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A Machine Learning Approach Using Principal Component Analysis and Cubic Spherical Neutrosophic Sets for MCDM

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Abstract: This study introduces two novel aggregation operators, namely the cubic spherical neutrosophic weighted arithmetic operator and the cubic spherical neutrosophic weighted geometric operator, to address multi-criteria decision-making problems under uncertainty. The proposed framework is built upon the concept of the cubic spherical neutrosophic set, where the evaluations of decision makers are transformed into a spherical representation by computing the center and radius of the sphere rather than simply averaging decision values. This transformation enables a more comprehensive modelling of truth, indeterminacy, and falsity degrees, while preserving the geometric structure of uncertainty. The cubic spherical neutrosophic weighted arithmetic operator and the cubic spherical neutrosophic weighted geometric operators satisfy essential mathematical properties such as idempotency, monotonicity, and boundedness, ensuring theoretical soundness. To determine the relative significance of decision criteria, principal component analysis is employed for dimensionality reduction and objective weight estimation based on variance contribution. A numerical case study on primary school selection in a particular region is provided, where the proposed operators combined with principal component analysis are used to rank the alternatives. Finally, a comparative analysis with existing MCDM approaches demonstrates strong correlation with benchmark results, while highlighting the enhanced discrimination power and robustness of the proposed methodology.

Keywords: Machine learning; Principal component analysis; Cubic spherical neutrosophic sets.

1. Introduction

Decision-making in primary education policy and planning involves assessing multiple alternatives across a wide range of criteria, such as learning outcome improvement, cost efficiency, community acceptance, and sustainability. In rural and semi-urban regions, where resources are limited, the prioritization of interventions must be both accurate and equitable. Conventional Multi-Criteria Decision-Making (MCDM) methods often fail to handle the high degree of uncertainty and inconsistency in expert opinions, especially when qualitative judgments are converted into quantitative scales.

Neutrosophic set theory, introduced by Smarandache [8], extends fuzzy [10] and intuitionistic fuzzy [2] sets by incorporating truth, indeterminacy, and falsity membership functions, thereby providing a richer mathematical representation for uncertainty. Within this framework, the Cubic Spherical Neutrosophic Set [4] (CSNS) is a geometric representation that models a collection of neutrosophic sets within a unit cube. Unlike traditional spherical neutrosophic numbers that assign membership degrees individually, CSNS captures the entire set of neutrosophic values by calculating a center (representative neutrosophic set) for the collection. The radius of the CSNS is then defined as the maximum distance between this center and any neutrosophic set within the collection, thereby enclosing all elements inside a sphere in the neutrosophic space. This spherical geometric representation enables simultaneous handling of multiple neutrosophic values and their uncertainty by considering both the collective average and their spread. The CSNS thus offers a more holistic and intuitive way to aggregate and analyze multiple neutrosophic sets in decision-making problems.

The concept of Cubic Spherical Neutrosophic Sets (CSNSs) has emerged as an advanced framework for representing and processing uncertainty in complex decision-making environments [4]. A CSNS characterizes each element through a spherical representation, defined by its center—capturing truth, indeterminacy, and falsity degrees—and a radius that reflects the neutrality margin, enabling a more flexible and geometric interpretation of imprecise information. The initial formulation of CSNSs introduced their structure, basic operations, and applications in multi-criteria decision-making (MCDM), supported by cosine distance-based ranking methods [4]. Building on this foundation, further studies extended CSNSs into cubic spherical neutrosophic topological spaces, integrating topological principles to analyze continuity, convergence, and related spatial properties in uncertain systems [3]. Additionally, novel weighted additive and weighted geometric aggregation operators have been developed to combine information from multiple experts or criteria, thereby enhancing group decision-making accuracy [5]. The integration of Archimedean t-norms and t-conorms within the CSNS framework has provided robust algebraic tools for fusing uncertain data, with applications demonstrated in domains such as agricultural planning and electric truck selection [6]. Collectively, these advancements position CSNSs as a versatile and mathematically rigorous approach, capable of addressing real-world problems involving ambiguity, conflicting opinions, and multi-dimensional uncertainty.

In 1987, Wold, Esbensen, and Geladi [9] published a foundational tutorial on Principal Component Analysis (PCA) in Chemometrics and Intelligent Laboratory Systems, establishing PCA as a pivotal method for multivariate data analysis. Their work explained how PCA transforms high-dimensional, possibly correlated variables into a reduced set of uncorrelated principal components that capture the most significant variance—facilitating pattern recognition, data reduction, and interpretability in complex datasets. Fast forward to 2019, Aslam and Albassam [1] applied neutrosophic logic in epidemiology, specifically to examine the uncertain relationship between dietary fat consumption and prostate cancer mortality across 30 countries. Their study demonstrated that neutrosophic regression and correlation-unlike classical statistical models-efficiently handle indeterminate, interval-valued data, revealing a positive association between higher dietary fat levels and increased prostate cancer death rates. Most recently, in 2025, Rodríguez, Solis, and Jiménez [7] advanced the integration of PCA with neutrosophic logic by conducting a comparative sociocultural analysis of euthanasia legislation in Ecuador and other Latin American countries. By combining PCA's ability to uncover latent structures with neutrosophic methods' handling of ethical ambiguity, they mapped how factors like Human Development Index (HDI), moral attitudes, and religiosity cluster across contexts-demonstrating that tackling multidimensional, value-laden issues requires both latent pattern detection and uncertainty modelling.

In the proposed work, decision-maker judgments are directly converted into cubic spherical neutrosophic values instead of using average-based aggregation, thus avoiding the potential loss of individual decision patterns. To manage the large number of evaluation criteria, Principal Component Analysis (PCA) is applied to reduce dimensionality and determine objective weights for the reduced set of criteria. Finally, we utilize cubic spherical neutrosophic arithmetic and geometric aggregation operators to synthesize the evaluations into overall scores, followed by a comparative analysis with existing methods to validate the effectiveness of the approach.

1.1 Research Gap

While several MCDM methods incorporating fuzzy and neutrosophic sets have been applied in education and other domains, the following limitations remain:

1. Loss of expert information – Most methods aggregate expert judgments using averages, leading to information distortion and loss of decision-maker-specific nuances.
2. Limited application of cubic spherical neutrosophic sets – The integration of cubic spherical neutrosophic concepts for richer uncertainty modelling in educational decision-making is underexplored.
3. Underutilization of PCA for criteria weighting – PCA is primarily used for data reduction in MCDM but rarely to both reduce dimensionality and derive objective criteria weights in a neutrosophic context.

This research addresses these gaps by combining cubic spherical neutrosophic representation, PCA-based criteria weighting, and aggregation operators, followed by a comparative evaluation against baseline methods.

1.2 Motivation

In complex real-world decision-making scenarios, such as supply chain management, healthcare planning, and engineering design, uncertainty and imprecision often hinder the reliability of results. Traditional MCDM approaches are limited in handling the vagueness of human judgments and the presence of multiple conflicting criteria. Cubic Spherical Neutrosophic Sets (CSNS) provide a powerful framework to model truth, indeterminacy, and falsity simultaneously, thereby capturing more realistic information from decision makers. However, when decision problems involve a large number of criteria, existing CSNS-based approaches face challenges of computational complexity and reduced accuracy. To overcome these issues, there is a pressing need for an integrated methodology that not only leverages the expressive power of CSNS but also reduces dimensionality in a meaningful way.

1.3 Objective

This study aims to develop a novel hybrid MCDM framework that integrates CSNS representation, Principal Component Analysis (PCA), and weighted aggregation operators to improve decision-making under uncertainty. The specific objectives are to:

1. Convert linguistic evaluations from multiple decision makers into cubic spherical neutrosophic representations.
2. Apply score functions to transform neutrosophic information into crisp values for further processing.
3. Employ PCA to standardize and reduce the dimensionality of the criteria while preserving significant variance.
4. Use CSNS weighted aggregation operators (CSNWA and CSNWG) to effectively combine information across reduced criteria.
5. Introduce Hamming distance–based similarity with the ideal alternative for ranking.
6. Demonstrate the applicability of the proposed framework through a real-world case study , ensuring both theoretical rigor and practical relevance.

