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Neutrosophic Probability and Measure for Resilient Supply Chain Management under Indeterminacy

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Abstract
Supply chains face two kinds of uncertainty: random variation and indeterminacy caused by missing, delayed, or conflicting information. This paper models both by using a neutrosophic representation that attaches to every key quantity a triplet for support, indeterminacy, and refutation. Building on this, we define neutrosophic measure and probability for supply-chain events, and we embed them in linear and mixed-integer programs that remain feasible under worst-case demand. The framework provides clear operators for aggregating evidence, converting triplets to actionable intervals, and ranking alternatives with a risk-averse score. We evaluate the approach in three studies. First, supplier selection with an explicit reliability share shifts volume toward trustworthy sources while meeting capacity and reducing total cost. Second, demand forecasting translates ambiguous signals into effective intervals that, when added to a simple baseline, lower mean absolute percentage error and maintain full coverage of realized demand. Third, transportation planning uses a tunable share on a high-availability path to trade cost for improved plan availability. Across all cases, enforcing robust balance removes indeterminacy from service, while neutrosophic metrics quantify where uncertainty remains and what it costs. The method is transparent, easy to audit, and compatible with standard optimization tools, making it practical for resilient planning.

Keywords: Neutrosophic measure; Neutrosophic probability; Indeterminacy; Supply chain management; Supplier selection; Demand forecasting; Transportation risk; Robust decision-making.

1. Introduction

Supply chains form the backbone of modern economies, connecting producers, distributors, and consumers across global networks. Yet, they increasingly face a barrage of uncertainties that go beyond simple randomness, such as fluctuating demand or variable lead times. Disruptions like pandemics, geopolitical conflicts, and climate events introduce indeterminacy situations where information is incomplete, conflicting, or simply unavailable at decision time. For instance, a supplier's reliability might appear strong based on one report but dubious in another due to missing data or contradictory audits [1][2]. Traditional models, which rely on probabilistic assumptions or worst-case bounds, often force planners to make arbitrary choices, resulting in plans that are either overly conservative, locking up resources in excess inventory, or insufficiently robust,

leading to costly shortages and delays [3][4]. This gap between modeled certainty and real-world ambiguity undermines supply chain resilience, defined as the capacity to anticipate, withstand, and recover from shocks while sustaining performance [5][6].

Recent studies underscore the scale of this challenge. In 2024, analyses revealed that up to 70% of supply chain disruptions stem from unmodeled information gaps rather than purely stochastic events, amplifying vulnerabilities in sourcing, logistics, and inventory management [7][8]. Classical approaches like stochastic programming and robust optimization perform well when data aligns with clear probability distributions or definable uncertainty sets [9][10]. However, they struggle with indeterminacy, where evidence resists reduction to a single value—think of conflicting market signals for demand forecasting or partial reports on route disruptions [11][12]. Planners thus resort to inflating uncertainty bounds or ignoring subtle but valuable indeterminate cues, which erodes decision quality and trust in the models [13][14].

1.1 Literature Review

The evolution of supply chain uncertainty modeling has progressed from deterministic frameworks to sophisticated stochastic and robust paradigms, reflecting the growing complexity of global operations. Early stochastic methods, such as scenario-based planning and Monte Carlo simulations, effectively captured demand variability and supply risks, enabling better hedging against probabilistic threats [15][16]. By 2023, resilience-focused research shifted toward integrated frameworks that blend network design, multi-tier sourcing, and adaptive inventory to counter shocks like those during the COVID-19 pandemic [17][18]. For example, vulnerability assessments across production and logistics stages have advocated for diversified networks that balance efficiency with flexibility, reducing recovery times by up to 40% in simulated disruptions [19][20].

Despite these advances, handling indeterminacy remains a persistent shortfall. Fuzzy set theory and interval arithmetic have emerged to address vagueness in parameters like quality scores or costs, providing ranges for imprecise inputs without assuming full consensus [21][22]. However, these tools still require agreed-upon boundaries, which prove elusive amid conflicting data such as one audit lauding a vendor's performance while another highlights unverified delays [23][24]. Neutrosophic logic addresses this by extending fuzzy sets to explicitly model truth (support), indeterminacy, and falsehood (refutation) via triplet structures, enabling paraconsistent reasoning that tolerates contradictions without premature resolution [25][26].

Applications of neutrosophic logic in supply chains have gained traction since 2023, particularly in multi-criteria decision-making under ambiguity. In supplier selection, single-valued neutrosophic sets integrated with VIKOR methods evaluate sustainable vendors by weighing vague environmental and cost criteria, yielding 25% higher robustness than traditional analytic hierarchy processes [27][28]. Demand forecasting has seen hybrid neutrosophic time series models that fuse historical data with external signals, cutting mean absolute errors by 15% in volatile sectors compared to ARIMA baselines

[29][30]. For transportation risks, interval-valued neutrosophic fuzzy EDAS prioritizes sustainability threats in automotive chains, incorporating partial sensor data to refine mitigation [31][32].

Transportation and broader risk management further illustrate neutrosophic potential. Neutrosophic extensions to management information systems model credit and investment risks in supply disruptions, using triplets to highlight unresolved gaps for clearer decision trees [33][34]. Green supply chain evaluations employ neutrosophic twofold algebra to assess eco-capabilities, revealing how indeterminacy in metrics influences overall resilience [35][36]. Risk assessment in halal supply chains applies neutrosophic theory to causal factors, illustrating methodologies with case studies that enhance transparency in ambiguous environments [37][38].

Yet, the literature reveals key gaps. Most neutrosophic applications remain isolated to ranking or scoring, lacking a cohesive measure-probability framework that propagates indeterminacy through full optimization pipelines [39][40]. Inventory and routing dynamics often default to arbitrary assumptions, inflating conservatism without quantifying data quality's impact [41][42]. No existing work derives tailored neutrosophic measure spaces for supply chain events, with operators that maintain indeterminacy in mixed-integer programs while nesting classical tools [43][44].

1.2 Our Approach and Contributions

To fill these voids, this paper proposes a comprehensive neutrosophic framework for resilient supply chain management, elevating indeterminacy to a fundamental component rather than a byproduct to average out. Core quantities demand, costs, and reliabilities adopt triplets (T, I, F), where T denotes supporting evidence, I unresolved ambiguity, and F contradictions. Neutrosophic measure spaces model events like timely deliveries, with normalized probabilities preserving I for downstream use. Aggregation, ranking, and optimization operators ensure tractable formulations, collapsing to probabilistic or fuzzy forms when I vanish [45].

This work delivers five pivotal contributions. First, it establishes neutrosophic measure and probability axioms for supply chain variables, complete with symbols and bounds for practical implementation [1]. Second, it introduces idempotent meet/join operators and risk-averse projections for triplet handling in constraints and objectives, facilitating direct MILP integration [2]. Third, the neutrosophic-robust model guarantees feasibility over plausibility intervals, with optional penalties for lingering ambiguity, producing traceable KPIs like service levels [3]. Fourth, three case studies: supplier selection with mixed evidence, demand forecasting via ambiguous signals, and routing under partial disruptions feature detailed computations, tables, and baseline comparisons, demonstrating 10-30% uplifts in robustness and accuracy [4][5]. Fifth, a comparative analysis against stochastic and interval methods quantifies cost-coverage trade-offs, spotlighting data levers for resilience gains [6].

1.3 Why Neutrosophic for Supply Chains?

Neutrosophic logic aligns seamlessly with supply chain realities, where evidence arrives piecemeal and rarely unanimous. A triplet encapsulates nuances like "high on-time support (T) but audit conflicts on defects (F) and missing records (I)," avoiding the pitfalls of averaged probabilities or blindly widened intervals [7]. This visibility builds planner confidence, as shrinking I through improved sensors directly tightens KPIs [8]. Moreover, the framework's generality embeds classical methods: with I=0 and F=1-T, it mirrors fuzzy or probabilistic paradigms, averting overhauls in data-rich scenarios [9]. In a landscape of noisy big data, this duality fosters antifragile chains that thrive on ambiguity [10].

1.4 Paper Organization

Section 2 details preliminaries: notation, triplets, measures, and projections. Section 3 constructs the model with balances, constraints, and the neutrosophic-robust MILP. Sections 4-6 explore case studies, each with annotated data, step-by-step results, and insights. Section 7 evaluates outcomes and sensitivities. Section 8 concludes with extensions, including multi-period lead times and triplet learning from data.

2. Preliminaries

2.1 Notation

$\mathbb{R}$ is the set of real numbers; $\mathbb{R}_{\geq 0} = \{x \in \mathbb{R}: x \geq 0\}$.

For a finite set S, $|S|$ is its cardinality.

For a vector $v \in \mathbb{R}^n$, $\|v\|_1 = \sum_{i=1}^{n} |v_i|$, $\|v\|_\infty = \max_i |v_i|$.

Bold lowercase (e.g., x) denotes vectors; uppercase (e.g., A) denotes matrices.

For a measurable space (Ω, Σ), $A^c = \Omega \setminus A$ is the complement of $A \in \Sigma$.

Intervals are written $[\underline{x}, \bar{x}] = \{x \in \mathbb{R}: \underline{x} \leq x \leq \bar{x}\}$.

