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Irregular Pythagorean Neutrosophic Graphs

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Abstract: Pythagorean Neutrosophic Fuzzy Graphs integrate the concepts of Pythagorean neutrosophic sets and graph theory to effectively represent imprecise, inconsistent, and vague information inherent in many real-world problems. This paper introduces and formalizes the concept of irregularity in Pythagorean Neutrosophic graphs by defining various classes such as neighbourly irregular, neighbourly totally irregular, highly irregular, and highly totally irregular Pythagorean Neutrosophic graphs. The study develops theoretical results related to these irregularities and provides suitable graphical representations for better understanding. The proposed irregular Pythagorean Neutrosophic graph models are demonstrated to be significant in areas like pattern recognition, biological networks, information systems, and other applications involving uncertainty and vagueness. This work extends the theoretical foundation and practical applicability of neutrosophic graph theory.

Keywords: Pythagorean Neutrosophic set, Irregular, Pythagorean Neutrosophic graphs, Neighbourhood

1. Introduction

Real life events with vagueness and insufficient information's can be very well modelled and solved with the help of one of the most attractive field of mathematics known as fuzzy set theory. Zadeh [1] pioneered the concept of Fuzzy sets whose membership grade assume values between 0 and 1. The study in [26] introduces the Wiener index and Wiener absolute index for bipolar fuzzy graphs to analyze connectivity, highlighting their properties and behavior across various structures like forests, bridges, and trees. A comparative analysis with connectivity index is presented, concluding with an application to regular travel routes between Paris and Brest. The concept of the complexity function in fuzzy graphs based on network load across structures like fuzzy cycles, trees, and complete graphs are developed in [27]. This article includes the application of analyzing COVID-19 transmission cycles among countries and identifying high-traffic nodes in wireless network systems. In [28], the authors explore the operations like Cartesian product, composition, and union, and demonstrates real-life applications in railway networks and medical science using the picture fuzzy min-tolerance competition graph. A comparative analysis with connectivity and Wiener indices is provided, along with an algorithm and real-world application in ranking key crossroads in Indonesia's tourism network between BKB and BKS are introduces in [29]. The applications demonstrated in modeling international relations during the Cold War and enhancing teacher-student communication for better decision-making using the Algorithms and flowcharts support methods are established in [30]. The application of the field can be encountered in statistics, computer science, mathematics, engineering, artificial intelligence, pattern recognition, image analysis and decision making [4 – 10, 21-25].

The extension of fuzzy sets by providing membership and non-membership grades was introduced by Atanassov in [2]. The fuzzy sets with the sum of membership and non-membership less than 1 is intuitionistic fuzzy set. Neutrosophic set investigated by Smarandache[3] in which the elements are characterised with truth, indeterminacy and false membership functions.Yager [11] developed the notion of Pythagorean sets which has relaxation in the condition as $\mu^2 + \sigma^2 \leq 1$. Likewise, picture and spherical sets have been developed from neutrosophic sets, and to these concepts, the Pythagorean Neutrosophic sets (PNS) were introduced.

Relationship between the objects are presented with vertices and edges together forming a graph. Not all real-life events can be give certain values resulting in uncertainties. This creates a need for new type of graphs to tackle this uncertainties known as fuzzy graphs. Based on Zadeh's [16] fuzzy relation, Kaufmann established the idea of fuzzy graphs in [17]. Rosenfeld in [18], have discussed and developed several basic graph theoretical concepts in fuzzy graph. Consequently, more theoretical concepts and operations in fuzzy graphs have been developed in [19,20].

Pythagorean Neutrosophic Graphs are developed by integrating the concept of PNSs and fuzzy graphs in [12]. The structural representation of the PNGs are similar to that of graphs with only disparity in summing up membership grades values being less than 2. Similar idea is followed for the edges. Main reason for studying Pythagorean neutrosophic graphs is due to their ability to increasing fuzziness of considered model or system and further theoretical concepts were developed in [13-15].

This article aims at introducing the concepts of neighbourly irregular Pythagorean neutrosophic graphs, neighbourly totally irregular Pythagorean neutrosophic graphs, highly irregular Pythagorean neutrosophic graphs, highly totally irregular Pythagorean neutrosophic graphs by providing suitable graphical representation. The article also focuses on providing some important results an irregularity.

