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An Algorithmic Approach to Bridge Detection in Bijective Neutrosophic Graph with Application in Cancer Diagnosis

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Abstract: This paper introduces an efficient algorithm for detecting bridges in Bijective neutrosophic graphs, an emerging approach in handling uncertainty in complex systems such as medical diagnostics. The algorithm identifies T-bridges, I- bridges and F-bridges in neutrosophic graphs, and incorporates a de neutrosophication method using score function to find neutrosophic bridges when needed. The application of the algorithm to cancer diagnosis is explored, demonstrating its potential to enhance disease identification and treatment planning based on symptoms. The result suggest that this method can improve the accuracy of medical decision- making in uncertain environments, thereby offering a promising tool for healthcare analysis. By providing a more precise understanding of medical data, this approach has the potential to optimize diagnostic processes and treatment strategies in uncertain and indeterminate contexts.

Keywords: Neutrosophic Graph; Bijective Neutrosophic graph; T-bridge, I-bridge, F-bridge; and Neutrosophic Bridge.

1. Introduction

Graph theory has emerged as a powerful mathematical tool for modeling complex systems in various domains, including biology, computer science, and medicine. Among its numerous applications, the analysis of graph structures offers valuable insights into the connectivity and critical components of networks. In this context, bridge detection plays a significant role in identifying the most sensitive or pivotal connections whose removal may

fragment the graph. To address the limitations of classical graph models in handling uncertainty, incompleteness and inconsistency inherent in real world data- particularly in medical diagnostics- the concept of neutrosophic graphs has gained traction. These graphs extend fuzzy and intuitionistic fuzzy models by incorporating degrees of truth, indeterminacy, and falsity. A further advancement, bijective neutrosophic graphs, ensures a one to one correspondence between graph elements, offering a more precise and structured representation of data. Akram, M et. al [1] introduced the concept of operations on Single Valued Neutrosophic Graphs (SVN-graphs) in 2017 Beaula Thangaraj et al. [2–4] contributed significantly to the field by applying various fuzzy numbers and ranking methods to solve critical path problems, exemplifying the expanding application of fuzzy logic in optimization and decision-making. It is important to acknowledge Broumi, S et al[5-6] studies about single valued neutrosophic graph in 2016. Further this concept take its shape as neutrosophic labelling graph in 2019, which was introduced by Gomathi et.al [7]. A.Hassan et.al [8] studied about single valued trees in 2018. Muthuraj et.al[9] studied about multi fuzzy graph in 2020. Rajalaxmi D et.al[10-11] studied about metric in fuzzy labeling graph and Bijective single valued in highlighting their structural properties and potential applications. Vijaya et al[12,13] have shaped the advancements of Neutrosophic graphs in find the solution of Decision making problem and critical path problems by using Pythagorean Fuzzy numbers and Neutrosophic Fuzzy numbers. Ye, J.,[14-15]studied Single-Valued Neutrosophic Minimum Spanning Tree in 2014. Finally, the pioneering work of Zadeh, L. [16] in 1965, who introduced the concept of fuzzy sets, laid the groundwork for the entire field of fuzzy and neutrosophic mathematics that followed.

An algorithmic method for detecting bridges in bijective neutrosophic graphs is presented in this article, with a focus on its use in the diagnosis of cancer. In addition to improving structural analysis of intricate networks, the algorithm offers a useful tool that can be applied to a range of real-world issues. The suggested techniques seek to assist oncology's early detection, risk assessment, and strategic intervention planning by locating important pathways or interactions in biological networks linked to cancer. Through this study, we show how algorithmic approaches and mathematical abstraction can greatly advance medical research and decision-making.

The neutrosophic framework permits the simultaneous representation of truth, indeterminacy, and falsity, the neutrosophic framework is especially useful in this situation. Incomplete, ambiguous, or contradicting information is frequently present in biological and medical networks, particularly those pertaining to the diagnosis of cancer. The ability of fuzzy graph models and even classical graph theory to handle such uncertainties is constrained. The suggested approach more successfully captures several aspects of uncertainty by using bijective neutrosophic graphs, producing robust analysis and more trustworthy results. This makes the neutrosophic approach especially important in advancing medical research and decision-making processes where ambiguity is inherent.

2. Preliminaries:

Definition 2.1[7]: A neutrosophic graph is of the form G = (V, σ,μ) where σ = (T_1, I_1, F_1) and μ = (T_2, I_2, F_2) (i) V = {v_1, v_2, v_3, ⋯, v_n} such that T_1: V → [0, 1], I_1: V → [0, 1] and F_1 : V → [0, 1] denote the degree of truth-membership function, indeterminacy-membership function and falsity-membership function of the vertex v_i ∈ V respectively, and $0 \leq T_1(v) + I_1(v) + F_1(v) \leq 3$ ∀ v_i ∈ V (i=1, 2, 3….n).
(ii) T_2 : V × V → [0, 1], I_2 : V × V → [0, 1] and F_2 : V × V → [0, 1], where $T_2(v_i, v_j)$, $I_2(v_i, v_j)$ and $F_2(v_i, v_j)$ denote the degree of truth-membership function, indeterminacy membership function and falsity-membership function of the edge (v_i, v_j) respectively such that for every (v_i, v_j), $T_2(v_i, v_j) \leq \min\{T_1(v_i), T_1(v_j)\}$, $I_2(v_i, v_j) \leq \min\{I_1(v_i), I_1(v_j)\}$, $F_2(v_i, v_j) \leq \max\{F_1(v_i), F_1(v_j)\}$, and $0 \leq T_2(v_i, v_j) + I_2(v_i, v_j) + F_2(v_i, v_j) \leq 3$.