All symbols introduced below are fully defined before use.

2.2 Neutrosophic triplets and their order

A neutrosophic triplet is a vector $u = (t, i, f) \in [0,1]^3$ whose components represent:
t : degree of support (truth),

i : degree of indeterminacy,

f : degree of refutation (falsehood).

We deliberately allow $t + i + f$ to be any value in [0,3]; this accommodates incomplete information ($t + i + f < 1$) and paraconsistent information ($t + i + f > 1$).

We use the following componentwise dominance on triplets:

$$u_1 = (t_1, i_1, f_1) \succeq u_2 = (t_2, i_2, f_2) \Leftrightarrow t_1 \geq t_2, i_1 \leq i_2, f_1 \leq f_2. \qquad \text{(Eq. 2.1)}$$

Intuitively, more support, and less indeterminacy/ refutation is better. We fix idempotent, commutative, associative meet/ join operators on triplets:

$$u_1 \sqcap u_2 := \big(\min(t_1, t_2), \max(i_1, i_2), \max(f_1, f_2)\big),$$
$$u_1 \sqcup u_2 := \big(\max(t_1, t_2), \max(i_1, i_2), \min(f_1, f_2)\big), \quad \text{(Eq. 2.2)}$$
$$\neg u := (f, i, t).$$

These choices are fixed for the remainder of the paper and will be used when aggregating evidence.

2.3 Neutrosophic sets

Given a universe U, a neutrosophic set A on U is specified by a membership map

$$\mu_A : U \to [0,1]^3, \mu_A(x) = (T_A(x), I_A(x), F_A(x)). \quad \text{(Eq. 2.3)}$$

For $x \in U, T_A(x)$ is support that " $x \in A$" , $F_A(x)$ is refutation, and $I_A(x)$ is indeterminacy. Set operations are defined pointwise using $\sqcap, \sqcup, \neg$:

$$\mu_{A\cap B}(x) = \mu_A(x) \sqcap \mu_B(x), \mu_{A\cup B}(x) = \mu_A(x) \sqcup \mu_B(x), \mu_{A^c}(x) = \neg\mu_A(x). \quad \text{(Eq. 2.4)}$$

2.4 Neutrosophic σ-algebras and measures

Let (Ω, Σ) be a measurable space (i.e., Σ is a σ-algebra on Ω). A neutrosophic measure on (Ω, Σ) is a triple of σ-additive set functions.

$$\nu = (\nu_T, \nu_I, \nu_F) : \Sigma \to [0, \infty]^3$$

satisfying, for any countable collection of pairwise disjoint sets $\{A_k\}_{k\geq 1} \subset \Sigma$:

$$\nu.(\bigcup_{k=1}^{\infty} A_k) = \sum_{k=1}^{\infty} \nu.(A_k), \cdot \in \{T, I, F\} \quad \text{(Eq. 2.5)}$$

and the null/total-mass conditions

$$\nu(\emptyset) = (0,0,0), \nu(\Omega) = (M_T, M_I, M_F) \in [0, \infty)^3.$$

We call ν finite if $M_T, M_I, M_F < \infty$. No other coupling between components is assumed; thus ν is a vector measure.

2.5 Neutrosophic probability

A neutrosophic probability (NP) on (Ω, Σ) is a neutrosophic measure $\mathrm{NP} = (\mathrm{NP}_T, \mathrm{NP}_I, \mathrm{NP}_F)$ with unit total mass:

$$\mathrm{NP}(\Omega) = (1,1,1)$$

For an event $A \in \Sigma$, we write

$$\mathrm{NP}(A) = (t_A, i_A, f_A) \in [0,1]^3$$

t_A is the amount of probabilistic support that A occurs, f_A is the amount of support that A does not occur, and i_A is the amount of indeterminacy remaining about A. Because the three components are recorded separately, $t_A + i_A + f_A$ need not equal 1 . Two immediate consequences of σ-additivity (with $\Omega = A \cup A^c, A \cap A^c = \emptyset$) are:

$$\mathrm{NP}.(A) + \mathrm{NP}.(A^c) = 1, \cdot \in \{T, I, F\}.$$

Hence $t_{A^c} = 1 - t_A, i_{A^c} = 1 - i_A, f_{A^c} = 1 - f_A$.
For events $A, B \in \Sigma$, we define conservative bounds using the meet/join on triplets:

$$\mathrm{NP}(A \cap B) \succeq \mathrm{NP}(A) \sqcap \mathrm{NP}(B), \mathrm{NP}(A \cup B) \succeq \mathrm{NP}(A) \sqcup \mathrm{NP}(B)$$

When independence is asserted (and only then), we will compute componentwise products/sums as:

$$\mathrm{NP}_T(A \cap B) = \mathrm{NP}_T(A) \cdot \mathrm{NP}_T(B)$$
$$\mathrm{NP}_F(A \cup B) = \mathrm{NP}_F(A) \cdot \mathrm{NP}_F(B)$$
$$\mathrm{NP}_I(A \cap B) = \min\{1, \mathrm{NP}_I(A) + \mathrm{NP}_I(B)\}$$

with dual formulas for $\cup$ and A^c. We always state any independence assumption explicitly before using these formulas.

2.6 Projections and scalarizations for decision-making
Optimization requires scalar scores and feasible intervals. From any triplet $u = (t, i, f)$ we define:

1. Plausibility interval for the underlying probability of occurrence:

$$\underline{p}(u) = t, \bar{p}(u) = 1 - f,$$

so $p \in [\underline{p}(u), \bar{p}(u)]$. The interval collapses to a point iff $i = 0$ and $t + f = 1$.

2. Center projection (mid-interval):

$$p^c(u) = t + \frac{i}{2} = \frac{\underline{p}(u) + \bar{p}(u)}{2}.$$

3. Risk-averse scalarization with parameter $\lambda \in [0,1]$ (fixed at $\lambda = 0.5$ in the case studies unless stated otherwise):

$$s_\lambda(u) = t - \lambda i - f.$$

Here $s_\lambda(u) \in [-1,1]$ ranks alternatives; larger is better.

4. Evidence fusion for m independent sources with weights $w_j \geq 0, \sum_{j=1}^m w_j = 1$:

$$\bigoplus_{j=1}^m w_j u^{(j)} := \left(\sum_j w_j t^{(j)}, \sum_j w_j i^{(j)}, \sum_j w_j f^{(j)} \right),$$

followed by clipping to $[0,1]^3$ if needed. When sources are not independent, we will use Π or $\sqcup$ as conservative aggregators.

2.7 Neutrosophic intervals for numerical parameters
A neutrosophic interval parameter is a pair

$$X = ([\underline{x}, \bar{x}], (t, i, f)),$$

meaning the scalar value lies in $[\underline{x}, \bar{x}]$ while the credibility of that bound is described by (t, i, f). For sums and products of such parameters, we use classical interval arithmetic on the interval part and $\Pi/\sqcup$ on the triplet part, declared explicitly each time.

Examples

Example 2.1: Event probability triplet and projections

A shipment is declared on time by a carrier (support), a sensor report is missing (indeterminacy), and a customer ticket alleges delay (refutation). Suppose the fused triplet is

$$u = (t, i, f) = (0.62, 0.21, 0.17)$$

Then the plausibility interval for the (classical) on-time probability is $[\underline{p}, \bar{p}] = [0.62, 1 - 0.17] = [0.62, 0.83]$. The center projection is $p^{c} = 0.62 + 0.5 \cdot 0.21 = 0.725$. With $\lambda = 0.5$, the risk-averse score is $s_{0.5} = 0.62 - 0.5 \cdot 0.21 - 0.17 = 0.345$.

Example 2.2: Neutrosophic measure on delivery time buckets

Let Ω be all shipments in a month; Σ is the σ-algebra generated by three disjoint buckets. $A_1 = \{0 - 2 \text{ days}\}, A_2 = \{3 - 5 \text{ days}\}, A_3 = \{> 5 \text{ days}\}$. Define a finite neutrosophic measure ν by:

$$\nu(A_1) = (430, 18, 7), \nu(A_2) = (150, 25, 9), \nu(A_3) = (40, 11, 12),$$

where the first component counts supported assignments, the second counts indeterminate assignments (e.g., incomplete scans), and the third counts refuted assignments, e.g., later corrections. Then:

$$\nu(\Omega) = \sum_{k=1}^{3} \nu(A_k) = (620, 54, 28).$$

Normalizing to a neutrosophic probability over Ω gives:

$$\mathrm{NP}(A_k) = \left(\frac{\nu_T(A_k)}{620}, \frac{\nu_I(A_k)}{54}, \frac{\nu_F(A_k)}{28} \right),$$

i.e.,

$$\mathrm{NP}(A_1) \approx (0.6935, 0.3333, 0.25)$$

$$\mathrm{NP}(A_2) \approx (0.2419, 0.4630, 0.3214),$$
$$\mathrm{NP}(A_3) \approx (0.0645, 0.2037, 0.4286).$$

These triplets, through $[\underline{p}, \bar{p}]$ and s_λ, will feed the optimization and ranking procedures in Sections 3-6.

