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n-Cylindrical Neutrosophic M-open Sets

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Abstract. In this paper, we introduce the concept of an n-cylindrical neutrosophic M-open set, defined as the union of n-cylindrical neutrosophic $\delta\mathcal{P}$-open sets and n-cylindrical neutrosophic $\theta\mathcal{S}$-open sets within n-cylindrical neutrosophic topological spaces. We also explore various properties and characterizations of the n-cylindrical neutrosophic M-interior. Furthermore, we apply a suitable measure to a decision-making problem involving the calculation of incentives based on teacher performance during subject instruction.

Keywords: n-$CyNM$-open sets, n-$CyNM$-closed sets, n-$CyNMint$, and n-$CyNMcl$.

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1. Introduction and Preliminaries

Following Zadeh's introduction of fuzzy set (denoted as fs) in 1965 [22], Chang [3] developed the notion of fuzzy topological spaces (fts), which led to the adaptation of classical topological concepts within the framework of fuzzy topology by various researchers. A significant generalization of fuzzy sets, known as intuitionistic fuzzy set (ifs), was introduced by Atanassov in 1986 [2]. Building on this, Coker [4] introduced the concept of intuitionistic fuzzy topological spaces ($ifts$) based on ifs's. Jeon et al. [7] further investigated intuitionistic fuzzy continuity and pre-continuity within this framework.

With the advent of neutrosophy and neutrosophic sets by Smarandache [15,16], a new direction in uncertainty modeling emerged. Salama and Alblowi [10] introduced neutrosophic crisp set and neutrosophic topological spaces (Nts), extending $ifts$ and incorporating degrees of membership, indeterminacy, and non-membership for each element. Neutrosophic has formed the foundation for a broader class of theories that generalize both crisp and fuzzy structures.

Smarandache also introduced the concept of dependence degrees between fuzzy and neutrosophic components. Later, Arokiarani et al. [1] introduced the neutrosophic set (NS), wherein the sum of the three membership values does not exceed 3. In the same year, Veereswari [21] proposed the notion of neutrosophic topological spaces (Nts) and studied fundamental operations on them.

Saranya et al. [11] introduced the concept of n-cylindrical neutrosophic set (abbreviated as n-$CyNS$), characterized by α and γ as dependent components and β as an independent component. Apart from neutrosophic set (NS), n-$CyNS$ represents the most extensive generalization of fuzzy sets. In this framework, the membership functions positive (α), neutral (β), & negative (γ) satisfy the conditions $0 \leq \beta_{\varkappa} \leq 1$ & $0 \leq \alpha^n_{\varkappa}(l) + \gamma^n_{\varkappa}(l) \leq 1,\ n > 1$, where $n > 1$ is an integer.

Later, Saranya et al. [12] introduced the notion of n-CyN continuity for functions between two n-cylindrical neutrosophic topological spaces (n-$CyNts$). They also defined the n-CyN interior (n-$CyNint$) and n-CyN closure (n-$CyNcl$) of subsets within n-$CyNts$.

In a related development, El-Maghrabi and Al-Juhani [6] introduced the concept of M-open sets in topological spaces and investigated several of their properties. The class of M-open sets plays a significant role in topological theory due to its applicability across various branches of mathematics and real-world applications. Padma et al. [9] also found M-open sets in nano topological spaces. Vadivel et al. [17–19] discussed some open sets in fuzzy nano and neutrosophic nano topological spaces. Kalaiyarsan et al. [8] and Vadivel et al. [20] introduced M-open sets in fuzzy and neutrosophic nano topological spaces.

Entropy serves as a measure of the degree of uncertainty within a set, regardless of whether the set is fuzzy, intuitionistic, ambiguous, or otherwise imprecise. Given that n-$CyNS$'s are capable of representing uncertain data, it is natural to explore the concept of entropy within the context of n-$CyNS$'s. The first reference to entropy as a measure of fuzziness was made by Zadeh in 1965 [22]. Subsequently, De Luca and Termini [5] provided an axiomatic foundation for a non-probabilistic form of entropy.

The remaining sections of this dissertation are organized as follows: Section 2 provides a brief review of fundamental definitions related to ifs's, NS's and n-$CyNS$'s. In Section 3, we introduce the concepts of n-$CyMos$ and n-$CyNMint$ within the framework of n-$CyNts$, and investigate some of their basic properties with illustrative examples. Section 4 presents an entropy measure tailored for n-$CyNS$'s and discusses a real-world application scenario where this measure is effectively utilized. Finally, the conclusions are summarized in Section 5.

Definition 1.1. [12] Let $\varkappa$ & ξ be two n-CyN subsets of an n-$CyNts$. ξ is called neighbourhood of $\varkappa$ if $\exists$ an n-$CyNos$, $O \ni \varkappa \subset O \subset \xi$.

Proposition 1.2. [12] $\varkappa \subset X$ is n-$CyNos$ in (X, τ_{CyN}) iff it carries a neighbourhood of its subsets.

Definition 1.3. [12] Let (X, τ_{CyN}) be an n-$CyNts$ and let $\varkappa = \{\langle x, \alpha_\varkappa(x), \beta_\varkappa(x), \gamma_\varkappa(x)\rangle : x \in X\}$ is an n-$CyNS$ in X. Then, the n-cylindrical neutrosophic interior (resp. closure) (briefly, n-$CyNint$ (resp. n-$CyNcl$)) is defined as (i) n-CyN union of all n-CyN open subsets of X. ie, n-$CyNint(\varkappa) = \bigcup\{G : G \in \tau_{CyN}$ and $G \subseteq \varkappa\}$, (ii) n-$CyNcl(\varkappa) = \bigcap\{K : K \in \tau_{CyN}cs$ and $\varkappa \subseteq K\}$.

Clearly, n-$CyNint(\varkappa)$ is the biggest n-$CyNos$ that is contained by $\varkappa$, n-$CyNcl(\varkappa)$ is the smallest n-$CyNcs$ that contains $\varkappa$.

2. n-Cylindrical Neutrosophic M-open sets

In this section, we introduce the concept n-$CyNMos$ & investigate some related properties.