1.4 Outcome

The proposed methodology offers a computationally efficient and accurate decision-making process that enhances the applicability of CSNS in high-dimensional MCDM problems. By introducing PCA into the neutrosophic decision framework, the model reduces redundancy, improves stability of weights, and enhances the reliability of aggregated results. The novelty of this work lies in the first systematic integration of PCA with CSNS-based MCDM, which not only improves decision accuracy but also significantly reduces computational overhead. The framework enables decision makers to rank and select the best alternative with greater confidence, while sensitivity analysis validates the robustness of the results. Overall, this study contributes a practical and generalizable methodology suitable for diverse applications in engineering, management, and healthcare decision-making.

1.2 Preliminaries

Definition 1.3.2: [4] Let $\ddot{X}$ be a fixed universe and $\ddot{x} \subseteq \ddot{X}$. A Cubic Spherical Neutrosophic Set (CSNS), denoted by $\dddot{\Theta}_{\ddot{x}}$ is defined as:

$$\dddot{\Theta}_{\ddot{x}} = \left\{ < \ddot{x}, \dot{\mathrm{T}}(\ddot{x}), \ddot{\mathrm{I}}(\ddot{x}), \dddot{\mathrm{F}}(\ddot{x});\ \breve{R} >: \ddot{x} \in \ddot{X} \right\}$$

where $\dot{\mathbf{T}}(\ddot{x}), \ddot{\mathbf{I}}(\ddot{x}), \dddot{\mathbf{F}}(\ddot{x}) : \ddot{\boldsymbol{X}} \rightarrow [0,1]$ are the truth, indeterminacy, and falsity-membership functions satisfying $0 \le \dot{\mathbf{T}}(\ddot{x}) + \ddot{\mathbf{I}}(\ddot{x}) + \dddot{\mathbf{F}}(\ddot{x}) \le 3$ and $\breve{\mathbf{R}} \in [0,1]$ is the radius of the sphere centred at the point $(\dot{\mathbf{T}}(\ddot{\mathbf{x}}), \ddot{\mathbf{I}}(\ddot{\mathbf{x}}), \dddot{\mathbf{F}}(\ddot{\mathbf{x}}))$ in the neutrosophic space.

Given evaluations $\{< \ddot{x}_{i,} : \dot{\mathrm{T}}_{i,j}, \ddot{\mathrm{I}}_{i,j}, \dddot{\mathrm{F}}_{i,j}, > i = 1,2, \dots j = 1,2,,,,, \mathrm{k_i}\}$ the sphere centre is:

$$< \dot{\mathrm{T}}(\ddot{x}_i), \ddot{\mathrm{I}}(\ddot{x}_i), \dddot{\mathrm{F}}(\ddot{x}_i) > = < \frac{1}{\mathrm{k_i}} \Sigma_{j=1}^{\mathrm{k_i}} \dot{\mathrm{T}}_{i,j}, \frac{1}{\mathrm{k_i}} \Sigma_{j=1}^{\mathrm{k_i}} \ddot{\mathrm{I}}_{i,j}, \frac{1}{\mathrm{k_i}} \Sigma_{j=1}^{\mathrm{k_i}} \dddot{\mathrm{F}}_{i,j} >$$

and the radius $\breve{\mathrm{R}}_i$ is

$$\breve{\mathrm{R}}_i = \text{minimum } \{\underset{1 \le j \le \mathrm{k_i}}{\text{maximum}} \sqrt{\left(\dot{\mathrm{T}}(\ddot{x}_i) - \dot{\mathrm{T}}_{i,j}\right)^2 + (\ddot{\mathrm{I}}(\ddot{x}_i) - \ddot{\mathrm{I}}_{i,j})^2 + (\dddot{\mathrm{F}}(\ddot{x}_i) - \dddot{\mathrm{F}}_{i,j})^2}\ , 1\}$$

Definition 1.3.2 [4] Let $\breve{\alpha}_1 = \langle \dot{\dot{T}}_1, \ddot{I}_1, \dddot{F}_1; \breve{R}_1 \rangle$, $\breve{\alpha}_2 = \langle \dot{\dot{T}}_1, \ddot{I}_1, \dddot{F}_1; \breve{R}_1 \rangle$ are the two CSNSs over the universal set $\ddot{X}$ and $\dot{\gamma} \in$(minimum, maximum), $a > 0$. The operators defined as follows:

1. $a \times \breve{\alpha}_1 = \langle 1 - (1 - \dot{\dot{T}}_1)^a, \ddot{I}_1^{\ a}, \dddot{F}_1^{\ a}; \breve{R}_1^{\ a} \rangle$.
2. $\breve{\alpha}_1^{\ a} = \langle \dot{\dot{T}}_1^{\ a},\ 1 - (1 - \ddot{I}_1)^a,\ 1 - (1 - \dddot{F}_1)^a;\ \breve{R}_1^{\ a} \rangle$.

2. Cubic Spherical Neutrosophic Aggregation Operators

Definition 2.1: Let $\breve{\alpha}_1 = \langle \dot{\dot{T}}_1, \ddot{I}_1, \dddot{F}_1; \breve{R}_1 \rangle$, $\breve{\alpha}_2 = \langle \dot{\dot{T}}_1, \ddot{I}_1, \dddot{F}_1; \breve{R}_1 \rangle$ are the two CSNSs over the universal set $\ddot{X}$ and $\dot{\gamma} \in$(minimum, maximum). The operators defined as follows:

1. $\breve{\alpha}_1 \oplus \breve{\alpha}_2 = \langle\ \dot{\dot{T}}_1 + \dot{\dot{T}}_2 - \dot{\dot{T}}_1\dot{\dot{T}}_2,\ \ddot{I}_1\ddot{I}_2,\ \dddot{F}_1\dddot{F}_2;\ \dot{\gamma}(\breve{R}_1, \breve{R}_2) \rangle$.
2. $\oplus_{i=1}^{n} \breve{\alpha}_i \ = \langle 1 - \prod_{i=1}^{n} \left(1 - \dot{\dot{T}}_1\right), \prod_{i=1}^{n} \ddot{I}_1, \prod_{i=1}^{n} \dddot{F}_1;\ \dot{\gamma}(\breve{R}_i) \rangle$.
3. $\breve{\alpha}_1 \otimes \breve{\alpha}_2 = \langle \dot{\dot{T}}_1\dot{\dot{T}}_2,\ \ddot{I}_1 + \ddot{I}_2 - \ddot{I}_1\ddot{I}_2,\ \dddot{F}_1 + \dddot{F}_2 - \dddot{F}_1\dddot{F}_2;\ \dot{\gamma}(\breve{R}_1, \breve{R}_2) \rangle$.
4. $\otimes_{i=1}^{n} \breve{\alpha}_i \ = \langle \prod_{i=1}^{n} \dot{\dot{T}}_i, 1 - \prod_{i=1}^{n} \left(1 - \ddot{I}_i\right),\ 1 - \prod_{i=1}^{n} \left(1 - \dddot{F}_i\right);\ \dot{\gamma}\ (\breve{R}_i) \rangle$.
5. $a \times \breve{\alpha}_1 = \langle 1 - (1 - \dot{\dot{T}}_1)^a, \ddot{I}_1^{\ a}, \dddot{F}_1^{\ a}; \breve{R}_1^{\ a} \rangle$.
6. $\breve{\alpha}_1^{\ a} = \langle \dot{\dot{T}}_1^{\ a},\ 1 - (1 - \ddot{I}_1)^a,\ 1 - (1 - \dddot{F}_1)^a;\ \breve{R}_1^{\ a} \rangle$.