1.1 Organization of the work

The organization of the work is structured to progressively introduce and develop the concept of Irregular Pythagorean Neutrosophic Graphs. It begins with an Introduction that outlines the background on fuzzy sets, neutrosophic sets, and Pythagorean neutrosophic theory, highlighting their significance in modeling uncertainty and motivating the need for new irregular graph structures. The Preliminaries section reviews fundamental concepts and related literature essential for understanding the proposed models. The core contribution is presented in the Development of Irregular Pythagorean Neutrosophic Graphs, where various irregular graph types-such as neighborly irregular, totally irregular, highly irregular, and highly totally irregular-are formally defined and illustrated. This is followed by Theoretical Results that establish key properties and theorems to support structural analysis. The Applications and Illustrations section demonstrates the practical utility of these graphs in handling real-world problems involving uncertain information, such as in pattern recognition, biological networks, and information systems. Finally, the Conclusion summarizes the findings, underscores the importance of the proposed models, and suggests directions for future research, with all supporting literature compiled in the References section.

2. Irregular Pythagorean Neutrosophic Fuzzy Graphs

This section discusses the fundamental principles and past study required to comprehend Pythagorean Neutrosophic Graphs. It follows the development of fuzzy sets to intuitionistic fuzzy, neutrosophic, and Pythagorean sets, highlighting their ability to describe uncertainty. To provide the groundwork for future research, the section also presents their definitions, characteristics, and integration with graph theory.

Definition 2.1. Let G be PNG. The neighbourhood (ngbd) of a vertex w in G is defined as
$N(w) = (N_\mu(w), N_\beta(w), N_\sigma(w))$ where,
$N_\mu(w) = \{o \in w: \mu_B(h_0) \leq \mu_A(w) \wedge \mu_A(o)\}$,
$N_\beta(w) = \{o \in w: \beta_B(h_0) \leq \beta_A(w) \wedge \beta_A(o)\}$,
$N_\sigma(w) = \{o \in w: \sigma_B(h_0) \leq \sigma_A(w) \wedge \sigma_A(o)\}$.

For instance, Consider a PNG G with vertex set $V=\{v_1,v_2,v_3,v_4\}$ and edge set $E=\{(v_1,v_2),(v_1,v_3),(v_2,v_4)\}$. The Pythagorean Neutrosophic membership values on vertices and edges specify degrees of truth, indeterminacy, and falsity, but for illustration of neighbourhood, these values are not immediately needed.

- The neighbourhood of vertex v_1 is: $N(v_1)=\{v_2,v_3\}$ since v_2 and v_3 are directly connected to v_1.
- Similarly, the neighbourhood of vertex v_2 is: $N(v_2)=\{v_1,v_4\}$
- The neighbourhood of vertex v_3 is: $N(v_3)=\{v_1\}$
- The neighbourhood of vertex v_4 is: $N(v_4)=\{v_2\}$

The above illustration demonstrates how neighbourhoods are defined within a PNG and forms a foundational concept for defining neighbourhood degrees and irregularities.

Definition 2.2. Let G be a PNG. The ngbd degree of a vertex w in G is defined by

$$deg_\mu(w) = \sum_{o \in N_\mu(x)} \mu_A(o),$$
$$deg_\beta(w) = \sum_{o \in N_\beta(x)} \beta_A(o),$$
$$deg_\sigma(w) = \sum_{o \in N_\sigma(x)} \sigma_A(o).$$

Definition 2.3. Let G be a PNG. The closed ngbd degree of a vertex w in G is defined as

$$deg_\mu[w] = \sum_{o \in N_\mu(x)} \mu_A(o) + \mu_A(w),$$
$$deg_\beta[w] = \sum_{o \in N_\beta(x)} \beta_A(o) + \beta_A(w),$$

$$deg_\sigma[w] = \sum_{o \in N_\sigma(x)} \sigma_A(o) + \sigma_A(w).$$

Definition 2.4. Let PNG G, is called regular if all the vertices has the same open neighbourhood degree.

Definition 2.5. Let G be a PNG. If there is a vertex which is adjacent to vertices with distinct neighbourhood degrees then G is called a irregular PNG.

