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Empowering Multigrade Interdisciplinary Assessment through Plithogenic Offsets and Artificial Intelligence for Dynamic and Personalized Education

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Abstract. This contribution addresses the issue of interdisciplinary assessment in multigrade classrooms: the diverse levels and the assessment path's unclear direction fail to render efficient learning. How can assessment be refined to avoid such discrepancies and establish a better path of equalized and contextualized learning? It's a pressing question for 21st-century learning—more so in rural areas where interdisciplinary attempts can create holistic and connected learning for the learners—but the literature fails to guide systematically, apart from rendering systematic solutions that rely only on tacit, non-explicit, technical interventions. Therefore this research contributes a systematic intervention that combines a Plithogenic Offset—neutrosophic attempts to neutralize the uncertainty of responses—with AI effort rendered that the AI's involvement is simplistic without programming—to assess endeavors that are dynamically responsive to need and culture. The open-ended responses suggest that such alignment increases motivation and engagement and interdisciplinary cooperation, improving formative assessment. Thus, the research presents a systematized contribution to the body of knowledge that expands education from practically from AI and neutrosophic integration for dynamic assessment of what's possible within educated, accessible resources for those educators able and positioned to provide consistent feedback integrated with reality on the ground.

Keywords: Interdisciplinary Assessment, Multigrade, Plithogenic Offsets, Artificial Intelligence, Contextualized Education, Educational Uncertainty, Formative Feedback.

1. Introduction

Multi-grade classroom education faces unique challenges due to the diversity of student levels and needs, making it essential to develop innovative assessment strategies that promote meaningful learning. This study explores how to enhance interdisciplinary assessment in these environments by integrating Plithogenic Offsets and accessible artificial intelligence (AI) tools, without resorting to programming, to achieve dynamic and personalized education. The relevance of this research lies in its capacity to transform teaching practice in rural contexts, where educational equity is crucial. According to Ceballos, curricular adaptations are essential to address diversity in multi-grade environments, highlighting the need for approaches that integrate disciplines and local contexts [1]. By combining AI with a neutrosophic approach, this work seeks to offer practical solutions that strengthen formative assessment, promoting student engagement and cultural connection.

Multigrade schools have been a practical response to ensuring education in rural areas, but their assessment has faced significant limitations. From traditional methods based on standardized tests to more recent approaches focused on formative assessment, education systems have sought to adapt to the particularities of these environments. In Latin America, as Freire points out, multigrade classrooms often lack resources and adequate teacher training, which hinders interdisciplinary integration [2]. Interdisciplinarity, advocated by, allows for connecting knowledge from diverse areas, fostering contextualized learning that reflects students' reality [3]. However, managing uncertainty in these assessments remains a persistent challenge.

Currently, multigrade classrooms face obstacles such as a lack of flexible assessment tools and limited personalized feedback, which limit the development of key competencies. Interdisciplinary assessment, although promising, does not always manage to address the variability in learning rates or the cultural diversity of students. Sandoval emphasizes that assessment should prioritize improving learning over simply assigning grades, a fundamental principle in multigrade environments [4]. The incorporation of accessible AI tools, along with approaches that manage indeterminacy, offers an opportunity to overcome these barriers, tailoring assessments to individual needs without requiring advanced technical knowledge.

The central problem of this research is formulated as: how to enhance interdisciplinary assessment in multi-grade classrooms to foster meaningful learning in contexts of uncertainty and diversity, using accessible tools? This challenge is particularly relevant in rural schools, where teachers must manage multiple grades in the same space, facing limitations in resources and training. The lack of methods that integrate uncertainty and personalize assessment exacerbates these difficulties, restricting educators' ability to promote equitable education. This study proposes an innovative approach that combines Plithogenic Offsets with AI, ensuring dynamic and contextualized assessments.

Interdisciplinarity in educational assessment seeks to integrate knowledge from diverse areas to build relevant and holistic learning. In multigrade contexts, where students interact with topics such as corn planting or local traditions, assessments must reflect this diversity. Aguinsaca highlights that disciplinary integration allows overcoming knowledge fragmentation, promoting a deeper understanding that is more connected to the environment [5]. However, the practical implementation of these approaches faces obstacles, such as the lack of tools adapted to student variability. This study addresses these limitations by proposing a framework that uses AI in an accessible way to personalize assessment activities.

The proposed methodology focuses on the design of interdisciplinary worksheets that integrate grade-differentiated activities, aligned with learning objectives, and contextualized to local reality. Plithogenic Offsets allow for modeling indeterminacy in student responses, while AI, used as an intuitive support tool, adapts these worksheets to individual and cultural needs without requiring programming. This approach, tested in a context such as the "Gonzalo Zaldumbide" School, seeks to improve evaluative equity and student motivation. According to Muñoz and Solís, formative approaches are key to fostering reflection and meaningful learning in complex environments [6]. This research offers a practical model for teachers in multigrade contexts.

The relevance of this study transcends the local level, as it proposes a replicable framework for other multi-grade schools in similar settings. By integrating accessible AI tools with a neutrosophic approach, it brings an innovative perspective to educational assessment, responding to the need for methods that address the complexity of rural contexts. The combination of Plithogenic Offsets and AI allows educators to design assessments that are not only dynamic but also culturally relevant, strengthening the connection between students and their environment. This work seeks to lay the foundation for more inclusive and equitable education in challenging environments.