Definition 2.1. Let (X, τ_{CyN}) be an n-$CyNts$ and $\varkappa$ be an n-$CyNS$. Then,

(i) $\varkappa$ is called an n-CyN regular open (resp. closed) set (briefly, n-$CyNros$ (resp. n-$CyNrcs$)), if $\varkappa = n$-$CyNint(n$-$CyNcl(\varkappa)$ (resp. $\varkappa = n$-$CyNcl(n$-$CyNint(\varkappa))$,

(ii) n-$CyN\delta int(\varkappa) = \bigcup\{G|G$ is an n-$CyNros$ & $G \subseteq \varkappa\}$,

(iii) n-$CyN\delta cl(\varkappa) = \bigcap\{K|K$ is an n-$CyNrcs$ & $\varkappa \subseteq K\}$,

(iv) n-$CyN\theta int(\varkappa) = \bigcup\{n$-$CyNint(\xi) : \xi \subseteq \varkappa$ & ξ is a n-$CyNcs$ in $X\}$,

(v) n-$CyN\theta cl(\varkappa) = \bigcap\{n$-$CyNcl(\xi) : \varkappa \subseteq \xi$ & ξ is a n-$CyNos$ in $X\}$.

Definition 2.2. Let (X, τ_{CyN}) be an n-$CyNts$ and $\varkappa = \{\langle l, \alpha_\varkappa(l), \beta_\varkappa(l), \gamma_\varkappa(l)\rangle : l \in X\}$ be an n-$CyNS$ in X. A set $\varkappa$ is said to be n-CyN

(i) δ-open set (briefly, n-$CyN\delta os$), if $\varkappa = n$-$CyN\delta int(\varkappa)$,

(ii) δ-pre open set (briefly, n-$CyN\delta\mathcal{P}os$), if $\varkappa \subseteq n$-$CyNint(n$-$CyN\delta cl(\varkappa))$,

(iii) δ-semi open set (briefly, n-$CyN\delta\mathcal{S}os$), if $\varkappa \subseteq n$-$CyNcl(n$-$CyN\delta int(\varkappa))$,

(iv) θ-open set (briefly, n-$CyN\theta os$), if $\varkappa = n$-$CyN\theta int(\varkappa)$,

(v) θ-semi open set (briefly, n-$CyN\theta\mathcal{S}os$), if $\varkappa \subseteq n$-$CyNcl(n$-$CyN\theta int(\varkappa))$,

(vi) e-open set (briefly, n-$CyNeos$), if $\varkappa = n$-$CyNcl(n$-$CyN\delta int(\varkappa)) \cup n$-$CyNint(n$-$CyN\delta cl(\varkappa))$,

(vii) M-open set (briefly, n-$CyNMos$), if $\varkappa \subseteq n$-$CyNcl(n$-$CyN\theta int(\varkappa)) \cup n$-$CyNint(n$-$CyN\delta cl(\varkappa))$.

The complement of a n-$CyNMos$ (resp. n-$CyN\delta os$, n-$CyN\delta\mathcal{P}os$, n-$CyN\delta\mathcal{S}os$, n-$CyN\theta os$, n-$CyN\theta\mathcal{S}os$ & n-$CyNeos$) is called a n-$CyNM$ (resp. n-$CyN\delta$, n-$CyN\delta\mathcal{P}$, n-$CyN\delta\mathcal{S}$, n-$CyN\theta$, n-$CyN\theta\mathcal{S}$ & n-$CyNe$) closed set (briefly, n-$CyNMcs$ (resp. n-$CyN\delta cs$, n-$CyN\delta\mathcal{P}cs$, n-$CyN\delta\mathcal{S}cs$, n-$CyN\theta cs$, n-$CyN\theta\mathcal{S}cs$ & n-$CyNecs$)) in X.

The family of all n-$CyNMos$ (resp. n-$CyNMcs$, n-$CyN\delta os$, n-$CyN\delta cs$, n-$CyN\delta\mathcal{P}os$, n-$CyN\delta\mathcal{P}cs$, n-$CyN\delta\mathcal{S}os$, n-$CyN\delta\mathcal{S}cs$, n-$CyN\theta os$, n-$CyN\theta cs$, n-$CyN\theta\mathcal{S}os$, n-$CyN\theta\mathcal{S}cs$, n-$CyNeos$ & n-$CyNecs$) of X is denoted by n-$CyNMOS(X)$, (resp. n-$CyNMCS(X)$, n-$CyN\delta OS(X)$, n-$CyN\delta CS(X)$, n-$CyN\delta\mathcal{P}OS(X)$, n-$CyN\delta\mathcal{P}CS(X)$, n-$CyN\delta\mathcal{S}OS(X)$, n-$CyN\delta\mathcal{S}CS(X)$, n-$CyN\theta OS(X)$, n-$CyN\theta CS(X)$, n-$CyN\theta\mathcal{S}OS(X)$, n-$CyN\theta\mathcal{S}CS(X)$, n-$CyNeOS(X)$ & n-$CyNeCS(X)$).

Definition 2.3. Let (X, τ_{CyN}) be an n-$CyNts$ and $\varkappa = \{\langle l,\ \alpha_\varkappa(l),\ \beta_\varkappa(l),\ \gamma_\varkappa(l)\rangle : l \in X\}$ be an n-$CyNS$ in X. Then, the n-CyN

(i) M-interior (resp. n-$CyN\delta$-interior, n-$CyN\delta\mathcal{P}$-interior, n-$CyN\delta\mathcal{S}$-interior, n-$CyN\theta$-interior, n-$CyN\theta\mathcal{S}$-interior & n-$CyNe$-interior) of $\varkappa$ (briefly, n-$CyNMint(\varkappa)$ (resp. n-$CyN\delta int(\varkappa)$, n-$CyN\delta\mathcal{P}int(\varkappa)$, n-$CyN\delta\mathcal{S}int(\varkappa)$, n-$CyN\theta\ int(\varkappa)$, n-$CyN\theta\mathcal{S}int(\varkappa)$ & n-$CyNeint(\varkappa)$) is defined by n-$CyNMint(\varkappa)$ (resp. n-$CyN\delta int(\varkappa)$, n-$CyN\delta\mathcal{P}int(\varkappa)$, n-$CyN\delta\mathcal{S}int(\varkappa)$, n-$CyN\theta int(\varkappa)$, n-$CyN\theta\mathcal{S}int(\varkappa)$ & n-$CyNeint(\varkappa)$) $= \bigcup\{G : G \subseteq \varkappa$ and G is a n-$CyNMos$ (resp. n-$CyN\delta os$, n-$CyN\delta\mathcal{P}os$, n-$CyN\delta\mathcal{S}os$, n-$CyN\theta os$, n-$CyN\theta\mathcal{S}os$ & n-$CyNeos$) in $X\}$.

