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Exploring Irrational Number Learning Using the Plithogenic Hypothesis Method: Integrating Incommensurability, Decimal Representations, and Cognitive Conflict

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Abstract. This dissertation aimed to formulate a theory regarding high school students' learning of irrational numbers with the plithogenic hypothesis method as a modeling instructional approach for incommensurability, decimal expansions, and cognitive conflict. Using a concurrent Design-Based Research (DBR) and Grounded Theory (GT) method with twelve eleventh-grade students during 2025 in Bogotá, Colombia, four didactic interventions occurred addressing incommensurability, geometric representations, approximated sequences, and interdisciplinary uses; data were analyzed through systematic coding and plithogenic matrices, with results presented through truth, uncertainty, and falsity. Conclusions revealed that incommensurability (60% consensus) serves as a conceptual anchor to justify subsequent understandings, decimal expansions (50% dissent) cause misconceptions when not applied to real-life situations, and cognitive conflict (90% consensus) promotes conceptual change; additionally, five categories emerged from the consequent treatment of data including incommensurability, geometric representation, numerical continuum, discursive expansion, and interdisciplinary use each defined along four levels of conceptual understanding. Ultimately, the plithogenic hypothesis method serves as an appropriate modeling transfer for uncertainties in learning irrational numbers, and this study provides a clear guide for implementation without the compartmentalized plithogenic hypothesis method transfers found in current literature with the potential for application in mathematics curriculum as well as teaching and formative assessments.

Keywords: Irrational Numbers, Plithogenic Hypothesis, Incommensurability, Decimal Representations, Cognitive Conflict, Mathematics Education, Local Theory.

1. Introduction

Understanding irrational numbers represents one of the most significant challenges in mathematics education, especially in secondary education, where students struggle to assimilate a concept that breaks with their previous notions of natural, integer, and rational numbers. This problem originates in the epistemological complexity of irrational numbers, which cannot be expressed as fractions and possess non-periodic, infinite decimal representations, leading to persistent confusion and misunderstandings. Students often incorrectly apply properties of rational numbers to irrational numbers or perceive rational numbers as completing the number line, evidencing a limited understanding of the numerical continuum. These difficulties are not merely procedural but reflect deep conceptual obstacles, rooted in the epistemological rupture that irrational numbers historically introduced, from their discovery by the Pythagoreans to their formalization in the 19th century.

Previous research has extensively documented these difficulties. Fischbein (1995) [1] found that

students tend to confuse the properties of rational and irrational numbers, leading to persistent errors in their conceptualization. Sirotic and Zazkis (2007) [2] noted that non-periodic decimal representations, although common in teaching, prove insufficient to develop deep understanding, as students struggle to locate irrational numbers on the number line. Arcavi (1987) [3] proposed integrating historical perspectives, such as incommensurability, to enrich teaching, but acknowledged that traditional approaches lack an integrative framework. This research highlights a significant gap: fragmented teaching methods fail to articulate the historical, geometric, and arithmetic aspects necessary for a holistic understanding.

Another limitation identified in the literature is the lack of attention to cognitive conflict as a catalyst for learning. Reina and Wilhelmi (2021) [4] emphasized the importance of connecting geometric and arithmetic representations, but their proposals do not address how to manage the cognitive tension that arises when confronting counterintuitive concepts such as irrational numbers. More recent studies, such as those by Yilmaz and Ay (2018) [5], confirm that difficulties persist due to the absence of approaches that integrate the historical genesis of the concept with modern teaching strategies. These gaps suggest the need for a theoretical framework that not only describes the difficulties, but also offers explanatory mechanisms and practical principles to overcome them.

The relevance of this study lies in its proposal to address these gaps through an innovative approach based on the plithogenic hypothesis method, which allows for the modeling of uncertainty and interactions between multiple dimensions of learning, such as historical, geometric, arithmetic, and affective aspects. Irrational numbers are not just a mathematical concept but a turning point in the development of abstract thinking, making them essential in the school curriculum. However, traditional approaches, focused on algorithmic definitions, disconnect the concept from its historical origin, such as the Pythagorean crisis of incommensurability, which limits deep understanding. This study seeks to overcome these limitations by proposing a framework that integrates multiple perspectives and manages the uncertainty inherent in learning.