The objectives of this research are clear and aligned with the research question. First, the aim is to design and validate interdisciplinary worksheets based on Plithogenic Offsets and accessible AI tools, promoting disciplinary integration and contextualization in multigrade classrooms. Second, the aim is to evaluate the effectiveness of these worksheets in terms of motivation, student engagement, and the development of key competencies. Finally, the study aims to propose a methodological framework adaptable to other multigrade contexts, contributing to the improvement of formative assessment and educational equity, without relying on complex technical knowledge.

2. Preliminaries.

This section recalls the main concepts of plithogenic assemblage, displacement, superposition, subposition and plithogenic displacement/superposition/ subposition.

A. Plithogenic set

Definition 1 ([7]). Let U be a universal set. A *plithogenic set* PS, where P is a subset of S, is defined as: $\mathrm{PS} = (\mathrm{P}, \mathrm{v}, \mathrm{Pv}, \mathrm{pdf}, \mathrm{pcf})$, such that:

- vis an attribute,
- Pvis the range of possible values for v,
- pdf: $\mathrm{P} \times \mathrm{Pv} \rightarrow [0,1]^s$is the *Degree of Relevance Function* (*DAF*),
- pcf: $\mathrm{Pv} \times \mathrm{Pv} \rightarrow [0,1]^t$is the *Degree of Contradiction Function* (*DCF)*).

So for all $a, b \in$ Pv, pcfsatisfies:

1. $\mathrm{pcf}(a,a) = 0$, DCF reflexivity.
2. $\mathrm{pcf}(a,b) = \mathrm{pcf}(b,a)$, commutativity of DCF.

They are classified as follows:

- For s = t = 1, it is a *plithogenic fuzzy set;*
- For s = 2, t = 1, it is an *intuitionistic plithogenic fuzzy set*;
- For s = 3, t = 1, it is a *Plithogenic Neutrosophic Set;*
- For s = 4, t = 1, it is a *plithogenic quadripartitioned neutrosophic set*;
- For s = 5, t = 1, it is a *plithogenic pentapartitioned neutrosophic set*;
- For s = 6, t = 1, it is a *plithogenic hexapartitioned neutrosophic set*;
- For s = 7, t = 1, it is a *plithogenic heptapartitioned neutrosophic ensemble*;
- For s = 8, t = 1, it is a *plithogenic octopartitioned neutrosophic set*;
- For s = 9, t = 1, it is a *plithogenic unpartitioned neutrosophic set;*

More about this theory and application can be read in [8-16].

B. Plithogenic displacement/displacement/undervaluation

Definition 2 ([7]). Let X be a universe of discourse. Furthermore, Ψrepresents true and Ωrepresents false. $A \subseteq X$It is a *Critical Shift* if there exists a characteristic function $\chi_A: X \rightarrow \{\Psi, \Omega\}$such that:

$$\chi_A(x) = \begin{cases} \Omega, \text{if x} \in \text{A}, \\ \Psi, \text{if x} \notin \text{A}. \end{cases}$$

Definition 3 ([7]). Let X be a universe of discourse. $\tilde{A}$It is a subset of X and is called *Fuzzy Shift* if it is defined as follows:

$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)): x \in X, \mu_{\tilde{A}}(x) \in [\Psi, \Omega]\}$, where $\Psi < 0$and $\Omega > 1$.

Definition 4 ([7]). Let X be a universe of discourse. A_{off}It is a subset of X and is called the *Univalued Neutrosophic Shift.* if defined as follows:

$A_{off} = \{(x, \langle T(x), I(x), F(x)\rangle): x \in X,\ s.t.\ \exists(T(x), I(x), F(x) < 0$ or $T(x), I(x), F(x) > 1)\}$, so:

- $T(x)$It is the function of belonging to truth, $I(x)$it is the function of belonging to indeterminacy and $F(x)$it is the function of belonging to falsehood.
- $T(x), I(x), F(x) \in [\Psi, \Omega]$, where $\Psi < 0$(called *UnderLimit*) and $\Omega > 1$(called *OverLimit*).
- When the interval is $[\Psi, 1]$and $\Psi < 0$, then it is an *UnderSet* .
- When the interval is $[0, \Omega]$and $\Omega > 1$, then it is an *OverSet* .

  Definition 5 ([7]). Let X be a universal set. A *plithogenic shift* PS_{off}, where P is a subset of S is defined as:

  $\mathrm{PS}_{off} = (\mathrm{P}, \mathrm{v}, \mathrm{Pv}, \mathrm{pdf}, \mathrm{pcf})$, such that:
- vis an attribute,
- Pvis the range of possible values for v,
- pdf: $\mathrm{P} \times \mathrm{Pv} \to [\Psi_v, \Omega_v]^s$is the *Degree of Relevance Function* (*DAF*),
- pcf: $\mathrm{Pv} \times \mathrm{Pv} \to [\Psi_v, \Omega_v]^t$is the *Degree of Contradiction Function* (*DCF)*).