Example 2.6. Let $G = (A, B)$ be the PNG with $A = \{x_1, x_2, x_3\}$, $B = \{(x_1, x_2), (x_2, x_3), (x_1, x_3)\}$. G is an irregular PNG. $\deg(x_1) = (.4, .7, .9), \deg(x_2) = (.4, .7, .1), \deg(x_3) = (.2, .6, .7)$. G is an irregular PNG.

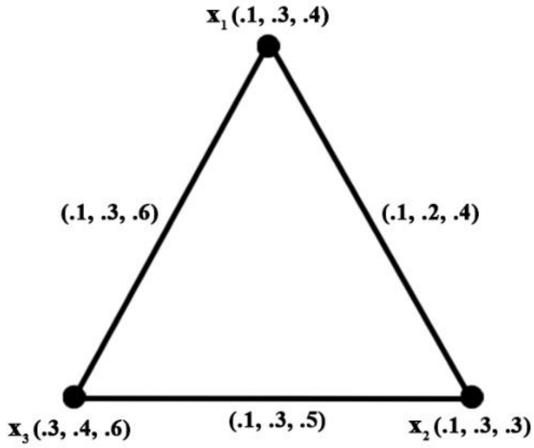

Table 1: Irregular PN Fuzzy Graph

Definition 2.7. If there is a vertex which is adjacent to vertices with distinct closed neighbourhood degrees, then PNG G is called a totally irregular PNG.

Example 2.8. Consider a PNG with $A = \{x_1, x_2, x_3, x_4, x_5\}, B = \{(x_1, x_2), (x_2, x_3), (x_3, x_4), (x_4, x_2),$
$(x_1, x_3), (x_4, x_5), (x_4, x_1)\}$.
Then, $\deg[x_1] = (1.4, .1.3, 2.7), \deg[x_2] = (1.4, 1.3, 2.7), \deg[x_3] = (1.4, 1.3, 2.7), \deg[x_4] = (2, 1.8, 3.1)$.
G is a totally irregular PNG.

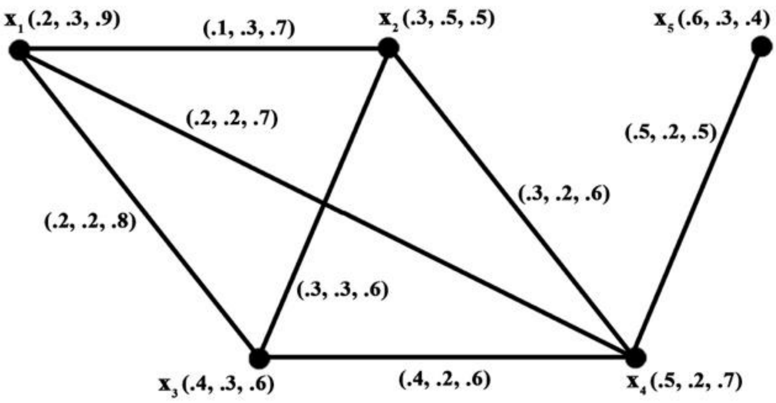

Table 2: Totally Irregular PN Fuzzy Graph

Definition 2.9. Let G be a connected PNG. If every two adjacent vertices of G have distinct open neighbourhood degrees, then G is called neighbourly irregular PNG.

Example 2.10. Let $G = (A, B)$ be a PNG with $A = \{x_1, x_2, x_3, x_4\}$, and $B = \{x_1, x_2, x_2, x_3, x_3, x_4, x_4, x_1\}$. The degrees of vertices are $\deg(x_1) = (.4, .9, 1.1), \deg(x_2) = (.4, .9, 1.1), \deg(x_3) = (.8, .8, .7), \deg(x_4) = (.4, .9, 1.1)$. G is a neighbourly irregular PNG.