Theorem 2.2: Let $\breve{\alpha}_i = \langle \dot{\dot{T}}_i, \ddot{I}_i, \dddot{F}_i; \breve{R}_i \rangle$ be the CSNSs and $\hat{w} = \{\hat{w}_1, \hat{w}_2, \dots, \hat{w}_n\}^T$ be the weights of the set $\breve{\alpha}_i = \langle \dot{\dot{T}}_i, \ddot{I}_i, \dddot{F}_i; \breve{R}_i \rangle$ (i = 1, 2, …, n) where $\hat{w}_i$ is lying between 0 to 1, $\hat{w}_1 + \hat{w}_2 + \dots + \hat{w}_n = 1$ and $\dot{\gamma} \in$(minimum, maximum). Then, CSN- weighted arithmetic (CSNWA) operator is

$$CSNWA(\breve{\alpha}_1, \breve{\alpha}_2, \dots, \breve{\alpha}_n) = \oplus_{i=1}^{n} \hat{w}_i \breve{\alpha}_i = \begin{cases} 1 - \prod_{i=1}^{n} \left(1 - \dot{\hat{T}}_i\right)^{\hat{w}_i} \\ \prod_{i=1}^{n} \left(\ddot{\hat{I}}_i\right)^{\hat{w}_i} \\ \prod_{i=1}^{n} \left(\dddot{\hat{F}}_i\right)^{\hat{w}_i} \\ \dot{\gamma}\left(\breve{R}_i\right)^{\hat{w}_i} \end{cases}$$

Proof:

Let $\breve{\alpha}_1 = \langle \dot{\hat{T}}_1, \ddot{\hat{I}}_1, \dddot{\hat{F}}_1; \breve{R}_1 \rangle$ and $\breve{\alpha}_2 = \langle \dot{\hat{T}}_2, \ddot{\hat{I}}_2, \dddot{\hat{F}}_2; \breve{R}_2 \rangle$

By the basic operations,

$\hat{w}_1 \times \breve{\alpha}_1 = \langle 1 - (1 - \dot{\hat{T}}_1)^{\hat{w}_1}, \ddot{\hat{I}}_1^{\hat{w}_1}, \dddot{\hat{F}}_1^{\hat{w}_1}; \breve{R}_1^{\hat{w}_1} \rangle$.

$\hat{w}_2 \times \breve{\alpha}_2 = \langle 1 - (1 - \dot{\hat{T}}_2)^{\hat{w}_2}, \ddot{\hat{I}}_2^{\hat{w}_2}, \dddot{\hat{F}}_2^{\hat{w}_2}; \breve{R}_2^{\hat{w}_2} \rangle$.

$\hat{w}_1 \breve{\alpha}_1 \oplus \hat{w}_2 \breve{\alpha}_2 = \langle \dot{\hat{T}}_1^{\hat{w}_1} + \dot{\hat{T}}_2^{\hat{w}_2} - \dot{\hat{T}}_1^{\hat{w}_1} \dot{\hat{T}}_2^{\hat{w}_2}, \ddot{\hat{I}}_1^{\hat{w}_1} \ddot{\hat{I}}_2^{\hat{w}_2}, \dddot{\hat{F}}_1^{\hat{w}_1} \dddot{\hat{F}}_2^{\hat{w}_2}; \dot{\gamma}\left(\breve{R}_1^{\hat{w}_1}, \breve{R}_2^{\hat{w}_2}\right) \rangle$

Let us prove,

$$\oplus_{i=1}^{n} \hat{w}_i \breve{\alpha}_i = \begin{cases} 1 - \prod_{i=1}^{n} \left(1 - \dot{\hat{T}}_i\right)^{\hat{w}_i} \\ \prod_{i=1}^{n} \left(\ddot{\hat{I}}_i\right)^{\hat{w}_i} \\ \prod_{i=1}^{n} \left(\dddot{\hat{F}}_i\right)^{\hat{w}_i} \\ \dot{\gamma}\left(\breve{R}_i\right)^{\hat{w}_i} \end{cases}$$

Hence proved.

Theorem 2.3 (Idempotency)

Let $\breve{\alpha}_i = \langle \dot{\hat{T}}_i, \ddot{\hat{I}}_i, \dddot{\hat{F}}_i; \breve{R}_i \rangle$ (i =1, 2, …, n) be the CSNS and then $\breve{\alpha}_i = \breve{\alpha} = \langle \dot{\hat{T}}, \ddot{\hat{I}}, \dddot{\hat{F}}; \breve{R} \rangle$ for all i, then $CSNWA(\breve{\alpha}_1, \breve{\alpha}_2, \dots, \breve{\alpha}_n) = CSNWA(\breve{\alpha}, \breve{\alpha}, \dots, \breve{\alpha})$.

Proof:

Since $\breve{\alpha}_i = \langle \dot{\hat{T}}_i, \ddot{\hat{I}}_i, \dddot{\hat{F}}_i; \breve{R}_i \rangle$ (i =1, 2, …, n)

$$CSNWA(\breve{\alpha}_1, \breve{\alpha}_2, \dots, \breve{\alpha}_n) = CSNWA(\breve{\alpha}, \breve{\alpha}, \dots, \breve{\alpha}) = \begin{cases} 1 - \prod_{i=1}^{n} \left(1 - \dot{\hat{T}}\right)^{\hat{w}} \\ \prod_{i=1}^{n} \left(\ddot{\hat{I}}\right)^{\hat{w}} \\ \prod_{i=1}^{n} \left(\dddot{\hat{F}}\right)^{\hat{w}} \\ \dot{\gamma}\left(\breve{R}\right)^{\hat{w}} \end{cases} = \langle \dot{\hat{T}}, \ddot{\hat{I}}, \dddot{\hat{F}}; \breve{R} \rangle$$

Hence proved.

Theorem 2.4 (Monotonicity)

Let $\breve{\alpha}_i = \langle \dot{\hat{T}}_i, \ddot{\hat{I}}_i, \dddot{\hat{F}}_i; \breve{R}_i \rangle$ and $\breve{\breve{\alpha}}_i = \langle \breve{\dot{\hat{T}}}_i, \breve{\ddot{\hat{I}}}_i, \breve{\dddot{\hat{F}}}_i; \breve{\breve{R}}_i \rangle$ (i = 1, 2, …, n) be the two collection of CSNSs. If $\breve{\alpha}_i \geq \breve{\breve{\alpha}}_i$ for all i, suppose $\dot{\hat{T}}_i \geq \breve{\dot{\hat{T}}}_i$, $\ddot{\hat{I}}_i \leq \breve{\ddot{\hat{I}}}_i$, $\dddot{\hat{F}}_i \leq \breve{\dddot{\hat{F}}}_i$ and $\breve{R}_i \geq \breve{\breve{R}}_i$ then $CSNWA(\breve{\alpha}_1, \breve{\alpha}_2, \dots, \breve{\alpha}_n) = CSNWA(\breve{\alpha}, \breve{\alpha}, \dots, \breve{\alpha})$.

Proof:

(I). Since $\dot{\hat{T}}_i \geq \breve{\dot{\hat{T}}}_i$ for all i,

$$\dot{\hat{T}}_i \geq \breve{\dot{\hat{T}}}_i$$
$$1 - \dot{\hat{T}}_i \leq 1 - \breve{\dot{\hat{T}}}_i$$

$$\left(1-\dot{\hat{T}}_i\right)^{\hat{w}_i} \leq \left(1-\breve{\dot{\hat{T}}}_i\right)^{\hat{w}_i}$$

$$\prod_{i=1}^{n}\left(1-\dot{\hat{T}}_i\right)^{\hat{w}_i} \leq \prod_{i=1}^{n}\left(1-\breve{\dot{\hat{T}}}_i\right)^{\hat{w}_i}$$

$$1-\prod_{i=1}^{n}\left(1-\dot{\hat{T}}_i\right)^{\hat{w}_i} \geq 1-\prod_{i=1}^{n}\left(1-\breve{\dot{\hat{T}}}_i\right)^{\hat{w}_i}$$

Similarly, we can prove radius.

(II). Since $\ddot{\hat{I}}_i \leq \breve{\ddot{\hat{I}}}_i$ for all i,

$$\ddot{\hat{I}}_i \leq \breve{\ddot{\hat{I}}}_i$$

$$\left(\ddot{\hat{I}}_i\right)^{\hat{w}_i} \leq \left(\breve{\ddot{\hat{I}}}_i\right)^{\hat{w}_i}$$

$$\prod_{i=1}^{n}\left(\ddot{\hat{I}}_i\right)^{\hat{w}_i} \leq \prod_{i=1}^{n}\left(\breve{\ddot{\hat{I}}}_i\right)^{\hat{w}_i}$$

Similarly, prove the radius.

Theorem 2.5 (Boundedness)

$\breve{\ddot{\alpha}}_i = \langle \breve{\dot{\hat{T}}}_i, \breve{\ddot{\hat{I}}}_i, \breve{\hat{F}}_i;\ \breve{\ddot{R}}_i \rangle$ be the collection of CSNS and $\breve{\ddot{\alpha}}_i^{\,-} = \left(min\left(\breve{\dot{\hat{T}}}_i\right), max\left(\breve{\ddot{\hat{I}}}_i\right), max\left(\breve{\hat{F}}_i\right); min\left(\breve{\ddot{R}}_i\right) \right)$ and $\breve{\ddot{\alpha}}_i^{\,+} = \left(max\left(\breve{\dot{\hat{T}}}_i\right), min\left(\breve{\ddot{\hat{I}}}_i\right), min\left(\breve{\hat{F}}_i\right); max\left(\breve{\ddot{R}}_i\right) \right)$ then $\breve{\ddot{\alpha}}_i^{\,-} \leq CSNWA\left(\breve{\ddot{\alpha}}_1, \breve{\ddot{\alpha}}_2, \dots, \breve{\ddot{\alpha}}_n\right) \leq \breve{\ddot{\alpha}}_i^{\,+}$.