  Where $\Psi_v < 0$and $\Omega_v > 1$.
- When the interval is $[\Psi_v, 1]$and $\Psi_v < 0$, then it is a *Plithogenic Subset* .
- When the interval is $[0, \Omega_v]$and $\Omega_v > 1$, then it is a *Plithogenic OverSet* .

  Equivalent to the Plithogenic Sets, we have the following classifications:
- For s = t = 1, it is a *plithogenic diffuse displacement* ;
- For s = 2, t = 1, it is an *intuitionistic plithogenic diffuse displacement* ;
- For s = 3, t = 1, it is a *Plithogenic Neutrosophic Offset* ;
- For s = 4, t = 1, it is a *Plithogenic Quadripartitioned Neutrosophic Offset* ;
- For s = 5, t = 1, it is a *Plithogenic Pentapartitioned Neutrosophic Offset* ;
- For s = 6, t = 1, it is a *Plithogenic Hexapartitioned Neutrosophic Offset* ;
- For s = 7, t = 1, it is a *Plithogenic Heptapartitioned Neutrosophic Offset* ;
- For s = 8, t = 1, it is a *Plithogenic Octopartitioned Neutrosophic Offset* ;
- For s = 9, t = 1, it is a *Plithogenic Unpartitioned Neutrosophic Offset* .

Example 1 ([7]): Let X be the set of suspected medical conditions. Then, for each medical condition, $x \in X$we have the following neutrosophic degrees $T(x), I(x), F(x) \in [\Psi, \Omega]$, extended beyond the classical interval $[0, 1]$.

$A_{off} = \{(Disease\ X, \langle T(X) = 1.1, I(X) = 0.4, F(X) = 0.2\rangle), (Disease\ Y, \langle T(Y) = 0.7, I(X) = 0.6, F(X) = 0.1\rangle)\}$.

So, while for Disease Y we have a standard assessment of truthfulness, indeterminacy, and falsity, for Disease X we have the following interpretation:

- $T(X) = 1.1$:There is a high probability of suffering from disease X due to advanced diagnostic tools,
- $I(X) = 0.4$:There is moderate uncertainty due to overlap of symptoms with other disorders,
- $F(X) = -0.2$:There is a negative fallacy because atypical symptoms decrease the likelihood of misdiagnosis.

Let X be a universe of discourse, $\mathrm{A} = \{(x, \langle \mathrm{T_A}(x), \mathrm{I_A}(x), \mathrm{F_A}(x)\rangle), x \in \mathrm{X}\}$and $\mathrm{B} = \{(x, \langle \mathrm{T_B}(x), \mathrm{I_B}(x), \mathrm{F_B}(x)\rangle), x \in \mathrm{X}\}$two single-valued neutrosophic Shifts/Shifts/Subshifts.

$T_A, I_A, F_A, T_B, I_B, F_B: X \to [\Psi, \Omega]$, where $\Psi \le 0 < 1 \le \Omega$, Ψis the lower limit, while T Ω$_A(x)$, $I_A(x)$, $F_A(x)$, $T_B(x)$, $I_B(x)$, $F_B(x)$ ∈is the upper limit *[* , Ψ] Ω.

Then the main operators are defined as follows [5]:

$\mathrm{A} \cup \mathrm{B} = \{(\mathrm{x}, \langle \max(\mathrm{T_A(x)}, \mathrm{T_B(x)}), \min(\mathrm{I_A(x)}, \mathrm{I_B(x)}), \min(\mathrm{F_A(x)}, \mathrm{F_B(x)})\rangle), \mathrm{x} \in \mathrm{X}\}$is the union.

$\mathrm{A} \cap \mathrm{B} = \{(\mathrm{x}, \langle \min(\mathrm{T_A(x)}, \mathrm{T_B(x)}), \max(\mathrm{I_A(x)}, \mathrm{I_B(x)}), \max(\mathrm{F_A(x)}, \mathrm{F_B(x)})\rangle), \mathrm{x} \in \mathrm{X}\}$is the intersection,

$\mathrm{C(A)} = \{(\mathrm{x}, \langle \mathrm{F_A(x)}, \Psi + \Omega - \mathrm{I_A(x)}, \mathrm{T_A(x)}\rangle), \mathrm{x} \in \mathrm{X}\}$It is the neutrosophic complement of the neutrosophic set.

A *unique negation* can be defined as in equation 1.

$$\underset{\mathrm{off}}{\neg}\langle \mathrm{T}, \mathrm{I}, \mathrm{F}\rangle = \langle \mathrm{F}, \Psi_\mathrm{I} + \Omega_\mathrm{I} - \mathrm{I}, \mathrm{T}\rangle \qquad (1)$$

Definition 6. Let *c be* a neutrosophic component (T_{Off}, I_{Off} or F_{Off}) . : $M_{Off} \to [\Psi, \Omega]$, where $\Psi \le 0$ and $\Omega \ge 1$. *The neutrosophic component N-OffNorm* N^{n}_{off}: $[\Psi,\Omega]^2 \to [\Psi,\Omega]$ satisfies the following conditions for any element x , y and $z \in M_{out}$:

i. $N^{n}_{off}(c(x),\Psi) = \Psi$, $N^{n}_{off}(c(x),\Omega) = c(x)$(Overpass conditions),
ii. $N^{n}_{off}(c(x),c(y)) = N^{n}_{off}(c(y),c(x))$(Commutativity),
iii. If $c(x) \le c(y)$then $N^{n}_{off}(c(x),c(z)) \le N^{n}_{off}(c(y),c(z))$(Monotonicity),
iv. $N^{n}_{off}(N^{n}_{off}(c(x),c(y)),c(z)) = N^{n}_{off}\big(c(x),N^{n}_{off}(c(y),c(z))\big)$(Associativity).