2.8 Fixed assumptions for the remainder of the paper

a) All neutrosophic sets and probability operations use the Π,⊔, ¬ operators defined in Section 2.2 unless stated otherwise.

b) Scalar decisions use p^{c} and s_{λ} with $\lambda = 0.5$ unless a different λ is explicitly specified.

c) Independence is never assumed silently; when required, it is stated, and only then are componentwise products used.

3. Neutrosophic supply-chain modeling and decision operators

3.1 Entities, indices, variables, and uncertain inputs

Indices and sets

$p \in \mathcal{P}$ (products), $s \in \mathcal{S}$ (suppliers), $l \in \mathcal{L}$ (transport lanes), $t = 1, \dots, T$ (periods).
Binary adjacency $A_{sl} \in \{0,1\}$ indicates whether supplier s can ship on lane l.

Decision variables (all for every applicable index unless stated otherwise)

$x_{lpt} \geq 0$: quantity of product p arriving via lane l in period t.
$I_{pt} \geq 0$: end-of-period on-hand inventory.
$B_{pt} \geq 0$: end-of-period backorders.
$y_{\text{spt}} \in \{0,1\}$: whether an order is placed to supplier s for p in t (fixed cost activation).

Uncertain inputs as neutrosophic intervals (Def. 2.7): for each scalar parameter X,

$$X = ([\underline{x}, \bar{x}], u), u = (t_X, i_X, f_X) \in [0,1]^3.$$

We use the following per-index parameters:

Demand $D_{pt} = \left([\underline{D}_{pt}, \bar{D}_{pt}], u^{D}_{pt}\right)$.
Per-unit procurement cost from s for p: $c_{spt} = \left([\underline{c}_{spt}, \bar{c}_{spt}], u^{c}_{spt}\right)$.
Per-unit transport cost on lane l : $k_{lpt} = \left([\underline{k}_{lpt}, \bar{k}_{lpt}], u^{k}_{lpt}\right)$.
Per-unit holding cost: $h_{pt} = \left([\underline{h}_{pt}, \bar{h}_{pt}], u^{h}_{pt}\right)$.
Per-unit shortage penalty: $p_{pt} = \left([\underline{p}_{pt}, \bar{p}_{pt}], u^{p}_{pt}\right)$.
Lane capacity $C_{lpt} = \left([\underline{C}_{lpt}, \bar{C}_{lpt}], u^{C}_{lpt}\right)$.
Supplier capacity $U_{spt} = \left([\underline{U}_{spt}, \bar{U}_{spt}], u^{U}_{spt}\right)$.
Fixed order cost $F_{spt} = \left([\underline{F}_{spt}, \bar{F}_{spt}], u^{F}_{spt}\right)$.

All triplets use the conventions and operations from Section 2 (meet/join Π,⊔, projections $p^{c}, [\underline{p}, \bar{p}]$). Table 1 lists the main symbols. As explained in the caption, we group them by role to make later formulas easy to parse.

Table 1. Core notation used throughout Section 3. The table groups indices, decisions,

Group	Symbol	Meaning
Indices	p, s, l, t	product, supplier, lane, period
Decisions	x_{lpt}	arrivals on lane l
	I_{pt}, B_{pt}	end-of-period inventory, backorder
	y_{spt}	order activation
Inputs (neutrosophic intervals)	D_{pt}	demand
	c_{spt}, k_{lpt}	procurement, transport cost
	h_{pt}, p_{pt}	holding, shortage penalty
	C_{lpt}, U_{spt}	lane, supplier capacities
	F_{spt}	fixed order cost

3.2 Inventory balance and robust satisfaction

We model receipts x_{lpt} as arrivals (lead times already resolved into the arrival schedule). The end-of-period material balance is:

$$I_{p,t} - B_{p,t} = I_{p,t-1} - B_{p,t-1} + \sum_{l \in \mathcal{L}} x_{lpt} - \widehat{D}_{pt} \mid \forall p, t, \text{b} \qquad \text{(Eq. 3.1)}$$

with $I_{p,0}, B_{p,0}$ given (deterministic). Here $\widehat{D}_{pt}$ is a deterministic projection of demand from the neutrosophic interval D_{pt}. We will use the robust projection:

$$\widehat{D}_{pt} := \bar{D}_{pt}, \qquad \text{(Eq. 3.2)}$$

So, the balance holds for all plausible demands $d \in \left[\underline{D}_{pt}, \bar{D}_{pt}\right]$. With this choice, the service event "no backorder" becomes certain in the neutrosophic sense (see Remark 3.1).

Capacity and activation constraints:

$$\sum_{p} x_{lpt} \leq \bar{C}_{lpt} \; \forall l, t, \sum_{l:A_{sl}=1} \sum_{p} x_{lpt} \leq \bar{U}_{spt} \; \forall s, t \qquad \text{(Eq. 3.3)}$$

$$\sum_{l:A_{sl}=1} x_{lpt} \leq M_{spt} y_{spt} \; \forall s, p, t, y_{spt} \in \{0,1\} \qquad \text{(Eq. 3.4)}$$

with $M_{spt} \geq \bar{U}_{spt}$ a known big-M.

Remark 3.1

If a constraint is enforced for all values in the plausibility interval of a parameter (e.g., (3.2) enforces balance at $\bar{D}_{pt}$), then the corresponding event (e.g., "balance holds" or "no backorder") has a neutrosophic triplet (1,0,0) regardless of the input's (t, i, f). This follows because the event becomes logically true under every admissible realization, eliminating indeterminacy in the statement itself.

3.3 Cost components and a neutrosophic-robust objective

For a plan (x, I, B, y), define the upper-bound (robust) cost using the upper endpoints of all cost intervals:

$$\bar{C}(x, I, B, y) = \sum_{t,p} \sum_{s,l:A_{sl}=1} \left(\bar{c}_{spt} + \bar{k}_{lpt}\right) x_{lpt} + \sum_{t,p} \bar{h}_{pt} I_{pt} + \sum_{t,p} \bar{p}_{pt} B_{pt} + \sum_{t,p,s} \bar{F}_{spt} y_{sp} \qquad (3.5)$$

Equation (3.5) upper bounds the realized cost for any parameter realization within the specified intervals (monotonicity).

Optional indeterminacy penalty:

Sometimes a planner wants to penalize how indeterminate the inputs are. Let's $\mathcal{Q}$ be the set of all scalar, per-unit cost parameters that multiply a quantity in $\bar{C}$ (entries of c, k, h, p, F). For $a \in \mathcal{Q}$, denote its interval width $w(a) = \bar{a} - \underline{a}$ and its indeterminacy $i(a)$. Define the usage of a in plan (x, I, B, y) as $\gamma_a(x, I, B, y)$ (e.g., for a per-unit transport cost on $(l, p, t), \gamma_a = x_{lpt}$; for holding on $(p, t), \gamma_a = I_{pt}$; for fixed order on $(s, p, t), \gamma_a = y_{spt}$). Then:

$\Phi_\rho(x, I, B, y) = \rho \sum_{a \in Q}\ i(a)w(a)\gamma_a(x, I, B, y), \rho \geq 0$ (Eq. 3.6)

is a linear indeterminacy surcharge. We set $\rho = 0$ unless otherwise specified in case studies (no ambiguity).
Primary objective, used by default:

$\min_{x,I,B,y} J(x, I, B, y) = \bar{C}(x, I, B, y) + \Phi_\rho(x, I, B, y), \rho = 0$ by default. (Eq. 3.7)

3.4 Neutrosophic service levels and KPIs
Define, for each (p, t), the on-time service event

$A_{pt} \equiv \{B_{pt} = 0\}$ (Eq. 3.8)

Given a scenario set Ξ with classical realizations of demands/capacities and a neutrosophic probability NP over $(\Xi, 2^\Xi)$ (Section 2.5) , compute:

$\mathrm{NP}(A_{pt}) = \left(\sum_{\xi \in \Xi}\ t_\xi 1\{B_{pt}(\xi) = 0\}, \sum_{\xi \in \Xi}\ i_\xi 1\{B_{pt}(\xi) = 0\}, \sum_{\xi \in \Xi}\ f_\xi 1\{B_{pt}(\xi) = 0\}\right)$,(Eq. 3.9)

where $(t_\xi, i_\xi, f_\xi) = \mathrm{NP}(\{\xi\})$. From (3.9) derive the plausibility interval and risk-averse score for service (Section 2.6):

$$\left[\underline{p}_{pt}^{\mathrm{srv}}, \bar{p}_{pt}^{\mathrm{srv}}\right] = \left[\mathrm{NP}_T(A_{pt}), 1 - \mathrm{NP}_F(A_{pt})\right], s_\lambda\left(\mathrm{NP}(A_{pt})\right) = \mathrm{NP}_T - \lambda \mathrm{NP}_I - \mathrm{NP}_F (.3.10)$$

Design option (neutrosophic service guarantees). Require thresholds $\tau_T \in [0,1], \tau_I, \tau_F \in [0,1]$:

$\mathrm{NP}_T(A_{pt}) \geq \tau_T, \mathrm{NP}_I(A_{pt}) \leq \tau_I, \mathrm{NP}_F(A_{pt}) \leq \tau_F, \forall p, t$ (Eq. 3.11)

When we choose the robust demand projection (3.2) and set $B_{pt} = 0$ by design, then A_{pt} becomes certain (triplet (1,0,0), Remark 3.1) and (3.11) is automatically satisfied for any $\tau_T \leq 1$.

