{wY}\,(h_I(x))^{wI}\,\,(h_F(x))^{wF} \tag{3}$$

where $w_T, w_I, w_F \in [0,1]$ are the weights, with $w_T + w_I + w_F = 1$. The resulting aggregated IVRNHFS is:

$$AO_{WG}(v)=\left\langle h_{T_{WG}}(v),\ h_{I_{WG}}(v),\ h_{F_{WG}}(v)\right\rangle$$

where,

$$h_{T_{WG}}(v)=\left[\prod_{j=1}^{mT}\left(T_j^L\right)^{wT},\ \prod_{j=1}^{mT}\left(T_j^U\right)^{wT}\right]$$

$$h_{I_{WG}}(v)=\left[1-\prod_{j=1}^{mI}\left(1-I_j^L\right)^{wI},1-\prod_{j=1}^{mI}\left(1-I_j^U\right)^{wI}\right]$$

$$h_{F_{WG}}(v)=\left[1-\prod_{j=1}^{mF}\left(1-F_j^L\right)^{wF},1-\prod_{j=1}^{mF}\left(1-F_j^U\right)^{wF}\right]$$

(c) Hybrid Aggregation Operator (HAO) for a IVRNHFS

The HAO combines the WAAM and WGAM with a parameter $\alpha \in [0,1]$ to balance their contributions, using individual weights for each membership component:

$$AO_{HA}(v)=\alpha\,AO_{WA}(v)+(1-\alpha)\,AO_{WG}(v) \tag{4}$$

(d) Hybrid Score Function (HSF) for IVRNHFS

The HSF evaluates an IVRNHFS or its aggregated form $AO_{HA}(v)$ by transforming the interval-valued membership degrees into a single numerical score for comparison:

$$S(A)=\frac{1}{3}\left[\frac{T^{U'}+T^{L'}}{2}+\left(1-\frac{I^{U'}+I^{L'}}{2}\right)+\left(1-\frac{F^{U'}+F^{L'}}{2}\right)\right] \tag{5}$$

6. Numerical Example

A hospital is tasked with selecting the most appropriate treatment plan for a patient diagnosed with a rare form of cancer, such as pancreatic cancer, where multiple treatment options (e.g., chemotherapy, immunotherapy or surgery) are available. The decision involves evaluating treatments based on refined levels like effectiveness and side effects. Each criterion is fraught with uncertainty due to incomplete clinical data, varying expert opinions, and patient-specific factors. The hospital uses IVRNHFS to model this complex decision-making process, as it can handle interval-based uncertainty, indeterminacy, and hesitancy more effectively than other fuzzy set frameworks.

i) Evaluation of Immunotherapy (v_1)

Truth Membership Intervals:

Level 1: [0.6, 0.8], [0.7, 0.9]
Level 2: [0.5, 0.7], [0.6, 0.75]
Then the Refined truth membership:

$$h_T(v_1) = \{[0.6, 0.8], [0.7, 0.9], [0.5, 0.7], [0.6, 0.75]\}$$

Indeterminacy Intervals:

Level 1: [0.2, 0.4], [0.1, 0.3]
Level 2: [0.3, 0.5], [0.2, 0.4]
Then the Refined Indeterminacy membership:

$$h_I(v_1) = \{[0.2, 0.4], [0.1, 0.3], [0.3, 0.5], [0.2, 0.4]\}$$

Falsity Intervals:

Level 1: [0.1, 0.2], [0.15, 0.25]
Level 2: [0.05, 0.15], [0.1, 0.2]
Then the Refined Falsity membership:

$$h_F(v_1) = \{[0.1, 0.2], [0.15, 0.25], [0.05, 0.15], [0.1, 0.2]\}$$

ii) Evaluation of Chemotherapy (v_2)

Truth:

Level 1: [0.4, 0.6], [0.5, 0.65]
Level 2: [0.45, 0.6], [0.5, 0.7]
Then the Refined truth membership:

$$h_T(v_2) = \{[0.4, 0.6], [0.5, 0.65], [0.45, 0.6], [0.5, 0.7]\}$$

Indeterminacy:

Level 1: [0.3, 0.5], [0.35, 0.6]
Level 2: [0.3, 0.4], [0.2, 0.4]
Then the Refined Indeterminacy membership:

$$h_I(v_2) = \{[0.3, 0.5], [0.35, 0.6], [0.3, 0.4], [0.2, 0.4]\}$$

Falsity:

