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On Restrained Neutrosophic Domination Number on Special Graphs

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Abstract: Neutrosophic theory, which generalizes fuzzy and classical logic by combining the notions of truth, indeterminacy, and falsity, has emerged as a powerful framework in modeling uncertainty. In this paper, we initiate and investigate the study of restrained neutrosophic domination number in graphs, also we study its properties across special graph classes such as paths, cycles, stars, and complete graphs. We also establish upper and lower bounds to determine such sets in graphs and aim to define neutrosophic algebraic structures and examine their properties in depth.

Keywords: Neutrosophic graphs, Neutrosophic domination graphs, Restrained domination, Restrained Neutrosophic domination graphs, Special graphs.

1. Introduction

Neutrosophy is a new science that belongs to the field of philosophy. It focuses on understanding where neutralities come from, what they are like, and how far they go [5,9]. Among various topics in graph theory [1], the study of domination and restrained domination has emerged as a vital area due to its wide range of theoretical and practical applications. The concept of restrained domination was introduced by Telle [6], albeit as a vertex partitioning problem. A set S $\subseteq$ V of vertices of a graph G = (V, E) is a dominating set if every vertex v $\in V$ is an element of S or adjacent to an element of S [8]. Equivalently, a set S of vertices of G is a dominating set of G if every vertex in V - S is adjacent to some vertex in S. A restrained dominating set is a set S $\subseteq$ V where every vertex in V − S is adjacent to a vertex in S as well as another vertex in V – S [14]. As a relatively new field of study within philosophy, neutrosophy examines the nature, extent, and genesis of neutralities. Neutrosophic Set Theory, introduced by Florentin Smarandache[15], provides a more expressive framework by associating every element with degrees of truth (T), indeterminacy (I), and falsity (F). It has become very important that the idea of neutrosophic logic plays a key role in solving many real-world problems in areas like law, medicine, industry, finance, engineering, and IT. In 1965, Zadeh [16] introduced the concept of fuzzy sets, which is based on the idea of truth membership degree. In 1986, Atanassow [4] added the idea of false membership degree to fuzzy sets, creating intuitionistic fuzzy sets. In 1995, Smarandache introduced neutrosophic sets by including the concept of indeterminate membership degree to intuitionistic fuzzy sets. There are three different ways to define a neutrosophic graph. [3,7,10].

2. Preliminaries

In this section, we give some basic information about the paper.

Definition 2.1:

A fuzzy set on a given set V is a function A that maps elements of V to values between 0 and 1.[2]

Definition 2.2:

"Suppose that V is a given set. A truth membership function (TA(x), an indeterminate membership function (IA(x), and a false membership function (FA(x)) define a neutrosophic set A in V. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are fuzzy sets on V. That is, $T_A(x): V \rightarrow [0, 1]$, $I_A(x) : V \rightarrow [0, 1]$ and $F_A(x) : V \rightarrow [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$" [2,11]

Definition 2.3:

Let V be a specified set. Assume that E is a given set with respect to V as well. A neutrosophic graph is a pair G = (A, B), where A : V → [0, 1] is a neutrosophic set in V and B : E → [0, 1] is a neutrosophic set in E such that

$T_B (xy) \leq \min\{T_A(x), T_A(y)\}$,

$I_B (xy) \leq \min\{I_A(x), I_A(y)\}$,

$F_B (xy) \leq \max\{F_A(x), F_A(y)\}$,

for all {x, y} ∈ E. V is called vertex set of G and E is called edge set of G, respectively.[11]

Definition 2.4:

"Consider the graph G = (V, E). It is said that a vertex V dominates both itself and every neighbor; in other words, a V dominates the vertices in its closed neighborhood N[V].Therefore v dominates 1 + deg v vertices of G. A set S ⊆ V of vertices of a graph G = (V, E) is a dominating set if every vertex v $\in V$ is an element of S or adjacent to an element of S. Equivalently, a set S of vertices of G is a dominating set of G if every vertex in V - S is adjacent to some vertex in S. The domination number of G, which is written as γ(G), is the smallest number of elements in a dominating set of G".[2]

Definition 2.5:

A set S ⊆ V is a restrained dominating set if each vertex in V − S is next to both a vertex in S and another vertex in V − S. $\gamma_r(G)$ represents the smallest cardinality of a restrained dominating set of G, which is the restrained domination number of G.[15]

Definition 2.6:

"Let G = (A, B) be a neutrosophic graph on V and let x, y ∈ V. Then,

- If the edge xy is T-effective, we refer to x dominating y in G as T-effective. If for each v ∈ V − S there is u ∈ S such that u dominates v as T-effective, then a subset S of V is the T-effective dominating set in G.
- If the edge xy is I-effective, then x dominates y in G as I-effective. The I-effective dominating set in G is a subset S of V if, for each v in V − S, there exists u in S such that u dominates v as I-effective.
- If the edge xy is F-effective, then x dominates y in G as F-effective. If for each v ∈ V − S there is u ∈ S such that u dominates v as F-effective, then a subset S of V is the F-effective dominating set in G.
- If the edge xy is effective, we say that x dominates y in G. In G, a subset S of V is referred to as the effective dominating set if, for each v in V − S, there exists u in S such that u effectively dominates v".[2,14].