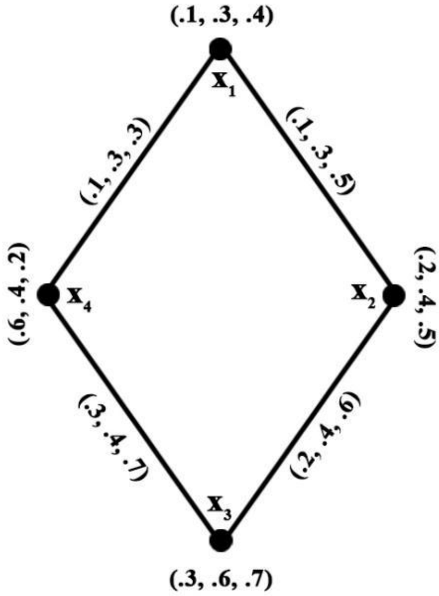

Table 3: Neighbourly Irregular PN Fuzzy Graph

Definition 2.11. A connected PNG is called neighbourly totally irregular PNG if every two adjacent vertices of G have distinct closed neighbourhood degree.

Example 2.12. Here the degree value for each vertex is, $\deg[x_1] = (1.1, .1.8, 1.7), \deg[x_2] = (1.2, 1.9, 1.9)$, $\deg[x_3] = (1.3, 2.1, 2.1), \deg[x_4] = (1.2, 2, 2)$.
Hence G is neighbourly totally irregular PNG.

Definition 2.13. A connected PNG, is called highly irregular PNG if every vertex of $\boldsymbol{G}$ is adjacent to vertices with distinct neighbourhood degrees

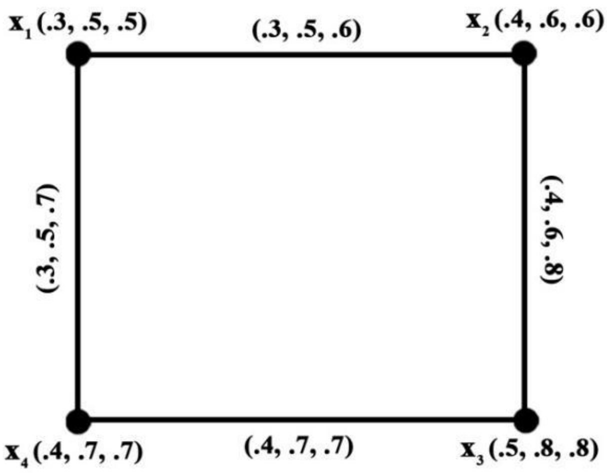

Table 4: Neighbourly Totally Irregular PN Fuzzy Graph

NOTE:

(i) A highly irregular PNG may not be neighbourly irregular PNG.
(ii) A neighbourly irregular PNG may not be highly irregular PNG.
(iii) A neighbourly irregular PNG may not be neighbourly totally irregular PNG.
(iv) A neighbourly totally irregular PNG may not be neighbourly irregular PNG.

Proposition 2.14. Let G be a PNG. Then G is highly irregular PNG and neighbourly irregular PNG iff the neighbourhood degrees of all the vertices of G are distinct.

Proof: Let $\mathcal{B}$ be a PNG with n-vertices $w_1, w_2, \dots, w_n$. Suppose G is both highly and neighbourly irregular PNG. We want to show the neighbourhood degrees of all vertices of G are distinct. Let $\deg(w_i) = (p_i, q_i, r_i), i = 1,2, \dots, n$.

Let the adjacent vertices of w_1 are $w_2, w_3, \dots, w_n$ with ngbd degrees $(p_2, q_2, r_2), (p_3, q_3, r_3), \dots, (p_n, q_n, r_n)$ respectively. Since G is highly irregular so $p_2 \neq p_3, \dots \neq p_n, q_2 \neq q_3, \dots \neq q_n, r_2 \neq r_3, \dots \neq r_n$. Also, $p_1 \neq p_2, \dots \neq p_n, q_1 \neq q_2, \dots \neq q_n, r_1 \neq r_2, \dots \neq r_n$, because G is neighbourly irregular.

So, $(p_1, q_1, r_1) \neq (p_2, q_2, r_2) \neq (p_3, q_3, r_3), \dots, (p_n, q_n, r_n)$. Hence the neighbourhood degrees of all vertices of G are distinct.
Conversely, suppose that the neighbourhood degrees of all the vertices are distinct.
Now we want to show that is highly irregular and neighbourly irregular PNG.

Let $\deg(w_i) = (p_i, q_i, r_i), i = 1,2, \dots, n$ given $p_1 \neq p_2, \dots \neq p_n, q_1 \neq q_2, \dots \neq q_n, r_1 \neq r_2, \dots \neq r_n$
Every two adjacent vertices have distinct neighbourhood degrees and to every vertex, the adjacent vertices have distinct neighbourhood degrees, which completes the proof.