Definition 2.2

A neutrosophic graph is said to be a bijective neutrosophic graph if $\sigma_T : V \to [0,1], \sigma_I : V \to [0,1], \sigma_F : V \to [0,1], \mu_T : V \times V \to [0,1], \mu_I : V \times V \to [0,1], \mu_F : V \times V \to [0,1]$ are bijective, such that the truth- membership function, Indeterminacy-membership function and Falsity- membership functions for every edge

$\mu_T(u,v) < \min(\sigma_T(u), \sigma_T(v))$

$\mu_I(u,v) < \min(\sigma_I(u), \sigma_I(v))$

$\mu_F(u,v) < \max(\sigma_F(u), \sigma_F(v))$ and $0 \leq \mu_T(u,v) + \mu_I(u,v) + \mu_F(u,v) \leq 3$

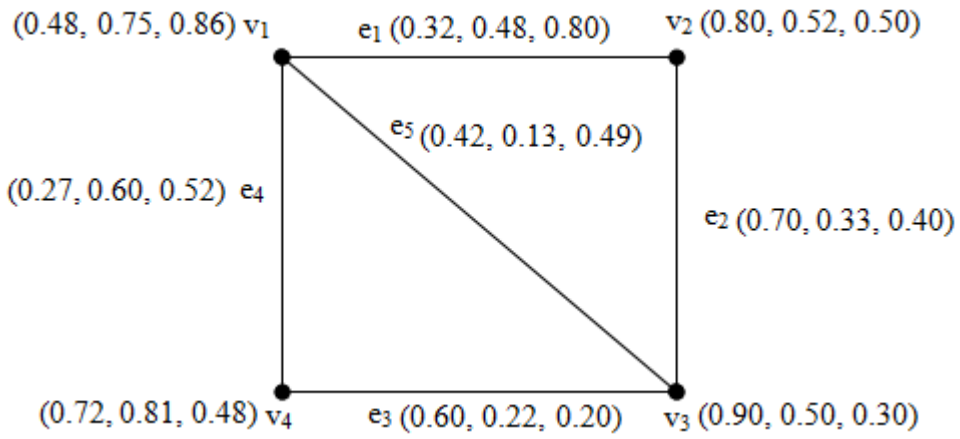

Figure 1:Bijective Neutrosophic Graph

Definition :2.3

The strength of the path with n edges is defined as S(P) = ($S(P_1)$, $S(P_2)$, $S(P_3)$) where

$S(P_1) = \bigwedge_{i=1}^{n} \mu_T(u,v)$, $S(P_2) = \bigwedge_{i=1}^{n} \mu_I(u,v)$, $S(P_3) = \bigvee_{i-1}^{n} \mu_F(u,v)$

Definition :2.4

Let G be a Bijective neutrosophic graph. The connected between any two vertices is defined by $\mu^{\infty}(u,v) = (\mu_T^{\infty}(u,v), \mu_I^{\infty}(u,v), \mu_F^{\infty}(u,v))$ where $\mu_T^{\infty}(u,v)$ = Max(S(P_1)),

$\mu_I^{\infty}(u,v) = \text{Max}(S(P_2))$, $\mu_F^{\infty}(u,v) = \text{Min}(S(P_3))$

Definition:2.5

Let G be a bijective neurtosophic graph. An edge of G is said to T-bridge

$$\text{if } \mu_T^{\infty}(u,v) < \mu_T^{'\infty}(u,v)$$

where $\mu_T^{'\infty}(u,v)$ is the connectedness between u and v by removing any edge.

An edge of G is said to I-bridge

$$\text{if } \mu_I^{\infty}(u,v) < \mu_I^{'\infty}(u,v)$$

where $\mu_I^{'\infty}(u,v)$ is the connectedness between u and v by removing any edge.

An edge of G is said to F-bridge

$$\text{if } \mu_F^{\infty}(u,v) > \mu_F^{'\infty}(u,v)$$

where $\mu_F^{'\infty}(u,v)$ is the connectedness between u and v by removing any edge.

Definition: 2.6

An edge of G is said to be a neutrosophic bridge if it is T- bridge, I-bridge and F- bridge.

3. An Algorithm for finding the bridges of any bijective neutrosophic graph

Input: A bijective neutrosophic graph G = (V, E, T, I, F), where each edge has associated truth-membership T, indeterminacy-membership I, and falsity-membership F.
Output: Classification of each bridge as T-Bridge, I-Bridge, or F-Bridge.

Step 1: Begin with a bijective neutrosophic graph G. Select an arbitrary crisp cycle C* consisting of n- edges, where n ≥ 3

Step 2: If T- bridge or I- bridge of G are required, then identify an edge

$\eta_T = \bigwedge_{i=1}^{n} \mu_T$ or $\eta_I = \bigwedge_{i=1}^{n} \mu_I$

by considering all the edges of C* respectively.

Step 3: Remove the identified edge η from G.

Step 4: Choose another cycle C* in G with any number of edges and repeat step 2 & step 3 until no such cycle remains in G.

Step 5: After removal of all η's from G, the resulting graph comprises the T- bridges or I – bridges of G respectively.

Step 6: If F- bridges is required for the chosen graph then find $\eta_F = \bigvee_{i-1}^{n} \mu_F$

by considering all the edges of C*.

Step 7: Repeat step 3 and step 4.

Step 8: Upon removal of all η_F's from the bijective neutrosophic graph G, the resulting graph comprises the F- bridges of G.

The T- bridges and I- bridges are the strongest connections between the vertices in the graph G and F – bridges is the weakest connections between the vertices in the graph G.
The resulting graph which is obtained is the T- maximum spanning sub graph or I- maximum spanning sub graph of G. Also one can obtain the F-minimum spanning subgraph of G.

Example:3.1

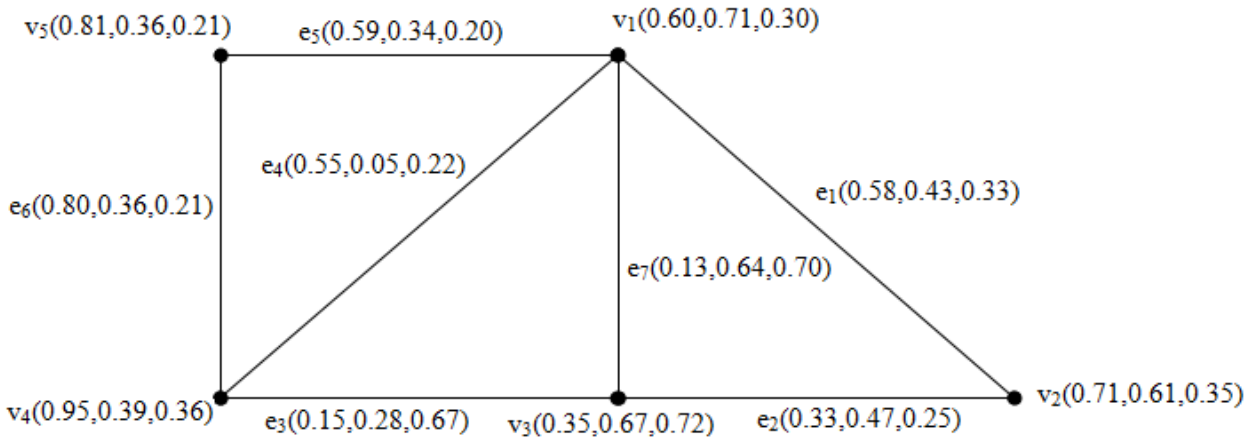

Figure 2:Bijective neutrosophic graph

Let us find the T- bridges of the above figure 2 by Considering the cycle C_1^* = v_1,v_3,v_4,v_5,v_1 of length 4.
Here η_1= min(0.13, 0.15, 0.80, 0.59)
η_1= 0.13

