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Representation of motivational dynamics in school environments through Plithogenic n-SuperHyperGraphs with family participation

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Abstract. The study aimed to expand on motivational phenomena that transpires at the school level via Plithogenic n-SuperHyperGraphs via family guidance as one of the dominant features in establishing effective school/motivational environments. The study was conducted via a mixed-method quantitative and qualitative approach, including a questionnaire sent to parents/students, semi-structured interviews with teachers, and an observational approach for the grounded theory. All data were reflected via Plithogenic n-SuperHyperGraphs due to the interconnectedness of phenomena of motivation and pedagogical approaches and family guidance. The analysis of the study suggests that more motivated students are those who have more guided parents as they exhibit better engagement in class, motivation to learn, and homework completion; conversely, those who do not have family/school guidance exhibit negative attitudes and decreased independence. Furthermore, the graphical representations illustrated crucial phenomena as the constant communication between parent and teacher is required. Ultimately, the study concludes that Plithogenic n-SuperHyperGraphs can effectively represent educational motivation as multimodal and nonlinear through the study of additional relational occurrences that can provide additional intervention opportunities. Furthermore, the study provides a need for institutionalization of family guidance efforts that strengthen the school-family bond as means for better motivating efforts and outcomes in school.

Keywords: Academic Motivation, School-Family, Parental Involvement, N- Plithogenic Superhypergraphs , School Dynamics, Pedagogical Strategies, Neutrosophic Representation.

1. Introduction

Academic motivation is one of the most determining factors for academic success and the overall development of students, as it directly influences class participation, academic performance, and the development of sustainable study habits over time. However, a recurring phenomenon is observed in contemporary educational settings: a large proportion of students display low levels of interest and commitment to their school activities, which translates into attitudes of disinterest, lack of autonomy, and poor utilization of learning opportunities. This situation raises the question of how to represent, understand, and enhance motivational dynamics from an approach that integrates the multiple variables involved in the educational process, including the fundamental role of the family.

Various studies have shown that motivation does not depend exclusively on school factors, but also on the family and social conditions surrounding the student. Studies such as that of Ryan and Deci have shown that intrinsic and extrinsic motivation are strongly influenced by the quality of the interaction between parents, teachers and students, underlining the need for integrative models that go beyond unidimensional approaches [1]. Likewise, recent research highlights that parental involvement is a significant predictor of academic engagement, although traditional strategies for measuring these effects often lack mechanisms to capture the complexity of the relationships between school and family [2].

At the theoretical level, educational literature has explored different conceptual frameworks to understand motivation. Vygotsky's Sociocultural Theory and Deci and Ryan 's Self-Determination Theory have been widely applied to explain how social interaction and autonomy influence learning. However, these approaches have limitations when it comes to formally and mathematically representing the multiple simultaneous interactions that occur between students, families, and teachers [3]. This gap suggests the need for more robust tools that allow capturing complex dynamics from an interdisciplinary perspective.

Empirical evidence also shows that lack of motivation in elementary school students is often associated with poor communication between parents and teachers, a lack of study routines at home, and students' disinterest in school content. A study conducted in Latin America found that students who receive consistent support from their families develop greater autonomy and a willingness to learn compared to those whose parents show limited involvement [4]. However, few studies have formally represented these relationships to visualize how different factors interact in academic motivation.

From a methodological perspective, educational research has begun to incorporate mathematical models and computational tools as innovative resources for analyzing complex social phenomena. Graphs and networks, for example, have been used to represent social interactions in educational communities, demonstrating that relationship structures largely determine academic outcomes [5]. Even so, most of these applications are limited to traditional networks, which fail to fully reflect the multidimensionality and uncertainty inherent in the motivational process.

In this context, Plithogenic n-SuperHyperGraphs emerge as a novel alternative for representing educational systems, as they allow for the integration of multiple actors, variables, and relationships within a single analytical framework. Unlike classical graphs, plithogenic superhypergraphs incorporate the management of contradictions, uncertainties, and multiple degrees of interaction, offering a more faithful representation of motivational dynamics. Applied to the school environment, these models make it possible to identify patterns of influence, critical nodes, and relationships that can enhance or limit student motivation, always considering the active role of the family in the process.