Proposition 2.15. Let G be a PNG. If G is neighbourly irregular PNG and $(\mu_A, \beta_A, \sigma_A)$ is a constant function, then G is a neighbourly totally irregular PNG.

Proof: Let $\mathcal{B}$ is a neighbourly irregular PNG. Let $w_i, w_j \in A$, where w_i and w_j are adjacent vertices with distinct and neighbourhood degrees (p_1, q_1, r_1) and (p_2, q_2, r_2) respectively. Let us assume that

$(\mu_A(w_i), \beta_A(w_i), \sigma_A(w_i)) = (\mu_A(w_j), \beta_A(w_j), \sigma_A(w_j)) = (k_1, k_2, k_3)$, where k_1, k_2, k_3 are constants and $k_1, k_2, k_3 \in [0,1]$. Therefore

$deg_\mu[w_i] = deg_\mu(w_i) + \mu_A(w_i) = p_1 + k_1,$
$deg_\beta[w_i] = deg_\beta(w_i) + \beta_A(w_i) = q_1 + k_2,$
$deg_\sigma[w_i] = deg_\sigma(w_i) + \sigma_A(w_i) = r_1 + k_3,$
$deg_\mu[w_j] = deg_\mu(w_j) + \mu_A(w_j) = p_2 + k_1,$
$deg_\beta[w_j] = deg_\beta(w_j) + \beta_A(w_j) = q_2 + k_2,$
$deg_\sigma[w_j] = deg_\sigma(w_j) + \sigma_A(w_j) = r_2 + k_3,$
$deg_\mu[w_i] = deg_\mu[w_j]$
$\Rightarrow p_1 + k_1 = p_2 + k_1 \Rightarrow p_1 - p_2 = 0 \Rightarrow p_1 = p_2$
which is a contradiction $(p_1 \neq p_2)$.
$deg_\beta[w_i] = deg_\beta[w_j]$
$\Rightarrow q_2 + k_2 = q_1 + k_2 \Rightarrow q_1 - q_2 = 0 \Rightarrow q_1 = q_2$
$\Rightarrow$ since $(q_1 \neq q_2)$.
$deg_\sigma[w_i] = deg_\sigma[w_j]$
$\Rightarrow r_1 + k_3 = r_2 + k_3 \Rightarrow r_1 - r_2 = 0 \Rightarrow r_1 = r_2$
Which is a contradiction $(r_1 \neq r_2)$.

Therefore G is neighbourly totally irregular PNG.

Proposition 2.16. Let G be a PNG. If G is neighbourly totally irregular PNG and $(\mu_A, \beta_A, \sigma_A)$ is a constant function, then G is a neighbourly irregular PNG.

Proof: Suppose G is a neighbourly totally irregular PNG. Then by definition, the closed neighbourhood degree of every two adjacent are distinct.

Let $w_i, w_j \in A$, where w_i and w_j are adjacent vertices with distinct degrees (p_1, q_1, r_1) and (p_2, q_2, r_2) respectively.

Let us assume that $(\mu_A(w_i), \beta_A(w_i), \sigma_A(w_i)) = (\mu_A(w_j), \beta_A(w_j), \sigma_A(w_j)) = (k_1, k_2, k_3)$,where k_1, k_2, k_3 are constants and $k_1, k_2, k_3 \in [0,1]$ and $deg[w_i] \neq deg[w_j]$, so $deg_\mu[w_i] \neq deg_\mu[w_j]$, $deg_\beta[w_i] \neq deg_\beta[w_j], deg_\sigma[w_i] \neq deg_\sigma[w_j]$.

Now $deg_\mu[w_i] \neq deg_\mu[w_j] \Rightarrow p_1 + k_1 = p_2 + k_1 \Rightarrow p_1 \neq p_2$

Similarly, $deg_\beta[w_i] = deg_\beta[w_j] \Rightarrow q_1 - q_2 \neq k_1 - k_2 = 0 \Rightarrow q_1 \neq q_2$

Similarly, $deg_\sigma[w_i] = deg_\sigma[w_j]$

$\Rightarrow r_1 + k_3 \neq r_2 + k_3 \Rightarrow r_1 \neq r_2$

Hence the degrees of $w_i, w_j \in A$ are distinct. This is true for every pair of adjacent vertices in $\mathcal{B}$. Therefore, G is neighbourly irregular PNG.

