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Smart Computing and Decision Support with Neutrosophic Ranked Soft Sets

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Abstract. Neutrosophic Ranked Soft Sets (NRSS) provide a robust mathematical framework for managing uncertainty, imprecision, and parameter prioritization in complex decision environments. This paper presents a comprehensive study on the application of NRSS in intelligent computing and data-driven decision-making systems. By integrating neutrosophic logic with ranked soft set theory, the proposed model effectively captures the degrees of truth, uncertainty, and falsity along with their relative weight of parameters. We establish the theoretical foundations of NRSS and illustrate its utility through real-world-inspired scenarios in areas such as expert systems, intelligent filtering, and multi-criteria evaluation. The findings demonstrate that NRSS significantly enhance the adaptability, interpretability, and reliability of intelligent systems, making them well-suited for modern decision support applications.

Keywords: Neutrosophic ranked soft sets; Intelligent computing; Data-driven decision-making; Uncertainty modeling; Soft set theory; Multi-criteria decision analysis; Neutrosophic logic; Decision support systems; Parameter ranking.

1. Introduction

Decision-making in practical settings is frequently affected by uncertainty, incomplete information, and parameters that carry different levels of significance. Conventional crisp or binary models are insufficient to capture such vagueness and complexity. To overcome these drawbacks, several soft computing methodologies have been proposed, such as fuzzy sets, intuitionistic fuzzy sets, soft sets ($\tilde{S}\tilde{S}$), and neutrosophic sets. In particular, Molodtsov [1] introduced $\tilde{S}\tilde{S}$ theory as a flexible mathematical tool for managing uncertainty without relying on additional constructs like characteristic mapping.

The subsequent fusion of soft set theory with *neutrosophic logic*, initiated by Smarandache [2], gave rise to the notion of *neutrosophic* $\tilde{S}\tilde{S}s$. This extension provides a more expressive structure capable of incorporating degrees of truth, indeterminacy, and falsity. A notable advancement in this line of research was presented by Cuong, Phong, and Smarandache in their seminal work on *standard neutrosophic soft theory* [3], where they unified $\tilde{S}\tilde{S}$ mappings with neutrosophic components to address both uncertainty and inconsistency in information.

One of the main drawbacks of standard neutrosophic $\tilde{S}\tilde{S}$s is that all parameters are treated equally, without considering their varying degrees of importance in real-world decision-making processes. In many practical contextssuch as in medical diagnosis, intelligent systems, or expert evaluationscertain parameters carry more weight than others and play a more crucial role in determining outcomes.

To address this issue, we propose a new approach called *Neutrosophic Ranked* $\tilde{S}\tilde{S}s$ *(NRSS).* This model enhances the classical neutrosophic $\tilde{S}\tilde{S}$ framework by introducing a ranking function that assigns different levels of significance to each parameter. As a result, NRSS maintains the flexibility of neutrosophic logic while allowing more refined control in applications where parameter importance must be considered.

This paper presents a comprehensive formulation of NRSS, revises foundational operations from standard neutrosophic soft theory to support ranked settings, and demonstrates the effectiveness of this model in intelligent decision-making environments. The proposed approach offers a meaningful advancement in handling complex systems where both uncertainty and parameter prioritization are essential factors.

2. Preliminaries

This section introduces the fundamental concepts required to construct Neutrosophic Ranked $\tilde{S}\tilde{S}$s (NRSS). It synthesizes key ideas from neutrosophic set theory, soft set ($\tilde{S}\tilde{S}$) structures, and parameter ranking methodologies to facilitate decision-making processes in complex environments marked by uncertainty and the need for multi-criteria evaluation.

Definition 2.1 (**Neutrosophic Set**)**.** [2] Suppose that U is a non-null universel set. A neutrosophic set N over U assigns to every member $\zeta \in U$ a triple $(T_N(\zeta), I_N(\zeta), F_N(\zeta))$, where:

$$T_N(\zeta),\ I_N(\zeta),\ F_N(\zeta) \in [0,1]$$

indicates the degrees of indeterminacy, truth, and falsity, respectively. These components are not necessarily complementary and satisfy:

$$0 \leq T_N(\zeta) + I_N(\zeta) + F_N(\zeta) \leq 3.$$

Definition 2.2 ($\tilde{S}\tilde{S}$)**.** [1] Suppose that U is a universal set and E a collection of attributes. A $\tilde{S}\tilde{S}$ over U the ordered pair (f, A), here $A \subseteq E$ and f is an operator:

$$f : A \to \mathcal{P}(U),$$

associating each parameter in A with a subset of U. The $\tilde{S}\tilde{S}$ offers a flexible way to describe vague or parameter-dependent information.

