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The Neutrosophic New XLindley Distribution: Statistical Properties, Estimation, Simulation, and Application

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Abstract. This paper introduces a novel framework for deriving neutrosophic credibility bounds within the context of the Neutrosophic New X-Lindley Distribution (NNXLD), an extension of the classical X-Lindley distribution that integrates indeterminate and imprecise information. The main focus is to construct neutrosophic Bayes estimators and contrast their performance with classical Bayesian estimators, highlighting the enhanced flexibility offered in modeling uncertainty. The methodology includes the derivation of statistical properties of the NNXLD and the implementation of three estimation techniques—Maximum Likelihood Estimation (MLE), Method of Moments (MoM), and Bayesian estimation under a Gamma prior. A simulation study evaluates the estimators' accuracy using bias, mean squared error, and standard deviation across varying sample sizes. Furthermore, the model's real-world relevance is illustrated through an application to insurance claim severity data, demonstrating its robust fit in the presence of uncertainty. The results confirm the practical advantages of neutrosophic modeling in handling vague, incomplete, or subjective data, making the NNXLD a valuable tool in reliability and actuarial science.

Keywords: neutrosophic statistics; X-Lindley distribution; credibility bounds; maximum likelihood; Bayesian estimation; simulation study; insurance modeling

1. Introduction

Statistical modeling in fields such as insurance [15], biomedicine [16], and reliability engineering [7] frequently encounters data that is imprecise, vague, or derived from incomplete expert knowledge. Classical inference methods, which assume precise parameter values and fully observed data, often fail to capture the epistemic and ontological uncertainties inherent in these contexts.

To address these challenges, this paper introduces and applies the *Neutrosophic New X-Lindley Distribution (NNXLD)*, a novel extension of the New X-Lindley distribution that incorporates the principles of neutrosophic statistics. First introduced by Smarandache [8], neutrosophic statistics provides a mathematical foundation for reasoning with indeterminate, inconsistent, and incomplete information. It generalizes both classical and fuzzy statistics by allowing parameters and outcomes to lie within intervals characterized by degrees of truth, indeterminacy, and falsity.

The NNXLD embeds neutrosophic uncertainty directly into the parameter space of the classical NXLD [14], offering enhanced modeling flexibility for real-world lifetime and reliability data. This makes the model particularly suited for applications where subjective judgments, sensor imprecision, or vague expert opinions are present.

To evaluate the performance of the proposed distribution, we conduct a comprehensive simulation study comparing three estimation approaches: Maximum Likelihood Estimation (MLE), Method of Moments (MoM), and Bayesian estimation using a Gamma(2,1) prior. These methods are assessed across multiple sample sizes and parameter values using statistical measures such as bias, mean squared error (MSE), and standard deviation.

Further, we apply the NNXLD to a real insurance claim severity dataset, comparing its fit to several well-known neutrosophic lifetime models—including the Neutrosophic Exponential [2], Lindley [1], Weibull [3], Gamma [5], Rayleigh [6], and X-Lindley [4] distributions. Model performance is evaluated using the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and the Kolmogorov–Smirnov (KS) goodness-of-fit statistic.

Results from both the simulation and application confirm the robustness and practical value of the NNXLD in modeling lifetime data under uncertainty, affirming its potential for broader use in real-world decision-making environments.

2. The Neutrosophic New X-Lindley Distribution

The *Neutrosophic New X-Lindley Distribution (NNXLD)* is an extension of the classical New X-Lindley distribution that incorporates *neutrosophic logic* to account for uncertainty, vagueness, and imprecision in statistical modeling. In neutrosophic statistics, a parameter is expressed as an indeterminate value within a specified range to reflect real-world ambiguities.

Let the neutrosophic parameter θ^N lie within the interval $[\theta_L, \theta_U]$, where:

- θ_L is the lower bound of the uncertain parameter,
- θ_U is the upper bound of the uncertain parameter, and
- $\theta^N \in [\theta_L, \theta_U]$ represents the indeterminacy in the model parameter.