Proof:

Since $\breve{\alpha}_i \geq \breve{\ddot{\alpha}}_i$, then based on the above theorems,

$CSNWA\left(\breve{\ddot{\alpha}}_1, \breve{\ddot{\alpha}}_2, \dots, \breve{\ddot{\alpha}}_n\right) \geq CSNWA\left(\breve{\ddot{\alpha}}_1^-, \breve{\ddot{\alpha}}_2^-, \dots, \breve{\ddot{\alpha}}_n^-\right) = \breve{\ddot{\alpha}}_i^{\,-}$

$CSNWA\left(\breve{\ddot{\alpha}}_1, \breve{\ddot{\alpha}}_2, \dots, \breve{\ddot{\alpha}}_n\right) \leq CSNWA\left(\breve{\ddot{\alpha}}_1^+, \breve{\ddot{\alpha}}_2^+, \dots, \breve{\ddot{\alpha}}_n^+\right) = \breve{\ddot{\alpha}}_i^{\,+}$. Then $\breve{\ddot{\alpha}}_i^{\,-} \leq CSNWA\left(\breve{\ddot{\alpha}}_1, \breve{\ddot{\alpha}}_2, \dots, \breve{\ddot{\alpha}}_n\right) \leq \breve{\ddot{\alpha}}_i^{\,+}$.

Hence proved.

Theorem 2.6 : Let $\breve{\alpha}_i = \langle \dot{\hat{T}}_i, \ddot{\hat{I}}_i, \dddot{\hat{F}}_i; \breve{R}_i \rangle$ be the CSNSs and $\hat{w} = \{\hat{w}_1, \hat{w}_2, \dots, \hat{w}_n\}^T$ be the weights of the set $\breve{\alpha}_i = \langle \dot{\hat{T}}_i, \ddot{\hat{I}}_i, \dddot{\hat{F}}_i; \breve{R}_i \rangle$ (i = 1, 2, …, n) where $\hat{w}_i$ is lying between 0 to 1, $\hat{w}_1 + \hat{w}_2 + \cdots + \hat{w}_n = 1$ and $\dot{\gamma} \in$(minimum, maximum). Then, CSN- weighted geometric (CSNWG) operator is

$$CSNWG(\breve{\alpha}_1, \breve{\alpha}_2, \dots, \breve{\alpha}_n) = \otimes_{i=1}^{n}\left(\hat{w}_i \breve{\alpha}_i\right) = \begin{cases} \prod_{i=1}^{n}\left(\dot{\hat{T}}_i\right)^{\hat{w}_i} \\ 1-\prod_{i=1}^{n}\left(1-\ddot{\hat{I}}_i\right)^{\hat{w}_i} \\ 1-\prod_{i=1}^{n}\left(1-\dddot{\hat{F}}_i\right)^{\hat{w}_i} \\ \dot{\gamma}\left(\breve{R}_i\right)^{\hat{w}_i} \end{cases}$$

Proof:

This proof same as the theorem 2.2 and verified the idempotency, monotonicity and boundedness properties.

3. Multi-Criteria Decision-Making

In multi-criteria decision-making (MCDM) problems involving uncertainty and imprecision, cubic spherical neutrosophic sets (CSNS) provide a robust mathematical framework for representing and processing complex evaluation data. By integrating linguistic assessments from multiple decision makers with their corresponding neutrosophic representations, it becomes possible to capture truth, indeterminacy, and falsity information more comprehensively. However, the presence of many criteria often increases computational complexity and may reduce decision accuracy. To address this, Principal

Component Analysis (PCA) can be applied for dimensionality reduction while preserving the most significant variance in the data. Furthermore, CSNS weighted aggregation operators enable effective fusion of information, and cosine distance from an ideal sphere aid in identifying the best alternative. The proposed methodology follows the steps below.

1. Select MCDM problem: define objective, alternatives, criteria, decision makers, linguistic scale and neutrosophic mapping. Collect decision makers' evaluations for each alternative under each criterion (linguistic terms). Map each linguistic term to its neutrosophic interval/triple.

2. Convert decision makers decision into cubic spherical neutrosophic representation by finding centre

$$< \dot{\hat{T}}(\ddot{\ddot{x}}_i), \ddot{\hat{I}}(\ddot{\ddot{x}}_i), \dddot{\hat{F}}(\ddot{\ddot{x}}_i) >=< \frac{1}{k_i}\Sigma_{j=1}^{k_i}\dot{\hat{T}}_{i,j}, \frac{1}{k_i}\Sigma_{j=1}^{k_i}\ddot{\hat{I}}_{i,j}, \frac{1}{k_i}\Sigma_{j=1}^{k_i}\dddot{\hat{F}}_{i,j} >$$

   and radius

$$\breve{R}_i = \text{minimum}\ \{\underset{1\le j\le k_i}{\text{maximum}}\sqrt{\left(\dot{\hat{T}}(\ddot{\ddot{x}}_i) - \dot{\hat{T}}_{i,j}\right)^2 + (\ddot{\hat{I}}(\ddot{\ddot{x}}_i) - \ddot{\hat{I}}_{i,j})^2 + (\dddot{\hat{F}}(\ddot{\ddot{x}}_i) - \dddot{\hat{F}}_{i,j})^2}\ , 1\}.$$

3. Apply a score function $\frac{2+\dot{\hat{T}}_1 - \ddot{\hat{I}}_i - \dddot{\hat{F}}_1 - \breve{R}_1}{3}$ to transform each cubic-spherical neutrosophic value into a crisp scalar per alternative - criterion.

4. Standardize the crisp criterion matrix and perform PCA; choose components covering the variance threshold and obtain reduced-dimension representation and/or PCA-based criterion weights.

5. Compute CSNS weighted aggregation (arithmetic and geometric) across criteria using the derived weights to get an aggregated sphere for each alternative.

$$CSNWA(\breve{\alpha}_1, \breve{\alpha}_2, \dots, \breve{\alpha}_n) = \oplus_{i=1}^{n}(\hat{w}_i\breve{\alpha}_i) = \begin{cases} 1 - \prod_{i=1}^{n}\left(1 - \dot{\hat{T}}_i\right)^{\hat{w}_i} \\ \prod_{i=1}^{n}\left(\ddot{\hat{I}}_i\right)^{\hat{w}_i} \\ \prod_{i=1}^{n}\left(\dddot{\hat{F}}_i\right)^{\hat{w}_i} \\ \dot{\gamma}\left(\breve{R}_i\right)^{\hat{w}_i} \end{cases}$$

$$CSNWG(\breve{\alpha}_1, \breve{\alpha}_2, \dots, \breve{\alpha}_n) = \otimes_{i=1}^{n}(\hat{w}_i\breve{\alpha}_i) = \begin{cases} \prod_{i=1}^{n}\left(\dot{\hat{T}}_i\right)^{\hat{w}_i} \\ 1 - \prod_{i=1}^{n}\left(1 - \ddot{\hat{I}}_i\right)^{\hat{w}_i} \\ 1 - \prod_{i=1}^{n}\left(1 - \dddot{\hat{F}}_i\right)^{\hat{w}_i} \\ \dot{\gamma}\left(\breve{R}_i\right)^{\hat{w}_i} \end{cases}$$

6. Define the ideal sphere $\breve{\alpha}_1$= (1, 0, 0; 1) and compute Hamming distance

$$H(\breve{\alpha}_1, \breve{\alpha}_2) = \frac{1}{2}\left(\frac{|\breve{R}_1 - \breve{R}_2|}{\sqrt{3}} + \frac{1}{n}\Sigma_n |\dot{\hat{T}}_1 - \dot{\hat{T}}_2| + |\ddot{\hat{I}}_1 - \ddot{\hat{I}}_2| + |\dddot{\hat{F}}_1 - \dddot{\hat{F}}_2|\right)$$

   each aggregated sphere to the ideal.