3.5 Supplier and route ranking under indeterminacy
For any alternative q (e.g., supplier s for product p, or lane l), define a neutrosophic performance triplet $u(q) = (t(q), i(q), f(q))$ by fusing evidence sources (quality audits, delays, claims) using the weighted sum from Section 2.6 (or $\Pi/\sqcup$ if dependence is suspected).

Rank by the risk-averse score:
Rank(q) descending by $s_{\lambda^\star}(u(q)) = t(q) - \lambda^\star i(q) - f(q), \lambda^\star \in [0,1]$ fixed (we use $\lambda^\star(3 = 1\mathbb{Q}, 5)$. (Eq. 3.12)

This ranking is used in heuristics (e.g., greedy fills) and for interpretability in the case studies; the optimization model remains the authoritative planner.

3.6 The neutrosophic-robust optimization model

Collecting (3.1)-(3.5) (and optionally (3.6), (3.11)), the default model is:

$$\begin{array}{lll} \min\limits_{x,I,B,y} & \bar{C}(x,I,B,y) + \Phi_\rho(x,I,B,y) & \text{(objective, (3.5) + (3.6))} \\ \text{s.t.} & I_{p,t} - B_{p,t} = I_{p,t-1} - B_{p,t-1} + \sum_l x_{lpt} - \bar{D}_{pt} & \forall p,t \ \text{(balance, robust)} \\ & \sum_p x_{lpt} \le \bar{C}_{lpt}, \sum_{l:A_{sl}=1} \sum_p x_{lpt} \le \bar{U}_{spt} & \forall l,s,t \ \text{(capacities)} \\ & \sum_{l:A_{sl}=1} x_{lpt} \le M_{spt} y_{spt}, y_{spt} \in \{0,1\} & \forall s,p,t \ \text{(activation)} \\ & x_{lpt}, I_{pt}, B_{pt} \ge 0 & \end{array} \quad \text{(Eq. 3.13)}$$

with $\rho = 0$ by default. The model is a MILP (binary y); if $F_{spt} \equiv 0$, set $y \equiv 0$ and the problem becomes an LP.

Example

Instance. One product ($p = 1$), one period ($t = 1$), two suppliers $s \in \{1,2\}$, two lanes $l \in \{1,2\}$ with $A_{sl} = 1\{s = l\}$. Initial $I_0 = B_0 = 0$.

Uncertain inputs (neutrosophic intervals; widths shown for clarity):

1. Demand $D_{11} = ([95,105],(0.70,0.20,0.10))$(width = 10).
2. Costs (unit):

Supplier 1: $c_{111} = [9.5,10.0], u^c_{111} = (0.80,0.10,0.10)$; lane 1: $k_{111} = [1.0,1.2], u^k_{111} =$ (0.70, 0.20, 0.10).

Supplier 2: $c_{211} = [10.5,11.5], u^c_{211} = (0.60,0.30,0.10)$; lane 2: $k_{211} = [0.6,0.8], u^k_{211} =$ (0.90, 0.05, 0.05).

Holding $h_{11} = [0.5,0.5]$ (crisp), Shortage $p_{11} = [4.0,4.0]$ (crisp).

1. Capacities (unit): $\bar{U}_{111} = \bar{C}_{111} = 80, \bar{U}_{211} = \bar{C}_{211} = 50$.
2. Fixed costs are zero; $y \equiv 0$. Use default $\rho = 0$.

Model. With (3.2), $\hat{D}_{11} = \bar{D}_{11} = 105$. Decision variables: arrivals x_{111}, x_{211}, inventory I_{11}, backorder B_{11}.

Balance (3.1):

$$I_{11} - B_{11} = x_{111} + x_{211} - 105 \quad \text{(Eq. 3.14)}$$

Capacities: $0 \le x_{111} \le 80, 0 \le x_{211} \le 50$.

Robust cost (3.5) uses upper unit costs:

Supplier 1 total unit = $\bar{c}_{111} + \bar{k}_{111} = 10.0 + 1.2 = 11.2$.

Supplier 2 total unit = $11.5 + 0.8 = 12.3$.

Thus,

$$\bar{C} = 11.2x_{111} + 12.3x_{211} + 0.5I_{11} + 4.0B_{11}. \quad \text{(Eq. 3.15)}$$

Because holding/shortage are strictly positive, optimal solutions avoid simultaneous positive I_{11} and B_{11}. To meet all plausible demands with no backorder, set $B_{11} = 0$ and enforce $x_{111} + x_{211} \ge 105$. The cheapest fill is greedy by unit cost:

Take $x_{111} = 80$ (capacity-tight), then $x_{211} = 25$ to reach $105, I_{11} = 0$.

Compute cost:
$\bar{C}^{\star} = 11.2 \cdot 80 + 12.3 \cdot 25 = 896.0 + 307.5 = 1203.5.$ (Eq. 3.16)

This plan guarantees $B_{11} = 0$ for all $d \in [95,105]$, so the service event A_{11} is certain with triplet (1,0,0) (Remark 3.1). For demands below 105 (e.g., 95), inventory $I_{11} = x_{111} + x_{211} - d$ may be positive in operation, but the robust objective (3.5) evaluates cost at the worst case and yields $I_{11} = 0$ there; this is consistent with minimizing the upper bound. Table 2 reports the optimal arrivals and the resulting robust cost upper bound. Because the balance is enforced at the demand upper endpoint, the on-time service event is neutrosophically certain, eliminating indeterminacy in this KPI.

Table 2. Results of Example 3.1 under robust projection $\widehat{D}_{11} = 105$.

Quantity	Value
$\boldsymbol{x_{111}}$ (units via lane 1)	80
$\boldsymbol{x_{211}}$ (units via lane 2)	25
$\boldsymbol{I_{11}, B_{11}}$ (units)	0,0
Robust unit costs $(\mathbf{1}, \mathbf{2})$	11.2, 12.3
$\boldsymbol{\bar{C}^{\star}}$	1203.5
Service triplet $\mathbf{NP}(\boldsymbol{A_{11}})$	(1, 0, 0)

The plan fills to the demand upper bound using the cheaper supplier first, then the second supplier up to the remaining requirement. As the balance is enforced at $\bar{D}_{11}$, service is guaranteed for all plausible demands, giving a neutrosophic certainty triplet (1,0,0) for on-time service.

3.7 How neutrosophy enters decisions

Our model brings neutrosophy into decisions in four connected ways. First, the robust constraints (3.1)-(3.4) use the upper ends of all input intervals to check feasibility against the worst case. This turns parameter indeterminacy into a guarantee: if the plan satisfies the constraints at those endpoints, the corresponding service event is certain, as noted in Remark 3.1. Second, the objective (3.7) values plans by their worst-case cost. When a planner wants to reflect data quality directly, the optional term Φ_{ρ}(3.6) adds a linear penalty for indeterminacy, scaled by both the width of each interval and how much the plan uses that parameter.

Key performance indicators are then read in neutrosophic form. When a scenario view is helpful, we evaluate events with the triplet calculus in (3.9)-(3.10); but if feasibility is enforced robustly, the event's triplet collapses to (1,0,0), removing any residual ambiguity for that KPI. Finally, the ranking rule (3.12) gives a clear order over suppliers or lanes under mixed evidence without changing the MILP in (3.13). All triplets, measures, probabilities, and the handling of independence or conservative bounds follow the definitions fixed in Section 2, ensuring consistency between the math and the decisions.

4. Case Study 1: Supplier Selection under Indeterminate Reliability

We consider a single product in one planning period with uncertain demand and indeterminate reliability across three candidate suppliers. Demand and all cost/ capacity parameters are modeled as neutrosophic intervals (Def. 2.7). Reliability evidence for each supplier is encoded as neutrosophic triplets and fused into a single performance triplet per supplier (Sec. 2.6). Selection is performed by solving the neutrosophic-robust MILP (Sec. 3.6) augmented with a reliability-share constraint that forces a minimum fraction of volume to come from "reliable" suppliers. The data used in this case are:

Indices and sets. Suppliers $s \in \{1,2,3\}$, lanes $l \in \{1,2,3\}$ with one lane per supplier, and $A_{sl} = 1\{s = l\}$. Single product and period; we drop (p, t) subscripts where obvious.

Demand (neutrosophic interval). $D = \left([\underline{D}, \bar{D}], u^D\right) = ([980,1020], (0.70,0.20,0.10))$.

Per-unit costs (intervals, treated as crisp with triplet (1,0,0)).

Procurement: $c_1 = [9.2,9.8], c_2 = [8.9,10.1], c_3 = [9.6,10.0]$.

Transport: $k_1 = [1.0,1.1], k_2 = [1.3,1.4], k_3 = [0.9,1.0]$.

Holding $h = [0.5,0.5]$ (crisp), shortage $p = [4.0,4.0]$ (crisp).

Capacities (crisp intervals). Supplier capacities $\bar{U}_1 = 500, \bar{U}_2 = 600, \bar{U}_3 = 400$. Lane capacities $\bar{C}_l = \bar{U}_s$ for the corresponding supplier.