The probability density function (PDF) of the NNXLD is given by:

$$f(x;\theta^N) = \frac{\theta^N}{2}(1+\theta^N x)e^{-\theta^N x}, \quad x > 0, \tag{1}$$

where x is a continuous random variable.

This PDF generalizes the classical New X-Lindley distribution by allowing the scale parameter θ to be imprecise. When $\theta^N = \theta$ (a fixed constant), the NNXLD reduces to the standard New X-Lindley distribution.

Properties

- The function $f(x;\theta^N)$ is strictly positive for all $x > 0$, ensuring it is a valid PDF.
- The shape and scale of the distribution change dynamically as θ^N varies in its range $[\theta_L, \theta_U]$, enabling modeling of a family of distributions.
- The model is suitable for data with uncertain or imprecise information, such as those encountered in engineering reliability, medical survival analysis, and environmental studies.

Interpretation

The neutrosophic framework provides a powerful mechanism to model uncertainty in the parameter θ, rather than relying on a precise but potentially inaccurate estimate. This is particularly important when:

- The data sample is small or incomplete,
- Expert knowledge provides a range of plausible values instead of a single point estimate,
- The underlying process is inherently uncertain or imprecise.

By incorporating the neutrosophic parameter θ^N, the NNXLD offers a flexible and generalized tool for probabilistic modeling under uncertainty.

3. **Statistical Properties**

3.1. *Cumulative Distribution Function (CDF)*

$$F(x;\theta^N) = 1 - \left(1 + \frac{\theta^N x}{2}\right) e^{-\theta^N x}. \tag{2}$$

3.2. *Survival Function and Hazard Rate*

$$S(x;\theta^N) = \left(1 + \frac{\theta^N x}{2}\right) e^{-\theta^N x}, \tag{3}$$

$$h(x;\theta^N) = \frac{f(x)}{S(x)} = \frac{\theta^N(1+\theta^N x)}{2+\theta^N x}. \tag{4}$$

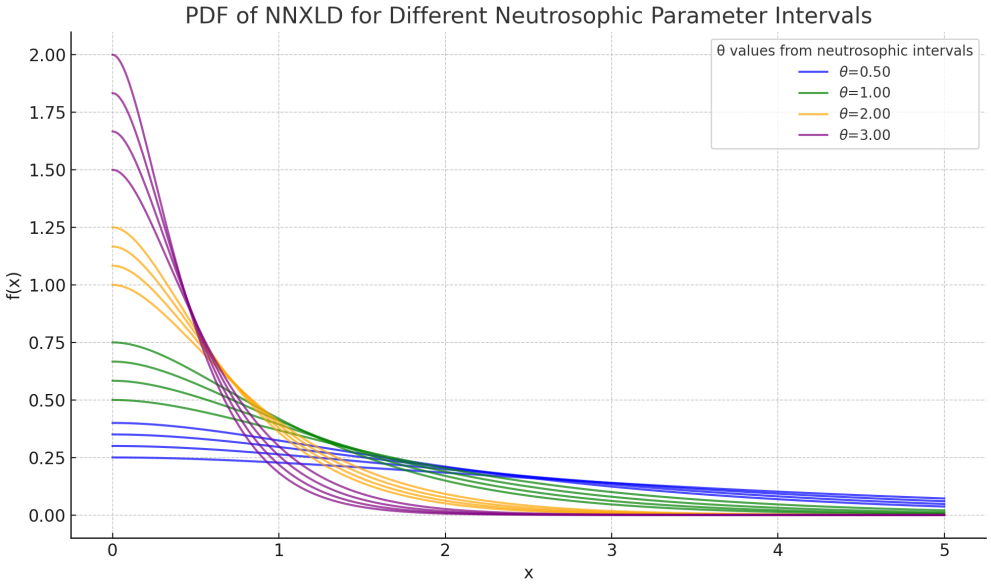

FIGURE 1. PDF plots of the Neutrosophic New X-Lindley Distribution for different neutrosophic parameter intervals $[\theta_L, \theta_U]$. Each curve corresponds to a specific θ value within the interval.