We can use the following simplified notation $\langle T_1, I_1, F_1\rangle \overset{\wedge}{_{off}} \langle T_2, I_2, F_2\rangle = \langle T_1 \overset{\wedge}{_{off}} T_2,\ I_1 \overset{\vee}{_{off}} I_2, F_1 \overset{\vee}{_{off}} F_2\rangle$.

Definition 7. Let c be a neutrosophic component (T_{Off}, I_{Off} or F_{Off}). c: $M_{Off} \to [\Psi, \Omega]$, where $\Psi \le 0$ and $\Omega \ge 1$. The neutrosophic component *N-OffConorm* N^{co}_{Off}: $[\Psi,\Omega]^2 \to [\Psi,\Omega]$satisfies the following conditions for any element x, y, z $\in M_{Off}$:

i. $N^{co}_{Off}(c(x),\Omega) = \Omega$, $N^{co}_{Off}(c(x),\Psi) = c(x)$(Overpass conditions),
ii. $N^{co}_{Off}(c(x),c(y)) = N^{co}_{Off}(c(y),c(x))$(Commutativity),
iii. If $c(x) \le c(y)$then $N^{co}_{Off}(c(x),c(z)) \le N^{co}_{Off}(c(y),c(z))$(Monotonicity),
iv. $N^{co}_{Off}(N^{co}_{Off}(c(x),c(y)),c(z)) = N^{co}_{Off}\big(c(x),N^{co}_{Off}(c(y),c(z))\big)$(Associativity).

For this we use the notation $\langle T_1, I_1, F_1\rangle \overset{\vee}{_{Off}} \langle T_2, I_2, F_2\rangle = \langle T_1 \overset{\vee}{_{Off}} T_2,\ I_1 \overset{\wedge}{_{Off}} I_2, F_1 \overset{\wedge}{_{Off}} F_2\rangle$.

3. Proposed model

Project: "EVAL-IA-MULTIGRADE"

This analysis assesses the feasibility and potential impact of the **"EVAL-IA-MULTIGRADO" project, which aims to enhance interdisciplinary assessment in multigrade classrooms through the use of Plithogenic Offsets and accessible Artificial Intelligence. The objective is to foster dynamic, personalized, and equitable education, especially in contexts with diverse learning levels. The mathematical model based on Plithogenic** Offsets is then applied to process the assessments of a panel of experts.

Project Structure and Evaluation Criteria

e1, e2 , e3) was convened to assess the project's potential effectiveness. The evaluations focus on a set of specific objectives and indicators, as well as the expected impacts.

Objectives and Indicators

- **Main Objective (vp):** Optimize interdisciplinary assessment in multigrade classrooms to promote equitable and contextualized education.
  - **Indicator (vp1):** Number of publications that document the methodological framework and its impact on meaningful learning.
- **Specific Objective 1 (v1):** Design and validate interdisciplinary evaluation tools based on Plithogenic Offsets and accessible AI.
  - **Indicator 1.1 (v11):** Number of validated interdisciplinary evaluation sheets that integrate Plithogenic Offsets.
  - **Indicator 1.2 (v12):** Level of adaptability of AI tools to customize assessment activities according to individual needs and cultural context.
  - **Indicator 1.3 (v13):** Degree of satisfaction of teachers with the accessibility of the tools (without requiring programming).
- **Specific Objective 2 (v2):** Evaluate the effectiveness of these cards in student motivation and participation.

- **Indicator 2.1 (v21):** Percentage increase measured in the motivation and active participation of students in assessment activities.
- **Specific Objective 3 (v3):** Propose a methodological framework adaptable to other multigrade contexts.
  - **Indicator 3.1 (v31):** Clarity and ease of use of the proposed methodological guide for teachers.
  - **Indicator 3.2 (v32):** Number of successful pilot implementations of the framework in other educational institutions.
- **Specific Objective 4 (v4):** Measure the impact on the development of key competencies.
  - **Indicator 4.1 (v41):** Demonstrable improvement in critical thinking and problem-solving skills in students.
- **Specific Objective 5 (v5):** Systematize the experience for its dissemination in the academic community.
  - **Indicator 5.1 (v51):** Number of papers accepted at national or international educational conferences.
  - **Indicator 5.2 (v52):** Number of scientific articles accepted in high-impact journals.

Potential Impacts

- **Scientific Impact (vi):** Contribution to knowledge about formative assessment in complex and highly diverse environments.
- **Social Impact (vii) :** Contribution to reducing educational gaps in rural or marginalized areas.
- **Economic Impact (viii) :** Optimization of educational resources by offering low-cost and highly effective tools.
- **Political Impact (viv) :** Influence on public curricular policies for multigrade education.
- **Influence on Educational Policies (vv) : (Dominant Criterion)** . The project's ability to serve as a basis for promoting policies that integrate innovative technologies into the national education system.
- **Strengthening the Inclusion Agenda (vvi) :** Alignment with national objectives for equitable access to education.
- **Cultural Impact (vvii) :** Promoting an assessment that respects and values the cultural diversity of students.
- **Technological Impact (vviii) :** Promoting the adoption of accessible and sustainable technologies in the educational field.
- **Ethical Impact (vix) :** Promoting a fair, transparent and bias-free evaluation.