Fixed order costs (crisp). $F_1 = 200, F_2 = 100, F_3 = 150$. Binary activation with $M_s = \bar{U}_s$.

Supplier reliability evidence. Three sources per supplier: on-time u^{on} , non-defect (the complement of defect) u^{ndef} , and no-disruption u^{nrisk} . Each is a neutrosophic triplet $(t, i, f) \in [0,1]^3$. We fuse them via a weighted sum with fixed weights $(w_{\text{on}}, w_{\text{ndef}}, w_{\text{nrisk}}) = (0.5,0.3,0.2)$ (Sec. 2.6):

$$u(s) = w_{\text{on}}\, u^{\text{on}}(s) + w_{\text{ndef}}\, u^{\text{ndef}}(s) + w_{\text{nrisk}}\, u^{\text{nrisk}}(s).$$

The risk-averse score (Sec. 2.6) is $s_{0.5}(u) = t - 0.5i - f$.

Table 3. Input data (interval endpoints and neutrosophic reliability evidence).

Parameter	$s = 1$	$s = 2$	$s = 3$
Procure c_s	[9.2, 9.8]	[8.9, 10.1]	[9.6, 10.0]
Transport k_s	[1.0, 1.1]	[1.3, 1.4]	[0.9, 1.0]
Capacity $\bar{U}_s$	500	600	400
Fixed F_s	200	100	150
$u^{\text{on}}(s)$	(0.78, 0.15, 0.07)	(0.65, 0.20, 0.15)	(0.75, 0.18, 0.07)
$u^{\text{ndef}}(s)$	(0.92, 0.05, 0.03)	(0.88, 0.07, 0.05)	(0.95, 0.03, 0.02)
$u^{\text{nrisk}}(s)$	(0.85, 0.10, 0.05)	(0.70, 0.20, 0.10)	(0.90, 0.07, 0.03)

Costs and capacities are crisp intervals with an implicit triplet (1,0,0). Reliability triplets reflect mixed evidence: support t, indeterminacy i, and refutation f (See Table 3).

4.1 Reliability fusion, scores, and "reliable" set

For each supplier, fuse and score:

$$u(s) = (t(s), i(s), f(s)) = 0.5u^{\text{on}} + 0.3u^{\text{ndef}} + 0.2u^{\text{nrisk}}, s_{0.5}(s) = t(s) - 0.5i(s) - f(s).$$

Computed results.

1. $u(1) = (0.8360, 0.1100, 0.0540), s_{0.5}(1) = 0.8360 - \frac{1}{2} \cdot 0.1100 - 0.0540 = 0.7270.$
2. $u(2) = (0.7290, 0.1610, 0.1100), s_{0.5}(2) = 0.7290 - \frac{1}{2} \cdot 0.1610 - 0.1100 = 0.5385.$
3. $u(3) = (0.8400, 0.1130, 0.0470), s_{0.5}(3) = 0.8400 - \frac{1}{2} \cdot 0.1130 - 0.0470 = 0.7365.$

Let the reliability threshold be $\theta = 0.70$ and the required reliable share be $\alpha = 0.60$. Define the reliable set $\mathcal{R} := \{s: s_{0.5}(s) \geq \theta\} = \{1,3\}$.

Table 4. Fused supplier triplets and risk-averse scores (descending).

Supplier	$u(s) = (t, i, f)$	$s_{0.5}(s) = t - 0.5i - f$	Rank
3	(0.8400, 0.1130, 0.0470)	0.7365	1
1	(0.8360, 0.1100, 0.0540)	0.7270	2
2	(0.7290, 0.1610, 0.1100)	0.5385	3

Suppliers 1 and 3 exceed the reliability threshold $\theta = 0.70$ and form $\mathcal{R}$. Supplier 2 is less reliable by the neutrosophic score as shown in Table 4.

4.2 Optimization model for this case

We use the robust demand projection $\widehat{D} = \bar{D} = 1020$ (Eq. 3.2). Let $x_s \geq 0$ be arrivals from supplier s_{a} and $y_s \in \{0,1\}$ the activation variables ($x_s \leq M_s y_s, M_s = \bar{U}_s$). Inventory and backorders are set to zero in the robust endpoint (they would only increase the objective). The unit robust cost for supplier s is

$$\bar{u}_s := \bar{c}_s + \bar{k}_s \Rightarrow \bar{u}_1 = 9.8 + 1.1 = 10.9, \bar{u}_2 = 10.1 + 1.4 = 11.5, \bar{u}_3 = 10.0 + 1.0 = 11.0.$$

The case-study MILP is:

$$\begin{aligned} \min_{x,y} \quad & \textstyle\sum_{s=1}^{3} \bar{u}_s x_s + \sum_{s=1}^{3} F_s y_s \\ \text{s.t.} \quad & \textstyle\sum_{s=1}^{3} x_s \geq 1020 \\ & \textstyle\sum_{s \in \mathcal{R}} x_s \geq \alpha \cdot 1020 = 612 \quad \text{(fill robust demand)} \\ & 0 \leq x_s \leq \bar{U}_s y_s, y_s \in \{0,1\} \quad \text{(reliable share)} \\ \text{for } s = 1,2,3 & \end{aligned} \qquad \text{(Eq. 4.1)}$$

4.3 Solving and verifying

Because $\bar{u}_1 < \bar{u}_3 < \bar{u}_2$, the cheapest policy is to:

1. saturate supplier 1 ,
2. then use supplier 3,
3. and use supplier 2 only for any residual needed to reach 1020.

Without the reliable-share constraint (4.1), suppliers 1 and 2 alone can meet demand: $x_1 = 500, x_2 = 520, x_3 = 0$, with fixed $y_1 = y_2 = 1, y_3 = 0$.
Cost: variable $= 10.9 \cdot 500 + 11.5 \cdot 520 = 11430.0$; fixed $= 200 + 100 = 300$; total =

11730.0.
Reliable share: volume from $\mathcal{R}$ is only $x_1 = 500$, i.e., $500/1020 \approx 49.0\% < 60\% \rightarrow$ violates (4.1).

With (4.1), require $x_1 + x_3 \geq 612$. The optimal allocation is:

$$x_1 = 500(\text{ cap }), x_3 = 400(\text{ cap }), x_2 = 1020 - 500 - 400 = 120,$$

with $y_1 = y_2 = y_3 = 1$.
Cost: variable $= 10.9 \cdot 500 + 11.0 \cdot 400 + 11.5 \cdot 120 = 11230.0$; fixed $= 200 + 100 + 150 = 450$; Total $= 11680.0$.

Reliable share: $(x_1 + x_3)/1020 = 900/1020 \approx 88.24\% \geq 60\% \rightarrow$ satisfies (4.1).
Because the fixed cost of supplier 3 is offset by replacing higher-cost units from supplier 2, the reliability-constrained plan is less expensive by $11730.0 - 11680.0 = 50.0$ units while meeting the reliability target. Enforcing a 60% reliable-share (with $\theta = 0.70$) improves robustness and reduces cost, as the additional fixed cost is outweighed by shifting volume from supplier 2 (11.5) to supplier 3 (11.0).

Table 5. Optimal allocations and costs with and without the reliability constraint.

Policy	x_1	x_2	x_3	Var. cost	Fixed cost	Total cost	Reliable share
Baseline (no 4.1)	500	520	0	11430.0	300	11730.0	49.0%
With (4.1)	500	120	400	11230.0	450	11680.0	88.24%

4.4 Plan-level neutrosophic performance

Aggregate the fused reliability triplets using shipment weights $w_s = x_s/1020$ (Sec. 2.6):

$$u_{\text{plan}} = \sum_{s=1}^{3} w_s u(s).$$

For the optimal reliability-constrained plan $(x_1, x_2, x_3) = (500,120,400)$,

$$w_1 = 500/1020 = 0.490196, w_2 = 120/1020 = 0.117647, w_3 = 400/1020 = 0.392157,$$
$$u_{\text{plan}} = (0.82498, 0.11718, 0.05784),$$
$$s_{0.5}(u_{\text{plan}}) = 0.82498 - \frac{1}{2} \cdot 0.11718 - 0.05784 = 0.70855,$$
$$[\underline{p}, \bar{p}]_{\text{plan}} = [t, 1 - f] = [0.82498, 0.94216], p^{c} = t + \frac{1}{2} i = 0.88357.$$

Table 6. Plan-level neutrosophic reliability for the selected allocation.

Quantity	Value
$u_{\text{plan}} = (t, i, f)$	(0.82498, 0.11718, 0.05784)
Risk-averse score $s_{0.5}$	0.70855
Plausibility interval $[\underline{p}, \bar{p}]$	[0.82498, 0.94216]
Center projection p^{c}	0.88357

The plan's neutrosophic reliability exceeds the supplier threshold in aggregate; the plausibility interval indicates that overall, on-time reliability lies between 82.5% and 94.2%.(See Table 6).

Service KPI under robust enforcement

The inventory balance is enforced at $\widehat{D} = \bar{D} = 1020$, and back orders are eliminated in the robust endpoint, so the on-time service event $A = \{B = 0\}$ is certain in the neutrosophic sense with $(T, I, F) = (1,0,0)$ (Remark 3.1). Hence, the service KPI has no residual indeterminacy under the chosen robust policy.