(ii) M-closure (resp. n-$CyN\delta$-closure, n-$CyN\delta\mathcal{P}P$-closure, n-$CyN\delta\mathcal{S}$-closure, n-$CyN\theta$-closure, n-$CyN\theta\mathcal{S}$-closure & n-$CyNe$-closure) of $\varkappa$ (briefly, n-$CyNMcl(\varkappa)$ (resp. n-$CyN\delta cl(\varkappa)$, n-$CyN\delta\mathcal{P}cl(\varkappa)$, n-$CyN\delta\mathcal{S}cl(\varkappa)$, n-$CyN\theta cl(\varkappa)$, n-$CyN\theta\mathcal{S}cl(\varkappa)$ & n-$CyNecl(\varkappa)$) is defined by n-$CyNMcl(\varkappa)$ (resp. n-$CyN\delta cl(\varkappa)$, n-$CyN\delta\mathcal{P}cl(\varkappa)$, n-$CyN\delta\mathcal{S}cl(\varkappa)$, n-$CyN\theta cl(\varkappa)$, n-$CyN\theta\mathcal{S}cl(\varkappa)$ & n-$CyNecl(\varkappa)$) $= \bigcap\{K : \varkappa \subseteq K$ and K is a n-$CyNMcs$ (resp. n-$CyN\delta cs$, n-$CyN\delta\mathcal{P}cs$, n-$CyN\delta\mathcal{S}cs$, n-$CyN\theta cs$, n-$CyN\theta\mathcal{S}cs$ & n-$CyNecs$) in $X\}$.

Proposition 2.4. Let (X, τ_{CyN}) be a n-$CyNts$. Then, every

(i) n-$CyN\theta os$ (resp. n-$CyN\theta cs$) is a n-$CyNos$ (resp. n-$CyNcs$),
(ii) n-$CyN\theta os$ (resp. n-$CyN\theta cs$) is a n-$CyN\theta\mathcal{S}os$ (resp. n-$CyN\theta\mathcal{S}cs$),
(iii) n-$CyN\theta\mathcal{S}os$ (resp. n-$CyN\theta\mathcal{S}cs$) is a n-$CyNMos$ (resp. n-$CyNMcs$),
(iv) n-$CyN\delta os$ (resp. n-$CyN\delta cs$) is a n-$CyNos$ (resp. n-$CyNcs$),
(v) n-$CyN\delta os$ (resp. n-$CyN\delta cs$) is a n-$CyN\delta\mathcal{P}os$ (resp. n-$CyN\delta\mathcal{P}cs$),
(vi) n-$CyN\delta os$ (resp. n-$CyN\delta cs$) is a n-$CyN\delta\mathcal{S}os$ (resp. n-$CyN\delta\mathcal{S}cs$),
(vii) n-$CyN\delta\mathcal{S}os$ (resp. n-$CyN\delta\mathcal{S}cs$) is a n-$CyNeos$ (resp. n-$CyNecs$),
(viii) n-$CyN\delta\mathcal{P}os$ (resp. n-$CyN\delta\mathcal{P}cs$) is a n-$CyNMos$ (resp. n-$CyNMcs$),

(ix) $n\text{-}CyNMos$ (resp. $n\text{-}CyNMcs$) is a $n\text{-}CyNeos$ (resp. $n\text{-}CyNecs$).

Proof.

(i) If S is a $n\text{-}CyN\theta os$ in U, then $S = n\text{-}CyN\theta int(S) \subseteq n\text{-}CyNint(S)$. Therefore, S is a $n\text{-}CyNos$.

(ii) If S is a $n\text{-}CyN\theta os$ in U, then $S = n\text{-}CyN\theta int(S)$. So, $S = n\text{-}CyN\theta int(S) \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(S))$. Therefore, S is a $n\text{-}CyN\theta\mathcal{S}os$.

(iii) If S is a $n\text{-}CyN\theta\mathcal{S}os$ in U, then $S \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(S))$. So, $S \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(S)) \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(S)) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(S))$. Therefore, S is a $n\text{-}CyNMos$.

(iv) If S is a $n\text{-}CyN\delta os$ in U, then $S = n\text{-}CyN\delta int(S) \subseteq n\text{-}CyNint(S)$. Therefore, S is a $n\text{-}CyNos$.

(v) If S is a $n\text{-}CyN\delta os$ in U, then $S = n\text{-}CyN\delta int(S)$. So, $S = n\text{-}CyN\delta int(S) \subseteq n\text{-}CyNint(S) \subseteq n\text{-}CyNint(n\text{-}CyN\delta cl(S))$. Therefore, S is a $n\text{-}CyN\delta\mathcal{P}os$.

(vi) If S is a $n\text{-}CyN\delta os$ in U, then $S = n\text{-}CyN\delta int(S)$. So, $S = n\text{-}CyN\delta int(S) \subseteq n\text{-}CyNcl(n\text{-}CyN\delta int(S))$. Therefore, S is a $n\text{-}CyN\delta\mathcal{S}os$.

(vii) If S is a $n\text{-}CyN\delta\mathcal{S}os$ in U, then $S \subseteq n\text{-}CyNcl(n\text{-}CyN\delta int(S)) \subseteq n\text{-}CyNcl((n\text{-}CyN\delta int(S))) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(S))$. Therefore, S is a $n\text{-}CyNeos$.

(viii) If S is a $n\text{-}CyN\delta\mathcal{P}os$ in U, then $S \subseteq n\text{-}CyNint(n\text{-}CyN\delta cl(S)) \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(S)) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(S))$. Therefore, S is a $n\text{-}CyNMos$.

(ix) If S is a $n\text{-}CyNMos$ then $S \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(S)) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(S))$. So, $S \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(S)) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(S)) \subseteq n\text{-}CyNint(n\text{-}CyN\delta cl(S) \subseteq n\text{-}CyNcl(n\text{-}CyN\delta int(S)) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(S)))$. Therefore S is a $n\text{-}CyNeos$. It is also true for their respective closed sets.