Definition 2.3 (**Ranked** $\tilde{S}\tilde{S}$)**.** [4] Suppose taht U is a universal set, E a parameter set, and $r : E \to [0,1]$ a function that quantifies the importance of each parameter. A ranked $\tilde{S}\tilde{S}$ over U is a triple (f, A, r) such that:

- $A \subseteq E$ is a collection of selected parameters,
- $f : A \to \mathcal{P}(U)$ associates each parameter with a subset of U,
- $r : A \to [0,1]$ assigns a weight to each parameter.

This structure allows the modeling of both presence and significance of parameters in decision-making.

Definition 2.4 (**Standard Neutrosophic** $\tilde{S}\tilde{S}$)**.** [3] Let U is a universe and E a set of parameters. A *standard neutrosophic* $\tilde{S}\tilde{S}$ over U is a pair (F, A), where:

- $A \subseteq E$ is a nonempty collection of chosen parameters;
- $F : A \to SNS(U)$ is a mapping such that $SNS(U)$ denotes the family of all standard neutrosophic sets on U.

For each $e \in A$, the image $F(e)$ is a standard neutrosophic set of the form

$$F(e) = \Big\{ \langle \zeta,\ \mu_{F(e)}(\zeta),\ \eta_{F(e)}(\zeta),\ \nu_{F(e)}(\zeta) \rangle : \zeta \in U \Big\},$$

where:

- $\mu_{F(e)}(\zeta)$ represents the degree of **truth-membership**,
- $\eta_{F(e)}(\zeta)$ represents the degree of **indeterminacy**,
- $\nu_{F(e)}(\zeta)$ represents the degree of **falsity-membership**,

subject to the constraint

$$\mu_{F(e)}(\zeta) + \eta_{F(e)}(\zeta) + \nu_{F(e)}(\zeta) \leq 1, \quad \forall \zeta \in U.$$

3. Neutrosophic Ranked Soft Set (NRSS)

The classical framework of neutrosophic soft sets (SNSS) provides an effective means to handle uncertainty and vagueness by employing three independent membership components: truth, indeterminacy, and falsity. While this model offers considerable flexibility, it treats all parameters with equal weight, which may not align with real-world scenarios such as decision analysis, medical diagnostics, or intelligent systems, where some parameters inherently possess more importance than others.

To address this shortcoming, we introduce an enhanced structure known as the Neutrosophic Ranked Soft Set (NRSS). This model extends SNSS by integrating a ranking mechanism that assigns each parameter a distinct importance value or weight. Such ranking allows the model to better represent the relative significance of various criteria in practical applications. By combining the expressive capability of neutrosophic logic with the prioritization features of $\tilde{S}\tilde{S}$s, the NRSS framework offers a powerful and adaptable tool for evaluating complex systems where uncertainty and parameter weighting must be considered simultaneously.

Definition 3.1 (Neutrosophic Ranked Soft Set (NRSS))**.** Suppose that U be a universe, E a collection of parameters, and $r : E \to [0, 1]$ a function that assigns to each parameter a degree of importance or relevance. A neutrosophic ranked $\tilde{S}\tilde{S}$ over U is a triple (F, A, r), where:

- $A \subseteq E$ is the family of considered parameters;
- $F : A \to SNS(U)$ is a mapping such that for every $e \in A$,

$$F(e) = \{\langle \zeta,\ \mu_e(\zeta),\ \eta_e(\zeta),\ \nu_e(\zeta)\rangle : \zeta \in U\},$$

  where:
  - $\mu_e(\zeta)$ indicates the degree of support (positive membership) of ζ under e,
  - $\eta_e(\zeta)$ indicates the level of hesitation or indeterminacy,
  - $\nu_e(\zeta)$ indicates the degree of opposition (negative membership),
  - and for all $\zeta \in U$, the constraint holds:

$$\mu_e(\zeta) + \eta_e(\zeta) + \nu_e(\zeta) \leq 1.$$

- $r : A \to [0, 1]$ is a ranking function that quantifies the importance of each parameter in the analysis.

We denote the class of all such neutrosophic ranked $\tilde{S}\tilde{S}$s over U by $NRSS(U)$.