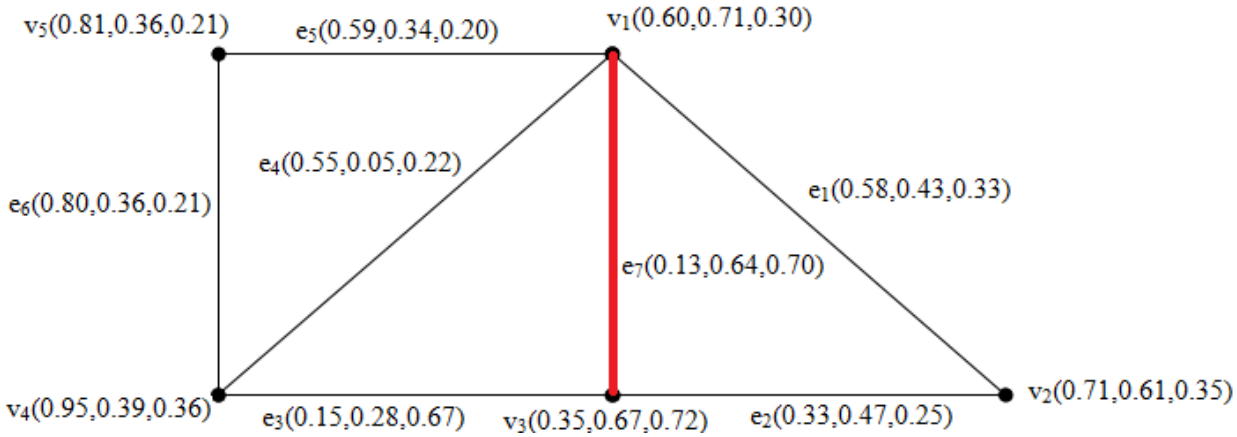

Figure 3

In the above figure 3 edge e_7 with 0.13 truth membership value has to be removed from G.

Let the next cycle be $C_2^* = v_1,v_4,v_5,v_1$ of length 3.
Here $\eta_2 = \min(0.55, 0.80, 0.59)$
$\eta_2 = 0.55$

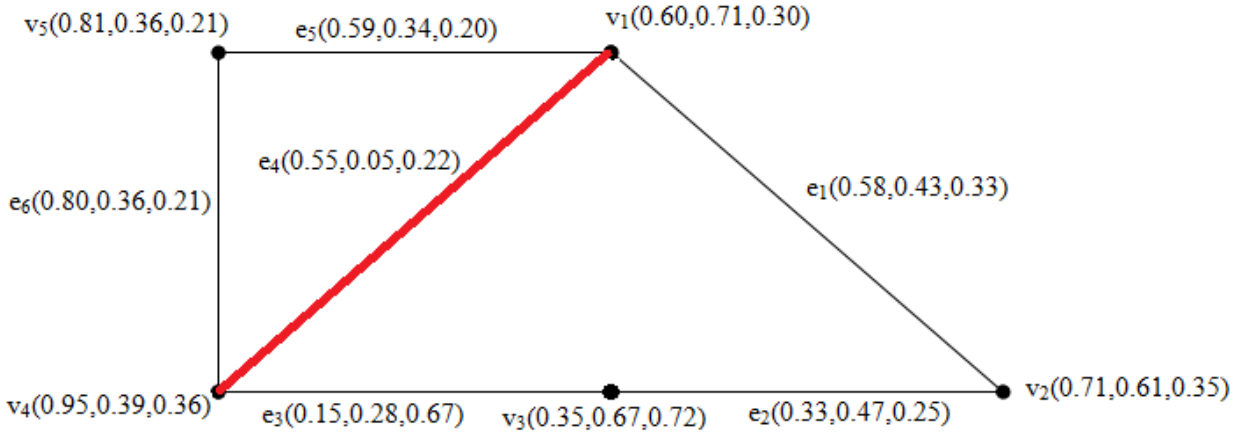

Figure 4

Clearly in the above figure 4 the edge e_4 with 0.55 truth membership value has to be removed from G. So Let the next cycle be $C_3^* = v_1,v_2,v_3,v_4,v_5,v_1$ of length 5.
Here $\eta_3 = \min(0.58, 0.33, 0.15, 0.80, 0.59)$
$\eta_3 = 0.15$

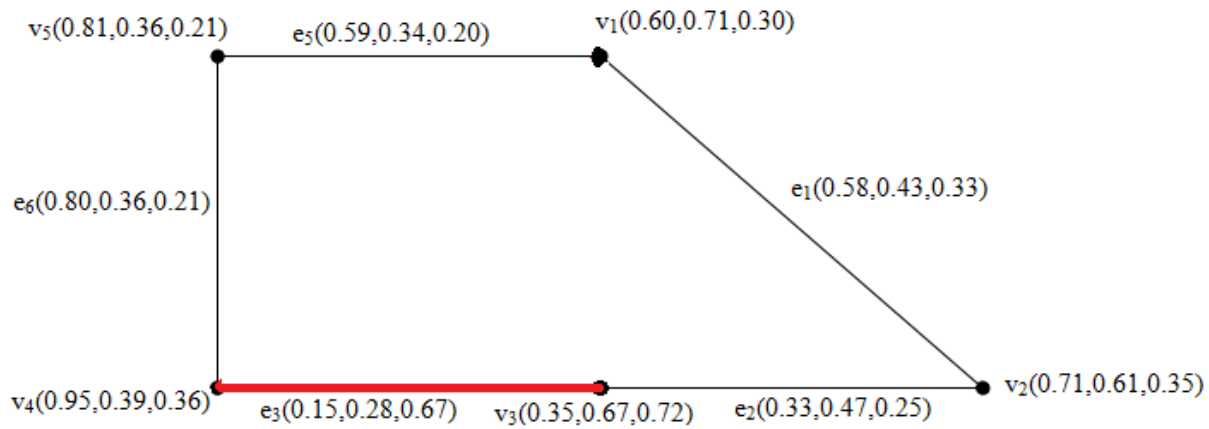

Figure 5

Clearly in the above figure 5 the edge e_3 with 0.15 as truth membership value has to be removed from G. Since no cycles remains after the removal of edges, the following resulting graph represents all the T-Bridges of G