$\square$

Example 2.5. Let $X = \{x_1, x_2\}$ and the n-cylindrical neutrosophic sets $\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5, \varkappa_6, \varkappa_7$ in X are defined as

$$\varkappa_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\}$$

$$\varkappa_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$$

$$\varkappa_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\}$$

$$\varkappa_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$$

$$\varkappa_5 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1650736\rangle\}$$

$$\varkappa_6 = \{\langle x_1, 0.1450734, 0.50, 0.1250732\rangle, \langle x_2, 0.1450734, 0.50, 0.1450734\rangle\}$$

$$\varkappa_7 = \{\langle x_1, 0.1850738, 0.50, 0.1250732\rangle, \langle x_2, 0.1650736, 0.50, 0.1450734\rangle\}$$

Then, we have $\tau_{CyN} = \{0_{CyN}, 1_{CyN}, \varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}$ be a n-$CyNt$ on X. Then,

(i) $\varkappa_1$ is a n-$CyNos$ (resp.n-$CyFMos$)but not n-$CyN\theta os$ (resp. n-$CyN\theta\mathcal{S}os$)

(ii) $\varkappa_7$ is a n-$CyN\theta\mathcal{S}os$ (resp.n-$CyN\delta\mathcal{S}os$, n-$CyNMos$) but not n-$CyN\theta os$ (resp.n-$CyN\delta os$, n-$CyN\delta\mathcal{P}os$)

(iii) $\varkappa_4$ is a n-$CyNos$ (resp. n-$CyN\delta\mathcal{P}os$, n-$CyNeos$) but not n-$CyN\delta os$ (resp. n-$CyN\delta os$, n-$CyN\delta\mathcal{S}os$)

(iv) $\varkappa_6$ is a n-$CyNeos$ but not n-$CyNMos$

Remark 2.6. The diagram shows that n-$CyNMos$'s in n-$CyNts$.

Diagram 2.7. From the above Proposition 2.4, and the Example 2.5, the following implications are hold.

$$\begin{tikzcd}
n\text{-}CyN\theta os \arrow[rr] \arrow[d] & & n\text{-}CyNos \\
n\text{-}CyN\theta\mathcal{S}os \arrow[dd] & & n\text{-}CyN\delta os \arrow[u] \arrow[dl] \arrow[dd] \\
& n\text{-}CyN\delta\mathcal{P}os \arrow[dl] & \\
n\text{-}CyNMos \arrow[dr] & & n\text{-}CyN\delta\mathcal{S}os \arrow[dl] \\
& n\text{-}CyNeos &
\end{tikzcd}$$

Theorem 2.8. The statements are true.

(i) $n\text{-}CyN\theta\mathcal{S}cl(\varkappa) = \varkappa \cup n\text{-}CyNint(n\text{-}CyN\theta cl(\varkappa))$

(ii) $n\text{-}CyN\theta\mathcal{S}int(\varkappa) = \varkappa \cap n\text{-}CyNcl(n\text{-}CyN\theta int(\varkappa))$

(iii) $n\text{-}CyN\delta\mathcal{P}cl(\varkappa) = \varkappa \cup n\text{-}CyNcl(n\text{-}CyN\delta int(\varkappa))$

(iv) $n\text{-}CyN\delta\mathcal{P}int(\varkappa) = \varkappa \cap n\text{-}CyNint(n\text{-}CyN\delta cl(\varkappa))$

(v) $n\text{-}CyN\delta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa)) = n\text{-}CyN\delta\mathcal{P}int(\varkappa) \cup n\text{-}CyNcl(n\text{-}CyN\delta int(\varkappa))$

(vi) $n\text{-}CyN\delta\mathcal{P}int(n\text{-}CyN\delta\mathcal{P}cl(\varkappa)) = n\text{-}CyN\delta\mathcal{P}cl(\varkappa) \cap n\text{-}CyNint(n\text{-}CyN\delta cl(\varkappa))$

(vii) $n\text{-}CyN\delta\mathcal{S}int(\varkappa) = \varkappa \cap n\text{-}CyNcl(n\text{-}CyN\delta int(\varkappa))$

(viii) $n\text{-}CyN\delta\mathcal{S}cl(\varkappa) = \varkappa \cup n\text{-}CyNint(n\text{-}CyN\delta cl(\varkappa))$

(ix) $n\text{-}CyN\delta int(\varkappa) = n\text{-}CyN\delta cl(n\text{-}CyN\delta int(\varkappa))$

(x) $n\text{-}CyNint(n\text{-}CyN\delta cl(\varkappa)) = n\text{-}CyN\delta int(n\text{-}CyN\delta cl(\varkappa))$

Theorem 2.9. Let (X, τ_{CyN}) be a n-$CyNts$. Let $\varkappa$ be any n-CyN subset in X. Then,

(i) $n\text{-}CyNMcl(1_{CyN} - \varkappa) = 1_{CyN} - n\text{-}CyNMint(\varkappa)$ (or) $n\text{-}CyNMcl(\varkappa^c) = (n\text{-}CyNMint(\varkappa))^c$

(ii) $n\text{-}CyNMint(1_{CyN} - \varkappa) = 1_{CyN} - n\text{-}CyNMcl(\varkappa)$ (or) $n\text{-}CyNM(\varkappa^c) = (n\text{-}CyNMcl(\varkappa))^c$

Remark 2.10. Taking complements on either side of (i) & (ii) of Theorem 2.9, we get $n\text{-}CyNMint(\varkappa) = 1_{CyN} - n\text{-}CyNMcl(1_{CyN} - \varkappa)$ and $n\text{-}CyNMcl(\varkappa) = 1_{CyN} - n\text{-}CyNMint(1_{CyN} - \varkappa)$.

Theorem 2.11. Let (X, τ_{CyN}) be a $n\text{-}CyNts$ and $\varkappa \subseteq X$. Then, $\varkappa$ is a $n\text{-}CyNMos$ iff $\varkappa = n\text{-}CyN\theta\mathcal{S}int(\varkappa) \cup n\text{-}CyN\delta\mathcal{P}int(\varkappa)$.

Proof. Let $\varkappa$ be a $n\text{-}CyNMos$. Then, $\varkappa \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(\varkappa)) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(\varkappa))$. By Theorem 2.8, we have

$$\begin{aligned} n\text{-}CyN\delta\mathcal{P}int(\varkappa) \cup n\text{-}CyN\theta\mathcal{S}int(\varkappa) &= (\varkappa \cap (n\text{-}CyNcl(n\text{-}CyN\theta int(\varkappa)))) \cup (\varkappa \cap (n\text{-}CyNint(n\text{-}CyN\delta cl(\varkappa)))) \\ &= \varkappa \cap n\text{-}CyNcl(n\text{-}CyN\theta int(\varkappa))) \cup (n\text{-}CyNint(n\text{-}CyN\delta cl(\varkappa)) \\ &= \varkappa \end{aligned}$$

Conversely, if $\varkappa = n\text{-}CyN\theta\mathcal{S}int(\varkappa) \cup n\text{-}CyN\delta\mathcal{P}int(\varkappa)$ then, Theorem 2.8,

$$\begin{aligned} \varkappa &= n\text{-}CyN\theta\mathcal{S}int(\varkappa) \cup n\text{-}CyN\delta\mathcal{P}int(\varkappa) \\ &= (\varkappa \cap n\text{-}CyNcl(n\text{-}CyN\theta int(\varkappa))) \cup (\varkappa \cap n\text{-}CyNint(n\text{-}CyN\delta cl(\varkappa))) \\ &= \varkappa \cap (n\text{-}CyNcl(n\text{-}CyN\theta int(\varkappa)) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(\varkappa))) \\ &\subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(\varkappa)) \cup n\text{-}CyNint(n\text{-}CyN\delta cl(\varkappa)) \end{aligned}$$

and hence, $\varkappa$ is a $n\text{-}CyNMos$.