3. Applications

Graph theory plays an important role in the application part of the networks that are used in real-life scenarios. The internet-related issues, road-transportation networks are few examples of technological networks. Various real-life networks, are vague and not well-defined in nature. The Pythagorean Neutrosophic graph is more compatible to model the ambiguous information in a network when compared to the fuzzy or vague graph. A graph is a way of modelling real-life networks, which has relationships between objects.

If there is uncertainties because of vague information about relations, or vertices then the Pythagorean neutrosophic graph model is very efficient in designing such networks. Pythagorean Neutrosophic graphs are important in the area of mathematical modelling, pattern recognition, biological networks and information systems. Pythagorean neutrosophic graphs and applied in information technology and computer science such that these graphs are used to illustrate data optimization, chip design and much more.

Moreover, the recent studies highlight the extended use of neutrosophic and fuzzy graphs in environmental and climatic analyses. For instance, interval intuitionistic neutrosophic sets and graphs have been employed to model climatic data with inherent uncertainties, enabling more accurate environmental assessments and forecasting. These models facilitate better understanding of complex ecological interactions where information is often incomplete or imprecise. Additionally, the flexibility of neutrosophic graphs in representing varying degrees of truth, indeterminacy, and falsehood makes them particularly suitable for applications in climate modeling, resource management, and ecological decision support systems, where uncertainty is a fundamental challenge. This demonstrates the broadening scope of neutrosophic graph theory, extending its applicability beyond traditional network and decision contexts to environmental science and sustainability studies.

Recent research highlights diverse applications of graph theory and fuzzy graphs across various domains:

- **Robotic Production Systems:** [21] Graph theory has been applied to design communication systems within robotic production cells, enhancing flexibility and adaptability in manufacturing processes.
- **Smart City Development:** [22] Fuzzy graph structures have been utilized to analyze the growth of smart cities in India, providing insights into urban infrastructure development and planning.
- **Traffic Management:** [23] Edge colouring of fuzzy graphs has been employed to model and improve traffic light systems, accounting for varying traffic conditions and congestion levels.
- **Manufacturing Industries:** [24] Extensions of fuzzy competition graphs have been used to model relationships and competition within manufacturing sectors, aiding in strategic decision-making.
- **Intelligent Transportation Systems:** [25] Graph-based machine learning approaches have been developed for smart urban transportation systems, facilitating efficient traffic management and route optimization.

4. Conclusion

Herein, we have defined the new concept of irregular Pythagorean Neutrosophic graphs. Also, we defined the concepts of neighbourly irregular Pythagorean neutrosophic graphs, neighbourly totally irregular Pythagorean neutrosophic graphs, highly irregular Pythagorean neutrosophic graphs, highly totally irregular Pythagorean neutrosophic graphs by providing suitable graphical representation.

The current study has certain limitations, as it primarily focuses on irregularity concepts within static Pythagorean Neutrosophic Graphs (PNGs), without addressing dynamic or evolving networks where vertex and edge membership values may vary over time. Theoretical results are derived from idealized graph structures, which may require modifications for application to large-scale or noisy real-world datasets. Additionally, while the proposed applications are conceptually outlined, empirical validations using real-world data remain limited, and computational complexity aspects related to detecting or constructing irregular PNGs are not extensively examined. Future work aims to extend irregularity concepts to dynamic or temporal PNGs to model time-varying uncertainties, and to develop efficient algorithms for detecting and measuring irregularities in large-scale graphs. Further directions include applying irregular PNG models to practical domains such as social network analysis, bioinformatics, and decision-making under uncertainty, supported by experimental validations. Moreover, exploring the relationship between irregularities in PNGs and other neutrosophic graph structures like picture fuzzy and interval-valued neutrosophic graphs, as well as investigating optimization techniques and machine learning frameworks that incorporate irregular PNGs, could enhance uncertainty management in data-driven applications.

Authors' contributions

All authors contributed equally in writing this article. All authors read and approved the final manuscript.

Conflicts of interest

The authors declare that they have no conflicts of interest.

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