Development of the Mathematical Model

The methodological procedure is then followed step by step.

Step 1: Gathering Expert Ratings

The experts provide their assessments for each criterion. These assessments consist of a triplet (T ,I,F) representing **Trueness** , **Indeterminacy** , and **Falsehood** , respectively. Scores are issued on a scale of -10 to 110, allowing for overconfirmation or overrefutation , and are summarized in **Table 1** .

Table 1. Expert Evaluation for each Criterion (Scale from -10 to 110)

Criterion/Expert	e1	e2	e3
v11	(85, 5, 20)	(65, 5, 15)	(50, -4, -5)
v12	(70, 4, -2)	(100, 10, 10)	(40, -5, 30)
v13	(72, 2, 5)	(105, 8, 10)	(45, -3, 5)
v21	(70, 1, 4)	(110, 5, -6)	(42, 8, 30)
v31	(68, 5, 2)	(106, 5, 30)	(40, 6, 35)
v32	(80, 5, -8)	(88, 10, 20)	(42, 10, 4)
v41	(102, 6, 25)	(78, 5, 18)	(45, 10, 22)
v51	(95, 2, -4)	(85, 4, -5)	(40, -5, 25)
v52	(82, 4, 3)	(92, 8, 8)	(52, -7, 10)
vi	(90, 5, -8)	(65, 12, -8)	(45, 10, 12)
vii	(88, 8, 10)	(110, 12, 10)	(48, -2, -3)
viii	(65, 10, 15)	(90, 4, -2)	(40, 2, 35)
viv	(90, 8, -5)	(65, 4, -8)	(50, 8, -10)
vv	(75, 2, 30)	(82, 8, 25)	(42, -3, 30)
vvi	(72, 4, -3)	(80, 12, 10)	(45, 8, 6)
vvii	(88, 2, 6)	(102, 8, 8)	(50, -3, 20)
vviii	(95, 10, -10)	(95, 9, 8)	(50, 6, 15)
vix	(100, 4, -7)	(105, 6, 3)	(45, -2, 30)

Step 2: Defining the Dominant Criterion

For this analysis, the criterion considered most relevant or dominant is **vv (Influence on Educational Policies)** . This criterion will serve as a reference for rescaling the scores for the other indicators.

Step 3: Rescaling Ratings and Calculating Dissimilarity

The evaluations in Table 2 are normalized to the interval [−0.1 ,1.1] by dividing each value by 100. The results are presented in **Table 2.**

Table 2. Expert Evaluation Rescaled to the Interval [-0.1, 1.1]

Criterion/Expert	e1	e2	e3
v11	(0.85, 0.05, 0.20)	(0.65, 0.05, 0.15)	(0.50, -0.04, -0.05)
v12	(0.70, 0.04, -0.02)	(1.00, 0.10, 0.10)	(0.40, -0.05, 0.30)
v11	(0.72, 0.02, 0.05)	(1.05, 0.08, 0.10)	(0.45, -0.03, 0.05)
v12	(0.70, 0.01, 0.04)	(1.10, 0.05, -0.06)	(0.42, 0.08, 0.30)
v13	(0.68, 0.05, 0.02)	(1.06, 0.05, 0.30)	(0.40, 0.06, 0.35)
v21	(0.80, 0.05, -0.08)	(0.88, 0.10, 0.20)	(0.42, 0.10, 0.04)
v31	(1.02, 0.06, 0.25)	(0.78, 0.05, 0.18)	(0.45, 0.10, 0.22)
v32	(0.95, 0.02, -0.04)	(0.85, 0.04, -0.05)	(0.40, -0.05, 0.25)
v41	(0.82, 0.04, 0.03)	(0.92, 0.08, 0.08)	(0.52, -0.07, 0.10)
v51	(0.90, 0.05, -0.08)	(0.65, 0.12, -0.08)	(0.45, 0.10, 0.12)
v52	(0.88, 0.08, 0.10)	(1.10, 0.12, 0.10)	(0.48, -0.02, -0.03)

Criterion/Expert	e1	e2	e3
vi	(0.65, 0.10, 0.15)	(0.90, 0.04, -0.02)	(0.40, 0.02, 0.35)
vii	(0.90, 0.08, -0.05)	(0.65, 0.04, -0.08)	(0.50, 0.08, -0.10)
viii	(0.75, 0.02, 0.30)	(0.82, 0.08, 0.25)	(0.42, -0.03, 0.30)
viv	(0.72, 0.04, -0.03)	(0.80, 0.12, 0.10)	(0.45, 0.08, 0.06)
vv	(0.88, 0.02, 0.06)	(1.02, 0.08, 0.08)	(0.50, -0.03, 0.20)
vvi	(0.95, 0.10, -0.10)	(0.95, 0.09, 0.08)	(0.50, 0.06, 0.15)
vvii	(1.00, 0.04, -0.07)	(1.05, 0.06, 0.03)	(0.45, -0.02, 0.30)

At the same time, the dissimilarity values (PCF) of each criterion with respect to the dominant criterion (vv) are established . These values, provided by the experts, are shown in **Table 3**.