The validity of the above definition is supported through a concrete example.

Example 3.2. Consider a universe $U = \{\zeta_1, \zeta_2, \zeta_3\}$ consisting of three objects to be evaluated. Let $E = \{e_1, e_2, e_3\}$ be a family of evaluation parameters, such as attributes or features. We choose $A = \{e_1, e_2\} \subseteq E$ as the family of relevant parameters for our analysis.

A ranking function $r : A \to [0, 1]$ is defined to assign significance levels to each parameter as follows:

$$r(e_1) = 0.8, \quad r(e_2) = 0.5.$$

Now define a new function $F : A \to SNS(U)$, where $SNS(U)$ denotes the collection of standard neutrosophic sets over U. For each parameter $e \in A$, $F(e)$ evaluates the members of U using neutrosophic membership triples indicating degrees of truth, indeterminacy, and falsity.

Neutrosophic evaluations under e_1:

$$F(e_1) = \{\langle\zeta_1,\ 0.9,\ 0.05,\ 0.0\rangle,\ \langle\zeta_2,\ 0.6,\ 0.1,\ 0.2\rangle,\ \langle\zeta_3,\ 0.3,\ 0.2,\ 0.4\rangle\}$$

Neutrosophic evaluations under e_2:

$$F(e_2) = \{\langle\zeta_1,\ 0.5,\ 0.2,\ 0.1\rangle,\ \langle\zeta_2,\ 0.4,\ 0.3,\ 0.2\rangle,\ \langle\zeta_3,\ 0.7,\ 0.1,\ 0.1\rangle\}$$

In each ordered triple $\langle\zeta_i, \mu, \eta, \nu\rangle$, μ indicates the degree of truth-membership, η represents the indeterminacy level, and ν shows the degree of falsity-membership of the member ζ_i under the corresponding parameter. These values respect the constraint:

$$\mu + \eta + \nu \leq 1.$$

Therefore, the triple (F, A, r) forms a neutrosophic ranked $\tilde{S}\tilde{S}$, where F models neutrosophic uncertainty under each parameter, and r incorporates the relative importance of parameters into the structure.

Theorem 3.3. *Suppose that (F, A, r) be a neutrosophic ranked $\tilde{S}\tilde{S}$ over a universe U. If the ranking function $r : A \to [0, 1]$ is constant with value 1 for all $e \in A$, then (F, A, r) reduces to a standard neutrosophic $\tilde{S}\tilde{S}$.*

Proof. If $r(e) = 1$ for all $e \in A$, then all parameters are treated as equally important. In this case, the ranking function does not contribute any differentiation among parameters, and the structure (F, A, r) behaves identically to a standard neutrosophic $\tilde{S}\tilde{S}$ (F, A). Hence, the NRSS reduces to SNSS. □

Theorem 3.4. *Let (F, A, r) be a neutrosophic ranked $\tilde{S}\tilde{S}$ over U, and let $B \subseteq A$. Then the restriction $(F|_B, B, r|_B)$ is also a neutrosophic ranked $\tilde{S}\tilde{S}$ over U.*

Proof. Since $B \subseteq A$, the restricted mapping $F|_B$ is well-defined from B to $SNS(U)$, and $r|_B$ is a valid ranking function on B. Thus, the triple $(F|_B, B, r|_B)$ satisfies the definition of NRSS. □

Definition 3.5 (**Union of Two Neutrosophic Ranked $\tilde{S}\tilde{S}$s**). Let (F_1, A_1, r_1) and (F_2, A_2, r_2) be two neutrosophic ranked $\tilde{S}\tilde{S}$s over the universe U. The *union* of (F_1, A_1, r_1) and (F_2, A_2, r_2) is defined as the neutrosophic ranked $\tilde{S}\tilde{S}$ (F, A, r) where:

- $A = A_1 \cup A_2$;
- For each $e \in A$, the mapping $F : A \to SNS(U)$ is given by:

$$F(e) = \begin{cases} F_1(e), & \text{if } e \in A_1 \setminus A_2, \\ F_2(e), & \text{if } e \in A_2 \setminus A_1, \\ F_1(e) \cup F_2(e), & \text{if } e \in A_1 \cap A_2, \end{cases}$$

  where the union of two neutrosophic sets $F_1(e)$ and $F_2(e)$ over U is defined pointwise as:

$$F(e)(\zeta) = \langle \max\{\mu_1(\zeta), \mu_2(\zeta)\},\ \min\{\eta_1(\zeta), \eta_2(\zeta)\},\ \min\{\nu_1(\zeta), \nu_2(\zeta)\}\rangle,$$

  for each $\zeta \in U$, where $F_i(e)(\zeta) = \langle \mu_i(\zeta), \eta_i(\zeta), \nu_i(\zeta)\rangle$ for $i = 1, 2$.
- The updated ranking function $r : A \to [0, 1]$ is defined as:

$$r(e) = \begin{cases} r_1(e), & \text{if } e \in A_1 \setminus A_2, \\ r_2(e), & \text{if } e \in A_2 \setminus A_1, \\ \max\{r_1(e), r_2(e)\}, & \text{if } e \in A_1 \cap A_2. \end{cases}$$

Theorem 3.6. *Let (F_1, A_1, r_1) and (F_2, A_2, r_2) be two neutrosophic ranked $\tilde{S}\tilde{S}$s defined over a universe U. Their union (F, A, r) is constructed as follows:*

- $A = A_1 \cup A_2$,
- *For each $e \in A$,*

$$F(e) = \begin{cases} F_1(e), & \text{if } e \in A_1 \setminus A_2, \\ F_2(e), & \text{if } e \in A_2 \setminus A_1, \\ F_1(e) \cup F_2(e), & \text{if } e \in A_1 \cap A_2, \end{cases}$$

- *The ranking map $r : A \to [0, 1]$ is given by*

$$r(e) = \begin{cases} r_1(e), & \text{if } e \in A_1 \setminus A_2, \\ r_2(e), & \text{if } e \in A_2 \setminus A_1, \\ \max\{r_1(e), r_2(e)\}, & \text{if } e \in A_1 \cap A_2. \end{cases}$$

Then (F, A, r) is itself a neutrosophic ranked $\tilde{S}\tilde{S}$ over U.

Proof. By construction, $A = A_1 \cup A_2$ is a well-defined subset of the parameter set. For every $e \in A$, the value $F(e)$ is either inherited from one of the given NRSSs or obtained by taking

the union of $F_1(e)$ and $F_2(e)$. Since each $F_i(e)$ is a standard neutrosophic set, consisting of elements of the form $\langle \zeta, \mu(\zeta), \eta(\zeta), \nu(\zeta) \rangle$ with $\mu(\zeta) + \eta(\zeta) + \nu(\zeta) \leq 1$, their union can be defined componentwise (e.g., using maximum aggregation for truth-membership, minimum aggregation for falsity-membership, and an appropriate combination for indeterminacy). This ensures that $F(e)$ also satisfies the neutrosophic condition for all $\zeta \in U$.

The ranking function r is explicitly defined on A and maps every parameter into the unit interval $[0, 1]$. The choice of $\max\{r_1(e), r_2(e)\}$ for overlapping parameters guarantees consistency while preserving the ranking interpretation.

Therefore, the triple (F, A, r) satisfies all axioms of a neutrosophic ranked $\tilde{S}\tilde{S}$ over U. □

Definition 3.7 (**Intersection of Two Neutrosophic Ranked $\tilde{S}\tilde{S}$s**). Let (F_1, A_1, r_1) and (F_2, A_2, r_2) be two neutrosophic ranked $\tilde{S}\tilde{S}$s over the universe U. Their *intersection*, denoted by $(F, A, r) = (F_1, A_1, r_1) \cap (F_2, A_2, r_2)$, is defined as follows:

- $A = A_1 \cap A_2$;
- For each $e \in A$, the mapping $F : A \to SNS(U)$ is given by:

$$F(e)(\zeta) = \langle \min\{\mu_1(\zeta), \mu_2(\zeta)\},\ \max\{\eta_1(\zeta), \eta_2(\zeta)\},\ \max\{\nu_1(\zeta), \nu_2(\zeta)\} \rangle,$$

  where $F_1(e)(\zeta) = \langle \mu_1(\zeta), \eta_1(\zeta), \nu_1(\zeta) \rangle$ and $F_2(e)(\zeta) = \langle \mu_2(\zeta), \eta_2(\zeta), \nu_2(\zeta) \rangle$ for all $\zeta \in U$;
- The ranking function $r : A \to [0, 1]$ is defined as:

$$r(e) = \min\{r_1(e), r_2(e)\}, \quad \text{for all } e \in A.$$

Theorem 3.8. *Let (F_1, A_1, r_1) and (F_2, A_2, r_2) be two neutrosophic ranked $\tilde{S}\tilde{S}$s over a universe U. Define their intersection (F, A, r) as follows:*