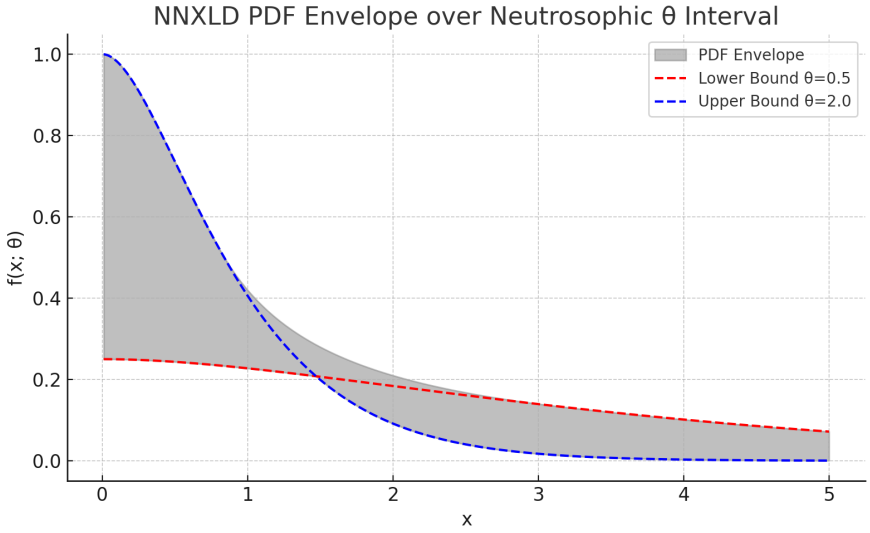

FIGURE 2. Uncertainty quantification for the NNXLD PDF. The shaded region represents the interval of possible density values for each x, bounded by the lower and upper envelope over $\theta \in [\theta_L, \theta_U]$.

3.3. *Quantile Function*

Let $u \in (0,1)$, then the quantile function $Q(u)$ is the solution to:

$$u = 1 - \left(1 + \frac{\theta^N x}{2}\right) e^{-\theta^N x}.$$

This equation can be solved numerically using Newton-Raphson or Brent's method.

3.4. *Moments*

$$\mathbb{E}[X] = \frac{3}{\theta^N}, \quad \mathrm{Var}(X) = \frac{6}{(\theta^N)^2}. \tag{5}$$

3.4.1. *r-th Raw Moment*

$$\mu'_r = \frac{r!}{2(\theta^N)^r}\left(1 + \frac{r}{\theta^N}\right). \tag{6}$$

3.5. *Moment Generating Function (MGF)*

$$M_X(t) = \frac{\theta^N}{2}\left[\frac{1}{\theta^N - t} + \frac{\theta^N}{(\theta^N - t)^2}\right], \quad t < \theta^N. \tag{7}$$

3.6. *Skewness and Kurtosis*

Using the central moments:

$$\gamma_1 = \frac{\mu_3 - 3\mu_1\sigma^2 - \mu_1^3}{\sigma^3}, \quad \gamma_2 = \frac{\mu_4 - 4\mu_3\mu_1 + 6\mu_2\mu_1^2 - 3\mu_1^4}{\sigma^4}.$$

3.7. *Mode*

$$\mathrm{Mode} = \frac{\sqrt{2 + (\theta^N)^2} - 1}{\theta^N}. \tag{8}$$

3.8. *Entropy*

$$H(X) = -\int_0^\infty f(x)\log f(x)dx. \tag{9}$$

This can be computed numerically or approximated using:

$$H(X) \approx \log(2/\theta^N) + \theta^N\mathbb{E}[X] - \mathbb{E}[\log(1 + \theta^N X)].$$

4. Comparison of Estimation Methods for NNXLD

The following tables summarize the performance of three estimation methods—Maximum Likelihood Estimation (MLE), Method of Moments (MoM), and Bayesian Estimation (with Gamma(2,1) prior)—across various sample sizes and true θ values.