Step 4: Aggregation of Average Values

Rescaled ratings of the three experts $x^{-} = 3x1 + x2 + x3$is calculated for each criterion. The formula applied to each component of the triplet is:

The results of this aggregation are found in **Table 3**.

Table 3. Average of the Evaluations of each Criterion for all Experts

Criterion	Average Value Added (T, I, F)
v11	(0.666667, 0.020000, 0.100000)
v12	(0.700000, 0.030000, 0.126667)
v13	(0.740000, 0.023333, 0.066667)
v21	(0.740000, 0.046667, 0.093333)
v31	(0.713333, 0.053333, 0.223333)
v32	(0.700000, 0.083333, 0.053333)
v41	(0.750000, 0.070000, 0.216667)
v51	(0.733333, 0.003333, 0.053333)
v52	(0.753333, 0.016667, 0.070000)
vi	(0.666667, 0.090000, -0.013333)
vii	(0.820000, 0.060000, 0.056667)
viii	(0.650000, 0.053333, 0.160000)
viv	(0.683333, 0.066667, -0.076667)
vv	(0.663333, 0.023333, 0.283333)
vvi	(0.656667, 0.080000, 0.043333)
vvii	(0.800000, 0.023333, 0.113333)
vviii	(0.800000, 0.083333, 0.043333)
vix	(0.833333, 0.026667, 0.086667)

Similarly, dissimilarity values (PCF) are aggregated by averaging the experts' ratings, as shown in **Table 4**.

Table 4. PCF Values for each Expert and Average

Criterion/Expert	PCF according to e1	PCF according to e2	PCF according to e3	Average PCF
v11	0.8	0.9	0.9	**0.866667**
v12	0.2	0.2	0.2	**0.200000**
v13	0.3	0.3	0.3	**0.300000**
v21	0.3	0.4	0.6	**0.433333**
v31	0.2	0.3	0.5	**0.333333**
v32	0.2	0.3	0.5	**0.333333**
v41	0.3	0.4	0.5	**0.400000**
v51	0.4	0.5	0.7	**0.533333**
v52	0.5	0.7	0.8	**0.666667**
vi	0.4	0.4	0.4	**0.400000**
vii	0.3	0.4	0.4	**0.366667**
viii	0.5	0.6	0.7	**0.600000**
viv	0.5	0.6	0.6	**0.566667**
vv	0.0	0.0	0.0	**0.000000**
vvi	0.3	0.5	0.6	**0.466667**
vvii	0.3	0.3	0.4	**0.333333**
vviii	0.2	0.3	0.3	**0.266667**
vix	0.2	0.2	0.4	**0.266667**

Step 5: Convert to Single Numeric Value (Score Function)

To convert the aggregated neutrosophic triplets in Table 4 into a crisp numerical value, Equation 5, the scoring function, is used:$S(vik) = 42 + Tik - Iik - Fik$

Example calculation for$\boldsymbol{v}$11: $S(v11) = 42 + 0.666667 - 0.020000 - 0.100000 = 42.546667 = 0.636667$

The results of applying this function to all criteria are presented in **Table 5**.

Table 5. Results of the Scoring Function

Criterion	Scoring Function Applied
v11	0.636667
v12	0.635833
v13	0.662500
v21	0.650000
v31	0.609167
v32	0.640833
v41	0.628333
v51	0.669167
v52	0.666667

Criterion	Scoring Function Applied
vi	0.645833
vii	0.675833
viii	0.609167
viv	0.685000
vv	**0.589167**
vvi	0.633333
vvii	0.665833
vviii	0.668333
vix	0.705000

The value of the dominant criterion v is **0.589167**.

Step 6: Rescaling Scores with the Dominant Criterion

Each value in Table 6 is rescaled using the dominant criterion value ($S(vv)$)and the average dissimilarity value (d¯ k) from Table 6. Equation 6 is applied:$vik' = S(vik) \times (1 - \bar{d}k)$

Example calculation for v11:

$$v11' = S(v11) \times (1 - \bar{d}11) = 0.636667 \times (1 - 0.866667) = 0.636667 \times 0.133333 = 0.084889$$

The final rescaled values are shown in **Table 6** .

Table 6. Rescaled Values using Equation 6

Criterion	Rescaled Values
v11	0.084889
v12	0.508667
v13	0.463750
v21	0.368333
v31	0.406111
v32	0.427222
v41	0.377000
v51	0.312278
v52	0.222222
vi	0.387500
vii	0.427889
viii	0.243667
viv	0.296444
vv	0.589167
vvi	0.337778
vvii	0.443889
vviii	0.489333
vix	0.517333

Step 7: Aggregation of Final Results by Criterion

Aggregate values are calculated for each main objective by averaging the rescaled values (v′) of its corresponding indicators.