Level 1: [0.1, 0.2], [0.15, 0.25]
Level 2: [0.1, 0.15], [0.1, 0.2]
Then the Refined Falsity membership:

$$h_F(v_2) = \{[0.1, 0.2], [0.15, 0.25], [0.1, 0.15], [0.1, 0.2]\}$$

iii) Evaluation of Sugeory (v_3)

Truth:

Level 1: [0.7, 0.9], [0.75, 0.95]

Level 2: [0.8, 0.9], [0.7, 0.85]
Then the Refined truth membership:

$$h_T(v_3)=\{[0.7,0.9],[0.75,0.95],[0.8,0.9],[0.7,0.85]\}$$

Indeterminacy:

Level 1: [0.1, 0.2], [0.15, 0.25]
Level 2: [0.05, 0.15], [0.1, 0.2]
Then the Refined Indeterminacy membership:

$$h_I(v_3)=\{[0.1,0.2],[0.15,0.25],[0.05,0.15],[0.1,0.2]\}$$

Falsity:

Level 1: [0.05, 0.1], [0.1, 0.15]
Level 2: [0.05, 0.1], [0.05, 0.1]
Then the Refined Falsity membership:

$$h_F(v_3)=\{[0.05,0.1],[0.1,0.15],[0.05,0.1],[0.05,0.1]\}$$

Thus, the **IVRNHFS set A** is:

$$A=\left\{\begin{array}{l}\left(\begin{array}{l}v_1,\{[0.6,0.8],[0.7,0.9],[0.5,0.7],[0.6,0.75]\},\ \{[0.2,0.4],[0.1,0.3],[0.3,0.5],[0.2,0.4]\},\\ \{[0.1,0.2],[0.15,0.25],[0.05,0.15],[0.1,0.2]\}\end{array}\right),\\ \left(\begin{array}{l}v_2,\{[0.4,0.6],[0.5,0.65],[0.45,0.6],[0.5,0.7]\},\ \{[0.3,0.5],[0.35,0.6],[0.3,0.4],[0.2,0.4]\},\\ \{[0.1,0.2],[0.15,0.25],[0.1,0.15],[0.1,0.2]\}\end{array}\right)\\ \left(\begin{array}{l}v_3,\{[0.7,0.9],[0.75,0.95],[0.8,0.9],[0.7,0.85]\},\{[0.1,0.2],[0.15,0.25],[0.05,0.15],[0.1,0.2]\},\\ \{[0.05,0.1],[0.1,0.15],[0.05,0.1],[0.05,0.1]\}\end{array}\right)\end{array}\right\}$$

By using the above proposed score function,
$S(v_1) = 0.6983$, $S(v_2) = 0.6282$, $S(v_3) = 0.8054$
IVRNHFS is particularly suited for this medical decision-making scenario because:
Interval-valued representation: Clinical data, such as treatment effectiveness, is often imprecise and expressed in ranges (e.g., a 60–80% success rate for chemotherapy).
Refined neutrosophic structure: Each criterion is evaluated for truth (how well it meets the goal, e.g., tumor reduction), indeterminacy (uncertainty due to limited trial data), and falsity (likelihood of failure, e.g., severe side effects). This is critical in cancer treatment, where data may be inconclusive.
Hesitant fuzzy component: Oncologists and specialists may hesitate when assigning a single value to a criterion due to conflicting evidence or patient variability. For example, immunotherapy's effectiveness might be rated as {[0.6, 0.7], [0.7, 0.8]} by different experts.

7. Conclusion

IVRNHFS is an effective concept that supports uncertainty, indeterminacy, and hesitation within the complex decision-making setting. IVRNHFS provides flexible aggregation of information via the clearly defined the operations. The score function is important in measuring the total assessment of each alternative by summing up the corrected truth, indeterminacy and falsity intervals. Depending on these scores, a ranking function allows alternatives to be compared and prioritized. This model finds particular application in expert systems and multi-criteria decision-making, where judgments can be inexact, or conflictual. IVRNHFS increases the expressiveness and correctness of decision models. This mathematical strength and flexibility qualifies it to be used in engineering, medicine and computational intelligence. . In the future, this research can be further extended by creating sophisticated aggregation operators and similarity measures, and by investigating real-world applications of IVRNHFS in domains like medical diagnosis, supply chain management, and risk analysis.

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