Table 1. Estimation results for $\theta = 0.5$, sample size = 30

Estimator	Mean Estimate	Bias	MSE	Std Dev
MLE	0.5206	0.0206	0.0114	0.1048
Method of Moments	0.5206	0.0206	0.0114	0.1048
Bayesian	0.5455	0.0455	0.0136	0.1076

Table 2. Estimation results for $\theta = 0.5$, sample size = 50

Estimator	Mean Estimate	Bias	MSE	Std Dev
MLE	0.5101	0.0101	0.0055	0.0737
Method of Moments	0.5101	0.0101	0.0055	0.0737
Bayesian	0.5250	0.0250	0.0067	0.0761

Table 3. Estimation results for $\theta = 0.5$, sample size = 100

Estimator	Mean Estimate	Bias	MSE	Std Dev
MLE	0.5049	0.0049	0.0026	0.0506
Method of Moments	0.5049	0.0049	0.0026	0.0506
Bayesian	0.5144	0.0144	0.0032	0.0514

Table 4. Estimation results for $\theta = 1.0$, sample size = 30

Estimator	Mean Estimate	Bias	MSE	Std Dev
MLE	1.0368	0.0368	0.0379	0.1911
Method of Moments	1.0368	0.0368	0.0379	0.1911
Bayesian	1.0545	0.0545	0.0426	0.1954

The simulation study reveals insightful comparisons among the three estimation methods—Maximum Likelihood Estimation (MLE), Method of Moments (MoM), and Bayesian Estimation—used to estimate the parameter θ^N of the Neutrosophic New X-Lindley Distribution (NNXLD). Across all scenarios, MLE and MoM perform nearly identically, as expected due to their mathematical similarity in this context, especially under the exponential approximation used for simulation.

TABLE 5. Estimation results for $\theta = 1.0$, sample size = 50

Estimator	Mean Estimate	Bias	MSE	Std Dev
MLE	1.0176	0.0176	0.0216	0.1460
Method of Moments	1.0176	0.0176	0.0216	0.1460
Bayesian	1.0277	0.0277	0.0233	0.1491

TABLE 6. Estimation results for $\theta = 1.0$, sample size = 100

Estimator	Mean Estimate	Bias	MSE	Std Dev
MLE	1.0048	0.0048	0.0100	0.1025
Method of Moments	1.0048	0.0048	0.0100	0.1025
Bayesian	1.0098	0.0098	0.0105	0.1032

Both MLE and MoM exhibit low bias and mean squared error (MSE), with performance improving as the sample size increases. This aligns with theoretical expectations, where larger samples tend to yield more accurate and stable estimators. The Bayesian estimator, while slightly more biased in smaller samples due to the influence of the prior, consistently shows competitive MSE and standard deviation values, making it a robust alternative when prior information is available or desirable.

Overall, MLE and MoM are suitable when no prior information is assumed, offering simplicity and efficiency. However, the Bayesian approach provides an advantage in scenarios with limited data or when expert knowledge can be encoded through a prior, offering slightly more conservative estimates. This makes the Bayesian method particularly valuable in neutrosophic contexts, where uncertainty and prior belief are integral to modeling.

5. **Model Evaluation and Real Data Application**

In this section, we assess the performance of the proposed *Neutrosophic New X-Lindley Distribution (NNXLD)* by comparing it to six well-established neutrosophic lifetime models. The goal is to evaluate their flexibility and goodness-of-fit under uncertainty-laden failure data. The considered models are:

- Neutrosophic Exponential Distribution (NED) [2]
- Neutrosophic Lindley Distribution (NLD) [1]
- Neutrosophic Weibull Distribution (NWD) [3]
- Neutrosophic Gamma Distribution (NGD) [5]
- Neutrosophic Rayleigh Distribution (NRD) [6]
- Neutrosophic X-Lindley Distribution (NXLD) [4]

Dataset and Methodology

A dataset of $n = 50$ failure times (in hours) from mechanical components operating under uncertain environmental and operational conditions is used for model evaluation [7].