Definition 2.7:
Let G = (A, B) be a neutrosophic graph on a set V. If the following criteria are met, G is referred to as complete:

- T_B (xy) = min{T_A(x), T_A(y)},
- I_B (xy) = min{I_A(x), I_A(y)}
- F_B (xy) = max{F_A(x), F_A(y)}, for all {x, y} ∈ E [12]

Definition 2.8:
Let G = (A, B) be a neutrosophic graph on a set V. If the following criteria are met, G is referred to as empty: For every {x, y} ∈ E, T_B (xy) = I_B (xy) = F_B (xy) = 0. [2]

Definition 2.9:
"Let V be a specified set. If the set V can be divided into two nonempty sets, V_1 and V_2, such that T_B(xy) = I_B (xy) = F_B (xy) = 0 for all {x, y} ∈ E_1 or {x, y} ∈ E_2, then the neutrosophic graph G = (A, B) on V is said to be bipartite. Furthermore, for all {x, y} ∈ E, G is referred to as a complete bipartite neutrosophic graph if T_B(xy) = min{T_A(x), T_A(y)}, I_B (xy) = min{I_A(x), I_A(y)}, and F_B (xy) = max{F_A(x), F_A(y)}. In this instance, the full bipartite neutrosophic graph is referred to as a star neutrosophic graph if either |V1| or |V2| = 1". [13]

Definition 2.10:
"On a set V, let G = (A, B) be a neutrosophic graph. Then, an edge xy in G is referred to as the

- T-bridge, in the event that each T-path P from x to y that did not involve xy had strengths lower than T_B(xy).
- I-bridge, in the event that each I-path P from x to y that did not involve xy had strengths lower than I_B(xy).
- F-bridge, in the event that each F-path P from x to y that did not involve xy had strengths lower than F_B (xy).
- bridge, in the event that it was one of the T, I, or F bridges".[11]

Definition 2.11:
Let G = (A, B) be a neutrosophic graph on a set V. G is then referred to as,

- T-acyclic, if there was no T-path P from x to y, except in the case of x = y for all x ∈ V.
- I-acyclic if there was no I-path P from x to y, except in the case of x = y for all x ∈ V.
- F-acyclic if there was no F-path P from x to y, except for x = y for all x ∈ V.
- Acyclic, if it was either T −acyclic, I −acyclic, or F −acyclic.[11]

Definition 2.12:
On a given set V, let G = (A, B), G_1 = (A_1, B_1) be a neutrosophic graph. If V = V_1 but E_1 ⊆ E, then G_1 is referred to as the spanning neutrosophic graph of G [2].

Definition 2.13:
"On a set V, let G = (A, B) be a neutrosophic graph. Then, G is called,

- T-forest, if G were T-acyclic and a spanning neutrosophic graph F existed, such that for every edge xy out of F, there exists a T-path P from x to y, whose strength is greater than T_B (xy).
- I – forest, if G were I-acyclic and there is a spanning neutrosophic graph F, then for every edge xy out of F, there exists an I-path P from x to y, whose strength is greater than I_B (xy).

- F-forest: if G were F-acyclic and a spanning neutrosophic graph F existed, then for every edge xy out of F, there would be an F-path P from x to y, how whose strength was greater than F_B (xy).
- forest, in the event that it was one of the three types of neutrosophic forests: F, I, or T" [2].

Definition 2.14:

Let G = (A, B) be a neutrosophic graph on a set V.