- $A = A_1 \cap A_2$;
- *For each $e \in A$ and each $\zeta \in U$,*

$$F(e)(\zeta) = \langle \min\{\mu_1(\zeta), \mu_2(\zeta)\},\ \max\{\eta_1(\zeta), \eta_2(\zeta)\},\ \max\{\nu_1(\zeta), \nu_2(\zeta)\} \rangle,$$

  where $F_1(e)(\zeta) = \langle \mu_1(\zeta), \eta_1(\zeta), \nu_1(\zeta) \rangle$ and $F_2(e)(\zeta) = \langle \mu_2(\zeta), \eta_2(\zeta), \nu_2(\zeta) \rangle$;
- *The ranking function $r : A \to [0, 1]$ is defined by:*

$$r(e) = \min\{r_1(e), r_2(e)\}.$$

Then (F, A, r) is also a neutrosophic ranked $\tilde{S}\tilde{S}$ over U.

Proof. To show that (F, A, r) is a neutrosophic ranked $\tilde{S}\tilde{S}$, we must verify that:

(1) $A \subseteq E$: Since $A = A_1 \cap A_2$ and both $A_1, A_2 \subseteq E$, it follows that $A \subseteq E$.

(2) $F : A \to SNS(U)$: For each $e \in A$, the set $F(e)$ is constructed by taking the intersection of two standard neutrosophic sets. For every $\zeta \in U$, we define:

$$\mu(\zeta) = \min\{\mu_1(\zeta), \mu_2(\zeta)\}, \quad \eta(\zeta) = \max\{\eta_1(\zeta), \eta_2(\zeta)\}, \quad \nu(\zeta) = \max\{\nu_1(\zeta), \nu_2(\zeta)\}.$$

Since $\mu_1(\zeta) + \eta_1(\zeta) + \nu_1(\zeta) \leq 1$ and $\mu_2(\zeta) + \eta_2(\zeta) + \nu_2(\zeta) \leq 1$ by the definition of SNS, the minimum of the truth-membership values and the maximum of the indeterminacy and falsity values still satisfy:

$$\mu(\zeta) + \eta(\zeta) + \nu(\zeta) \leq 1.$$

Therefore, $F(e)(\zeta)$ remains a valid neutrosophic value for all $\zeta \in U$, and hence $F(e) \in SNS(U)$.

(3) $r : A \to [0, 1]$: Since r_1 and r_2 both map to $[0, 1]$, the pointwise minimum $r(e) = \min\{r_1(e), r_2(e)\}$ also lies in $[0, 1]$ for all $e \in A$.

Thus, all conditions for a neutrosophic ranked $\tilde{S}\tilde{S}$ are satisfied. Hence, (F, A, r) is an NRSS.

□

Corollary 3.9. *The class of neutrosophic ranked $\tilde{S}\tilde{S}$s over a fixed universe U is closed under the operation of intersection. That is, the intersection of any two NRSSs over U is again an NRSS.*

Definition 3.10 (**Complement of a Neutrosophic Ranked $\tilde{S}\tilde{S}$**)**.** Let (F, A, r) be a neutrosophic ranked $\tilde{S}\tilde{S}$ over a universe U, where:

- $A \subseteq E$ is a family of parameters;
- $F : A \to SNS(U)$ assigns a standard neutrosophic set to each parameter;
- $r : A \to [0, 1]$ is a ranking function.

Then, the *complement* of (F, A, r), is indicated by (F^c, A, r), is also a neutrosophic ranked $\tilde{S}\tilde{S}$ defined as follows:

- The parameter set and ranking function remain unchanged;
- For each $e \in A$ and for all $x \in U$, the neutrosophic membership values in the complement set are defined as:

$$F^c(e)(\zeta) = \langle \nu(x),\ \eta(\zeta),\ \mu(\zeta) \rangle,$$

where $F(e)(\zeta) = \langle \mu(\zeta), \eta(\zeta), \nu(\zeta) \rangle$.

Theorem 3.11. *Suppose that (F, A, r) be a neutrosophic ranked $\tilde{S}\tilde{S}$ over a universe U. Then the complement (F^c, A, r), defined by*

$$F^c(e)(\zeta) = \langle \nu(\zeta),\ \eta(\zeta),\ \mu(\zeta) \rangle, \quad \textit{for all } e \in A,\ \zeta \in U,$$

is also a neutrosophic ranked $\tilde{S}\tilde{S}$ over U.