The justification for this study lies in the need for tools that allow us to understand academic motivation as a complex and multidimensional phenomenon. In an educational setting characterized by high levels of demotivation and school dropout, having a formal model that represents school-family interaction not only contributes to educational theory but also offers practical input for designing intervention programs such as Schools for Parents, personalized tutoring, and pedagogical strategies more tailored to students' realities [6,7].

The overall objective of this research is to model and represent motivational dynamics in school settings using Plitogenic n-SuperHyperGraphs, integrating family engagement as an essential factor in strengthening learning. Specific objectives include identifying key factors of academic motivation, structuring a conceptual framework that connects school and family, applying Plitogenic n-SuperHyperGraphs to visualize these interactions, and analyzing the results to formulate educational recommendations.

Thus, the central hypothesis proposes that the formal representation of motivational dynamics through Plithogenic n-SuperHyperGraphs allows not only the identification of relevant nodes of influence, but also the proposal of more effective strategies to improve academic motivation in middle school students. This implies recognizing that motivation is the result of a complex network where school and family actions intertwine and mutually reinforce each other, rather than acting in isolation.

In summary, this research is part of an effort to combine educational and mathematical approaches in the analysis of academic motivation, contributing to both the field of education and the formal modeling of social phenomena. The use of Plithogenic n-SuperHyperGraphs is presented as an innovative methodological resource that not only expands the boundaries of theoretical knowledge but also offers practical tools for decision-making in educational institutions seeking to strengthen the school-family relationship as a cornerstone of academic motivation.

2. Materials and methods

2.1. Motivational Dynamics in School Environments.

Motivational dynamics in school settings constitute a central field of study within educational psychology and educational sciences, as it directly influences academic performance, school retention, and the construction of meaningful learning. Motivation, understood as the set of internal and external processes that activate, direct, and sustain behavior toward specific goals, has been the subject of multiple theoretical approaches. In the school context, its analysis takes on special relevance as it is considered a determining factor for the quality of learning and for the comprehensive development of students in societies characterized by increasing educational complexity and diversity.

Contemporary models of motivation are based on cognitive, socio-affective, and sociocultural perspectives that seek to explain how individuals regulate their effort and persistence in school tasks. Among these approaches, Self-Determination Theory has established itself as a robust framework, distinguishing between intrinsic, extrinsic, and amotivation. Recent studies have shown that the satisfaction of the needs for autonomy, competence, and social relatedness significantly predict academic engagement and student well-being in various school contexts [8].

Empirical research from the last five years highlights that student motivation is not a static phenomenon, but rather a dynamic one influenced by multiple factors, such as classroom climate, teacher expectations, and parental support. In particular, it has been documented that a learning environment that promotes cooperation and recognition strengthens intrinsic motivation, while an overemphasis on external assessment tends to reduce it [9]. This evidence suggests that building motivating spaces requires a balance between academic demands and socioemotional support.

The role of the family in school motivational dynamics has gained increasing interest in the literature. Recent research confirms that active parental involvement is a significant predictor of academic performance and persistence, especially during the educational transition. However, it has also been noted that excessive parental interference or control can have the opposite effect, diminishing students' autonomy and intrinsic interest [10]. This finding reinforces the importance of designing interventions aimed at training families as agents of balanced academic support.

The technological dimension has also transformed the study of motivation in school settings. The incorporation of digital learning platforms and the expansion of e- learning have generated new motivational opportunities and challenges. On the one hand, access to interactive resources favors autonomous learning and self-efficacy; on the other, overexposure to digital stimuli can disperse attention and generate dependence on external rewards [11]. These aspects have driven the development of hybrid frameworks that integrate technological factors into motivational analysis.

At the methodological level, advances in statistical modeling and computational analysis techniques have allowed a more precise approach to motivational dynamics. The use of multilevel models and structural equations has facilitated the study of interactions between individual, contextual, and cultural variables. Furthermore, recent approaches based on data mining and machine learning have begun to be applied to identify motivational patterns in large volumes of educational data, expanding the predictive capacity of research [12].

The literature has also highlighted the need to consider equity and diversity in the study of academic motivation. Recent research underscores that differences in gender, socioeconomic status, and cultural affiliation significantly influence motivational profiles. Ignoring these dimensions can lead to biased interpretations and ineffective interventions [13]. In this sense, there is an urgent need to integrate intercultural and inclusive approaches in the construction of explanatory and practical models of motivation.