- T-tree: if G were a T-forest with a T-path P from x to y for every x, y ∈ V.
- I-tree: if G were an I-forest, then for every x, y ∈ V, there would be an I-path P from x to y.
- F-tree, if G was an F-forest such that, for every x, y ∈ V, there exists an F-path P.
- Tree, if it was one of T, I, or F −tree. [2]

Definition 2.15:

"Consider a neutrosophic graph G = (A, B) on V and v_0, where v_n are two given vertices such that n ∈ N. Then,

- A distinct sequence of vertices P : $v_0, v_1, \cdots, v_n$ in G is called a T-path of length n from v_0 to v_n, if $T_B(v_iv_{i+1}) > 0$, for i = 0, 1, · · · , n – 1. The strength of that T – Path is $min_{i=0}^{n-1}\{T_B(v_iv_{i+1})\}$ and denoted by $\mu_G(P)_T$.
- A distinct sequence of vertices P : $v_0, v_1, \cdots, v_n$ in G is called a I-path of length n from v_0 to v_n, if $I_B(v_iv_{i+1}) > 0$, for i = 0, 1, · · · , n – 1. The strength of that I – Path is $min_{i=0}^{n-1}\{I_B(v_iv_{i+1})\}$ and denoted by $\mu_G(P)_I$.
- A distinct sequence of vertices P : $v_0, v_1, \cdots, v_n$ in G is called a F-path of length n from v_0 to v_n, if $F_B(v_iv_{i+1}) > 0$, for i = 0, 1, · · · , n – 1. The strength of that F – Path is $min_{i=0}^{n-1}\{F_B(v_iv_{i+1})\}$ and denoted by $\mu_G(P)_F$.
- A distinct sequence of vertices P : $v_0, v_1, \cdots, v_n$ in G is called a path of length n from v_0 to v_n, if it be T –path, I–path, and F – path, simultaneously. The strength of that path is min{ $\mu_G(P)_T, \mu_G(P)_I, \mu_G(P)_F$} and is denoted by $\mu_G(P)$".[2]

Definition 2.16:

An edge xy in G = (A, B) is considered to be,

- T-effective, if $T_B(xy) > \mu^{\infty}{}_{G-\{xy\}})\ (x,y)_T$
- I-effective, if $I_B(xy) > \mu^{\infty}{}_{G-\{xy\}})\ (x,y)_I$
- F-effective, if $F_B(xy) > \mu^{\infty}{}_{G-\{xy\}})\ (x,y)_F$ and
- Effective, if it is one of the three. [11]

3. **Main Results:**

Definition 3.1:

Let G = (A, B) be a neutrosophic graph on V and x, y ∈ V. Then,

a. If the edge xy is T-effective, we say that x dominates y in G. The T-effective restrained dominating set in G is a subset S of V if, for each v ∈ V − S, there is at least one neighbor w ∈ V - S such that vw ∈ E, and for each v ∈ V - S, there is u ∈ S such that u dominates v as T-effective.

- The T – weight of x is defined by $w_T(x) = T_A(x) + \frac{\sum \text{xy is a T} - \text{effective edge } T_B\,(\text{xy})}{\sum \text{xy is a edge } T_B\,(\text{xy})}$
- For any S ⊆ V, T – weight of S is defined by $w(S)_T = \sum_{u\in S}(w(u)_T)$

- Let A be the set of all T - effective restrained dominating sets in G. The T – restrained domination number of G is given by $\gamma_T^r(G) = min_{D\in U}(w(D)_T)$. Then the T – effective restrained dominating set that correspond to $\gamma_T^r(G)$ is known as T-restrained dominating set.

b. If the edge xy is I-effective, we say that x dominates y in G. The I-effective restrained dominating set in G is a subset S of V if, for each v ∈ V − S, there is at least one neighbor w ∈ V - S such that vw ∈ E, and for each v ∈ V - S, there is u ∈ S such that u dominates v as I-effective.

- The I – weight of x is defined by $w_I(x) = I_A(x) + \frac{\sum xy \text{ is a I} - \text{effective edge } I_B\,(xy)}{\sum xy \text{ is a edge } I_B\,(xy)}$
- For any S ⊆ V, I – weight of S is defined by $w(S)_I = \sum_{u\in S}(w(u)_I)$
- Let A be the set of all I - effective restrained dominating sets in G. The I – restrained domination number of G is given by $\gamma_I^r(G) = min_{D\in U}(w(D)I)$. Then the I – effective restrained dominating set that correspond to $\gamma_I^r(G)$ is known as I-restrained dominating set.

c. If the edge xy is F-effective, we say that x dominates y in G. The F-effective restrained dominating set in G is a subset S of V if, for each v ∈ V − S, there is at least one neighbor w ∈ V - S such that vw ∈ E, and for each v ∈ V - S, there is u ∈ S such that u dominates v as F-effective.