5. Case Study 2: Demand Forecasting with Ambiguous Market Signals

We forecast short-horizon demand under ambiguous signals (marketing, web activity, macro conditions). Each signal contributes a nominal effect interval (units) and a neutrosophic triplet $u = (t, i, f)$. We transform triplets to plausibility multipliers and propagate them to an interval forecast with an associated forecast triplet. Accuracy is evaluated against realized demand and compared to a classical baseline. Horizon $t = 1,2,3,4$ (four future periods). Units are identical across all quantities.

Let the last six realized demands be

$$(980,1000,990,1010,995,1025).$$

Their sum is 6000, and the simple moving-average baseline is

$$b := \frac{6000}{6} = 1000$$

The classical point forecast for each future period is therefore $b_t \equiv 1000$. For the neutrosophic model, we also attach a baseline interval and triplet (reflecting statistical and data-quality uncertainty):

$$B_t = [995,1005], u^B = (0.80,0.15,0.05) \text{ for all } t.$$

5.1 Neutrosophic signal-to-forecast transformation

For each period t and signal $j \in$ {M (marketing), W (web), E (macro) } :
Nominal effect interval (units): $[\underline{A}_{jt}, \bar{A}_{jt}]$.

Triplet: $u_{jt} = (t_{jt}, i_{jt}, f_{jt}) \in [0,1]^3$.

Plausibility multiplier interval from the triplet (Sec. 2.6):

$$m_{jt} \in [t_{jt}, 1 - f_{jt}]. \qquad \text{(Eq. 5.1)}$$

Effect interval $\mathcal{E}_{jt}$ (units), computed by interval arithmetic with nonnegative multipliers:

$$\mathcal{E}_{jt} = \begin{cases} [\underline{A}_{jt} t_{jt}, \bar{A}_{jt}(1 - f_{jt})], & \underline{A}_{jt} \geq 0, \\ [\underline{A}_{jt}(1 - f_{jt}), \bar{A}_{jt} t_{jt}], & \bar{A}_{jt} \leq 0, \\ [\underline{A}_{jt}(1 - f_{jt}), \bar{A}_{jt}(1 - f_{jt})], & \underline{A}_{jt} < 0 < \bar{A}_{jt}. \end{cases} \qquad \text{(Eq. 5.2)}$$

Forecast interval:

$F_t = B_t + \mathcal{E}_{\mathrm{M}t} + \mathcal{E}_{\mathrm{W}t} + \mathcal{E}_{\mathrm{E}t}$ (endpoint-wise sum). (Eq. 5.3)

Forecast triplet (conservative fusion) via meet Π (Sec. 2.2):

$$u_t^F = u^B \sqcap u_{\mathrm{M}t} \sqcap u_{\mathrm{W}t} \sqcap u_{\mathrm{E}t} = (\min t, \max i, \max f). \quad \text{(Eq. 5.4)}$$

Point extraction for accuracy reporting:

$$\mathrm{center}(F_t) = \frac{1}{2}\left(\underline{F_t} + \bar{F}_t\right), [\underline{p}, \bar{p}](u_t^F) = [t_t^F, 1 - f_t^F] \quad \text{(Eq. 5.5)}$$

Data: nominal signal intervals and triplets

All values are per period and in units.

Table 7. Nominal signal intervals and neutrosophic triplets; plausibility multipliers $[t, 1-f]$.

Period t	Signal	Nominal $[\underline{A}, \bar{A}]$	Triplet $u = (t, i, f)$	Multiplier $[t, 1-f]$
1	M (marketing)	[40, 60]	(0.70, 0.20, 0.10)	[0.70, 0.90]
1	W (web)	[10,20]	(0.60, 0.30, 0.10)	[0.60, 0.90]
1	E (macro)	[-30, -10]	(0.50, 0.30, 0.20)	[0.50, 0.80]
2	M	[20,40]	(0.60, 0.25, 0.15)	[0.60, 0.85]
2	w	[5,15]	(0.55, 0.35, 0.10)	[0.55, 0.90]
2	E	[-20,0]	(0.45, 0.35, 0.20)	[0.45, 0.80]
3	M	[0,10]	(0.30, 0.50, 0.20)	[0.30, 0.80]
3	w	[-5,5]	(0.40, 0.40, 0.20)	[0.40, 0.80]
3	E	[-25, -15]	(0.70, 0.20, 0.10)	[0.70, 0.90]
4	M	[30,50]	(0.80, 0.15, 0.05)	[0.80, 0.95]
4	w	[20,30]	(0.75, 0.20, 0.05)	[0.75, 0.95]
4	E	[-10,0]	(0.50, 0.40, 0.10)	[0.50, 0.90]

Each signal's nominal effect interval is multiplied by a plausibility multiplier interval derived from its triplet. For intervals crossing zero, Eq. (5.2) uses the upper multiplier $1 - f$ on both endpoints (maximal magnitude), producing a symmetric or asymmetric effect interval that includes 0.

5.2 Computations and results

Effect intervals $\mathcal{E}_{jt}$ from Eq. (5.2):

$t = 1$:

$$\mathcal{E}_{\mathrm{M1}} = [40 \cdot 0.70, 60 \cdot 0.90] = [28,54],$$
$$\mathcal{E}_{\mathrm{W1}} = [10 \cdot 0.60, 20 \cdot 0.90] = [6,18],$$
$$\mathcal{E}_{\mathrm{E1}} = [-30 \cdot 0.80, -10 \cdot 0.50] = [-24, -5].$$

$t = 2$:

$$\mathcal{E}_{\mathrm{M2}} = [20 \cdot 0.60, 40 \cdot 0.85] = [12,34],$$
$$\mathcal{E}_{\mathrm{W2}} = [5 \cdot 0.55, 15 \cdot 0.90] = [2.75, 13.5],$$
$$\mathcal{E}_{\mathrm{E2}} = [-20 \cdot 0.80, 0 \cdot 0.45] = [-16, 0].$$

$t = 3$:

$$\mathcal{E}_{\mathrm{M3}} = [0 \cdot 0.30, 10 \cdot 0.80] = [0,8],$$
$$\mathcal{E}_{\mathrm{W3}} = [-5 \cdot 0.80, 5 \cdot 0.80] = [-4,4],$$
$$\mathcal{E}_{\mathrm{E3}} = [-25 \cdot 0.90, -15 \cdot 0.70] = [-22.5, -10.5].$$

$t = 4$:

$$\mathcal{E}_{M4} = [30 \cdot 0.80, 50 \cdot 0.95] = [24, 47.5],$$
$$\mathcal{E}_{W4} = [20 \cdot 0.75, 30 \cdot 0.95] = [15, 28.5],$$
$$\mathcal{E}_{E4} = [-10 \cdot 0.90, 0 \cdot 0.50] = [-9, 0].$$

Forecast intervals F_t from Eq. (5.3) using $B_t = [995, 1005]$:

$t = 1$: $[995, 1005] + [28, 54] + [6, 18] + [-24, -5] = [1005, 1072]$.

$t = 2$: $[995, 1005] + [12, 34] + [2.75, 13.5] + [-16, 0] = [993.75, 1052.5]$

$t = 3$: $[995, 1005] + [0, 8] + [-4, 4] + [-22.5, -10.5] = [968.5, 1006.5]$.

$t = 4$: $[995, 1005] + [24, 47.5] + [15, 28.5] + [-9, 0] = [1025, 1081]$.

Forecast triplets u_t^F via Eq. (5.4) (meet of u^B and period- t signal triplets):
$t = 1$: $(\min t, \max i, \max f) = (0.50, 0.30, 0.20)$.

$t = 2$: $(0.45, 0.35, 0.20)$.

$t = 3$: $(0.30, 0.50, 0.20)$.

$t = 4$: $(0.50, 0.40, 0.10)$.

Centers from Eq. (5.5):

$$\text{center}(F_1) = \frac{1005 + 1072}{2} = 1038.5$$
$$\text{center}(F_2) = \frac{993.75 + 1052.5}{2} = 1023.125$$
$$\text{center}(F_3) = \frac{968.5 + 1006.5}{2} = 987.5$$
$$\text{center}(F_4) = \frac{1025 + 1081}{2} = 1053$$

Realized demand for evaluation (exogenous):

$$d_1 = 1050, d_2 = 1018, d_3 = 990, d_4 = 1068.$$

Table 8. Forecast outcomes by period: intervals, triplets, centers, realizations, and errors.

t	$F_t = [\underline{F}, \bar{F}]$	$u_t^F = (t, i, f)$	center(F_t)	**Realized** d_t	$\text{APE}_{\text{baseline (vs 1000)}}$	$\text{APE}_{\text{neutro}}$ **(vs center)**
1	[1005, 1072]	(0.50, 0.30, 0.20)	1038.5	1050		1050-1000
2	[993.75, 1052.5]	(0.45, 0.35, 0.20)	1023.125	1018	18/1018 = 0.017688 → 1.7688%	5.125/1018 = 0.005036 → 0.5036%
3	[968.5, 1006.5]	(0.30, 0.50, 0.20)	987.5	990	10/990 = 0.010101 → 1.0101%	2.5/990 = 0.002525 → 0.2525%
4	[1025, 1081]	(0.50, 0.40, 0.10)	1053	1068	68/1068 ≈ 0.063662 → 6.3662%	15/1068 ≈ 0.014044 → 1.4044%

Forecast intervals F_t are constructed by adding the baseline interval to the signal effect intervals; all realized values d_t fall inside their respective intervals (100% coverage). The forecast triplet reflects conservative fusion of baseline and signals; lower t and higher i, f indicate periods where ambiguity is higher.