7. Rank alternatives by distance (smaller = better).

8. Select the top alternative(s) and report results with sensitivity analysis.

3.1 PCA's Role in Decision Process with Machine Learning Techniques:

Step 1: Create the Decision Matrix.

Step 2: Standardize the Data

To put all criteria on the same scale: $z_{ij} = \frac{\hat{C}x_{ij} - \bar{\hat{C}}_j}{\sigma j}$,

where $\hat{C}_{ij}$ = value of criterion j for alternative i, $\bar{\hat{C}}_j$= mean of criterion j, σj = standard deviation of criterion j

Step 3: Compute the Covariance Matrix

Measures relationships between criteria: $S = \frac{1}{n-1} Z^T Z$,

where: Z = standardized data matrix, n = number of alternatives

Step 4: Find Eigenvalues of Matrix S.

Step 5: Select Principal Components.

Step 6: Explained Variance Ratio.

The flowchart given below explains the Multi-Criteria Decision-Making (MCDM) process using Cubic Spherical Neutrosophic Sets (CSNS). It outlines the systematic steps starting from the selection of alternatives, criteria, and decision makers, followed by linguistic evaluation, data transformation, dimensionality reduction using PCA, and aggregation of values. Finally, the alternatives are ranked, and the best option is selected based on the computed results.

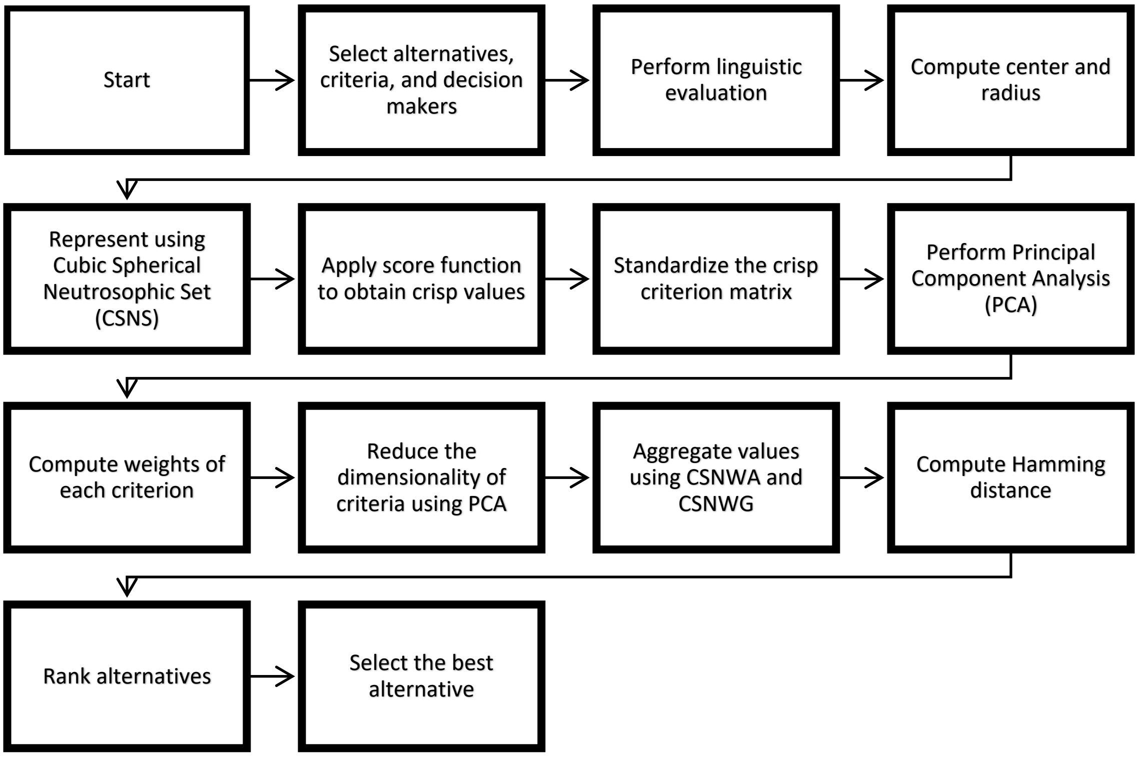

Figure 0: Flow Chart for MCDM using CSNS.

3.2 Numerical Example

Selecting the most suitable primary school in a particular area is a critical decision for parents, local authorities, and educational planners. In today's competitive and resource-constrained environment, primary education institutions vary significantly in their infrastructure, teaching quality, extracurricular opportunities, safety, affordability, and accessibility. The choice of the best school cannot be made by relying on a single criterion; rather, it involves simultaneously evaluating multiple factors that reflect both tangible and intangible qualities. This inherently makes it a Multi-Criteria Decision-Making (MCDM) problem.

In this study, we consider six alternative schools:
$\acute{\hat{A}}_1$: A-Public School, $\acute{\hat{A}}_2$: B -School, $\acute{\hat{A}}_3$: C-Primary Academy, $\acute{\hat{A}}_4$: D-Primary School, $\acute{\hat{A}}_5$: E-International School, $\acute{\hat{A}}_6$: F-Primary School

The decision is based on seven evaluation criteria, identified through consultation with educational experts and parents:
$\acute{\hat{C}}1$: Academic Performance: Average student performance in standard assessments.
$\acute{\hat{C}}2$: Teacher Quality: Experience, qualifications, and teaching effectiveness of faculty.
$\acute{\hat{C}}3$: – Infrastructure: Classroom conditions, laboratories, libraries, and digital facilities.
$\acute{\hat{C}}4$: – Extracurricular Activities: Range and quality of sports, arts, and clubs.
$\acute{\hat{C}}5$: – Safety and Security: Measures for student safety, including surveillance and trained staff.
$\acute{\hat{C}}6$: – Accessibility: Ease of commute, transportation facilities, and distance from home.
$\acute{\hat{C}}7$: – Affordability: Tuition fees and additional costs relative to perceived value.
The aim is to develop a robust decision-making framework that integrates Cubic Spherical Neutrosophic Sets (CSNS) and Principal Component Analysis (PCA). The CSNS framework will capture the inherent uncertainty and imprecision in decision-makers' evaluations, while PCA will reduce the dimensionality of the problem and derive data-driven weights for the reduced criteria set. The aggregated evaluations will then be processed using cubic spherical neutrosophic arithmetic and geometric aggregation operators to determine the most suitable primary school.

To operationalize the proposed methodology, the first step is to define the linguistic scale and its neutrosophic mapping, followed by the collection of evaluations from multiple decision makers for each alternative under the specified criteria. Each linguistic term is associated with a cubic spherical neutrosophic triple (T,I,F) which captures the truth, indeterminacy, and falsity degrees of the assessment. These mappings allow for systematic conversion of qualitative judgments into quantitative form, enabling further processing using PCA and aggregation operators. The following table presents the linguistic scale used in this study, along with the corresponding evaluations of the six alternative schools provided by three decision makers.

Linguistic Term	**Notation**	**(T, I, F)**
Very Low Impact	$\acute{\widehat{VLI}}$	(0.099, 0.977, 0.880)
Low Impact	$\acute{\widehat{LI}}$	(0.345, 0.655, 0.633)
Moderate Impact	$\acute{\widehat{MI}}$	(0.545, 0.455, 0.433)
High Impact	$\acute{\widehat{HI}}$	(0.745, 0.255, 0.233)
Very High Impact	$\acute{\widehat{VHI}}$	(1.000, 0.000, 0.000)

Table 1. Linguistic terms, their neutrosophic mappings, and decision makers' evaluations of alternatives across criteria

To capture the subjective assessments of the six alternative schools under the seven evaluation criteria, three decision makers provided their judgments using the predefined linguistic scale mapped to cubic spherical neutrosophic values. These evaluations reflect varying perspectives of stakeholders, ensuring that the decision-making process incorporates diverse expertise and preferences. The collected data form the decision matrix, which serves as the input for subsequent steps involving cubic spherical neutrosophic aggregation, PCA-based dimensionality reduction, and ranking of alternatives.