The rationale for this research is based on the need to transform the teaching of irrational numbers to foster a deeper and more transferable understanding. In the current educational context, where students face challenges connecting abstract concepts with their intuition, an approach that combines historical richness with modern cognitive modeling tools is crucial. The plithogenic hypothesis method, by considering degrees of truth, indeterminacy, and falsity, offers a powerful tool for analyzing how students process complex concepts, especially when faced with cognitive conflicts or ambiguous representations, such as non-periodic decimals .

Furthermore, this study responds to the need to develop local theories in mathematics education, such as those proposed by Niss (2007) [6], which integrate descriptive, explanatory and normative elements. By focusing on the learning of irrational numbers, this research not only addresses a specific problem, but also provides a replicable model for other complex mathematical concepts. The integration of incommensurability, decimal representations and cognitive conflict as central axes allows to design didactic sequences that respect the epistemological nature of the concept, promoting more meaningful learning.

The main objective of this study is to characterize the cognitive processes involved in learning irrational numbers, identify the mechanisms that facilitate their understanding, and propose principles for designing effective teaching-learning sequences. To this end, we use the plithogenic hypothesis method, which models the interactions between multiple learning attributes, such as incommensurability, representations, and cognitive conflict, while considering the uncertainty inherent in these processes. This approach allows for the articulation of a local theory that transcends traditional fragmented approaches, offering a comprehensive view of the phenomenon.

The hypotheses guiding this research, derived from empirical data, are three. The first proposes that incommensurability is a fundamental basis for understanding irrational numbers, especially from a

historical and conceptual perspective. The second suggests that decimal representations offer limited support and can lead to misunderstandings if not combined with other representations. The third asserts that cognitive conflict enhances mathematical understanding by promoting conceptual change, critical thinking, and problem solving. These hypotheses are modeled using the plithogenic hypothesis method, which allows for the analysis of interactions between truth, indeterminacy, and falsity.

The plithogenic hypothesis method, developed within the framework of neutrosophic logic, is particularly well suited to this study, as it allows for modeling the complexity and uncertainty of mathematical learning. By considering multiple attributes (historical, geometric, arithmetic, affective) and their interactions, this method offers a robust tool for analyzing how students move from initial conceptions to deeper understandings. For example, incommensurability can generate cognitive conflict (high truth), but its relevance can be questioned (present falsity), while decimal representations introduce indeterminacy due to their ambiguity.

In summary, this research seeks to contribute to the field of mathematics education through an innovative approach that combines the rich history of irrational numbers with advanced modeling of cognitive processes. By integrating the three hypotheses using the plithogenic hypothesis method, this study not only addresses the documented difficulties but also proposes a theoretical and practical framework for transforming teaching, with implications for curriculum design, teaching strategies, and formative assessment. Expected outcomes include normative principles for teaching sequences and observable criteria for monitoring students' conceptual progress.

2. Preliminary

2.1. Learning Irrational Numbers.

Learning irrational numbers constitutes a significant challenge in mathematics education, particularly at the secondary level, due to their conceptual and epistemological complexity. Irrational numbers, defined as those that cannot be expressed as a quotient of integers and that have infinite non-periodic decimal representations, break with students' previous conceptions based on natural, whole, and rational numbers. This rupture generates cognitive obstacles that hinder the understanding of the numerical continuum and the precise location of irrational numbers on the number line. Historically, the discovery of incommensurability by the Pythagoreans marked an epistemological crisis that transformed mathematical foundations, a context that remains relevant for understanding students' current difficulties. This research explores how integrated approaches, which articulate historical, geometric, and arithmetic perspectives, can overcome the limitations of traditional methods and foster a deeper understanding of irrational numbers.