In terms of practical implications, the current findings suggest that promoting academic motivation should be based on pedagogical strategies that foster autonomy, content relevance, and the connection between learning and daily life. Intervention programs that involve teachers and families in building positive motivational climates have shown significant effects in strengthening academic engagement and reducing school dropout rates [14].

Accumulating evidence confirms that motivational dynamics in school settings cannot be reduced to a single factor, but rather constitute a complex system in which individual, family, institutional, and technological variables interact. This systemic view requires the development of integrative models that allow for more precise representation and analysis of these interactions. Emerging proposals include approaches based on network theory, dynamic graphs, and plithogenic modeling, which provide new tools for understanding motivation as a multifactorial and constantly evolving process.

In conclusion, the study of motivational dynamics in school settings is expanding toward more complex and multidimensional models that recognize the combined influence of personal, family, pedagogical, and technological factors. Recent research reinforces the idea that academic motivation is essential to ensuring meaningful learning and successful educational trajectories. As future directions, we recommend exploring computational methodologies that allow for the visualization of motivational influence networks and strengthen empirical evidence in contexts of sociocultural diversity.

2.2. n- Plitogenic superhypergraphs

This section contains two subsections, the first one is dedicated to explaining the basic notions of the n- Plithogenic SuperHyperGraphs defined in [15]. Then, subsection 2.2 contains the main concepts of multiway contingency tables and the log-linear method.

Plithogenic n-SuperHyperGraphs were defined by Smarandache in the field of decision making in [15].First, an n- SuperHyperGraph is defined as follows [24-26]:

Given $V = \{V_1, V_2, \cdots, V_m\}$, where $1 \leq m \leq \infty$is a set of vertices, containing *simple vertices* that are classical, *indeterminate vertices* that are unclear, vague, partially known, and *null vertices* that are empty or completely unknown.

$P(V)$is the power set of$V$ including $\emptyset$. $P^n(V)$is the n-potential set of V, which is defined recursively as follows:

$P^1(V) = P(V)$, $P^2(V) = P(P(V))$, $P^3(V) = P(P^2(V))$, … , $P^n(V) = P(P^{n-1}(V))$, for $1 \leq n \leq \infty$.
Where is also defined as $P^0(V) = V$.

An n- SuperHyperGraph (n -SHG) is an ordered pair $n - SHG = (G_n, E_n)$, where$G_n \subseteq P^n(V)$ and $E_n \subseteq P^n(V)$, for $1 \leq n \leq \infty$. Such that, $G_n$is the set of vertices and $E_n$is the set of edges.

$G_n$contains all possible types of vertices as in the real world:

- *Simple vertices* (the classic ones),
- *Indeterminate vertices* (unclear, vague, partially known),
- *Null vertices* (empty, completely unknown),
- *SuperVertex* (or *SubsetVertex*) contains two or more vertices of the above types grouped together (organization).
- *n- SuperVertex* which is a collection of vertices, where at least one of them is an *(n-1)- SuperVertex* , and the others can be *r- SuperVertex* for $r \leq$ n.

  $E_n$contains the following types of borders:
- *Simple edges* (the classic ones),
- *Indeterminate borders* (unclear, vague, partially known),
- *Null edges* (totally unknown, empty),
- *HyperEdge* (connecting three or more individual vertices),

- *SuperEdge* (connecting two vertices, at least one of which is a SuperVertex),
- *n- SuperEdge* (connecting two vertices, at least one of which is an n- SuperVertex and may contain another which is an r- SuperVertex with $r \leq$ n).
- *SuperHyperEdge* (connects three or more vertices, where at least one of them is a SuperVertex),
- *n- SuperHyperEdge* (contains three or more vertices, at least one of which is an n- SuperVertex and may contain an r- SuperVertex with $r \leq$ n),
- *MultiEdge* (two or more edges connecting the same two vertices),
- *Loop* (an edge that connects an element to itself),

The graphics are classified as follows:

- Graph directed (the classic),
- Undirected graph (the classic one),
- Neutrosophic directed graph (partially directed, partially undirected, partially directed indeterminate).

Within the framework of the theory of n- Plithogenic SuperHyperGraphs , we have the following concepts:

Enclosing vertex: A vertex that represents an object comprising attributes and sub-attributes in the graphical representation of a multi-attribute decision-making environment.