- v1=average$(v11', v12', v13') = \frac{0.084889+0.508667+0.463750}{3} = 0.352435$
- $v2 = v21' = 0.368333$
- v3=average$(v31', v32') = \frac{0.406111+0.427222}{2} = 0.416667$
- $v4 = v41' = 0.377000$
- v5=average$(v51', v52') = \frac{0.312278+0.222222}{2} = 0.267250$

The impact values (vi to vix) remain as their rescaled values from Table 7. The results are summarized in **Table 7**.

Step 8: Calculating the Overall Project Outcome

Finally, the total value of the project is obtained by averaging all the values in Table 7 (the 5 aggregate objectives and the 9 impacts).$vTotal = 14\sum v'$

Table 7. Aggregation Results and Total Value

Criterion	Added Value
v1	0.352435
v2	0.368333
v3	0.416667
v4	0.377000
v5	0.267250
vi	0.387500
vii	0.427889
viii	0.243667
viv	0.296444
vv	0.589167
vvi	0.337778
vvii	0.443889
vviii	0.489333
vix	0.517333
vTotal	**0.393909**

Total Sum: 5.514735 **Total Value (Average):** $\frac{5.514735}{14} = 0.393909$

Verification of Calculations

- **Average Check (Table 4, v11):**
  - T: $(0.85 + 0.65 + 0.50)/3 = 2.00/3 = 0.666667$ $(Correcto)$
  - I: $(0.05 + 0.05 - 0.04)/3 = 0.06/3 = 0.020000$ $(Correcto)$
  - F: $(0.20 + 0.15 - 0.05)/3 = 0.30/3 = 0.100000$ $(Correcto)$
- **Scoring Function Verification (Table 6, vv):**
  - $S(vv) = \frac{2+0.663333-0.023333-0.283333}{4} = \frac{2.356667}{4} = 0.589167$ $(Correcto)$

- **Rescaling Verification (Table 7, v12):**
  - $v12' = S(v12) \times (1 - \bar{d}12) = 0.635833 \times (1 - 0.200000) = 0.635833 \times 0.8 = 0.508667\ (Correcto)$
- **Total Check:** The sum of the 14 values in Table 7 is 5.514735. The average is$\frac{5.514735}{14} = 0.393909$.(Correct)

All calculations have been verified and are accurate according to the methodology provided.

Conclusion of the Analysis

EVAL-IA-MULTIGRADO project aims to revolutionize assessment in multigrade classrooms through an innovative framework that integrates Plithogenic Offsets and Artificial Intelligence. This approach seeks to overcome the barriers of traditional assessment by offering a dynamic, equitable, and contextualized system that manages the uncertainty inherent in student responses.

The applied mathematical model, based on plithogenic theory, processed the assessments of a panel of experts on 18 key indicators. The use of an expanded scale (from -0.1 to 1.1) made it possible to capture complex nuances in the assessments, such as results that exceed expectations or, conversely, generate unforeseen adverse effects.

The quantitative analysis yielded a **final evaluation score for the project of 0.393909** . This value, although positive, indicates moderate viability with significant areas for improvement. Looking at the individual results, it stands out that the criterion with the highest score is **Influence on Educational Policies (vv =0.589167)** , suggesting that experts see great potential in the project to impact educational guidelines at the macro level. However, other objectives, such as **Systematization and Dissemination (v5=0.267250)** , obtained lower scores, pointing to the need to strengthen strategies to communicate and transfer the project results to the academic and professional community.

In short, this model not only provides a final rating, but also functions as a diagnostic tool, allowing the project's strengths and weaknesses to be identified from a multidimensional and expert perspective.

4. Discussion

The analysis of the "EVAL-IA-MULTIGRADO" project using the Plithogenic Offsets model has yielded a multifaceted assessment that goes beyond a simple "success" or "failure" verdict. The overall score of 0.393909 places the project within the threshold of moderate viability, indicating that, while its proposal is fundamentally sound, there are critical areas that require strategic attention to maximize its impact and ensure its success.

Strengths Analysis: The Potential for Political and Social Impact

The most notable finding is the high rating of the "Influence on Educational Policies" criterion (vv = 0.589167), which served as the pillar of the evaluation. This score suggests that experts perceive the project not only as a pedagogical intervention, but as a potential catalyst for systemic change.

The reason for this strength likely lies in the innovative and timely nature of the proposal. In an educational landscape urgently seeking solutions for equity and personalization, especially in underserved contexts such as rural multigrade classrooms, EVAL-AI-MULTIGRADO offers a structured and technologically advanced methodological framework. The combination of accessible Artificial Intelligence and neutrosophic logic for managing uncertainty is a direct response to the shortcomings of traditional assessment systems. Policymakers value solutions that are scalable, data-driven, and address equity issues—core features of this project.

Similarly, the social impact (vii) and technological impact (vviii) received relatively high scores, reinforcing the idea that the project is well aligned with current social needs and technological trends.

Weakness Analysis: The Challenge of Practice and Dissemination

The complexity of the underlying theoretical framework— Plithogenic Offsets and Neutrosophic Logic—can be a double-edged sword. While it lends mathematical robustness to the model, it also makes it difficult to communicate to a non-specialist audience, such as the academic education community. Experts may anticipate difficulties in getting the project accepted in traditional forums if its results and methodology do not translate effectively.

Likewise, the Economic Impact (viii = 0.243667) was perceived as low. Although the project promotes low-cost tools, its economic benefits are indirect and difficult to quantify in the short term. The project needs to better articulate its value proposition in terms of resource optimization and long-term efficiency.