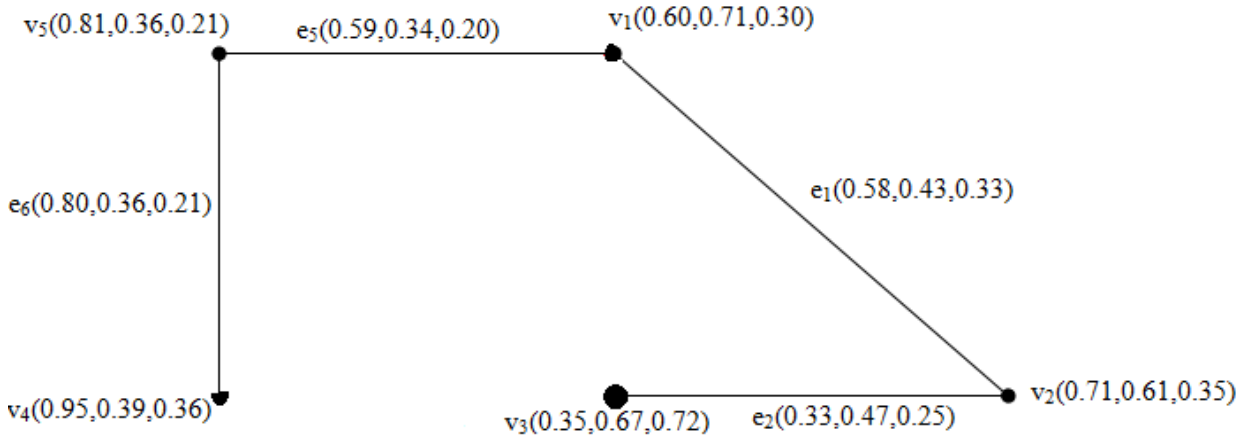

Figure 6: T-Bridges of G

Hence the T- Bridges of G are (v_1,v_2) , (v_2,v_3) , (v_4,v_5) and (v_5,v_1)

If the above algorithm is applied for Indeterminancy then the I- Bridge of G can be obtained.

Hence the I- Bridges of G are (v_1,v_3) , (v_1,v_5) , (v_2,v_3) and (v_3,v_4)

Similarly F-Bridges of G are (v_1,v_5) , (v_1,v_4) , (v_1,v_2) and (v_2,v_3).

In some practical situation if it is necessary to consider all the membership functions in the same time then we can use score function to find the bridges of bijective neutrosophic graph.

Definition:[12]

The score function is defined as

$$S(u) = \frac{\sigma_T(u) + \sigma_I(u) + \sigma_F(u)}{3}$$

The same algorithm can be used to find the bridges of the bijective neutrosophic graph after find the score function for all the vertices and edges.

Example: Now let us find the bridges of the graph given in figure after finding the score function

$S(v_1) = \frac{\sigma_T(v_1)+\sigma_I(v_1)+\sigma_F(v_1)}{3} = \frac{0.60+0.71+0.30}{3} = 0.54$

$S(v_2) = 0.56$, $S(v_3) = 0.58$ and so on.

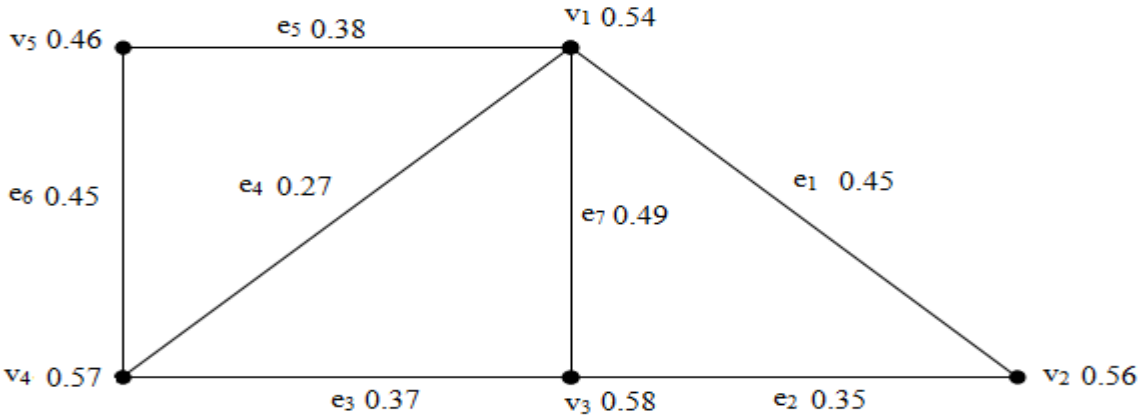

Figure 7

For finding the bridges of G let us first consider a cycle $C_1^* = v_1,v_2,v_3,v_4,v_5,v_1$ of length 5.