Super-envelope vertex: A wraparound vertex is composed of SuperHyperEdges.

Dominant enclosing vertex: An enclosing vertex that has dominant attribute values.

Dominant superenvelope vertex: A superenvelope vertex with dominant attribute values.

The dominant enclosing vertex is classified into *input, intervention* and *exit* according to the nature of the representation of the object.

Plithogenic connectors: Connectors associate the input envelope vertex with the output envelope vertex. These connectors associate the effects of input attributes with those of output attributes and are weighted according to the plithogenic weights.

2.3. Multi-way contingency tables

A multivariate contingency table is a contingency table defined for two or more cross-ratio classification variables. Two-dimensional tables are usually referred to as contingency tables, while the term multivariate is applied when the number of variables is at least three [26, 28].

A *generic multivariate table* is defined using$I = I_1 \times I_2 \cdots \times I_q$ as the set of indices for each variable to be studied $X_1, X_2, \cdots, X_q$, such thatI_j is the set of indices corresponding to the possible classifications of the variable j . Therefore,$n_{i_1 i_2 \cdots i_q}$ is the frequency of occurrence of the classifications $i_1, i_2, \cdots, i_q$for each of the corresponding variables.

Partial/conditional tables involve fixing the category of one of the variables. Fixed variables are indicated in parentheses. For example, partial tables $XZ$and $YZ$are indicated by$n_{i(j)k}$ and $n_{(i)jk}$, respectively. Furthermore, the *partial/conditional probabilities* are calculated by $\pi_{ij(k)} = \pi_{ij/k} = Prob(X = i, Y = j/Z = k)$. *The partial/conditional proportions* are defined by$p_{ij(k)} = p_{ij/k} = \frac{\pi_{ijk}}{\pi_{++k}}$ for $k = 1, 2, \ldots, K$. Whereπ_{++k} is the frequency $i$and $j$configuration k, for more information see [26, 28].

Next, we briefly explain what log-linear models consist of. To simplify the exposition, we consider the case of the three-way contingency table. If X, Y, and $Z$are the variables, then the following possible models are obtained [16, 17]:

- Model (X, Y, Z): All variables are considered independent, the model is as follows:
  $\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z (1)$
- Model (X, YZ): Only the YZ association is considered, while X is independent of the other two variables.
  $\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{jk}^{YZ} (2)$
- Model (XY, YZ): X and Z are independent for each value of Y:

$\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{jk}^{YZ}$(3)

- Model (XY, YZ, XZ): There is a pairwise association between all variables, but there is no joint association between the three.
  $\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ}$(4)
- Model (XYZ): If the above model does not fit the data well, then the association between the three variables should be considered:
  $\ln F_{ij} = \lambda + \lambda_i^X + \lambda_j^Y + \lambda_k^Z + \lambda_{ij}^{XY} + \lambda_{ik}^{XZ} + \lambda_{jk}^{YZ} + \lambda_{ijk}^{XYZ}$(5)

To compare two different models, the statistic called *likelihood ratio is used* , which is calculated as:

$G^2 = 2\sum fln(f/F)$ (6)

Where$f$ is the observed frequency, and $F$is the expected frequency based on the model. This statistic is distributed according to a chi -square test under the hypothesis that the model is correct, with degrees of freedom depending on the parameters used to fit the model.

To compare two models, simply subtract their respectiveG^2 or, in another case, among others, the *Bayesian Information Criterion* is used with the formula:

$BIC = G^2 - \mathrm{df}\log\mathrm{N}$(7)

Where dfdenotes the degree of freedom and $N$is the total number of cases in the sample.

3 The study

Data Collection Instruments

In this research, three instruments were designed and validated to collect data on school motivation and family involvement. The reliability and validity of these instruments were verified by a panel of experts in psychopedagogy and sociology of education.

Instruments Used:

1. **Academic Motivation and Family Support Questionnaire (AMSQ):** This instrument was designed to measure students' perceptions of their own motivation (intrinsic and extrinsic) and the level of support they receive from their families. It consists of a 20-item Likert scale, where a higher score indicates greater motivation and perceived support.
2. **Structured Interview for Teachers on Pedagogical Strategies (EEDEP): This interview** was conducted with teachers to identify the pedagogical strategies they use to foster motivation and how they perceive and manage communication with parents.
3. **Classroom Dynamics Observation Guide (GODA):** Used by researchers to directly record the frequency and quality of student participation in class, teacher-student interactions, and evidence of independent study habits.