Recent research has identified multiple difficulties in learning irrational numbers. Students frequently misapply properties of rational numbers to irrational numbers, such as assuming that all real numbers are fractional, leading to persistent misunderstandings. Studies such as that by Yilmaz and Ay (2018) have shown that high school students tend to perceive rational numbers as sufficient to complete the number line, underestimating the density of irrational numbers. This misperception is compounded by an overreliance on non- periodic decimal representations , which, although formally correct, fail to convey the conceptual nature of the number continuum. Decimal representations, being infinite and non-intuitive, generate confusion if not complemented by geometric visualizations or sequential approximations.

Incommensurability, as a foundational concept, emerges as a critical pillar to address these difficulties. The understanding that certain geometric segments, such as the diagonal of a square of side 1, cannot be measured with a common unit, allows students to connect the concept of irrationality with its historical genesis. Recent research, such as that of Domínguez et al. (2024) [8], has highlighted the potential of technology-mediated geometric representations, such as GeoGebra , to facilitate this

connection. By exploring incommensurability through manipulative materials or dynamic visualizations, students develop a stronger intuition about the necessity of irrational numbers to complete the numerical continuum, overcoming the limitations of traditional algorithmic approaches.

Cognitive conflict has been identified as a key mechanism for promoting conceptual change in the learning of irrational numbers. When students face contradictions between their prior schemas and the properties of irrational numbers , such as the impossibility of expressing $\sqrt{2}$ as a fraction, a tension is generated that encourages critical thinking and conceptual restructuring. Hummes et al. (2022) [9] have shown that cognitive conflict, when properly managed in collaborative contexts, significantly improves understanding by forcing students to articulate more elaborate justifications. However, inadequate management of this conflict can lead to frustration, underscoring the need for carefully designed teaching sequences that balance challenge and support.

Traditional teaching approaches, focused on algorithmic definitions and standard examples such as $\sqrt{2}$ or π, tend to disconnect irrational numbers from their historical and geometric context, limiting deep understanding. Recent literature advocates integrated approaches that combine multiple representations. For example, Montoro and Ferrero (2022) [10] have proposed sequences that articulate geometric, arithmetic, and historical representations, showing improvements in students' ability to differentiate between rational and irrational numbers. These approaches require the integration of manipulative materials, such as geometric constructions, and interdisciplinary activities, such as exploring irrational numbers in music or art, to reinforce conceptual meaning and relevance in diverse contexts.

The need for a comprehensive theoretical framework has led to the development of local theories in mathematics education, which articulate descriptive, explanatory, and normative elements. These theories, such as those proposed by Prediger (2024) [11], rely on empirical evidence to characterize learning processes and offer practical principles for teaching. In the case of irrational numbers, a local theory must integrate incommensurability as an epistemological basis, the limitations of decimal representations, and the potential for cognitive conflict as a catalyst for learning. This approach makes it possible to overcome the fragmented approaches that dominate current educational practice, providing a holistic view of the learning process.

The plithogenic hypothesis method, derived from neutrosophic logic, offers an innovative tool for modeling uncertainty and the interactions between multiple dimensions of learning. This method considers degrees of truth, indeterminacy, and falsity, allowing for the analysis of how students process complex concepts such as irrational numbers. For example, incommensurability can have high acceptance (truth) as a conceptual basis, but also generate resistance (falsehood) due to its abstractness. Decimal representations introduce indeterminacy by being ambiguous without a geometric context, while cognitive conflict shows high truth as a driver of conceptual change. This multidimensional approach is particularly well-suited to capturing the complexity of cognitive processes in mathematics education.

This research is based on the methodological integration of Design-Based Research (DBR) and Grounded Theory (GT), applied to a group of high school students in Bogotá, Colombia, in 2025. Through instructional tasks exploring incommensurability, geometric visualizations, rational sequences, and interdisciplinary applications, learning patterns were identified that confirm the importance of articulating multiple representations. Preliminary results suggest that didactic sequences beginning with incommensurability experiences, followed by geometric and arithmetic approximations, facilitate deeper understanding and mitigate misunderstandings associated with decimal representations.