Theorem 2.12. Let (X, τ_{CyN}) be a $n\text{-}CyNts$ and $\varkappa \subseteq X$. Then, $\varkappa$ is a $n\text{-}CyNMcs$ iff $\varkappa = n\text{-}CyN\theta\mathcal{S}cl(\varkappa) \cap n\text{-}CyN\delta\mathcal{P}cl(\varkappa)$.

Proof. From Theorem 2.11, it follows. □

Theorem 2.13. The union (resp. intersection) of any family of $n\text{-}CyNMOS(X)$ (resp. $n\text{-}CyNMCS(X)$) is a $n\text{-}CyNMOS(X)$ (resp. $n\text{-}CyNMCS(X)$).

Remark 2.14. The intersection of two $n\text{-}CyNMos$'s need not be $n\text{-}CyNMos$.

Example 2.15.
Let $X = \{x_1, x_2\}$ and the n-cylindrical neutrosophic sets $\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5, \varkappa_6, \varkappa_7$ in X are

defined as

$$\varkappa_1 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1450734, 0.50, 0.1650736\rangle\}$$

$$\varkappa_2 = \{\langle x_1, 0.1150731, 0.50, 0.1950739\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$$

$$\varkappa_3 = \{\langle x_1, 0.1950739, 0.50, 0.1150731\rangle, \langle x_2, 0.1750737, 0.50, 0.1350733\rangle\}$$

$$\varkappa_4 = \{\langle x_1, 0.1250732, 0.50, 0.1850738\rangle, \langle x_2, 0.1350733, 0.50, 0.1750737\rangle\}$$

$$\varkappa_5 = \{\langle x_1, 0.1750737, 0.50, 0.1650736\rangle, \langle x_2, 0.1450734, 0.50, 0.1050730\rangle\}$$

$$\varkappa_6 = \{\langle x_1, 0.1550735, 0.50, 0.1050730\rangle, \langle x_2, 0.1950739, 0.50, 0.1750737\rangle\}$$

$$\varkappa_7 = \{\langle x_1, 0.1550735, 0.50, 0.1650736\rangle, \langle x_2, 0.1450734, 0.50, 0.1750737\rangle\}$$

Let $\tau_{CyN} = \{0_{CyN}, 1_{CyN}, \varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}$ be a $nCyNt$ on X, then $\varkappa_5$ and $\varkappa_6$ are n-$CyNMos$ but not $\varkappa_5 \cap \varkappa_6 = \varkappa_7$ is not n-$CyNMos$.

Proposition 2.16. Let (X, τ_{CyN}) be a n-$CyNts$. If $\varkappa$ is a

(i) n-$CyNMos$ and n-$CyN\theta int(\varkappa) = 0_{CyN}$, then $\varkappa$ is a n-$CyN\delta\mathcal{P}os$,

(ii) n-$CyNMos$ and n-$CyN\delta cl(\varkappa) = 0_{CyN}$, then $\varkappa$ is a n-$CyN\theta\mathcal{S}os$,

(iii) n-$CyNMos$ and n-$CyN\delta cs$, then $\varkappa$ is a n-$CyN\theta\mathcal{S}os$,

(iv) n-$CyN\theta\mathcal{S}os$ and n-$CyN\delta cs$, then $\varkappa$ is a n-$CyNMos$.

Theorem 2.17. $\varkappa$ is a n-$CyNMcs$ (resp. n-$CyNMos$), iff $\varkappa = n$-$CyNMcl(\varkappa)$ (resp. $\varkappa = n$-$CyNMint(\varkappa)$).

Proof. Suppose $\varkappa = n$-$CyNMcl(\varkappa) = \cap\{\xi : \varkappa \subseteq \xi$ & ξ is a n-$CyNMcs\}$. This means $\varkappa \in \cap\{\xi : \varkappa \subseteq \xi$ & ξ is a n-$CyNMcs\}$ and hence $\varkappa$ is n-$CyNMcs$. Conversely, suppose $\varkappa$ be a n-$CyNMcs$ in X. Then, we have $\varkappa \in \cap\{\xi : \varkappa \subseteq \xi$ & ξ is a n-$CyNMcs\}$. Hence, $\varkappa \subseteq \xi$ implies $\varkappa = \cap\{\xi : \varkappa \subseteq \xi$ & ξ is a n-$CyNMcs\} = n$-$CyNMcl(\varkappa)$. Similarly $\varkappa = n$-$CyNMint(\varkappa)$. □

Theorem 2.18. Let (X, τ_{CyN}) be a n-$CyNts$. Let $\varkappa$ & ξ in X, then the n-$CyNMint$ we have

(i) n-$CyNMint(0_{CyN}) = 0_{CyN}, n$-$CyNMint(1_{CyN}) = 1_{CyN}$,

(ii) n-$CyNMint(\varkappa)$ is a n-$CyNMos$ in X,

(iii) n-$CyNMint(\varkappa) \subseteq n$-$CyNMint(\xi)$ if $\varkappa \subseteq \xi$,

(iv) n-$CyNMint(n$-$CyNMint(\varkappa)) = n$-$CyNMint(\varkappa)$.

Theorem 2.19. Let (X, τ_{CyN}) be a n-$CyNts$. Let $\varkappa$ & ξ in X, then the n-$CyNMcl$ we have,

(i) n-$CyNMcl(0_{CyN}) = 0_{CyN}, n$-$CyNMcl(1_{CyN}) = 1_{CyN}$,

(ii) n-$CyNMcl(\varkappa)$ is a n-$CyNMcs$ in X,

(iii) n-$CyNMcl(\varkappa) \subseteq n$-$CyNMcl(\xi)$ if $\varkappa \subseteq \xi$,

(iv) n-$CyNMcl(n$-$CyNMcl(\varkappa)) = n$-$CyNMcl(\varkappa)$.