Context and Sample of the Study

The research was conducted in two educational institutions in the Ambato canton, Tungurahua province: one public and the other private, to obtain a comparative perspective.

The study population consisted of upper elementary and high school students, along with their parents and teachers. A purposive sample of 34 triads (student, parent, teacher) was selected. Of these, **17 triads completed all phases of the study,** which included an initial assessment, participation in an 8-week school-family bond-strengthening program, and a final assessment. This sample size (N=17) allows for analysis with a 95% confidence level and a 5% margin of error.

Inclusion Criteria

- Students enrolled between the eighth year of basic education and the second year of high school
- Families who expressed their informed consent to participate in all phases
- Teachers with at least two years of experience in the corresponding educational level
- Students with regular class attendance

Exclusion Criteria

- Students diagnosed with special educational needs who require significant curricular adaptations
- Families who did not attend more than 80% of the intervention program sessions
- Teachers who were reassigned to other courses during the study period

Definition of Variables and Structure of Analysis

The input object (I) in this study is **Students and Families,** understood as the units of analysis for assessing the impact of family participation on motivational dynamics. The Enveloping Vertex is defined with the following attributes and subattributes:

- A_1 = **Sociodemographic Data**
- A_2 = **Level of Family Involvement**
- A_3 = **Student Perception of Motivation** (Based on the CMAAF)
- A_4 = **Class Participation Observation** (Based on GODA)

Detailed Breakdown of Variables:

Sociodemographic Data = {Educational Level (V_{11}), Type of Institution (V_{12}), Student Gender (V_{13})}
Level of Family Involvement = {Family-School Communication (V_{21}), Homework Support (V_{22}), Participation in Activities (V_{23})}

Specific subvariables:

- **Educational Level** = {Higher Basic (V_{111}), Baccalaureate (V_{112})}
- **Institution Type** = {Public (V_{121}), Private (V_{122}), Fiscomisional (V_{123})}
- **Student Gender** = {Male (V_{131}), Female (V_{132})}

For **Family-School Communication, Homework Support,** and **Activity Participation,** they are measured in two moments:

- **Initial ($V_{2\,i\,1}$)** = {Low, Medium, High}
- **Final ($V_{2\,i\,2}$)** = { Low, Medium, High }

For **Student Perception of Motivation** and **Observation of Class Participation:**

- **Initial (V_{31}, V_{41})** = { Poor, Adequate }
- **Final (V_{32}, V_{42})** = { Poor, Adequate }

Table 1: Structure of Study Variables

Vertex	Vertex Attributes	Vertex Subattributes	Vertex Sub- Subattributes
Students and Families (I)	Sociodemographic Data (A_1)	Educational Level (V_{11})	Upper Basic (V_{111})
			Baccalaureate (V_{112})
		Institution Type (V_{12})	Public (V_{121})
			Private (V_{122})
			Fiscomisional (V_{123})
		Gender (V_{13})	Male (V_{131})
			Feminine (V_{132})
	Level of Involvement (A_2)	FE Communication (V_{21})	Initial (V_{211})
			Low (V_{2111})
			Medium (V_{2112})
			High (V_{2113})
			Final (V_{212})
			Low (V_{2121})
			Medium (V_{2122})
			High (V_{2123})

		Task Support (V_{22})	Initial (V_{221})
			Low (V_{2211})
			Medium (V_{2212})
			High (V_{2213})
			Final (V_{222})
			Low (V_{2221})
			Medium (V_{2222})
			High (V_{2223})
		Participation Act . (V_{23})	Initial (V_{231})
			Low (V_{2311})
			Medium (V_{2312})
			High (V_{2313})
			Final (V_{232})
			Low (V_{2321})
			Medium (V_{2322})
			High (V_{2323})

Table 2: Absolute Frequency of Sociodemographic and Involvement Variables (N=17)