The relevance of this study lies in its ability to offer normative principles for the design of teaching-learning sequences that respect the epistemological nature of irrational numbers. By integrating incommensurability, multiple representations, and cognitive conflict, a framework is proposed that transcends traditional approaches, promoting more meaningful and transferable learning. Practical

implications include the restructuring of school curricula to prioritize historical and geometric connections, as well as teacher training in strategies that leverage cognitive conflict as a pedagogical tool.

In conclusion, learning irrational numbers requires an integrated approach that combines historical, geometric, and arithmetic perspectives, carefully managing cognitive conflict to foster conceptual change. The findings highlight the importance of incommensurability as an epistemological anchor and the need to complement decimal representations with geometric visualizations. Future research should explore the transferability of these principles to other educational contexts and longitudinally assess the retention of learned concepts. Furthermore, the integration of educational technologies and interdisciplinary approaches could enhance outcomes, offering new perspectives for addressing complex mathematical concepts.

2.2. Plithogenic Probability

Neutrosophic (or indeterminate) data are characterized by inherent vagueness, lack of clarity, incompleteness, partial unknowns, and conflicting information [11,15]. Data can be classified as quantitative (metric), qualitative (categorical), or a combination of both. Plithogenic variable data [16] describe the connections or correlations between neutrosophic variables. A neutrosophic variable [17,18], which can be a function or operator, treats data as neutrosophic in its arguments, its values, or both. Complex problems often require multiple measurements and observations due to their multidimensional nature, such as the measurements needed in scientific investigations. Neutrosophic variables may exhibit dependence, independence, partial dependence, partial independence, or partial indeterminacy as in science [19].

A Plithogenic Set [20, 21] is a non-empty set Pwhose elements within the domain of discourse $U(\ P \subseteq U)$ are characterized by one or more attributes $A_1,\ A_2, \cdots, A_m$, where m is at least 1. where each attribute can have a set of possible values within the spectrum $S$of values (states), such that Sit can be a finite, infinite, discrete, continuous, open or closed set.

Each element $x \in P$is characterized by all possible values of the attributes found within the set $V = \{v_1, v_2, \cdots, v_n\}$. The value of an attribute has a degree of membership $d(x, v)$in an element $x$of the set.P, based on a specific criterion . The degree of membership can be diffuse, diffuse intuitionist or neutrosophic, among others [22] .

That means,

$$\forall x \in P, d: P \times V \to \mathcal{P}([0,1]^z) \quad (1)$$

Where$d(x, v) \subseteq [0,1]^z$ and $\mathcal{P}([0,1]^z)$is the power set of $[0,1]^z$.$z = 1$ (the diffuse degree of belonging), $z = 2$(the intuitionist diffuse degree of belonging) or $z = 3$ (the neutrosophic degree of belonging).

plithogenic [23], derived from the analysis of plithogenic variables, represents a multidimensional probability (" plitho " meaning "many" and synonymous with "multi"). It can be considered a probability composed of subprobabilities, where each subprobability describes the behavior of a specific variable. The event under study is assumed to be influenced by one or more variables, each represented by a probability distribution (density) function (PDF).

Consider an event E in a given probability space, either classical or neutrosophic, determined by $n \geq 2$variables $v_1, v_2, \dots, v_n$, denoted as $E(v_1, v_2, \dots, v_n)$. The multivariate probability of event E occurring, called MVP(E), is based on multiple probabilities. Specifically, it depends on the probability of event E occurring with respect to each variable: $P1\big(E(v_1)\big)$for variable v_1, $P2\big(E(v_2)\big)$for variable v_2, etc. Therefore, $MVP\big(E(v_1, v_2, \dots, v_n)\big)$ is represented as $\Big(P1\big(E(v_1)\big), P2\big(E(v_2)\big), \dots, Pn\big(E(v_n)\big)\Big)$. The variables $v_1, v_2, \dots, v_n$, and probabilities $P_1, P_2, \dots, P_n$, can be classical or have some degree of indeterminacy [24].