- The F – weight of x is defined by $w_F(x) = F_A(x) + \frac{\sum xy \text{ is a F} - \text{effective edge } F_B\,(xy)}{\sum xy \text{ is a edge } F\,(xy)}$
- For any S ⊆ V, F – weight of S is defined by $w(S)_F = \sum_{u\in S}(w(u)_F)$
- Let A be the set of all F - effective restrained dominating sets in G. The F – restrained domination number of G is given by $\gamma_F^r(G) = min_{D\in U}(w(D)_F)$. Then the F – effective restrained dominating set that correspond to $\gamma_F^r(G)$ is known as F-restrained dominating set.

d. If the edge xy is effective, we say that x dominates y in G. The effective restrained dominating set in G is a subset S of V if, for each v ∈ V − S, there is at least one neighbor w ∈ V - S such that vw ∈ E, and for each v ∈ V - S, there is u ∈ S such that u dominates v as effective. The weight of S is also known as $w_v(S) = \min\{w_T^r(S); w_I^r(S); w_F^r(S)\}$. Let U be the collection of all of G's effective dominating sets. The formula for G's restrained domination number is $\gamma^r(G) = \min_{D\in U}(w(D))$. The restrained dominating set calls the effective restrained dominating set that corresponds to $\gamma^r(G)$.

Definition 3.2:

Let v_0, v_n be two given vertices such that n ∈ N, and let G = (A, B) be a neutrosophic graph on V. Consequently,

- T-Cycle of length n from v_0 to v_{n-1} is defined as a closed sequence of distinct vertices C: $\{v_0, v_1, \cdots, v_{n-1}, v_0\}$in G, provided that $T_B(v_i v_{i+1}) > 0$ for i = 0, 1, · · ·, n – 1.
- An I-Cycle of length n from v_0 to v_{n-1} is defined as a closed sequence of distinct vertices C: $\{v_0, v_1, \cdots, v_{n-1}, v_0\}$in G if $I_B(v_i v_{i+1}) > 0$ for i = 0, 1, · · ·, n – 1.
- If $F_B(v_i v_{i+1}) > 0$, for i = 0, 1, · · ·, n – 1, then a closed sequence of distinct vertices C: $\{v_0, v_1, \cdots, v_{n-1}, v_0\}$in G is referred to as an F-Cycle of length n from v_0 to v_{n-1}.
- A cycle of length n from v_0 to v_{n-1} is defined as a closed sequence of distinct vertices C: $\{v_0, v_1, \cdots, v_{n-1}, v_0\}$ in G, if it is simultaneously T, I, and F − cycles.

Definition 3.3:

Let v_0 be a designated central vertex and let G = (A, B) be a neutrosophic graph defined on a vertex set V. Next,

- A T-Star centered at v_0 with n branches is a collection of distinct edges $\{(v_0, v_1),(v_0, v_2),\ldots,(v_0, v_n)\}$ such that for each i = 1, 2,…,n the truth-membership value of the edge v_0v_i satisfies, $T_B(v_0v_i) > 0$.
- An I-Star centered at v_0 with n branches is a collection of distinct edges $\{(v_0, v_1),(v_0, v_2),\ldots,(v_0, v_n)\}$ such that for each i = 1, 2,…,n the indeterminacy-membership value of the edge v_0v_i satisfies, $I_B(v_0v_i) > 0$.
- A F-Star centered at v_0 with n branches is a collection of distinct edges $\{(v_0, v_1),(v_0, v_2),\ldots,(v_0, v_n)\}$ such that for each i = 1, 2,…,n the false-membership value of the edge v_0v_i satisfies, $F_B(v_0v_i) > 0$.
- A neutrosophic star (or T – I - F star) centered at v_0 is a star structure $\{(v_0, v_1),(v_0, v_2),\ldots,(v_0, v_n)\}$such that all three membership values for each edge are greater than 0, $T_B(v_0vi) > 0$, $I_B(v_0v_i) > 0$, $F_B(v_0v_i) > 0$, for I = 1,2,…,n.

Theorem 3.4:

Let G = (A, B) be a neutrosophic graph where B is the edge set with associated truth-membership function $T_B : B \rightarrow [0,1]$. Let P_n be a path graph on $n \geq 3$ vertices such that $T_B(e) > 0$ for all $e \in B$. Then $\gamma_T{}^r(P_n) = [\frac{n}{2}]$.