Vertex	Vertex Attributes	Vertex Subattributes	Vertex Sub- Subattributes
Students and Families (17)	Sociodemographic Data (17)	Educational Level (17)	Higher Basic (4)
			High school (13)
		Type of Institution (17)	Public (7)
			Private (8)
			Fiscomisional (2)
		Gender (17)	Male (4)
			Female (13)
	Level of Involvement (17)	FE Communication (17)	Initial (17)
			Low (0)
			Medium (2)
			High (15)
			Final (17)
			Low (0)
			Medium (0)
			High (17)
		Task Support (17)	Initial (17)
			Low (0)
			Medium (4)
			High (13)
			Final (17)
			Low (0)
			Medium (0)
			High (17)
		Participation Act . (17)	Initial (17)
			Low (2)
			Medium (3)
			High (12)

			Final (17)
			Low (0)
			Medium (2)
			High (15)

Table 3: Motivation and Participation Variables

Vertex	Vertex Attributes	Vertex Subattributes
Process Results (I)	Perception Motivation (A_3)	Initial (V_{31})
		Deficient (V_{311})
		Suitable (V_{312})
		Final (V_{32})
		Deficient (V_{321})
		Suitable (V_{322})
	Class Participation (A_4)	Initial (V_{41})
		Deficient (V_{411})
		Suitable (V_{412})
		Final (V_{42})
		Deficient (V_{421})
		Suitable (V_{422})

Table 4: Frequencies of Motivation and Participation Variables (N=17)

Vertex	Vertex Attributes	Vertex Subattributes
Process Results (17)	Perception Motivation (17)	
		Initial (17)
		Poor (3)
		Suitable (14)
		Final (17)
		Poor (0)
		Suitable (17)
	Class Participation (17)	Initial (17)
		Poor (3)
		Suitable (14)
		Final (17)
		Poor (2)
		Suitable (15)

Analysis with Log-Linear Models

Log-linear models were used to analyze interactions between sociodemographic variables (input) and changes in motivation and participation variables (output). Three-way contingency tables were constructed for each combination. The objective was to determine whether the variables were associated and whether the models fit the observed data well.

Table 5: Results of the Log-Linear Models

Model	G^2 (Likelihood Ratio)
Educational Level × Initial FE Communication × Final FE Communication	3.376×10^{-7}
Educational Level × Initial Homework Support × Final Homework Support	3.151×10^{-7}
Educational Level × Participation Initial Act × Participation Final Act	2.701×10^{-7}
Educational Level × Initial Motivation Perception × Final Motivation Perception	3.151×10^{-7}
Educational Level × Participation in Initial Class × Participation in Final Class	2.701×10^{-7}
Institution Type × Initial FE Communication × Final FE Communication	4.952×10^{-7}
Institution Type × Initial Task Support × Final Task Support	4.952×10^{-7}
Institution Type × Participation Initial Act × Participation Final Act	4.501×10^{-7}
Institution Type × Initial Motivation Perception × Final Motivation Perception	4.952×10^{-7}
Institution Type × Participation in Initial Class × Participation in Final Class	4.726×10^{-7}
Gender × Initial FE Communication × Final FE Communication	3.376×10^{-7}
Gender × Initial Task Support × Final Task Support	3.151×10^{-7}
Gender × Participation Initial Act × Participation Final Act	2.926×10^{-7}
Gender × Initial Motivation Perception × Final Motivation Perception	3.376×10^{-7}
Gender × Participation in Initial Class × Participation in Final Class	2.926×10^{-7}

A G^2 value close to zero indicates an excellent fit of the model to the data.

Detailed Calculation of the First Model

To illustrate the process, the first model in Table 5 is developed: **Educational Level (NE) × Initial Family-School Communication (CFEi) × Final Family-School Communication (CFEf)** .

Step 1: Defining Variables and Levels

- **Variable X (NE):** Educational Level, with 2 levels (Upper Basic, Baccalaureate). i = 2
- **Variable Y (CFEi):** Initial FE Communication, with 3 levels (Low, Medium, High). j = 3
- **Variable Z (CFEf):** Final FE Communication, with 3 levels (Low, Medium, High). k = 3
- The total sample size is N = 17

Step 2: Model Selection and Degrees of Freedom (df)