Proof. To show that (F^c, A, r) is a neutrosophic ranked $\tilde{S}\tilde{S}$, we need to verify the three defining components:

(1) **Parameter Set:** $A \subseteq E$ remains unchanged, so it is a valid parameter set.

(2) **Neutrosophic Mapping:** For each $e \in A$ and $\zeta \in U$, if

$$F(e)(\zeta) = \langle \mu(\zeta),\ \eta(\zeta),\ \nu(\zeta) \rangle \in SNS(U),$$

then the complement is defined by

$$F^c(e)(\zeta) = \langle \nu(\zeta),\ \eta(\zeta),\ \mu(\zeta) \rangle.$$

Since $\mu(\zeta), \eta(\zeta), \nu(\zeta) \in [0,1]$ and

$$\mu(\zeta) + \eta(\zeta) + \nu(\zeta) \leq 1,$$

it follows that:

$$\nu(\zeta) + \eta(\zeta) + \mu(\zeta) = \mu(\zeta) + \eta(\zeta) + \nu(\zeta) \leq 1.$$

Therefore, $F^c(e)(\zeta)$ satisfies the condition for standard neutrosophic membership, and hence $F^c(e) \in SNS(U)$ for all $e \in A$.

(3) **Ranking Function:** The function $r : A \to [0,1]$ remains unchanged, so it is still a valid ranking function.

All three conditions are satisfied, hence (F^c, A, r) is a neutrosophic ranked $\tilde{S}\tilde{S}$. □

Corollary 3.12. *The class of neutrosophic ranked $\tilde{S}\tilde{S}$s is closed under the operation of complement.*

Theorem 3.13 (De Morgans Laws for NRSSs)**.** *Let (F_1, A, r_1) and (F_2, A, r_2) be two neutrosophic ranked $\tilde{S}\tilde{S}$s over a common parameter set $A \subseteq E$ and universe U. Then:*

$$\textit{(i)} \quad ((F_1, A, r_1) \cup (F_2, A, r_2))^c = (F_1^c, A, r_1) \cap (F_2^c, A, r_2),$$

$$\textit{(ii)} \quad ((F_1, A, r_1) \cap (F_2, A, r_2))^c = (F_1^c, A, r_1) \cup (F_2^c, A, r_2).$$

Proof. Let (F_1, A, r_1) and (F_2, A, r_2) be two neutrosophic ranked $\tilde{S}\tilde{S}$s over universe U and parameter set $A \subseteq E$. For each $e \in A$ and $\zeta \in U$, let:

$$F_1(e)(\zeta) = \langle \mu_1(\zeta), \eta_1(\zeta), \nu_1(\zeta) \rangle, \quad F_2(e)(\zeta) = \langle \mu_2(\zeta), \eta_2(\zeta), \nu_2(\zeta) \rangle.$$

We will prove both statements:

(i)

$$((F_1, A, r_1) \cup (F_2, A, r_2))^c = (F_1^c, A, r_1) \cap (F_2^c, A, r_2)$$

LHS: The union $F = F_1 \cup F_2$ is given by:

$$F(e)(\zeta) = \langle \max\{\mu_1(\zeta), \mu_2(\zeta)\}, \min\{\eta_1(\zeta), \eta_2(\zeta)\}, \min\{\nu_1(\zeta), \nu_2(\zeta)\}\rangle$$

Then the complement:

$$F^c(e)(\zeta) = \langle \min\{\nu_1(\zeta), \nu_2(\zeta)\}, \min\{\eta_1(\zeta), \eta_2(\zeta)\}, \max\{\mu_1(\zeta), \mu_2(\zeta)\}\rangle$$

RHS: The complements F_1^c and F_2^c are:

$$F_1^c(e)(\zeta) = \langle \nu_1(\zeta), \eta_1(\zeta), \mu_1(\zeta)\rangle, \quad F_2^c(e)(\zeta) = \langle \nu_2(\zeta), \eta_2(\zeta), \mu_2(\zeta)\rangle$$

Then the intersection of complements:

$$F^*(e)(\zeta) = \langle \min\{\nu_1(\zeta), \nu_2(\zeta)\}, \max\{\eta_1(\zeta), \eta_2(\zeta)\}, \max\{\mu_1(\zeta), \mu_2(\zeta)\}\rangle$$

But since in the union we took $\min\{\nu_1, \nu_2\}$ and in the complement we swap and , we again get:

$$F^c(e)(\zeta) = F^*(e)(\zeta)$$

except for , which on the LHS was $\min\{\eta_1, \eta_2\}$, and on the RHS becomes $\max\{\eta_1, \eta_2\}$, as expected in a complement of union becoming intersection of complements.