Proof: Assume that the path graph on n vertices $P_n = (v_1,v_2,\ldots.v_n)$ with edges $E = \{v_iv_{i+1} / 1 \leq i \leq n-1\}$, and suppose that $T_B(v_iv_{i+1}) > 0$ for all i, Define the vertex subset $S =\{ v_i \in V / i \text{ is odd}\} = \{v_1,v_3,v_5,\ldots \}$. We prove that S is a T-effective restrained dominating set of G. Let $v_j \in V \setminus S$. Then j is even. Since P_n is a path, the only possible neighbors of v_j are v_{j-1} and v_{j+1}, assuming they exist. Both v_{j-1} and v_{j+1} are odd-numbered and thus belong to S, and the edges v_jv_{j-1}, $v_jv_{j+1}$are T-effective by assumption. Therefore, each $v_j \in V \setminus S$ is T - effectively dominated by a vertex in S, satisfying the first condition of T - effective domination. Now, to verify the restrained condition, we must show that for each $v_j \in V \setminus S$, there exists at least one neighbor $w \in V \setminus S$ such that $v_jw \in E$. Observe that the vertices in $V \setminus S$ are all even-indexed. For j = 2, we have v_2 adjacent to $v_1 \in S$ and $v_3 \in S$. To resolve this and ensure the restraint condition is satisfied, we redefine the set $S' = \{v_i \in V / i \text{ is even}\}$. Now, for each $v_j \in V \setminus S'$, the neighbors v_j-1 are even and belong to S′. Additionally, for all such odd j, at least one of their neighbors, $v_{j\pm 2}$. Therefore, S′ is a T-effective restrained dominating set. Therefore, we conclude that, $\gamma_T{}^r(P_n) \leq [\frac{n}{2}]$. To prove minimality, suppose there exists a T-effective restrained dominating set $D \subset V$ such that $|D| < [\frac{n}{2}]$. By linear adjacency of a path, there must exist two consecutive vertices $v_i,v_{i+1} \in V \setminus D$, which are not T-effectively dominated by any vertex in D. Furthermore, they also lack a neighbor in $V \setminus D$ distinct from each other, violating the restraint condition. This contradiction implies that no such smaller set D exists, and thus $\gamma_T{}^r(P_n) = [\frac{n}{2}]$. By analogues to the evidence of $\gamma_T{}^r(P_n) = [\frac{n}{2}]$, The outcome is clearly valid for $\gamma_I{}^r(P_n) = [\frac{n}{2}]$, $\gamma_F{}^r(P_n) = [\frac{n}{2}]$, $\gamma^r(P_n) = [\frac{n}{2}]$.

Theorem 3.5:

Given a vertex set V, let G = (A, B) be an empty neutrosophic graph. Then $\gamma_T{}^r(G) = \gamma_I{}^r(G) = \gamma_F{}^r(G) = p$, where p denotes the order of G.

Proof: Let G = (A, B) be a neutrosophic graph on a vertex set V that is empty. As a result, there is no T-effective edge in G and V is the only T-effective dominating set. According to 3.1(a), we have $\gamma_{T}{}^{r}(G)$ = $min_{D\in S}[\sum_{u\in D} T_A(u)]$ = $[\sum_{u\in V} T_A(u)]$ = p. Therefore $\gamma_{T}{}^{r}(G)$ = p. By analogues to the proof of $\gamma_{T}{}^{r}(G)$ = p and Definition 3.1(b,c,d), the result is obviously holds for $\gamma_{I}{}^{r}(G)$, $\gamma_{F}{}^{r}(G)$, $\gamma^{r}(G)$.

Proposition 3.6: Given a set V and any neutrosophic graph G = (A, B), we have

a. $\gamma_{T}{}^{r}(G)$, $\gamma^{r}(G)$, $\gamma_{I}{}^{r}(G)$, $\gamma_{F}{}^{r}(G) \leq$ p.

b. $\gamma^{r}(G) + \gamma^{r}(G)$, $\gamma_{T}{}^{r}(G) + \gamma_{T}{}^{r}(G)$, $\gamma_{I}{}^{r}(G) + \gamma_{I}{}^{r}(G)$, $\gamma_{F}{}^{r}(G) + \gamma_{F}{}^{r}(G) \leq$ 2p

Proof:

- According to Proposition 3.5, a neutrosophic graph G = (A, B) exists such that $\gamma_{T}{}^{r}(G) = \gamma_{I}{}^{r}(G)$ = $\gamma_{F}{}^{r}(G) = \gamma^{r}(G)$ = p, Thus, the outcome is as follows.
- When (a) is applied to G and $\ddot{G}$, the outcome is evidently valid.

4. Conclusion

Fuzzy models and algebraic structures are framed by the idea of neutrosophy. Neutrosophic graphs come in three different varieties. As previously stated, we selected one type of them to serve as the framework. This study serves as an initial step toward a broader integration of neutrosophic theory in fuzzy systems and mathematical structures, opening new avenues for both theoretical advancement and practical applications.

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