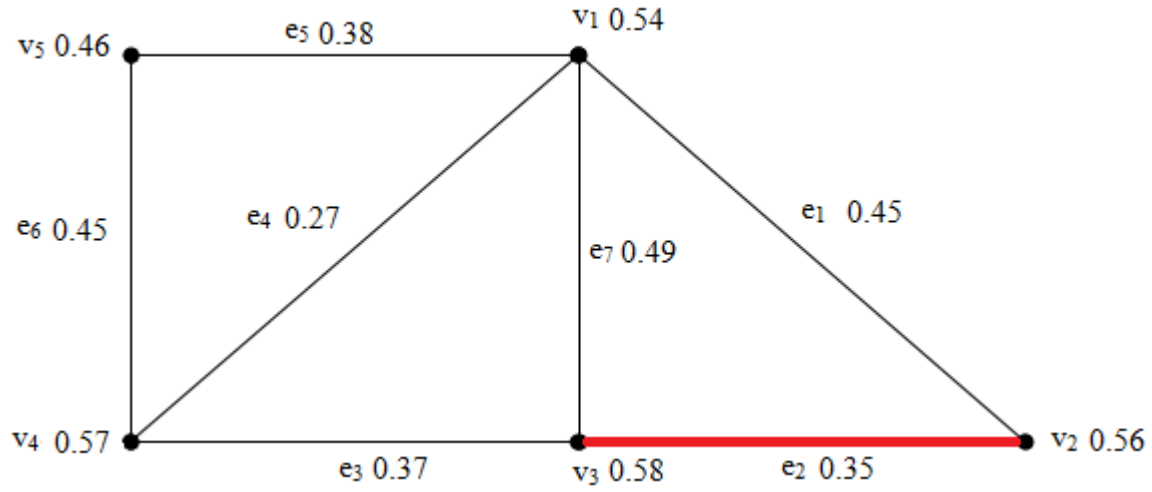

Figure 8

Clearly the edge e_2 is to be removed as it has minimum value among the other edge values in the considered cycle. Now let us choose another cycle $C_2^* = v_1,v_3,v_4,v_5,v_1$ of length 4.

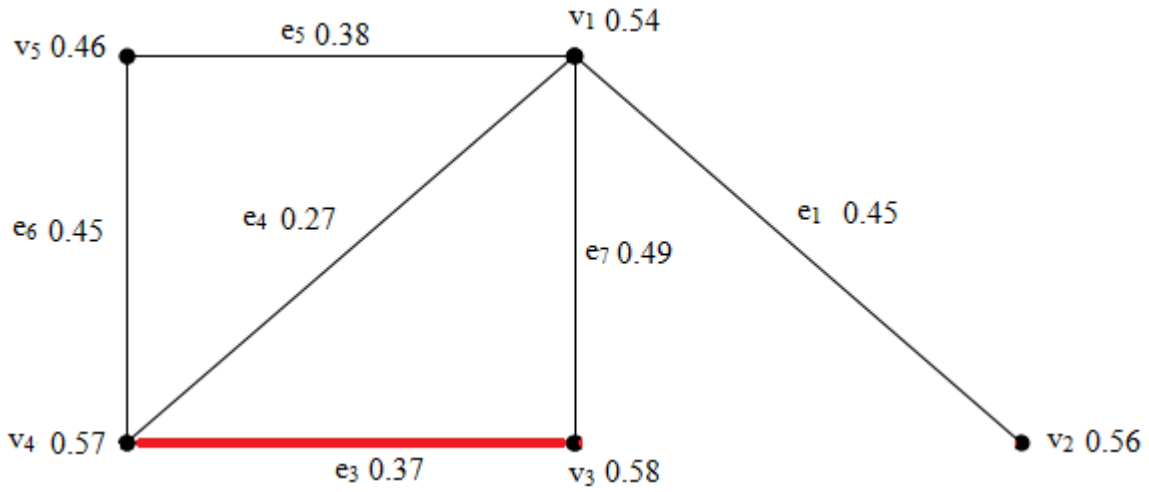

Figure 9

The above marked edge e_3 is the next edge with minimum score value which has to be removed next. Now consider another cycle $C_3^* = v_1,v_4,v_5,v_1$ of length 3.

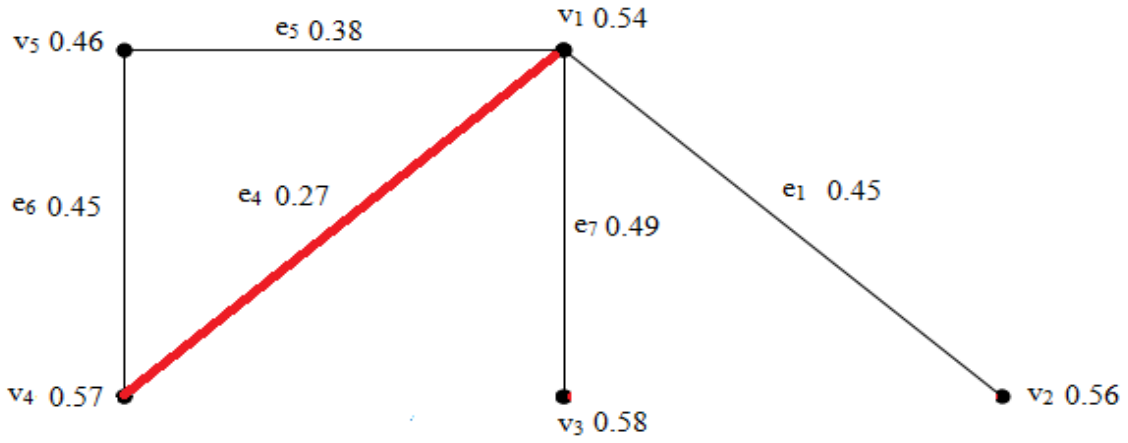

Figure 10

Clearly if the above marked edge is removed, then the bridges of G are (v_1,v_2) , (v_1,v_3) , (v_4,v_5) and (v_5,v_1).

Note: The score function is one of the methods of de neutrosophication. So score function need not be bijective.

4. Applications

Now, Let's use the above discussed algorithm to find a better diagnosis for cancer. Let's consider a specific case in which we have a patient with the more or less equal symptoms of Inflammation of Gastrointestinal Tract (GT), Chronic Cough(C) and Fatigue(F).**i.e. As per the data, he exhibits 48% of the typical symptoms associated with Inflammation of Gastrointestinal Tract, 40% clinical symptoms for chronic cough and 42% of Fatigue.** And we suspect that the patient might be suffering from Colorectal Cancer(CRC), Lung Cancer(LC), Tuberculosis(TB) and Inflammatory Bowel Disease(IBD).**i.e. There is approximately equal chance for him to suffer from all the above mentioned disease. In**

detail, clinical data suggests that there is 55% likelihood for him to suffer from CRC, 52% to present with Lung cancer, 45% exhibits Tuberculosis and 50% chance to experience IBD. But still we need a better diagnostic approach to enhance a safe and effective treatment outcomes by reducing side effects. In this Crucial case, Neutrosophic Graph ensures effective Cancer-focused framework since fatigue is one of the most common seen symptoms for many types of cancer,it helps to capture indeterminacy value for better diagnosis. In our use case, it is significant to highlight that 39% of fatigue could linked to Colorectal Cancer (CRC), 38% manifests Lung cancer(LC), there is 31% and 34% probability that fatigue could cause TB and IBD respectively. Therefore, Bijective NeutrosophicGraph lends us a helping hand since the given patient data is more accurate. It is also necessary to recall that the bridges are the crucial links that would helps to find the underlying issue. Finally, we can say that the **Bijective Neutrosophic Graph** arise as a natural response to this particular use case. To construct a Bijective Neutrosophic Framework for this cancer diagnosis, let's take the symptoms and suspected diseases as vertices.