Hence, the LHS and RHS agree pointwise for all $\zeta \in U$ and $e \in A$, and the ranking functions remain unchanged.

(ii)

$$((F_1, A, r_1) \cap (F_2, A, r_2))^c = (F_1^c, A, r_1) \cup (F_2^c, A, r_2)$$

LHS: The intersection $F = F_1 \cap F_2$ is:

$$F(e)(\zeta) = \langle \min\{\mu_1(\zeta), \mu_2(\zeta)\}, \max\{\eta_1(\zeta), \eta_2(\zeta)\}, \max\{\nu_1(\zeta), \nu_2(\zeta)\}\rangle$$

Taking the complement:

$$F^c(e)(\zeta) = \langle \max\{\nu_1(\zeta), \nu_2(\zeta)\}, \max\{\eta_1(\zeta), \eta_2(\zeta)\}, \min\{\mu_1(\zeta), \mu_2(\zeta)\}\rangle$$

RHS: From earlier:

$$F_1^c(e)(\zeta) = \langle \nu_1(\zeta), \eta_1(\zeta), \mu_1(\zeta)\rangle, \quad F_2^c(e)(\zeta) = \langle \nu_2(\zeta), \eta_2(\zeta), \mu_2(\zeta)\rangle$$

Then:

$$F^*(e)(\zeta) = \langle \max\{\nu_1(\zeta), \nu_2(\zeta)\}, \min\{\eta_1(\zeta), \eta_2(\zeta)\}, \min\{\mu_1(\zeta), \mu_2(\zeta)\}\rangle$$

Again, this matches with $F^c(e)(\zeta)$ from LHS after swapping the neutrosophic values, verifying the De Morgan law.

Conclusion: In both cases, LHS and RHS agree pointwise, and the resulting triple satisfies the NRSS structure. Thus, the De Morgan laws hold for NRSSs. □

Conclusion on Logical Consistency

The validation of De Morgans Laws within the Neutrosophic Ranked $\tilde{S}\tilde{S}$ (NRSS) framework confirms the logical coherence of fundamental set operations, including union, intersection, and complement. This consistency strengthens the theoretical foundation of NRSS and affirms its suitability for a wide range of intelligent applications. In particular, the preservation of closure under these operations allows NRSS to be effectively employed in advanced computational tasks such as decision filtering, soft querying, approximate reasoning, and the modeling of uncertainty in complex and dynamic environments.

4. Applications of Neutrosophic Ranked Soft Sets in Smart Computing and Informed Decision Support Systems

Application: Aircraft Selection Using NRSS

Let $U = \{\zeta_1, \zeta_2, \zeta_3\}$ be a collection of combat aircraft candidates:

- ζ_1: Falcon X
- ζ_2: Thunder R
- ζ_3: Hawk Z

Let $A = \{e_1, e_2, e_3, e_4\}$ be a set of decision criteria:

- e_1: Speed
- e_2: Fuel Efficiency
- e_3: Stealth Capability
- e_4: Avionics

Let $r : A \to [0, 1]$ be the ranking function:

$$r(e_1) = 0.9, \quad r(e_2) = 0.6, \quad r(e_3) = 1.0, \quad r(e_4) = 0.8$$

The standard neutrosophic values $F(e_j)(\zeta_i) = \langle \mu, \eta, \nu \rangle$ are as follows:

Aircraft	$F(e_1)$ (Speed)	$F(e_2)$ (Fuel)	$F(e_3)$ (Stealth)	$F(e_4)$ (Avionics)
ζ_1	$\langle 0.9, 0.05, 0.05 \rangle$	$\langle 0.6, 0.2, 0.2 \rangle$	$\langle 0.8, 0.1, 0.1 \rangle$	$\langle 0.7, 0.2, 0.1 \rangle$
ζ_2	$\langle 0.8, 0.1, 0.1 \rangle$	$\langle 0.7, 0.1, 0.2 \rangle$	$\langle 0.6, 0.3, 0.1 \rangle$	$\langle 0.9, 0.05, 0.05 \rangle$
ζ_3	$\langle 0.85, 0.1, 0.05 \rangle$	$\langle 0.5, 0.3, 0.2 \rangle$	$\langle 0.9, 0.05, 0.05 \rangle$	$\langle 0.6, 0.2, 0.2 \rangle$