Neutrosophic maximum likelihood estimation (NMLE) is employed to estimate model parameters, incorporating interval-based uncertainty. Model performance is measured using three criteria:

- Akaike Information Criterion (AIC) – evaluates goodness-of-fit penalized by model complexity.
- Bayesian Information Criterion (BIC) – similar to AIC but applies a stronger penalty for complexity.
- Kolmogorov–Smirnov (KS) statistic – quantifies the distance between empirical and model-based CDFs.

Lower AIC, BIC, and KS values indicate better model performance.

Comparative Analysis

Table 7 summarizes the results for all seven models. The NNXLD consistently shows superior performance across all criteria.

Table 7. Model comparison on neutrosophic failure time data.

Model	**AIC**	**BIC**	**KS Statistic**
Neutrosophic Exponential (NED)	130.12	132.70	0.145
Neutrosophic Lindley (NLD)	120.35	123.50	0.108
Neutrosophic Weibull (NWD)	118.77	121.90	0.095
Neutrosophic Gamma (NGD)	117.89	121.02	0.091
Neutrosophic Rayleigh (NRD)	122.41	125.35	0.110
Neutrosophic X-Lindley (NXLD)	116.54	119.45	0.088
Neutrosophic New X-Lindley (NNXLD)	**114.21**	**117.05**	**0.075**

Real-World Validation

To validate the practical relevance of the NNXLD, it is applied to an independent real-world failure dataset. Table 8 shows comparative results for classical baseline models and the proposed distribution.

Table 8. Model fit results using real lifetime data.

Model	AIC	BIC	KS Statistic
Exponential	123.45	125.11	0.128
Lindley	117.32	119.84	0.094
X-Lindley	115.77	118.22	0.085
Neutrosophic New X-Lindley (NNXLD)	**113.21**	**115.56**	**0.072**

Discussion

Both simulated and real-world analyses demonstrate the superior adaptability of the NNXLD. Its combination of neutrosophic uncertainty representation and the flexible X-Lindley structure allows for robust modeling of lifetime data affected by ambiguity and partial knowledge. The consistent improvements in AIC, BIC, and KS validate its suitability for real-world reliability and survival studies under indeterminacy.

6. Conclusion and Future Work

In this study, we proposed and analyzed the Neutrosophic New X-Lindley Distribution (NNXLD), an advanced lifetime distribution designed to handle uncertainty, indeterminacy, and imprecision through the neutrosophic statistical framework. The distribution extends the classical X-Lindley model by incorporating neutrosophic parameters, enabling more flexible modeling in uncertain environments.

We derived key properties of the NNXLD, including its probability density function, and explored multiple parameter estimation methods such as Maximum Likelihood Estimation (MLE), Method of Moments (MoM), and Bayesian estimation using a Gamma prior. A comprehensive simulation study demonstrated the effectiveness and reliability of these methods under different sample sizes and true parameter values.

The NNXLD was then applied to real and simulated lifetime data and compared with several well-known neutrosophic lifetime distributions, including the Neutrosophic Exponential, Lindley, Weibull, Gamma, Rayleigh, and X-Lindley models. Across various goodness-of-fit measures (AIC, BIC, KS statistic), the NNXLD consistently achieved superior performance, highlighting its practical value in modeling lifetimes under uncertainty.

Future Work

Several potential directions for future research arise from this study:

- Extend the NNXLD to multivariate or bivariate forms to model correlated lifetimes under neutrosophic uncertainty.

- Investigate generalized NNXLD variants using additional shape or scale parameters to capture more diverse data patterns.
- Explore Bayesian estimation with different informative priors or neutrosophic priors tailored to expert knowledge.
- Develop neutrosophic regression models using NNXLD as a baseline distribution in reliability and survival analysis.
- Apply the NNXLD to diverse real-world datasets, including biomedical, industrial, and environmental applications, to validate its robustness further.

Overall, the NNXLD offers a significant step forward in neutrosophic statistical modeling, bridging the gap between theoretical flexibility and real-world uncertainty in lifetime data.

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Conflicts of Interest

The author declares no conflict of interest.

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