Membership value for vertices:

1. Truth membership value(T) ensure the degree of confirmed occurrence of symptoms/diseases,
2. Indeterminacy value(I) tells us the uncertainty due to conflict reports or incomplete data and
3. Falsity-membership value(F) ensure the degree of belief that there is no such occurrence exists.

Edges represent the relations such as symptoms correlation, biological contact, cooccurrence of diseases etc., It is always necessary to consider Symptom-Symptom relationships, since it adds a deeper layer for medical insight especially when symptoms Co-occurrence, contradict occurs. Similarly, we need to consider Disease-Disease relationship, as in many cases one or more disease appears together. In Particular, since liver and lungs are the most common sites for colorectal cancer, there is a possibility that 18% of patients affected with Colorectal Cancer(CRC) may be presents with lung cancer. Therefore, it becomes significant to consider disease-disease relationships to have a refined diagnostic perspective.(In our use case, there is 50% chance of patient suffering from Colorectal cancer to get suffer from lung cancer as per the clinical data).In addition to that, TB cannot directly cause lung cancer, but it can increase the risk of developing lung cancer. Therefore, there is 35% likelihood of him to develop lung cancer as per the patient's data.

Membership value for edges:

1. Truth membership value (T) exhibits the degree of correlation. 2. Indeterminacy value (I) tells the degree of uncertainty arises due to incomplete data (may be due to ongoing investigation).3. Falsity value ensures the belief that no occurrence exists.

Analysis for cancer diagnosis:

It is significant to remember the following remarks to give membership values for the disease-disease relations in the graphical representation for cancer diagnosis.

1. Generally, Colorectal Cancer (CRC) does not cause tuberculosis (TB). It is almost uncommon in most of the cases.
2. Tuberculosis (TB) is one of the risk factor for lung cancer.
3. Inflammation in Gastro intestinal tract (GT) in addition with the Inflammatory Bowl Disease (IBD) provides a high risk factor for Colorectal Cancer (CRC). And this relation is bidirectional.
4. There is high possibility for a person getting affected with Colorectal Cancer (CRC) to get suffer with Lung cancer. Since Liver and lungs are the common sites for cancer development.
5. Chronic cough(C) is one of the risk factors for lung cancer (LC), Tuberculosis (TB).
6. Fatigue (F) is one of the common symptoms for many of the cancers.

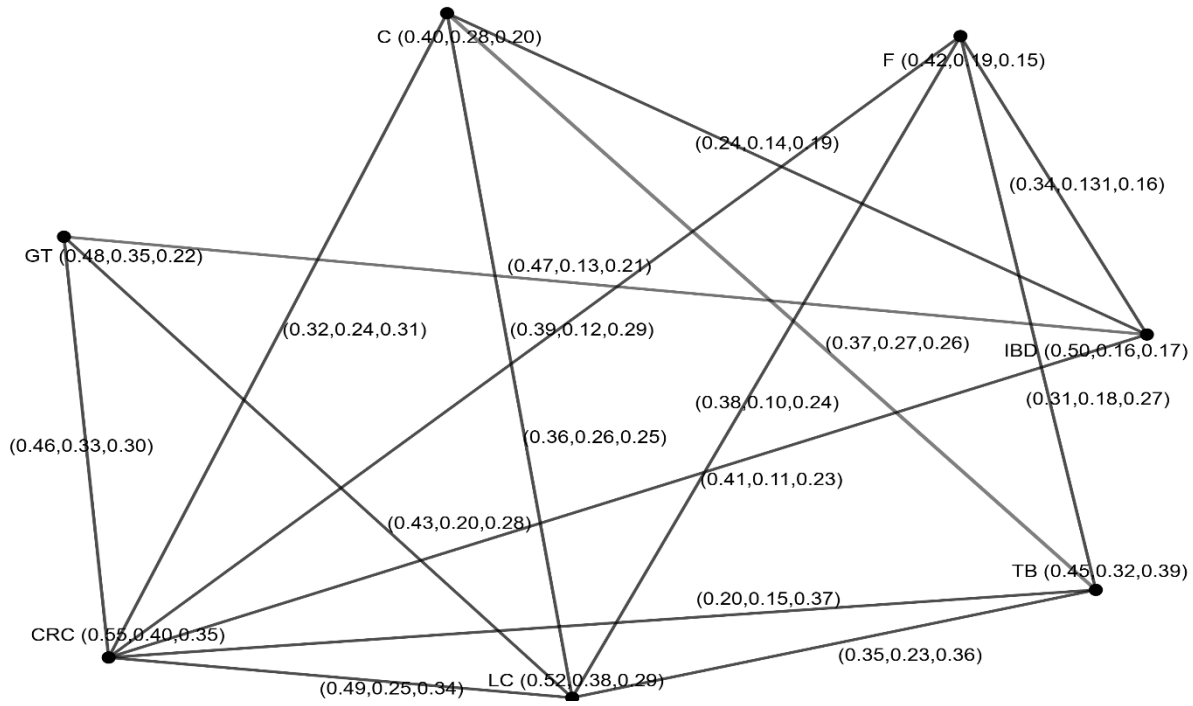

Figure 11

Graphical representation of the Symptoms and Disease relations for cancer diagnosis

This research aims to determine the clinically significant disease amongst four suspected ones, enabling the most prioritization of essential treatment interventions, thereby optimising treatment outcomes by enhancing patient's safety.

In pursuit of this goal, this work adopts the concept of "Bridges" for disease identification and effective treatment. As the given clinical data is distinct, Bijective neutrosophic Bridges shines here. Now, to achieve this objective, an algorithm to find the Bijective neutrosophic Bridges serves as a catalyst to connect diagnostic precision with timely, ensured treatment strategies.