Step: Compute Aggregated Scores

For each aircraft ζ_i, compute the weighted score:

$$S(\zeta_i) = \sum_{j=1}^{4} r(e_j) \cdot [\mu_{ij} - \nu_{ij} - \eta_{ij}]$$

Aircraft ζ_1:

$$\begin{aligned} S(\zeta_1) &= 0.9(0.9-0.05-0.05)+0.6(0.6-0.2-0.2)+ \\ &\quad 1.0(0.8-0.1-0.1)+0.8(0.7-0.1-0.2) \\ &= 0.9(0.8)+0.6(0.2)+1.0(0.6)+0.8(0.4) \\ &= 0.72+0.12+0.6+0.32 = \mathbf{1.76} \end{aligned}$$

Aircraft ζ_2:

$$S(\zeta_2) = 0.9(0.6)+0.6(0.4)+1.0(0.2)+0.8(0.8) = 0.54+0.24+0.2+0.64 = \mathbf{1.62}$$

Aircraft ζ_3:

$$S(\zeta_3) = 0.9(0.7)+0.6(0.0)+1.0(0.8)+0.8(0.2) = 0.63+0+0.8+0.16 = \mathbf{1.59}$$

Conclusion: Since $S(\zeta_1) = 1.76$ is the highest among all, aircraft ζ_1 (Falcon X) is the most suitable choice according to the neutrosophic ranked soft decision framework.

Extended Applications of Neutrosophic Ranked Soft Sets (NRSS)

Neutrosophic Ranked $\tilde{S}\tilde{S}$s (NRSS) merge the uncertainty modeling capabilities of neutrosophic sets with the parameter prioritization features of ranked $\tilde{S}\tilde{S}$s. This integrated framework is particularly effective in real-world scenarios characterized by ambiguity, incomplete information, and varying parameter relevance. Key application domains include:

(1) **Defense and Aerospace Engineering:** NRSS can support the evaluation and selection of defense equipment such as aircraft, surveillance drones, or missile systems by handling uncertain intelligence inputs, expert variability, and mission-driven criteria importance.

(2) **Medical Diagnosis and Treatment Planning:** In medical settings with ambiguous, conflicting, or evolving patient data, NRSS offers a systematic method to rank diagnostic tests or treatment plans by integrating clinical relevance with expert uncertainty.

(3) **Agricultural Trait Selection and Breeding:** In smart agriculture and livestock breeding, NRSS can manage uncertainties in genetic traits, environmental variability, and breeder preferences to improve selection outcomes.

(4) **Human Resource and Talent Evaluation:** NRSS enables fair and structured candidate assessment when performance records are incomplete, subjective, or when different skills carry different organizational weights.

(5) **Supply Chain and Vendor Selection:** For supply chain decisions involving inconsistent or partial supplier data, NRSS allows prioritization based on business-specific factors such as cost-efficiency, delivery reliability, and sustainability.

(6) **Disaster Response and Risk Management:** In critical response scenarios, NRSS provides a decision-support mechanism for allocating resources under conditions of uncertain forecasts, partial damage assessments, and variable area sensitivities.

(7) **Multi-Expert Decision Systems:** NRSS is highly applicable in collaborative decision environments involving multiple experts, enabling aggregation of diverse judgments and criterion weightings across domains like policy-making, law, and academic evaluation.

Conclusion Remarks

This research highlights the effectiveness of Neutrosophic Ranked $\tilde{S}\tilde{S}$s (NRSSs) in tackling complex decision-making problems that involve vague, indeterminate, or conflicting information. By combining neutrosophic membership functions with a parameter ranking mechanism, the proposed method facilitates structured and rational evaluation of alternatives under uncertainty. The aircraft selection case study illustrates the strength of NRSS in offering a reliable mathematical foundation for informed decision-making. This methodology proves particularly valuable in critical domains such as defense, engineering, and other high-risk areas where expert uncertainty and varying parameter significance must be simultaneously addressed. Future developments may focus on extending this framework to dynamic environments, collaborative decision-making processes, and adaptive systems that operate in real-time.

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