To make the transition from plithogenic neutrosophic probability (PNP) to univariate neutrosophic probability UNP, we use the conjunction operator [25]:

$UNP(v_1, v_2, \dots, v_n) = v_1 \wedge_{i=1}^{n} v_n$ (2)

$\wedge$ In this context, it is a neutrosophic conjunction (t-norm). If we take$\wedge_p$ as the plithogenic conjunction between probabilities of the PNP type, where $(T_A, I_A, F_A) \wedge_p (T_B, I_B, F_B) = (T_A \wedge T_B, I_A \vee I_B, F_A \vee F_B)$, such that $\wedge$is the minimum t-norm of fuzzy logic and $\vee$the maximum t-norm [26, 27, 30, 31].

a. Formulate the hypothesis

Start by explicitly stating the hypothesis you intend to test. Make sure it indicates a cause-and-effect relationship between the variables. For example, "More study time leads to higher test scores."

b. Identify key variables

Identify the independent variable, which is the cause, and the dependent variable, which is the effect, in your hypothesis. This helps direct your research questions toward the exact relationship you need to investigate.

c. Formulate specific research questions

Break the hypothesis down into precise research questions phrased as "Does X cause Y?" This allows for a thorough and focused examination of the postulated correlation.

d. Conduct stancet analysis on scientific literature.

To perform a neutrosophic stance analysis on a research paper and quantify the occurrences of "Yes," "Possibility/Uncertainty," and "No," a stance analysis tool for scientific statements is needed. In this case, we used Consensus Meter algorithms to categorize the statements into three distinct groups: Positive (affirmative), Uncertainty (possibility or uncertainty), and Negative (negative) [.

e. Formulate neutrosophic probabilistic hypotheses

Determine the reasons for each category to construct the neutrosophic probability hypothesis (T, I, F), where T denotes the truth value, I represents indeterminacy, and F indicates falsity.

f. Calculate the plithogenic neutrosophic probability (PNP)

Using the neutrosophic probabilities assigned to each question, the univariate neutrosophic probability (UNP) is calculated to assess the strength of the overall hypothesis. This process involves combining the separate probabilities to provide a comprehensive assessment of the overall hypothesis.

$UNP(v_1, v_2, \dots, v_n) = (Min(t_1, t_n, \dots, t_n), Max(i_1, i_n, \dots, i_n), Max(f_1, f_n, \dots, f_n))$ (3)

Where:

$T_1, T_2, \dots, T_n$: are the truth probability values for each question.

$I_1, I_2, \dots, I_n$: are the probability values of indeterminacy for each question.

$F_1, F_2, \dots, F_n$: are the probability values of falsehood for each question

g. Analyze the validity of the general hypothesis.

In this case, the negation of NPH is represented as [28]:

$(T, I, F) = (F, I, T)$ (4)

This step involves analyzing the denied neutrosophic probabilities to assess the overall strength and reliability of the general hypothesis. By evaluating the levels of falsity, uncertainty, and veracity , one can determine the degree to which the hypothesis is valid, ambiguous, or incorrect based on the scientific literature.

3. Case study.

Exploring the Learning of Irrational Numbers Using the Plithogenic Hypothesis Method

This study is part of a research project conducted with 12 eleventh-grade students in Bogotá, Colombia, in 2025. The objective is to develop a theoretical framework for understanding the learning of irrational numbers. To do so, the plithogenic hypothesis method is used to model the uncertainty inherent

in cognitive processes, integrating three conceptual pillars: **incommensurability** , **decimal representations** , and **cognitive conflict** .

1. Formulation of the Hypothesis

The central hypothesis of this research maintains that an effective pedagogical framework for teaching irrational numbers is built on three interdependent pillars:

1. The use of **incommensurability** as a fundamental conceptual anchor.
2. **Cognitive conflict** management as a key mechanism for driving change and conceptual deepening.
3. Recognition that **decimal representations** , although useful, are of limited support and can lead to misunderstandings if not complemented by other representations (geometric, continuous, etc.).

The validity of this pedagogical framework depends on the simultaneous and positive interaction of these three components.