Using an algorithm,

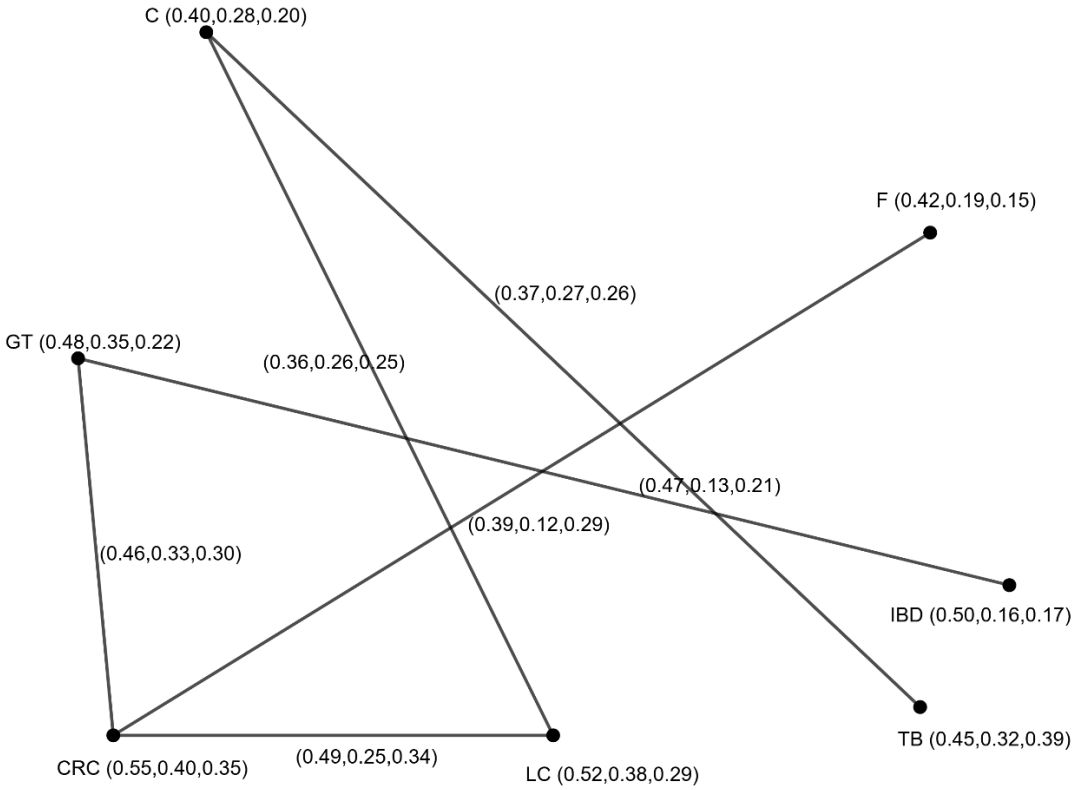

Figure:12 Graphical representation of T-bridges

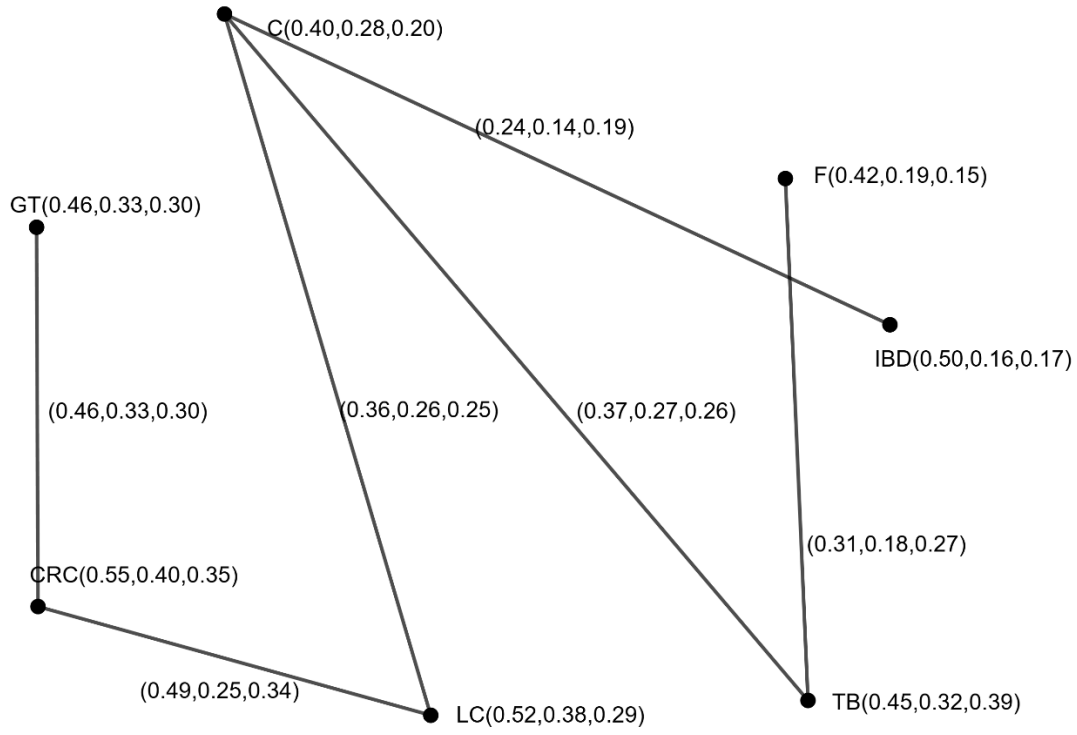

Figure: 13 Graphical representation of I-bridges

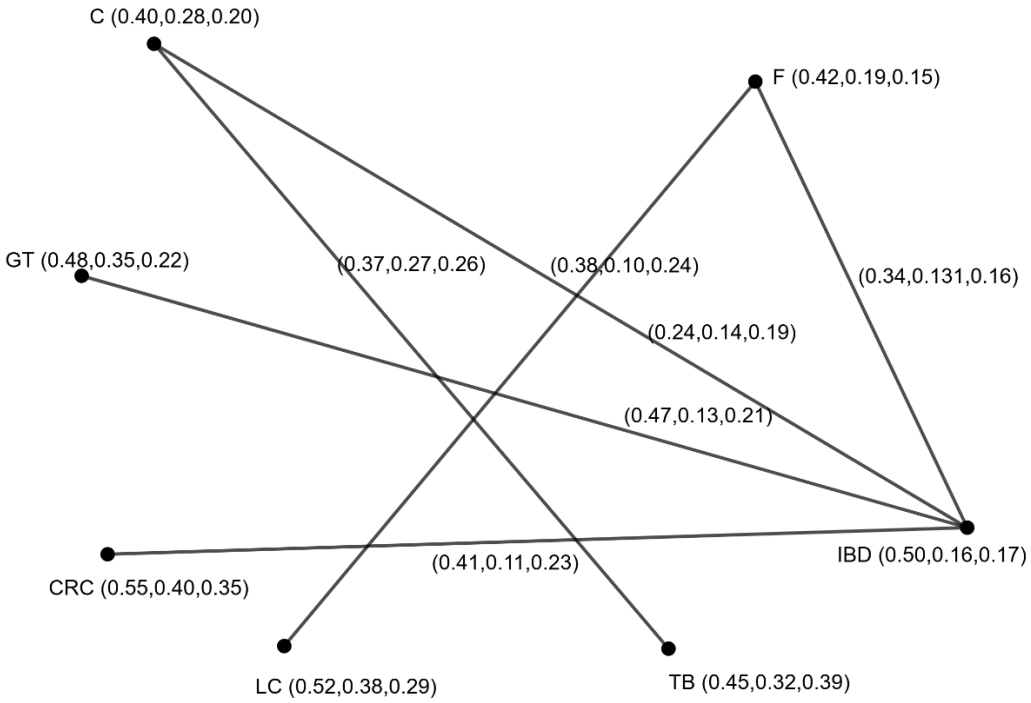

Figure:14 Graphical representation of F-bridges

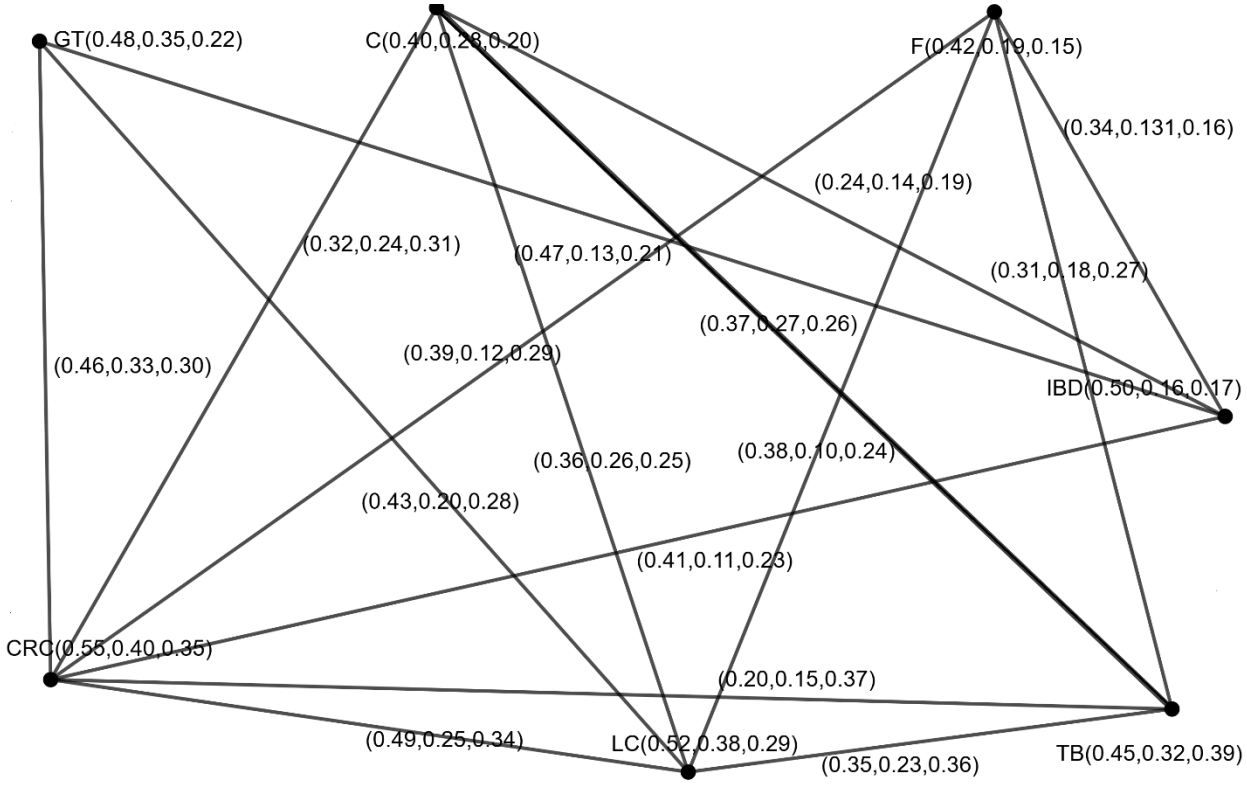

Figure:15 Neutrosophic bridge

From this, we can ensure that Chronic Cough(C) to Tuberculosis(TB) is the crucial relation in this representation.i.e) C-TB is the bridge(crucial relation) in this Bijective Neutrosophic

graphical Framework. It can be concluded that, Although, there is chance of 55% that the patient may be presented with Colorectal Cancer (CRC) and 50% likelihood of Inflammatory Bowl disease (IBD), Tuberculosis(TB) is considered as the primary underlying condition. In this context, the presence of Chronic cough (C) should be prioritized as the primary key symptoms guiding further effective clinical assessment.
Hence, the diagnosis for tuberculosis is the enhanced and safe treatment for this scenario.

5. Conclusion:

A new concept of neutrosophic graph called Bijective neutrosophic graph has been introduced in this paper. An algorithm has been proposed to effectively identify different bridges of any Bijective neutrosophic graph. Due to this unique property of one to one correspondence it finds its application in analysis of cancer detection where precision and structural insights are crucial. Furthermore, by employing a de-neutrosophication function—specifically the score function—the same algorithm can be extended to identify neutrosophic bridges. However, certain limitations remain. The proposed approach is computationally intensive for large-scale networks, and its effectiveness depends on the accuracy of assigned neutrosophic membership values. Future work may focus on optimizing the algorithm, validating it on larger datasets, and exploring broader applications.

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Conflicts of Interest
The authors declare no conflict of interest.

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