5.3 Accuracy summary

Define absolute percentage error (APE) and mean absolute percentage error (MAPE):

$$\mathrm{APE}_t(\hat{y}_t) = \frac{|d_t - \hat{y}_t|}{d_t}, \mathrm{MAPE} = \frac{1}{4}\sum_{t=1}^{4} \mathrm{APE}_t. \qquad \text{(Eq. 5.6)}$$

Baseline MAPE (point forecast 1000 each period):

$$\frac{1}{4}(0.047619 + 0.017688 + 0.010101 + 0.063662) = 0.034768 \Rightarrow 3.4768\%.$$

Neutrosophic center MAPE:

$$\frac{1}{4}(0.010952 + 0.005036 + 0.002525 + 0.014044) = 0.008139 \Rightarrow 0.8139\%$$

Table 9. Summary metrics.

Metric	Baseline	Neutrosophic (center)
Coverage (actual $\in F_t$)	-	$4/4 = 100\%$
MAPE	3.4768%	0.8139%

The neutrosophic method reduces MAPE by approximately 76.6% relative (from 3.4768% to 0.8139%) and provides calibrated uncertainty intervals that fully cover realized demand in all four periods.

5.4 Discussion

The map from a triplet $u = (t, i, f)$ to the interval $[t, 1 - f]$ (Eq. 5.1) turns ambiguity into a clean multiplier range. Applying this range to each nominal signal produces an effect interval (Eq. 5.2). As uncertainty grows, these intervals widen. Weak evidence often raises the lower bound less than it raises the upper bound, so intervals become asymmetric. This gives a direct, numerical view of how credible each signal is.

To combine sources, we use conservative fusion (Eq. 5.4). The fused triplet takes the smallest support and the largest indeterminacy and refutation across the baseline and signals. In practice, this means the combined forecast inherits the most cautious reading of the evidence. The resulting triplet aligns with the width of the forecast interval: lower support and higher ambiguity lead to wider bounds.

When signals point in different directions, the fused triplet makes that clear. In period $t = 3$, for example, the value of t falls and i rises, reflecting disagreement and negative macro pressure. The forecast interval then shifts downward and becomes wider on the low side. Even so, the interval's center remains close to realized demand, providing a single point that is informative while still consistent with the uncertainty shown by the bounds.

6. Case Study 3: Transportation Risk Analysis under Indeterminacy

We study a single-origin, single-destination network with an intermediate hub, where lanes may be disrupted (partially observed) and lane availability is represented by

neutrosophic triplets. We compare a direct path and a two-leg hub path, optimizing the shipment split to (i) meet robust demand, (ii) respect capacities, and (iii) ensure that a required share is routed through a high-availability path. All symbols and operators follow Sections 2-3.

Network nodes: origin O, hub H, destination D.
Lanes: $l_1: O \to H, l_2: O \to D$ (direct), $l_3: H \to D$.
Two candidate paths:
1. $P_1 = \{l_2\}$ (direct).
2. $P_2 = \{l_1, l_3\}$ (via hub).

We ship nonperishable units in one period; procurement at the origin is deterministic data are

Demand (neutrosophic interval).

$$D = \left([\underline{D}, \bar{D}], u^D\right) = ([480,520], (0.75,0.20,0.05)).$$

We use the robust projection $\widehat{D} = \bar{D} = 520$ (Eq. 3.2).

Procurement (crisp).

Per-unit procurement cost at origin: $c_0 = [9.5,9.5]$ with implicit triplet (1,0,0).

Transport cost intervals (per unit).

$$k_{l_1} = [0.9,1.1], k_{l_2} = [1.4,1.6], k_{l_3} = [0.8,1.0].$$

Capacities (units).

Lane capacities: $\bar{C}_{l_1} = 500, \bar{C}_{l_2} = 350, \bar{C}_{l_3} = 500$.
Path capacity: $\bar{C}_{P_1} = 350$ and $\bar{C}_{P_2} = \min\left(\bar{C}_{l_1}, \bar{C}_{l_3}\right) = 500$.

Lane availability triplets (event "lane is available"):

$$u_{\text{av}}(l_1) = (0.88,0.08,0.04), u_{\text{av}}(l_2) = (0.70,0.20,0.10), u_{\text{av}}(l_3) = (0.92,0.05,0.03).$$

Assumption (stated once): lane availabilities are independent for composition. Transport cost intervals are combined with the deterministic procurement cost $c_0 = 9.5$ to form path costs. Capacities include a direct lane and a two-leg hub route with higher aggregate capacity. Neutrosophic availability is the event that the lane is usable; independence is used only for path composition below.

Table 10. Network, costs, capacities, and lane availability.

Item	$l_1: O \to H$	$l_2: O \to D$	$l_3: H \to D$
Cost interval k_l	[0.9, 1.1]	[1.4, 1.6]	[0.8, 1.0]
Capacity $\bar{C}_l$	500	350	500
Availability u_{av} (l)	(0.88, 0.08, 0.04)	(0.70, 0.20, 0.10)	(0.92, 0.05, 0.03)

6.1 Path availability and path costs

Path availability triplets.

P_1 (direct): $u_{\text{av}}(P_1) = u_{\text{av}}(l_2) = (0.70,0.20,0.10)$.

P_2 (hub): availability is the intersection of l_1 and l_3. With independence (Sec. 2.5):

$$\text{NP}_T\left(A_{P_2}\right) = 0.88 \times 0.92 = 0.8096$$
$$\text{NP}_I\left(A_{P_2}\right) = \min\{1, 0.08 + 0.05\} = 0.13$$
$$\text{NP}_F\left(A_{P_2}\right) = 1 - (1 - 0.04)(1 - 0.03) = 1 - 0.9312 = 0.0688$$

so $u_{\text{av}}(P_2) = (0.8096,0.13,0.0688)$.

Robust per-unit path costs (upper endpoints; Eq. 3.5):

$$\bar{c}(P_1) = 9.5 + \bar{k}_{l_2} = 9.5 + 1.6 = 11.1, \bar{c}(P_2) = 9.5 + \bar{k}_{l_1} + \bar{k}_{l_3} = 9.5 + 1.1 + 1.0 = 11.6.$$

Table 11. Path availability and robust per-unit costs.

Path	Availability $u_{\text{av}} = (t, i, f)$	Plausibility $[\underline{p}, \bar{p}] = [t, 1 - f]$	$s_{0.5} = t - 0.5i - f$	$\bar{c}(\cdot)$
$P_1 = \{l_2\}$	(0.70, 0.20, 0.10)	[0.70, 0.90]	0.50	11.1
$P_2 = \{l_1, l_3\}$	(0.8096, 0.13, 0.0688)	[0.8096, 0.9312]	0.6758	11.6

The hub path P_2 is more available (higher t, lower f) but more expensive under robust costs. Scores $s_{0.5}$ emphasize both higher support and lower indeterminacy/ refutation.

6.2 Optimization model with availability-share requirement

Decision variables (path-level): $z_1 \geq 0$ units via $P_1, z_2 \geq 0$ units via P_2.

Constraints:

$$\begin{aligned} & z_1 + z_2 && \geq \widehat{D} = 520 && \\ & z_1 \leq \bar{C}_{P_1} = 350, z_2 \leq \bar{C}_{P_2} = 500 && && \text{(meet robust demand)} \\ & z_2 \geq \alpha \widehat{D} && && \text{(apacities)} \\ & && && \text{availability-share)} \end{aligned} \quad \text{(Eq. 6.1)}$$

where $\alpha \in [0,1]$ is a design parameter specifying the minimum share routed on the high-availability path P_2.

Objective (robust; Eq. 3.7 with $\rho = 0$):

$$\min_{z_1, z_2} J(z_1, z_2) = \bar{c}(P_1) z_1 + \bar{c}(P_2) z_2 = 11.1 z_1 + 11.6 z_2. \quad \text{(Eq. 6.2)}$$

This is a small LP; the extreme-point solution uses as little P_2 as allowed by (6.1), filling the remainder with P_1 (cheaper) within capacities.

6.3 Solutions for three α-policies

We evaluate $\alpha \in \{0,0.60,0.80\}$.

(i) $\alpha = 0$ (no availability-share constraint).

Capacity of P_1 bounds $z_1 \leq 350$, so the cheapest fill is $z_1 = 350, z_2 = 520 - 350 = 170$.

Cost:

$$J = 11.1 \cdot 350 + 11.6 \cdot 170 = 3885.0 + 1972.0 = 5857.0.$$

(ii) $\alpha = 0.60$.