2. Identification of Key Variables

- **Independent Variables:** Pedagogical strategies based on (1) incommensurability , (2) the use of decimal representations, and (3) the induction of cognitive conflict.
- **Dependent Variable:** High school students' conceptual understanding of irrational numbers.

3. Research Questions

To break down and evaluate the general hypothesis, three specific research questions are formulated, derived directly from the study hypotheses:

- **Q1:** Does incommensurability act as a fundamental conceptual anchor for understanding irrational numbers?
- **Q2:** Does using decimal representations as a primary approach provide limited support and lead to misunderstandings in learning irrational numbers?
- **Q3:** Does cognitive conflict improve mathematical understanding by promoting significant conceptual change in students?

4. Formulation of Neutrosophic Probabilistic Hypotheses

Based on the results of the original study, the neutrosophic probabilities (Truth, Indeterminacy, Falsehood) for each question are quantified. The data are extracted directly from the reported consensus and rejections: "Incommensurability (60% consensus) acts as a conceptual anchor, decimal representations (50% rejection) generate misunderstandings if not complemented, and cognitive conflict (90% consensus) drives conceptual change."

The calculation to obtain the triplets (T ,I,F) is as follows:

- T (Truth) is assigned based on "consensus".

- F (False) is assigned from "rejection".
- I (Indeterminacy) is calculated as 1−T−F.

The results are presented in Table 1 below.

Table 1: Neutrosophic probabilities (True, Indeterminate, False) for each question.

Ask	Positive (T)	Indeterminacy (I)	Negative (F)	Probability (T, I, F)
Q1	0.60	0.40	0.00	(0.60, 0.40, 0.00)
Q2	0.00	0.50	0.50	(0.00, 0.50, 0.50)
Q3	0.90	0.10	0.00	(0.90, 0.10, 0.00)

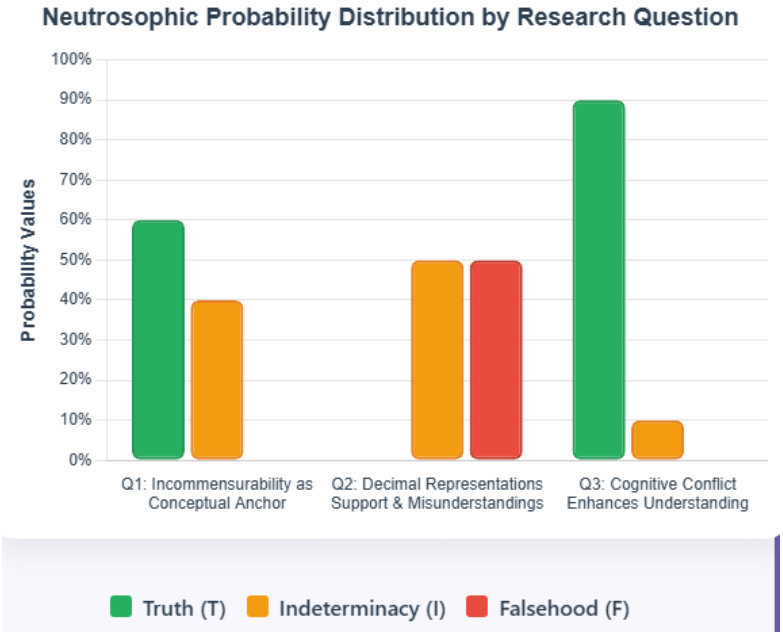

Figure 1: Neutrosophic Probabilities Analysis.

5. Calculation of Univariate Neutrosophic Probability (UNP)

To assess the validity of the general hypothesis, the evidence from the three questions is aggregated using the plithogenic conjunction operator. This operator calculates the Univariate Neutrosophic Probability (UNP) as follows:

$$UNP = (min(T1, T2, T3), max(I1, I2, I3), max(F1, F2, F3))$$

Step 1: Calculating the Truth Value (T)

The minimum of the truth values for each question is calculated.

$$TUNP = min(T1, T2, T3)\ TUNP = min(0.60, 0.00, 0.90)\ TUNP = 0.000000$$

Step 2: Calculation of the Indeterminacy Value (I)