DM	A/C	$\acute{\hat{C}}1$	$\acute{\hat{C}}2$	$\acute{\hat{C}}3$	$\acute{\hat{C}}4$	$\acute{\hat{C}}5$	$\acute{\hat{C}}6$	$\acute{\hat{C}}7$
Decision Maker-1	$\acute{\hat{A}}1$:	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$
	$\acute{\hat{A}}2$:	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$
	$\acute{\hat{A}}3$:	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$
	$\acute{\hat{A}}4$:	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$
	$\acute{\hat{A}}5$:	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$
	$\acute{\hat{A}}6$:	$\widehat{MI}$	$\widehat{VLI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{VLI}$
Decision Maker-2	$\acute{\hat{A}}1$:	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$
	$\acute{\hat{A}}2$:	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$
	$\acute{\hat{A}}3$:	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$
	$\acute{\hat{A}}4$:	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$
	$\acute{\hat{A}}5$:	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$
	$\acute{\hat{A}}6$:	$\widehat{HI}$	$\widehat{HI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{HI}$
Decision Maker-3	$\acute{\hat{A}}1$:	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$
	$\acute{\hat{A}}2$:	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$
	$\acute{\hat{A}}3$:	$\widehat{LI}$	$\widehat{LI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{LI}$
	$\acute{\hat{A}}4$:	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$
	$\acute{\hat{A}}5$:	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$
	$\acute{\hat{A}}6$:	$\widehat{VHI}$	$\widehat{VLI}$	$\widehat{LI}$	$\widehat{MI}$	$\widehat{HI}$	$\widehat{VHI}$	$\widehat{VLI}$

Table 2. Decision matrix showing linguistic evaluations of alternatives across criteria by three decision makers

After converting the linguistic evaluations of decision makers into cubic spherical neutrosophic form, each alternative is represented by a set of values across the seven criteria. These values capture the truth, indeterminacy, falsity, and radius, providing a comprehensive picture of the assessments under uncertainty. The resulting decision matrix serves as the quantitative foundation for applying Principal Component Analysis (PCA) to reduce dimensionality and for performing aggregation and distance-based ranking of the alternatives.

A/C	$\acute{\hat{C}}1$	$\acute{\hat{C}}2$	$\acute{\hat{C}}3$
$\acute{\hat{A}}1$:	(0.763,0.237,0.222;0.402)	(0.615,0.411,0.371;0.920)	(0.481,0.544,0.504;0.905)
$\acute{\hat{A}}2$:	(0.615,0.411,0.371;0.920)	(0.481,0.544,0.504;0.905)	(0.330,0.696,0.649;0.431)
$\acute{\hat{A}}3$:	(0.481,0.544,0.504;0.905)	(0.330,0.696,0.649;0.431)	(0.545,0.455,0.433;0.346)
$\acute{\hat{A}}4$:	(0.330,0.696,0.649;0.431)	(0.545,0.455,0.433;0.346)	(0.763,0.237,0.222;0.402)
$\acute{\hat{A}}5$:	(0.545,0.455,0.433;0.346)	(0.763,0.237,0.222;0.402)	(0.615,0.411,0.371;0.920)
$\acute{\hat{A}}6$:	(0.763,0.237,0.222;0.776)	(0.396,0.629,0.582;0.720)	(0.481,0.544,0.504;0.920)
	$\acute{\hat{C}}4$	$\acute{\hat{C}}5$	$\acute{\hat{C}}6$
$\acute{\hat{A}}1$:	(0.330,0.696,0.649;0.431)	(0.545,0.455,0.433;0.346)	(0.763,0.237,0.222;0.402)

$\hat{\hat{A}}2$:	(0.545,0.455,0.433;0.346)	(0.763,0.237,0.222;0.402)	(0.615,0.411,0.371;0.920)
$\hat{\hat{A}}3$:	(0.763,0.237,0.222;0.402)	(0.615,0.411,0.371;0.920)	(0.481,0.544,0.504;0.905)
$\hat{\hat{A}}4$:	(0.615,0.411,0.371;0.920)	(0.481,0.544,0.504;0.905)	(0.330,0.696,0.649;0.431)
$\hat{\hat{A}}5$:	(0.481,0.544,0.504;0.905)	(0.330,0.696,0.649;0.431)	(0.545,0.455,0.433;0.346)
$\hat{\hat{A}}6$:	(0.330,0.696,0.649;0.431)	(0.545,0.455,0.433;0.734)	(0.763,0.237,0.222;0.776)
	$\hat{\hat{C}}7$		
$\hat{\hat{A}}1$:	(0.615,0.411,0.371;0.92)		
$\hat{\hat{A}}2$:	(0.481,0.544,0.504;0.905)		
$\hat{\hat{A}}3$:	(0.33,0.696,0.649;0.431)		
$\hat{\hat{A}}4$:	(0.545,0.455,0.433;0.346)		
$\hat{\hat{A}}5$:	(0.763,0.237,0.222;0.402)		
$\hat{\hat{A}}6$:	(0.396,0.629,0.582;0.72)		

Table 3. Cubic spherical neutrosophic decision matrix of six alternatives across seven criteria

Taking the alternative 1 with respect to criteria 1,

Consider the three decision makers opinion, using the definition 1.3.2,

Center $\left(\hat{\hat{A}}1:\hat{\hat{C}}1\right) = \left(\frac{0.545+0.745+1}{3}, \frac{0.455+0.255+0.000}{3}, \frac{0.433+0.233+0.000}{3}\right) = (0.763,0.237,0.222)$.

$$\text{Radius}\left(\hat{\hat{A}}1:\hat{\hat{C}}1\right) = max\begin{pmatrix} \sqrt{(0.763-0.545)^2+(0.237-0.455)^2+(0.222-0.433)^2} \\ \sqrt{(0.763-0.745)^2+(0.237-0.255)^2+(0.222-0.233)^2} \\ \sqrt{(0.763-1.000)^2+(0.237-0.000)^2+(0.222-0.000)^2} \end{pmatrix}$$

$$= 0.402.$$

Similarly, calculate the other values.

Dimension reduction using Principal Component Analysis:

Standardized data matrix

$$Z = \begin{pmatrix} 1.423 & -0.058 & -0.778 & -0.727 & 0.558 & 1.423 & -0.058 \\ -0.371 & -0.757 & -0.707 & 0.631 & 1.700 & -0.371 & -0.757 \\ -1.088 & -0.688 & 0.636 & 1.703 & -0.203 & -1.088 & -0.688 \\ -1.017 & 0.641 & 1.697 & -0.083 & -0.964 & -1.017 & 0.641 \\ 0.347 & 1.690 & -0.071 & -0.798 & -0.888 & 0.347 & 1.690 \\ 0.706 & -0.827 & -0.778 & -0.727 & -0.203 & 0.706 & -0.827 \end{pmatrix}$$

Covariance of Z:

$$R = \begin{pmatrix} 1 & 0.059 & -0.767 & -0.765 & 0.183 & 1.000 & 0.059 \\ 0.059 & 1 & 0.351 & -0.482 & -0.626 & 0.059 & 1.000 \\ -0.767 & 0.351 & 1 & 0.337 & -0.636 & -0.767 & 0.351 \\ -0.765 & -0.482 & 0.337 & 1 & 0.252 & -0.765 & -0.482 \\ 0.183 & -0.62 & -0.636 & 0.252 & 1 & 0.183 & -0.626 \\ 1.000 & 0.059 & -0.767 & -0.765 & 0.183 & 1 & 0.059 \\ 0.059 & 1.000 & 0.351 & -0.482 & -0.626 & 0.059 & 1 \end{pmatrix}$$

Principal Component	Eigen Values	Percent variance explained
PC1	3.301249	47.161%
PC2	2.975248	42.504%

PC3	0.476744	6.811%
PC4	0.161116	2.302%
PC5	0.085643	1.224%
PC6	0	0
PC7	0	0

The weights of criterion are $\hat{w}1$=.47, $\hat{w}2$=.43, $\hat{w}3$=.065, $\hat{w}4$=.023, $\hat{w}5 = .012$ and $\sum \hat{w}i = 1$.