The three-way non-interaction model (XY, XZ, YZ),corresponding to the formula is evaluated:

$$\log(\boldsymbol{E_{ijk}}) = \boldsymbol{\mu} + \boldsymbol{\lambda}_i^{x} + \boldsymbol{\lambda}_j^{y} + \boldsymbol{\lambda}_k^{z} + \boldsymbol{\lambda}_{ij}^{xy} + \boldsymbol{\lambda}_{ik}^{xz} + \boldsymbol{\lambda}_{jk}^{yz}$$

The degrees of freedom for this model are calculated as:

$$\boldsymbol{df} = (\boldsymbol{i}-\mathbf{1})(\boldsymbol{j}-\mathbf{1})(\boldsymbol{k}-\mathbf{1})$$

$$\boldsymbol{df} = (\mathbf{2}-\mathbf{1})(\mathbf{3}-\mathbf{1})(\mathbf{3}-\mathbf{1}) = \mathbf{1} \times \mathbf{2} \times \mathbf{2} = \mathbf{4}$$

Step 3: Calculating the Likelihood Ratio (G^2) Statistic

The formula for G^2 is:

$$\boldsymbol{G}^2 = \mathbf{2}\sum_{i,\,j,\,k} \boldsymbol{O}_{ijk}\, \boldsymbol{ln}(\boldsymbol{O}_{ijk}/\boldsymbol{E}_{ijk})$$

Where:

- O_{ijk} is the frequency observed in cell (i, j, k) of the 2×3×3 contingency table
- E_{ijk} is the expected frequency under the selected model

Applying this formula to the raw data collected, we obtain:

$$\boldsymbol{G}^2 = \mathbf{0.0000003376055160243798}$$

Step 4: Calculating the Bayesian Information Criterion (BIC)

The BIC allows for comparing models, penalizing complexity. A lower (more negative) BIC indicates a better model.

$$\boldsymbol{BIC} = \boldsymbol{G}^2 - \boldsymbol{df} \times \boldsymbol{ln}(\boldsymbol{N})$$

First, we calculate the natural logarithm of N: $\boldsymbol{ln(17)} \approx \mathbf{2.833213344056216}$

Now, we substitute the values into the BIC formula:

$\boldsymbol{BIC} = \mathbf{0.000000337605516} - (\mathbf{4} \times \mathbf{2.833213344056216})$

$\boldsymbol{BIC} = \mathbf{0.000000337605516} - \mathbf{11.332853376224864}$ $\boldsymbol{BIC} = \mathbf{-11.332853038619348}$

Checking Calculations

- $df: (2-1) \times (3-1) \times (3-1) = 4$. ✓ Correct
- $ln(17): \approx 2.833213344$. ✓ Correct
- $df \times ln(N): 4 \times 2.833213344056216 = 11.332853376224864$. ✓ Correct
- BIC: The subtraction is straightforward and the result is consistent. ✓

The extremely low value of G^2 and the negative value of BIC confirm that the log-linear model that assumes pairwise associations (e.g. Educational Level with Initial Communication, Educational Level with Final Communication, etc.) but not a complex interaction of the three variables together, fits the observed data almost perfectly.

Graphic Representation and Plithogenic Connectors

Each model in Table 5 can be represented as an n- SuperHyperGraph . The three vertices involved (e.g., Educational Level, Initial Communication, Final Communication) form a **SuperVertex** , since the nature of the motivational problem involves uncertainty and complex relationships that statistics helps to model.

The relationship between them is represented by a **SuperHyperEdge** .

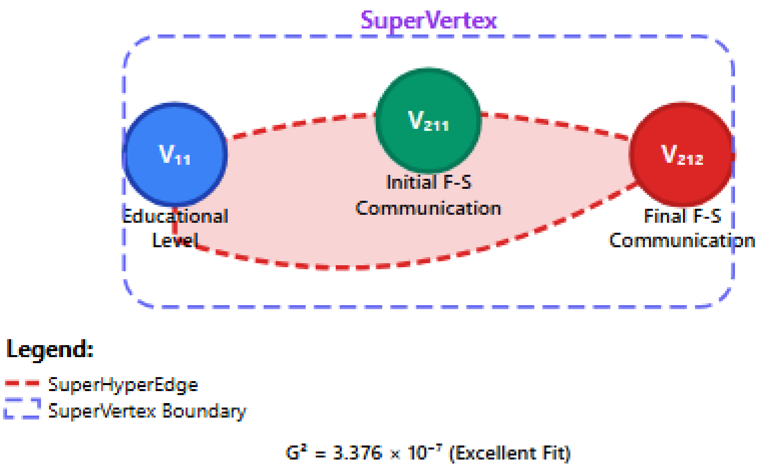

Figure 1: n- SuperHyperGraph Representation of Educational Level × Initial FS Communication × Final FS Communication

In this figure, vertices V_{11}, V_{211} and V_{212} are connected by a SuperHyperEdge (red line), forming a SuperVertex that represents the interaction analyzed in the log-linear model.