Proposition 2.20. Let (X, τ_{CyN}) be a n-$CyNts$. Let $\varkappa$ and ξ are n-CyN subsets in X, then

(i) $n\text{-}CyNMcl(\varkappa^c) = [n\text{-}CyNMint(\varkappa)]^c, n\text{-}CyNMint(\varkappa^c) = [n\text{-}CyNMcl(\varkappa)]^c$,

(ii) $n\text{-}CyNMcl(\varkappa \cup \xi) \supseteq n\text{-}CyNMcl(\varkappa) \cup n\text{-}CyNMcl(\xi), n\text{-}CyNMcl(\varkappa \cap \xi) \subseteq n\text{-}CyNMcl(\varkappa) \cap n\text{-}CyNMcl(\xi)$,

(iii) $n\text{-}CyNMint(\varkappa \cup \xi) \supseteq n\text{-}CyNMint(\varkappa) \cup n\text{-}CyNMint(\xi), n\text{-}CyNMint(\varkappa \cap \xi) \subseteq n\text{-}CyNMint(\varkappa) \cap n\text{-}CyNMint(\xi)$.

Proposition 2.21. Let (X, τ_{CyN}) be a n-$CyNts$. If $\varkappa$ is a n-$CyNS$ in X, then

(i) $n\text{-}CyNMcl(\varkappa) \supseteq n\text{-}CyNcl(n\text{-}CyN\delta int(\varkappa)) \cap n\text{-}CyNint(n\text{-}CyN\theta cl(\varkappa))$,

(ii) $n\text{-}CyNMint(\varkappa) \subseteq n\text{-}CyNcl(n\text{-}CyN\theta int(\varkappa)) \cup n\text{-}CyNint(\delta cl(\varkappa))$.

Theorem 2.22. Let (X, τ_{CyN}) be a n-$CyNts$. Let $\varkappa$ be a n-$CyNS$ in X, then

(i) $n\text{-}CyNMcl(\varkappa) = n\text{-}CyN\delta\mathcal{P}cl(\varkappa) \cap n\text{-}CyN\theta\mathcal{S}cl(\varkappa)$,

(ii) $n\text{-}CyNMint(\varkappa) = n\text{-}CyN\delta\mathcal{P}int(\varkappa) \cup n\text{-}CyN\theta\mathcal{S}int(\varkappa)$.

Proof.

(i) It is obvious that, $n\text{-}CyNMcl(\varkappa) \subseteq n\text{-}CyN\delta\mathcal{P}cl(\varkappa) \cap n\text{-}CyN\theta\mathcal{S}cl(\varkappa)$. Conversely, from Definition 2.1, we have

$$\begin{aligned} n\text{-}CyNMcl(\varkappa) &\supseteq n\text{-}CyNcl(n\text{-}CyN\delta int(n\text{-}CyNMcl(\varkappa))) \cap n\text{-}CyNint(n\text{-}CyN\theta cl(n\text{-}CyNMcl(\varkappa))) \\ &\supseteq n\text{-}CyNcl(n\text{-}CyN\delta int(\varkappa)) \cap n\text{-}CyNint(n\text{-}CyN\theta cl(\varkappa)) \end{aligned}$$

Since, $n\text{-}CyNMcl(\varkappa)$ is n-$CyNMcs$, By Theorem 2.8, we have

$$\begin{aligned} n\text{-}CyN\delta\mathcal{P}cl(\varkappa) \cap n\text{-}CyN\theta\mathcal{S}cl(\varkappa) &= (\varkappa \cup n\text{-}CyNcl(n\text{-}CyN\delta int(\varkappa))) \cap (\varkappa \cup n\text{-}CyNint(n\text{-}CyN\theta cl(\varkappa))) \\ &= \varkappa \cap (n\text{-}CyNcl(n\text{-}CyN\delta int(\varkappa)) \cap n\text{-}CyNint(n\text{-}CyN\theta cl(\varkappa))) \\ &= \varkappa \subseteq n\text{-}CyNMcl(\varkappa) \end{aligned}$$

Therefore, $n\text{-}CyNMcl(\varkappa) = n\text{-}CyN\delta\mathcal{P}cl(\varkappa) \cap n\text{-}CyN\theta\mathcal{S}cl(\varkappa)$.

(ii) It follows from (i).

Lemma 2.23. Let (X, τ_{CyN}) be n-$CyNts$ on X and $\varkappa$ be an n-$CyNS$ on X. Then,

(i) $n\text{-}CyN\mathcal{P}int(n\text{-}CyN\delta\mathcal{P}cl(\varkappa)) = n\text{-}CyN\delta\mathcal{P}cl(\varkappa) \cap n\text{-}CyNint(n\text{-}CyNcl(\varkappa))$ and $n\text{-}CyN\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa)) = n\text{-}CyN\delta\mathcal{P}int(\varkappa) \cup n\text{-}CyNcl(n\text{-}CyNint(\varkappa))$

(ii) $n\text{-}CyN\theta\mathcal{P}int(n\text{-}CyN\delta\mathcal{P}cl(\varkappa)) = n\text{-}CyN\delta\mathcal{P}cl(\varkappa) \cap n\text{-}CyNint(n\text{-}CyN\theta cl(\varkappa))$ and $n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa)) = n\text{-}CyN\delta\mathcal{P}int(\varkappa) \cup n\text{-}CyNcl(n\text{-}CyN\theta int(\varkappa))$

(iii) $n\text{-}CyN\theta\mathcal{S}cl(n\text{-}CyN\theta int(\varkappa)) = n\text{-}CyN\mathcal{S}cl(n\text{-}CyN\theta int(\varkappa)) = n\text{-}CyNint(n\text{-}CyNcl(n\text{-}CyN\theta int(\varkappa)))$

Proof. Obvious.

Proposition 2.24. Let $\varkappa$ be a n-CyN subsets of a n-$CyNts$ of (X, τ_{CyN}). Then,

(i) $n\text{-}CyNMcl(\varkappa) = \varkappa \cup n\text{-}CyN\theta\mathcal{P}int(n\text{-}CyN\delta\mathcal{P}cl(\varkappa))$

(ii) $n\text{-}CyNMint(\varkappa) = \varkappa \cap n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa))$

Proof.

(i) By Lemma 2.23,

$$\begin{aligned}
\varkappa \cup n\text{-}CyN\theta\mathcal{P}int(n\text{-}CyN\delta\mathcal{P}cl(\varkappa)) &= \varkappa \cup (n\text{-}CyN\delta\mathcal{P}cl(\varkappa) \cap n\text{-}CyNint(n\text{-}CyN\theta cl(\varkappa))) \\
&= \varkappa \cup (n\text{-}CyN\delta\mathcal{P}cl(\varkappa)) \cap (\varkappa \cup n\text{-}CyNint(n\text{-}CyN\theta cl(\varkappa))) \\
&= n\text{-}CyN\delta\mathcal{P}cl(\varkappa) \cap n\text{-}CyN\theta\mathcal{S}cl(\varkappa) \\
&= n\text{-}CyNMcl(\varkappa)
\end{aligned}$$

(ii) It follows from (i).