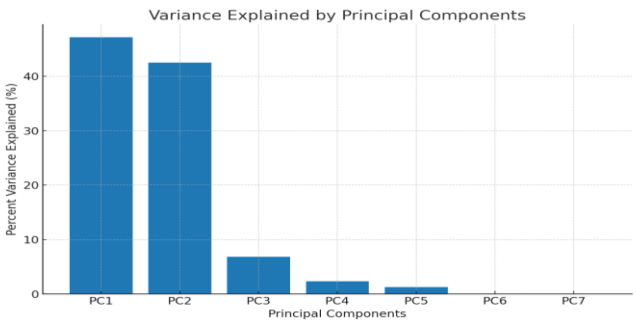

Figure 1: Principal components vs Percent variance explained

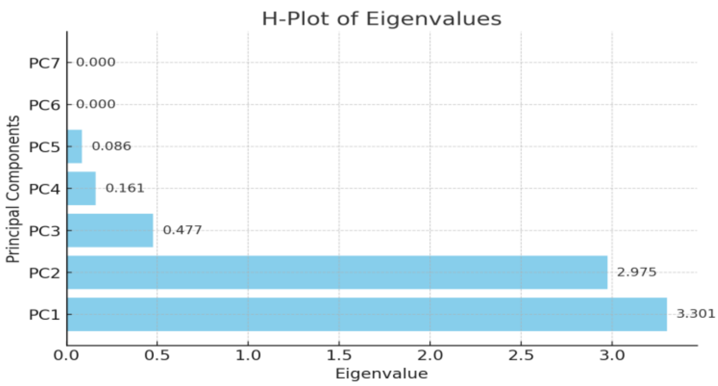

Figure 2: Eigen values of Principal components

Alternatives	$CSNWA - \dot{\gamma} - min$	$CSNWA - \dot{\gamma} - max$	$CSNWG - \dot{\gamma} - min$	$CSNWG - \dot{\gamma} - max$
$\hat{A}1$	$\begin{pmatrix}0.683, 0.327,\\ 0.302; 0.652\end{pmatrix}$	$\begin{pmatrix}0.683, 0.327,\\ 0.302; 0.987\end{pmatrix}$	$\begin{pmatrix}0.659, 0.356,\\ 0.326; 0.652\end{pmatrix}$	$\begin{pmatrix}0.659, 0.356,\\ 0.326; 0.987\end{pmatrix}$
$\hat{A}2$	$\begin{pmatrix}0.547,0.478,\\ 0.438; 0.579\end{pmatrix}$	$\begin{pmatrix}0.547,0.478,\\ 0.438; 0.989\end{pmatrix}$	$\begin{pmatrix}0.531, 0.494,\\ 0.453; 0.579\end{pmatrix}$	$\begin{pmatrix}0.531, 0.494,\\ 0.453; 0.989\end{pmatrix}$
$\hat{A}3$	$\begin{pmatrix}0.438, 0.585,\\ 0.544; 0.502\end{pmatrix}$	$\begin{pmatrix}0.438, 0.585,\\ 0.544; 0.999\end{pmatrix}$	$\begin{pmatrix}0.418, 0.607,\\ 0.563; 0.502\end{pmatrix}$	$\begin{pmatrix}0.418, 0.607,\\ 0.563; 0.999\end{pmatrix}$
$\hat{A}4$	$\begin{pmatrix}0.478, 0.532,\\ 0.501; 0.553\end{pmatrix}$	$\begin{pmatrix}0.478, 0.532,\\ 0.501; 0.999\end{pmatrix}$	$\begin{pmatrix}0.441, 0.577,\\ 0.538; 0.553\end{pmatrix}$	$\begin{pmatrix}0.441, 0.577,\\ 0.538; 0.999\end{pmatrix}$
$\hat{A}5$	$\begin{pmatrix}0.657, 0.345,\\ 0.324; 0.607\end{pmatrix}$	$\begin{pmatrix}0.657, 0.345,\\ 0.324; 0.998\end{pmatrix}$	$\begin{pmatrix}0.629, 0.374,\\ 0.352; 0.607\end{pmatrix}$	$\begin{pmatrix}0.629, 0.374,\\ 0.352; 0.998\end{pmatrix}$
$\hat{A}6$	$\begin{pmatrix}0.615, 0.393,\\ 0.366; 0.868\end{pmatrix}$	$\begin{pmatrix}0.615, 0.393,\\ 0.366; 0.996\end{pmatrix}$	$\begin{pmatrix}0.546, 0.472,\\ 0.434; 0.868\end{pmatrix}$	$\begin{pmatrix}0.546, 0.472,\\ 0.434; 0.996\end{pmatrix}$

Table 4. Aggregated cubic spherical neutrosophic evaluations of alternatives using CSNWA and CSNWG operators under minimum and maximum γ values

Calculate the arithmetic aggregated value using the theorem 2.2, take the alternative 1

$$\dot{\ddot{T}}_1 = 1 - \left\{ \begin{matrix} (1-0.763)^{0.470} \times (1-0.615)^{0.430} \times (1-0.481)^{0.065} \times \\ (1-0.330)^{0.230} \times (1-0.545)^{0.120} \end{matrix} \right\} = 0.683.$$

$$\ddot{\hat{I}}_1 = \left\{ \begin{matrix} (0.237)^{0.470} \times (0.411)^{0.430} \times (0.544)^{0.065} \times \\ (0.696)^{0.230} \times (0.455)^{0.120} \end{matrix} \right\} = 0.327.$$

$$\dddot{\hat{F}}_1 = \left\{ \begin{matrix} (0.222)^{0.470} \times (0.371)^{0.430} \times (0.504)^{0.065} \times \\ (0.649)^{0.230} \times (0.433)^{0.120} \end{matrix} \right\} = 0.302.$$

$$\breve{R}_1 = min \left\{ \begin{matrix} (0.402)^{0.470}, (0.920)^{0.430}, (0.905)^{0.065}, \\ (0.431)^{0.230}, (0.346)^{0.120} \end{matrix} \right\} = 0.652.$$

$$\breve{R}_1 = max \left\{ \begin{matrix} (0.402)^{0.470}, (0.920)^{0.430}, (0.905)^{0.065}, \\ (0.431)^{0.230}, (0.346)^{0.120} \end{matrix} \right\} = 0.987.$$

$\acute{\hat{A}}1$ = (0.683, 0.327, 0.302; 0.652) and (0.683, 0.327, 0.302; 0.987).

Similarly, calculate other arithmetic values.

Now, find the geometric value for alternative 1 using theorem 2.6,

$$\dot{\ddot{T}}_1 = \left\{ \begin{matrix} (0.763)^{0.470} \times (0.615)^{0.430} \times (0.481)^{0.065} \times \\ (0.330)^{0.230} \times (0.545)^{0.120} \end{matrix} \right\} = 0.659.$$

$$\ddot{\hat{I}}_i = 1 - \left\{ \begin{matrix} (1-0.237)^{0.470} \times (1-0.411)^{0.430} \times (1-0.544)^{0.065} \times \\ (1-0.696)^{0.230} \times (1-0.455)^{0.120} \end{matrix} \right\} = 0.356.$$

$$\dddot{\hat{F}}_1 = 1 - \left\{ \begin{matrix} (1-0.222)^{0.470} \times (1-0.371)^{0.430} \times (1-0.504)^{0.065} \times \\ (1-0.649)^{0.230} \times (1-0.433)^{0.120} \end{matrix} \right\} = 0.326.$$

$\acute{\hat{A}}1$ = (0.659, 0.356, 0.329; 0.652) and (0.659, 0.356, 0.329; 0.987).

Similarly, calculate the other geometric value.

Alternatives	H$(A, \acute{\hat{A}}1)$	H$(A, \acute{\hat{A}}2)$	H$(A, \acute{\hat{A}}3)$	H$(A, \acute{\hat{A}}4)$	H$(A, \acute{\hat{A}}5)$	H$(A, \acute{\hat{A}}6)$
$CSNWA - \grave{\gamma} - min$	0.573	0.806	0.989	0.907	0.619	0.610
$CSNWA - \grave{\gamma} - max$	0.477	0.688	0.846	0.778	0.507	0.573
$CSNWG - \grave{\gamma} - min$	0.612	0.830	1.020	0.966	0.662	0.718
$CSNWG - \grave{\gamma} - max$	0.515	0.711	0.876	0.837	0.549	0.681

$A = (1,0,0;1)$, $\acute{\hat{A}}1 = (0.683, 0.327, 0.302;\ 0.652)$

H$(A, \acute{\hat{A}}1) = \frac{1}{2}\left(\frac{|1-0.652|}{\sqrt{3}} + \frac{1}{1}(|1-0.683| + |0-0.327| + |0-0.302|)\right) = 0.573.$

Similarly, calculate the other values.