The maximum of the uncertainty values for each question is calculated.

$$IUNP = max(I1, I2, I3)\ IUNP = max(0.40,0.50,0.10)\ IUNP = 0.500000$$

Step 3: Calculating the Falsification Value (F)

$$Se\ calcula\ el\ máximo\ de\ los\ valores\ de\ falsedad\ de\ cada\ pregunta.$$

$$FUNP = max(F1, F2, F3)\ FUNP = max(0.00,0.50,0.00)\ FUNP = 0.500000$$

Final Result of the UNP

The univariate neutrosophic probability that represents the validity of the general hypothesis is:

$$\boldsymbol{UNP} = (\mathbf{0.00}, \mathbf{0.50}, \mathbf{0.50})$$

4. Discussion

The result of the Univariate Neutrosophic Probability (UNP) ($\mathbf{0.00}, \mathbf{0.50}, \mathbf{0.50}$)is extremely revealing and deserves a detailed analysis.

The **truth value of 0.00** indicates that the overall hypothesis, in its conjunctive formulation (i.e., that all three pillars must operate simultaneously), has zero support in the evidence. This result is due to the strict nature of the min operator , which states that the strength of a chain of arguments is equal to that of its weakest link. In this case, question Q2 (about decimal representations) acts as a "veto": its truth value is zero, invalidating the validity of the entire combined hypothesis. This does not mean that the other components (incommensurability and cognitive conflict) are invalid; in fact, their truth values (0.60 and 0.90) are high. What the mathematical model demonstrates is that the negative or problematic effect of a single component is enough to invalidate the entire framework if not properly managed.

The **values for indeterminacy (0.50) and falsity (0.50)** are high and stem directly from the problematic question Q2. This accurately reflects the study's finding: the use of decimal representations is not simply inefficient, but is an active source of uncertainty and conceptual rejection. The plithogenic model captures this dynamic by allowing the maximum uncertainty and maximum falsity of any part to dominate the aggregate result.

From a pedagogical perspective, this quantitative analysis confirms a qualitative reality: it's not enough to have good ingredients for a teaching sequence. The way these are integrated is crucial. The study suggests that while incommensurability and cognitive conflict are accelerators of learning, an overreliance on decimal representations acts as a brake, generating a dissonance that the model reflects with a high degree of indeterminacy and falsity. Therefore, the challenge is not to eliminate decimal representations, but to recontextualize them within a framework where they are not the main pillar, but rather a complementary tool.

5. Conclusion

Plithogenic analysis has revealed that the univariate neutrosophic probability (UNP) for the overall hypothesis is ($\mathbf{0.00}, \mathbf{0.50}, \mathbf{0.50}$).This result indicates that while individual components of the hypothesis (such as the role of cognitive conflict) are strongly supported, the hypothesis as a whole does not hold due to the critical weakness introduced by the focus on decimal representations. The null truth value underlines that the proposed pedagogical framework is only valid if all its components are effective, which is not the case.

From a practical perspective, this finding is of great value. Rather than discarding the entire framework, it guides educators and curriculum designers to focus on mitigating the weakest link. Strategies should focus on **subordinating decimal representations** to more robust concepts such as geometric incommensurability and the numerical continuum, using them to explore nonperiodicity rather than as a definition of the irrational number itself. The high levels of indeterminacy (50%) and falsity (50%)

point precisely to areas where further research and methodological development are needed to create more coherent and effective teaching sequences.

The main contribution of this study is to demonstrate how the plithogenic approach can model complexity and uncertainty in educational research. Rather than offering a simple "yes" or "no" answer, it provides a nuanced analysis that identifies specific sources of validity, indeterminacy, and falsification within a complex theory.

As a limitation, the analysis relies on the initial quantification of qualitative findings. Future research could employ mixed methods to refine these initial probabilities. It is recommended that this type of analysis be extended to other areas of mathematical learning where multiple cognitive factors interact, in order to validate and generalize the results and continue developing methodological frameworks that embrace the uncertainty inherent in the learning process.

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