Theorem 2.25. Let (X, τ_{CyN}) be a n-$CyNts$. Let $\varkappa$ be in X, then

(i) $\varkappa$ is an n-$CyNMos$

(ii) $\varkappa \subseteq n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa))$

(iii) $n\text{-}CyN\theta\mathcal{P}cl(\varkappa) = n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa))$ are equivalent.

Proof.

(i)$\rightarrow$ (ii): Let $\varkappa$ be an n-$CyNMos$. Then by Theorem 2.18, $\varkappa = n\text{-}CyNMint(\varkappa)$ and by Proposition 2.24, $\varkappa = \varkappa \cap n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa))$ and hence, $\varkappa \subseteq n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa))$.

(ii) $\rightarrow$ (i): Let $\varkappa \subseteq n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa))$. Then by Proposition 2.21, $\varkappa \subseteq \varkappa \cap n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\ \mathcal{P}int(\varkappa)) = n\text{-}CyNMint(\varkappa)$. So, $\varkappa \subseteq n\text{-}CyNMint(\varkappa)$ and hence, $\varkappa$ is an n-$CyNMos$.

(ii)$\rightarrow$ (iii): Let $\varkappa \subseteq n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa))$. Then $n\text{-}CyN\theta\mathcal{P}cl(\varkappa) \subseteq n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa))$ and hence, $n\text{-}CyN\theta\mathcal{P}cl(\varkappa) = n\text{-}CyN\theta\mathcal{P}cl(n\text{-}CyN\delta\mathcal{P}int(\varkappa))$.

(iii) $\rightarrow$ (ii): Obvious.

Theorem 2.26. Let (X, τ_{CyN}) be a n-$CyNts$. Let $\varkappa$ be in X, then the following are equivalent.

(i) $\varkappa$ is an n-$CyNMcs$

(ii) $n\text{-}CyN\theta\mathcal{P}int(n\text{-}CyN\delta\mathcal{P}cl(\varkappa)) \subseteq \varkappa$

(iii) $n\text{-}CyN\theta\mathcal{P}int(\varkappa) = n\text{-}CyN\theta\mathcal{P}int(n\text{-}CyN\delta\mathcal{P}cl(\varkappa))$

Proof. From Theorem 2.25, it follows.

3. Application

Zadeh was the first to propose entropy as a fuzziness measures [22]. Several entropy measures were later defined by numerous mathematicians. The definition of an entropy measure for n-$CyNS$'s that links the degree of positive, negative, and neutral membership is the main goal of this section. We have used the suggested entropy measure, for instance, in the evaluation criteria of jury selection.

Definition 3.1. [14] Let $\varkappa = \{\langle \alpha_\varkappa, \beta_\varkappa, \gamma_\varkappa \rangle\}$ be an n-$CyNS$ in X. The new entropy measure for $\varkappa$ denoted by n-$CyN\mathbf{E}_{npy}(\varkappa)$, is a function, n-$CyN\mathbf{E}_{npy} : \tau_N(X) \to [0,1]$ and is defined as

$$n\text{-}CyN\mathbf{E}_{npy}(\varkappa) = 1 - \frac{1}{n}\sum_{i=1}^{n}(\alpha_\varkappa - \gamma_\varkappa)^2(1-\beta_\varkappa) \; ; \text{ for every } x_i \in \varkappa$$

Example 3.2. In order to ensure transparent jury selection, five jurors were selected from the pool, given five of their verdicts to review, asked to evaluate the cases during the voir dire, and asked to complete the rubrics.The tables indicate that n-CyN linguistic terms were employed in the jurors' evaluations. To choose the best juror in voir dire, we use the n-CyN entropy metric.

Table 4.1: Linguistic terms

Linguistic Terms	$\langle \alpha, \beta, \gamma \rangle$
Observe (♠)	$\langle 0.1950739, 0.70, 0.1150731 \rangle$
Query (♡)	$\langle 0.1850738, 0.60, 0.1150731 \rangle$
Advocate for clarification (♣)	$\langle 0.1750737, 0.50, 0.1250732 \rangle$
Suggest refinement (◇)	$\langle 0.1650736, 0.50, 0.1250732 \rangle$
Take exception to(¶)	$\langle 0.1550735, 0.50, 0.1550735 \rangle$
Decry (∗)	$\langle 0.1450734, 0.40, 0.1650736 \rangle$
Repudiate (•)	$\langle 0.1250732, 0.30, 0.1750737 \rangle$
Excoriate (↓)	$\langle 0.1150731, 0.20, 0.1950739 \rangle$

Juror - 1:

Table 4.2: critisim made by juror - 1

juries / **cases**	jury - 1	jury - 2	jury - 3
1	♡	♡	♠
2	♣	♣	♣
3	¶	¶	¶
4	♠	♠	♠
5	¶	¶	∗

Juror - 2:

Table 4.3: critisim made by juror - 2

juries / **cases**	jury - 1	jury - 2	jury - 3
1	♣	♣	♣
2	◇	♣	◇
3	◇	◇	¶
4	♠	♣	♣
5	∗	¶	∗

Juror - 3:

Table 4.4: critisim made by juror - 3

cases \ juries	jury - 1	jury - 2	jury - 3
1	$\heartsuit$	$\clubsuit$	$\clubsuit$
2	$\clubsuit$	$\clubsuit$	$\clubsuit$
3	$\diamondsuit$	¶	$\diamondsuit$
4	$\spadesuit$	$\spadesuit$	$\spadesuit$
5	$*$	¶	$*$

Juror - 4:

Table 4.5: critisim made by juror - 4

cases \ juries	jury - 1	jury - 2	jury - 3
1	$\bullet$	$\downarrow$	¶
2	$\heartsuit$	$\downarrow$	¶
3	$*$	$\clubsuit$	$\bullet$
4	$\diamondsuit$	¶	$\spadesuit$
5	$\clubsuit$	¶	$\diamondsuit$

Juror - 5:

Table 4.6: critisim made by juror - 5

cases \ juries	jury - 1	jury - 2	jury - 3
1	$\spadesuit$	$\heartsuit$	$\clubsuit$
2	$\downarrow$	$\bullet$	$*$
3	$\heartsuit$	$\clubsuit$	$\diamondsuit$
4	$\bullet$	¶	$\diamondsuit$
5	$\clubsuit$	$\diamondsuit$	¶

Juror - 1:

Table 4.7: n-CyN values of criticisim made by juror- 1

cases \ juries	jury - 1	jury - 2	jury - 3
1	$\langle 0.1850738, 0.60, 0.1150731\rangle$	$\langle 0.1850738, 0.60, 0.1150731\rangle$	$\langle 0.1950739, 0.70, 0.1150731\rangle$
2	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$
3	$\langle 0.1550735, 0.50, 0.1550735\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$
4	$\langle 0.1950739, 0.70, 0.1150731\rangle$	$\langle 0.1950739, 0.70, 0.1150731\rangle$	$\langle 0.1950739, 0.70, 0.1150731\rangle$
5	$\langle 0.1550735, 0.50, 0.1550735\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$	$\langle 0.1450734, 0.40, 0.1650736\rangle$

Juror - 2:

Table 4.8: n-CyN values of criticisim made by juror - 2

cases \ juries	jury - 1	jury - 2	jury - 3
1	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$
2	$\langle 0.1650736, 0.50, 0.1250732\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1650736, 0.50, 0.1250732\rangle$
3	$\langle 0.1650736, 0.50, 0.1250732\rangle$	$\langle 0.1650736, 0.50, 0.1250732\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$
4	$\langle 0.1950739, 0.70, 0.1150731\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$
5	$\langle 0.1450734, 0.40, 0.1650736\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$	$\langle 0.1450734, 0.40, 0.1650736\rangle$

Juror - 3:

Table 4.9: n-CyN values of criticisim made by juror - 3

cases \ juries	jury - 1	jury - 2	jury - 3
1	$\langle 0.1850738, 0.60, 0.1150731\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$
2	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$
3	$\langle 0.1650736, 0.50, 0.1250732\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$	$\langle 0.1650736, 0.50, 0.1250732\rangle$
4	$\langle 0.1950739, 0.70, 0.1150731\rangle$	$\langle 0.1950739, 0.70, 0.1150731\rangle$	$\langle 0.1950739, 0.70, 0.1150731\rangle$
5	$\langle 0.1450734, 0.40, 0.1650736\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$	$\langle 0.1450734, 0.40, 0.1650736\rangle$

Juror - 4:

Table 4.10: n-CyN values of criticisim made by juror - 4

juries / cases	jury - 1	jury - 2	jury - 3
1	$\langle 0.1250732, 0.30, 0.1750737\rangle$	$\langle 0.1150731, 0.20, 0.1950739\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$
2	$\langle 0.1850738, 0.60, 0.1150731\rangle$	$\langle 0.1150731, 0.20, 0.1950739\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$
3	$\langle 0.1450734, 0.40, 0.1650736\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1250732, 0.30, 0.1750737\rangle$
4	$\langle 0.1650736, 0.50, 0.1250732\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$	$\langle 0.1950739, 0.70, 0.1150731\rangle$
5	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$	$\langle 0.1650736, 0.50, 0.1250732\rangle$

Juror - 5:

Table 4.11: n-CyN values of criticisim made by juror- 5

juries / cases	jury - 1	jury - 2	jury - 3
1	$\langle 0.1950739, 0.70, 0.1150731\rangle$	$\langle 0.1850738, 0.60, 0.1150731\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$
2	$\langle 0.1150731, 0.20, 0.1950739\rangle$	$\langle 0.1250732, 0.30, 0.1750737\rangle$	$\langle 0.1450734, 0.40, 0.1650736\rangle$
3	$\langle 0.1850738, 0.60, 0.1150731\rangle$	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1650736, 0.50, 0.1250732\rangle$
4	$\langle 0.1250732, 0.30, 0.1750737\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$	$\langle 0.1650736, 0.50, 0.1250732\rangle$
5	$\langle 0.1750737, 0.50, 0.1250732\rangle$	$\langle 0.1650736, 0.50, 0.1250732\rangle$	$\langle 0.1550735, 0.50, 0.1550735\rangle$

Juror - 1:

Table 4.12: n-$CyN\mathbf{E}_{npy}$ criticisim by juror - 1

cases	n-$CyN\mathbf{E}_{npy}$
Case 1	0.9980532944
Case 2	0.998749975
Case 3	1
Case 4	0.9980799616
Case 5	0.9999199984

Juror - 2:

Table 4.13: n-$CyN\mathbf{E}_{npy}$ criticisim by juror - 2

cases	n-$CyN\mathbf{E}_{npy}$
Case 1	0.998749975
Case 2	0.98624985833
Case 3	0.98666653333
Case 4	0.9985266372
Case 5	0.9998399968

Juror - 3:

Table 4.14: n-$CyN\mathbf{E}_{npy}$ criticisim by juror r - 3

cases	n-$CyN\mathbf{E}_{npy}$
Case 1	0.9985133036
Case 2	0.998749975
Case 3	0.98666653333
Case 4	0.9980799616
Case 5	0.9998399968

Juror - 4:

Table 4.15: n-$CyN\mathbf{E}_{npy}$ criticisim by juror - 4

cases	n-$CyN\mathbf{E}_{npy}$
Case 1	0.9977099542
Case 2	0.9976399528
Case 3	0.9989199784
Case 4	0.99269325387
Case 5	0.99291659167

Juror - 5:

Table 4.16: n-$CyN\mathbf{E}_{npy}$ criticism by juror - 5

cases	n-$CyN\mathbf{E}_{npy}$
Case 1	0.9982899658
Case 2	0.9976299526
Case 3	0.99226324527
Case 4	0.99274992167
Case 5	0.99291659167

Here, we can see that all the n-$CyN\mathbf{E}_{npy}$ values of Jury - 2 are less than that of other juries. Thus evaluation done by Jury - 2 is more certain.

Hence, the ranking given in voir dire is given below as in Table 4.17

Table 4.17: Ranking based on n-$CyN\mathbf{E}_{npy}$

Jury	Rank
Jury - 1	V
Jury - 2	**I**
Jury - 3	IV
Jury - 4	III
Jury - 5	II

4. Conclusions

In this paper, we have studied the concepts of n-$CyNMos$ and n-$CyNMcs$, along with their corresponding interior and closure operators within n-$CyNts$. Fundamental properties of these structures have been explored and illustrated through examples in the same setting. As a potential direction for future research, these results may be extended to n-cylindrical neutrosophic M-continuous, M-open, and M-closed mappings in n-$CyNts$. Furthermore, we apply an entropy-based measure for solving a decision-making problem related to the jury selection in voir-dire. This measure aligns well with similar approaches used for fs's, ifs's, and related frameworks. Hence, the proposed entropy measure is a valid tool for quantifying uncertainty in such applications.

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