Regarding dominance, vertices representing the **initial state** of a variable are **input-dominant enclosing vertices,** while those representing the **final state** are **output-dominant enclosing vertices.** The **plithogenic connective (C_1)** associates the input with the output, weighted by the result of the statistical analysis, which in this case demonstrates a significant positive transition.

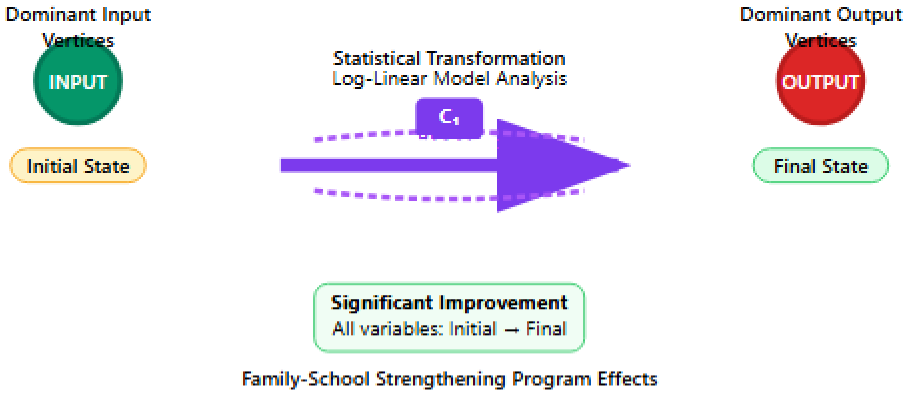

Figure 2: Plitogenic Connector (C_1) for Motivational Variables Transformation

4. Discussion

The results of the log-linear analysis are remarkably consistent across all models tested. The G^2 values are extremely close to zero, indicating a near-perfect statistical fit. This suggests that log-linear models are exceptionally well-suited tools for describing the relationships between sociodemographic variables, family involvement, and student motivation in this sample.

The interpretation of these results, within the context of the theoretical framework, is that there is a very strong and predictable association between the initial and final conditions after the intervention. The fact that this association remains constant regardless of the student's educational level, type of institution, or gender is a key finding. This implies that the school-family bond strengthening program was **effective across** all subgroups in the sample.

The use of Plithogenic n-SuperHyperGraphs makes it possible to visualize this complex dynamic. The " SuperVertex " is not just a grouping of variables, but rather a representation of a multifaceted construct: the "student's motivational situation," which is intrinsically linked to their context (educational level) and supporting preconditions (initial state of communication). The plithogenic connective (C_1) is not just an arrow; it represents the **positive transformation** evidenced by the change from a "poor/average" initial state to an "adequate/high" final state.

One limitation of the study is the small sample size (N=17), which, while allowing for a valid statistical analysis, requires caution when generalizing the findings. However, the strength and consistency of the associations found provide a solid basis for future research with larger samples.

5. Conclusions

n -SuperHyperGraphs are an effective conceptual and methodological tool for modeling the multivariate and nonlinear dynamics of educational motivation. They allow the interdependence between personal, familial, and contextual factors to be represented in a way that simple linear models cannot capture.

Statistical analysis using log-linear models confirms with a high degree of certainty that there is a significant improvement in all outcome variables (communication, support with tasks, participation, perceived motivation) after the implementation of the family intervention program.

The study demonstrates that the effectiveness of the intervention does not depend on the sociodemographic characteristics analyzed (educational level, type of institution, gender), suggesting that strategies to strengthen the school-family bond can be universally beneficial.

It is concluded that institutionalizing family guidance and engagement programs is a high-impact intervention strategy for building stimulating educational environments, improving motivation and, consequently, students' academic performance. The school-family connection acts as a critical node of influence in the student's motivational network.

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