Constraint $z_2 \geq 0.60 \cdot 520 = 312$. Cheapest is $z_2 = 312$, $z_1 = 520 - 312 = 208$.
Cost:

$$J = 11.1 \cdot 208 + 11.6 \cdot 312 = 2308.8 + 3619.2 = 5928.0.$$

(iii) $\alpha = 0.80$.

Constraint $z_2 \geq 0.80 \cdot 520 = 416$. Cheapest is $z_2 = 416, z_1 = 520 - 416 = 104$.
Cost:

$$J = 11.1 \cdot 104 + 11.6 \cdot 416 = 1154.4 + 4825.6 = 5980.0.$$

Table 12. Optimal routing and robust costs across availability-share policies.

α	z_1 via P_1	z_2 via P_2	Feasible?	Robust cost J
0.00	350	170		5857.0
0.60	208	312		5928.0
0.80	104	416		5980.0

Higher α (more volume forced onto the higher-availability path) increases robust cost as expected. All solutions respect capacities $z_1 \leq 350, z_2 \leq 500$ and meet robust demand 520 as illustrated above in Table 12.

6.4 Plan-level neutrosophic availability and KPIs

For each policy, aggregate path availability using shipment weights $w_j = z_j/520$:

$$u_{\text{plan}} = w_1 u_{\text{av}}(P_1) + w_2 u_{\text{av}}(P_2) = (t_{\text{plan}}, i_{\text{plan}}, f_{\text{plan}}).$$

Compute u_{plan}, risk score $s_{0.5}$, plausibility interval $[t, 1-f]$, and center projection $p^c = t + \frac{1}{2}i$:

$\alpha = 0: w_1 = 350/520 = 0.6730769, w_2 = 0.3269231.$

$$t = 0.6730769 \cdot 0.70 + 0.3269231 \cdot 0.8096 = 0.73515$$
$$i = 0.6730769 \cdot 0.20 + 0.3269231 \cdot 0.13 = 0.17712$$
$$f = 0.6730769 \cdot 0.10 + 0.3269231 \cdot 0.0688 = 0.08981$$
$$s_{0.5} = 0.73515 - 0.5 \cdot 0.17712 - 0.08981 = 0.55679$$
$$[\underline{p}, \bar{p}] = [0.73515, 0.91019], p^c = 0.73515 + 0.5 \cdot 0.17712 = 0.82371$$

$\alpha = 0.60: w_1 = 208/520 = 0.4, w_2 = 0.6.$

$$t = 0.4 \cdot 0.70 + 0.6 \cdot 0.8096 = 0.76576$$
$$i = 0.4 \cdot 0.20 + 0.6 \cdot 0.13 = 0.15800$$
$$f = 0.4 \cdot 0.10 + 0.6 \cdot 0.0688 = 0.08128$$
$$s_{0.5} = 0.76576 - 0.5 \cdot 0.15800 - 0.08128 = 0.60548$$
$$[\underline{p}, \bar{p}] = [0.76576, 0.91872], p^c = 0.84476$$

$\alpha = 0.80: w_1 = 104/520 = 0.2, w_2 = 0.8.$

$$t = 0.2 \cdot 0.70 + 0.8 \cdot 0.8096 = 0.78768$$
$$i = 0.2 \cdot 0.20 + 0.8 \cdot 0.13 = 0.14400$$
$$f = 0.2 \cdot 0.10 + 0.8 \cdot 0.0688 = 0.07504$$
$$s_{0.5} = 0.78768 - 0.5 \cdot 0.14400 - 0.07504 = 0.64064$$
$$[\underline{p}, \bar{p}] = [0.78768, 0.92496], p^{c} = 0.85968$$

Table 13. Plan-level availability and risk scores vs α.

α	t_{plan}	i_{plan}	f_{plan}	$s_{0.5}$	$[\underline{p}, \bar{p}]$	p^{c}
0.00	0.73515	0.17712	0.08981	0.55679	[0.73515, 0.91019]	0.82371
0.60	0.76576	0.15800	0.08128	0.60548	[0.76576, 0.91872]	0.84476
0.80	0.78768	0.14400	0.07504	0.64064	[0.78768, 0.92496]	0.85968

Increasing α moves more volume onto the higher-availability path P_2, improving the plan's neutrosophic availability (higher t, lower f) and risk score $s_{0.5}$, at the expense of robust cost (Table 13). The plausibility interval $[t, 1 - f]$ tightens at the upper end as f decreases.

Service KPI. As in Sections 3-4, the inventory balance is enforced at $\widehat{D} = 520$ with no backorders in the robust endpoint, so the fill event $A = \{B = 0\}$ is neutrosophically certain, i.e., $\text{NP}(A) = (1,0,0)$ (Remark 3.1). Route availability affects risk exposure, not the feasibility of the material balance under the robust plan.

7. Comparative evaluation and discussion

The three case studies show how the neutrosophic framework changes planning when information is incomplete or conflicting. In the supplier selection study, enforcing a reliability share produced a plan that met the robust demand while shifting volume to the more trustworthy suppliers. This shift did not raise total cost; on the contrary, the reliability-constrained plan achieved a small but clear saving because it replaced a portion of higher unit-cost volume from the less reliable source with units from a moderately cheaper and more reliable source. The final allocation satisfied the reliability target by a wide margin, and the on-time service event became certain under the robust projection (no backorders at the demand upper bound). This confirms a practical point: when two suppliers are close in unit cost, adding a reliability share can improve both robustness and cost once fixed ordering costs and lane differences are accounted for by the optimization.

In the forecasting study, ambiguous signals were translated into plausible effect intervals using their triplets, then combined with a short moving-average baseline. Each period's forecast interval included the realized demand, and the center of each interval produced a much lower error than a constant classical baseline. The mean absolute percentage error dropped from about three and a half percent to under one percent, and this improvement held across all four periods. The key reason is that indeterminacy was not forced into a single probability or a single adjustment; instead, the multiplier range reflected the credibility of each signal, so positive effects were damped when refutation or

indeterminacy was large, and negative effects were given room when macro conditions were plausibly adverse.

In the transportation study, the planner adjusted a simple policy knob: the minimum share that must travel on the more available path. As this share increased, the plan's neutrosophic availability improved, support increased, and refutation decreased—while robust cost rose in predictable steps because the hub path had the higher unit cost at the upper endpoints. The increase in cost was moderate, and every policy maintained a guaranteed service outcome at the robust demand level. This shows how a planner can "buy" availability with a transparent parameter and see the trade clearly in both money and neutrosophic terms.

A unifying observation across the three studies is that robust feasibility at upper endpoints collapses the associated service event to certainty, removing indeterminacy from that KPI and leaving cost and risk as the active trade-offs. When data quality is poor, this is often the right choice operationally: guarantee the flow first, then reason about what margin to pay for reliability or availability. Another consistent finding is that neutrosophic scores and plausibility intervals give interpretable diagnostics. In the supplier plan, the weighted triplet showed high support with modest residual indeterminacy, helping justify the reliability share. In the routing plan, the same view showed monotone improvement in availability as the policy share increased. In the forecast, conservative fusion correctly signaled the periods with weaker evidence by lowering support and widening the effect interval.

If the planner increased the penalty on indeterminacy in the objective, the optimization would naturally favor suppliers, lanes, and periods whose costs are both low and well-determined; this can be done without changing the feasibility model. If the planner preferred a different attitude to uncertainty in ranking, for example, penalizing indeterminacy more heavily than refutation, the scoring parameter can be adjusted, and the ranks would change while the underlying MILP or LP remains the anchor for decisions. Because the models here are linear, solutions were unique at the reported endpoints; when ties arise in larger instances, the triplet-based ranks provide a clean tiebreak that is aligned with data quality.

8. Conclusion and future work

This paper presented a supply-chain modeling approach that treats indeterminacy as a first-class quantity rather than a nuisance to be averaged away. By pairing every uncertain input with a triplet that records support, indeterminacy, and refutation, the models remained readable and auditable while still fitting into standard linear and mixed-integer programs. The supplier study showed that reliability shares can improve robustness and even reduce costs when they steer volume to sources that are both credible and competitive. The forecasting study showed that translating ambiguous signals into multiplier intervals reduces systematic error while offering calibrated coverage. The

routing study showed how to trade robust cost against plan-level availability in a way that can be tuned by policy and explained to nontechnical stakeholders. Across all studies, enforcing feasibility at the upper demand bound provided guaranteed service; neutrosophic analysis then clarified where uncertainty still mattered and how much it was worth to reduce it.

There are several natural extensions. A first direction is a multi-period model with explicit lead times and order crossovers, still carrying triplets forward so that late or conflicting scans appear as indeterminacy rather than noise. A second direction is a multi-echelon network with capacity coupling and transshipment, where path triplets would be built from arc triplets with either conservative bounds or explicit dependence models. A third direction is statistical: learn triplets directly from data by combining labeled events (support and refutation) with quality tags or missingness indicators (indeterminacy), then update them online as new evidence arrives. A fourth direction is decision policy design: automate the choice of reliability and availability shares by solving a small parametric program that targets a given budget or a desired risk score. Finally, from a theoretical point of view, it would be valuable to study the stability of optimal solutions under small changes in the triplets and to derive bounds that connect interval width and indeterminacy with worst-case regret, so planners can quantify the value of better data before launching a data-quality initiative.

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