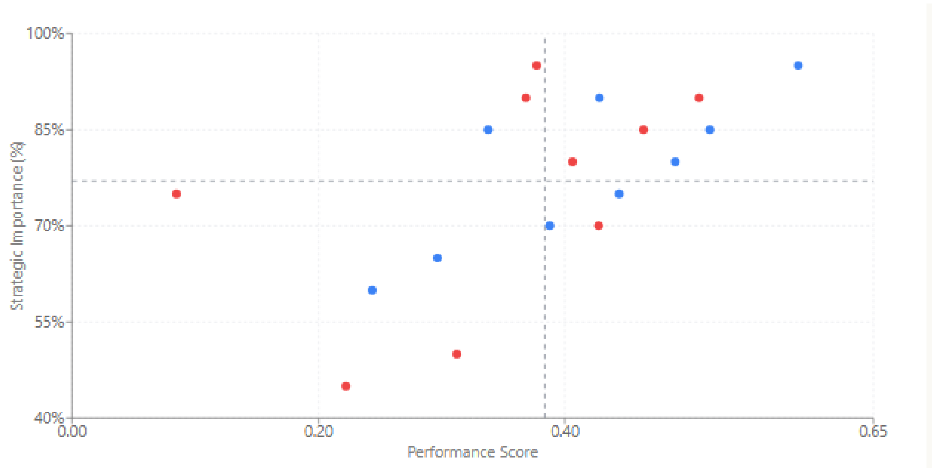

Figure 1: EVAL-IA-MULTIGRADE: Performance vs Strategic Importance Matrix.

Implications of the Methodological Model

The application of the Plithogenic Offsets model was in itself a key finding. This method proved exceptionally useful in capturing the complexity and uncertainty inherent in evaluating an innovative educational project. Unlike traditional Likert scales, the neutrosophic approach (Truthfulness, Indeterminacy, Falsehood) allowed experts to express doubt and ambiguity.

Furthermore, the extended scale [-0.1, 1.1] proved valuable when modeling realistic scenarios. For example, the ability to assign a negative falsity value (e.g., -0.05 in v11) means that a feature is not only "false" (non-compliant), but also has an unexpected beneficial effect. On the other hand, a trueness value above 1 (e.g., 1.10 in v21) captures compliance that radically exceeds expectations. This granularity is critical for authentic and formative evaluation of the project itself.

Limitations and Future Directions

It is crucial to recognize the limitations of this analysis. The evaluation is based on the perceptions of a small panel of three experts. A larger, more diverse group could have offered different perspectives, potentially altering the results. Furthermore, this is an ex ante evaluation; the results are based on the project's promise, not its actual performance.

Based on this discussion, the following strategic recommendations are proposed:

Strengthen the Dissemination Strategy: The project should invest resources in translating its complex methodology. This could include creating practical manuals, workshops for educators, and data visualizations that clearly and convincingly demonstrate the model's impact.

Conduct a Pilot Study: Implementing the project on a small scale is the next logical step. This will generate crucial empirical data to validate experts' perceptions, fine-tune the methodology, and provide solid evidence for publication and funding.

Quantifying Economic Value: A more explicit cost-benefit analysis is needed to demonstrate how streamlining assessment and improving learning translates into more efficient use of educational resources over the long term.

5. Conclusions

1. **The Project is Viable but Requires Strategic Adjustments :** The final score of **0.393909** indicates that the project's conceptual proposal is sound and addresses a real need in the educational field. However, it is not without significant risks. The project's viability should not be taken for granted; its success will depend on the team's ability to proactively address the areas of weakness identified during the assessment.
2. **The project's greatest strength is its potential for systemic impact :** The evaluation highlights that the project's main strength is its ability to **influence educational policy** . Experts believe that its innovative framework, which combines accessible AI and sophisticated uncertainty management, can serve as a model for reforming assessment systems on a scale larger than that of the individual classroom. This is the project's main asset and should be the cornerstone of its communication strategy.
3. **The main challenge is the translation and dissemination of knowledge:** The most critical weakness lies in the **systematization and dissemination** of its complex methodology. There is a real risk that the mathematical sophistication of the model could become a barrier to its adoption and understanding by the educational community. Without a clear strategy to make its concepts and results accessible, the project could remain a theoretical exercise with limited practical impact.
4. **The Plithogenic Methodology is a Powerful Diagnostic Tool :** Beyond evaluating the project itself, this study demonstrates the exceptional value of the Plithogenic Offsets model as an evaluation tool for complex educational initiatives. Its ability to handle **uncertainty** , **ambiguity** , and **unexpected outcomes** (both positive and negative) offers a much richer and more realistic view than traditional evaluation methods. The model not only gives a grade, but also provides a detailed roadmap for continuous improvement.

Ultimately, the **EVAL-IA-MULTIGRADO project** is emerging as a high-risk, high-reward initiative. Its approach is both innovative and necessary. If it succeeds in overcoming the challenge of making its complexity accessible and demonstrating its practical value through pilot studies, it has the potential not only to transform assessment in multigrade classrooms but also to establish a new paradigm for the integration of technology and advanced logic into pedagogy.

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