Alternatives	H$(A, \acute{\hat{A}}1)$	H$(A, \acute{\hat{A}}2)$	H$(A, \acute{\hat{A}}3)$	H$(A, \acute{\hat{A}}4)$	H$(A, \acute{\hat{A}}5)$	H$(A, \acute{\hat{A}}6)$
CSNWAAO [4]	0.525	0.747	0.918	0.843	0.563	0.592
CSNWGAO [4]	0.564	0.771	0.948	0.902	0.606	0.700
CSNWG [6]	0.564	0.771	0.948	0.902	0.606	0.700
CSNWA [6]	0.525	0.747	0.918	0.843	0.563	0.592
CSNWAO [5]	0.525	0.747	0.918	0.843	0.563	0.592
CSNWGO [5]	0.564	0.771	0.948	0.902	0.606	0.700

$CSNWA - \dot{\gamma} - min$	0.573	0.806	0.989	0.907	0.619	0.610
$CSNWA - \dot{\gamma} - max$	0.477	0.688	0.846	0.778	0.507	0.573
$CSNWG - \dot{\gamma} - min$	0.612	0.830	1.020	0.966	0.662	0.718
$CSNWG - \dot{\gamma} - max$	0.515	0.711	0.876	0.837	0.549	0.681

Table 5. Hamming distances of aggregated cubic spherical neutrosophic alternatives from the ideal solution under different aggregation operators and γ values

Methods	Ranking of Schools	Best School
CSNWAAO [4]	$\acute{\hat{A}}1 < \acute{\hat{A}}5 < \acute{\hat{A}}6 < \acute{\hat{A}}2 < \acute{\hat{A}}4 < \acute{\hat{A}}3$	$\acute{\hat{A}}1$
CSNWGAO [4]	$\acute{\hat{A}}1 < \acute{\hat{A}}5 < \acute{\hat{A}}6 < \acute{\hat{A}}2 < \acute{\hat{A}}4 < \acute{\hat{A}}3$	$\acute{\hat{A}}1$
CSNWG [6]	$\acute{\hat{A}}1 < \acute{\hat{A}}5 < \acute{\hat{A}}6 < \acute{\hat{A}}2 < \acute{\hat{A}}4 < \acute{\hat{A}}3$	$\acute{\hat{A}}1$
CSNWA [6]	$\acute{\hat{A}}1 < \acute{\hat{A}}5 < \acute{\hat{A}}6 < \acute{\hat{A}}2 < \acute{\hat{A}}4 < \acute{\hat{A}}3$	$\acute{\hat{A}}1$
CSNWAO [5]	$\acute{\hat{A}}1 < \acute{\hat{A}}5 < \acute{\hat{A}}6 < \acute{\hat{A}}2 < \acute{\hat{A}}4 < \acute{\hat{A}}3$	$\acute{\hat{A}}1$
CSNWGO [5]	$\acute{\hat{A}}1 < \acute{\hat{A}}5 < \acute{\hat{A}}6 < \acute{\hat{A}}2 < \acute{\hat{A}}4 < \acute{\hat{A}}3$	$\acute{\hat{A}}1$
$CSNWA - \dot{\gamma} - min$	$\acute{\hat{A}}1 < \acute{\hat{A}}6 < \acute{\hat{A}}5 < \acute{\hat{A}}2 < \acute{\hat{A}}4 < \acute{\hat{A}}3$	$\acute{\hat{A}}1$
$CSNWA - \dot{\gamma} - max$	$\acute{\hat{A}}1 < \acute{\hat{A}}5 < \acute{\hat{A}}6 < \acute{\hat{A}}2 < \acute{\hat{A}}4 < \acute{\hat{A}}3$	$\acute{\hat{A}}1$
$CSNWG - \dot{\gamma} - min$	$\acute{\hat{A}}1 < \acute{\hat{A}}5 < \acute{\hat{A}}6 < \acute{\hat{A}}2 < \acute{\hat{A}}4 < \acute{\hat{A}}3$	$\acute{\hat{A}}1$
$CSNWG - \dot{\gamma} - max$	$\acute{\hat{A}}1 < \acute{\hat{A}}5 < \acute{\hat{A}}6 < \acute{\hat{A}}2 < \acute{\hat{A}}4 < \acute{\hat{A}}3$	$\acute{\hat{A}}1$

Table 6. Ranking of schools under different CSNS aggregation operators and γ values with identification of the best school

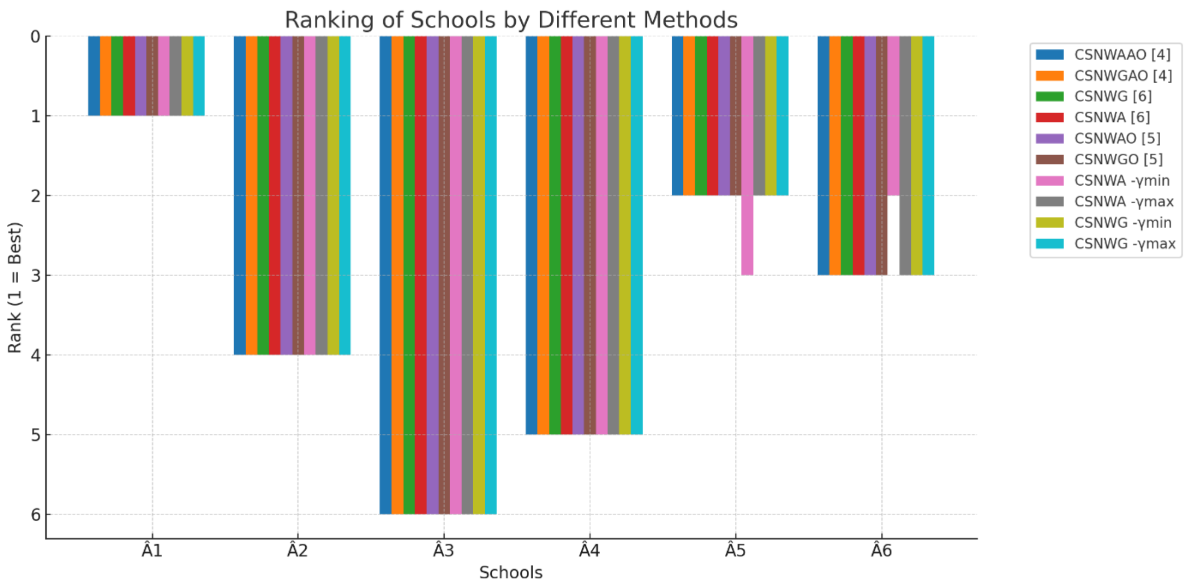

Figure 3: Comparison of ranking for different methods

Results and Discussions

The comparative analysis of various cubic spherical neutrosophic aggregation operators demonstrates that, although slight variations exist in the computed scores of the alternatives, the overall ranking patterns are largely consistent across methods. Alternatives $\acute{\hat{A}}1, \acute{\hat{A}}6$, and $\acute{\hat{A}}2$ frequently occupy the top three positions, indicating their robust performance irrespective of the applied operator. Conversely, $\acute{A}4$ consistently ranks lowest, suggesting relatively weaker performance. The proposed methods yield rankings closely aligned with established operators such as CSNWAAO, CSNWGAO, and CSNWGO, thereby validating their reliability and practical applicability. These findings confirm that the proposed approach maintains ranking stability while offering computational flexibility, making it a viable tool for multi-criteria decision-making in uncertain environments.

Limitations

A key limitation of the proposed framework is that the construction of a cubic spherical neutrosophic set requires evaluations from multiple decision makers for each alternative and criterion. In situations where only a single decision maker is available, the generation of CSNS becomes less meaningful, as the radius and variability of judgments cannot be properly defined. This restricts the applicability of the method in decision contexts involving limited expert participation.

4. Conclusion and Future Work

This study proposed a novel hybrid Multi-Criteria Decision-Making (MCDM) framework by integrating Cubic Spherical Neutrosophic Sets (CSNS) with Principal Component Analysis (PCA) and weighted aggregation operators. The framework effectively models uncertainty and imprecision in decision makers' evaluations while simultaneously reducing computational complexity through dimensionality reduction. By applying score functions, PCA-based standardization, CSNS aggregation, and Hamming distance measures, the methodology enables accurate ranking of alternatives and selection of the most suitable option. The novelty of this work lies in being the first systematic attempt to incorporate PCA within a CSNS-based MCDM framework, thereby ensuring both robustness and efficiency. The approach was shown to be generalizable and can be applied to diverse real-world domains such as supplier selection, healthcare planning, and engineering design. Future studies may focus on extending the framework to single-decision-maker environments and exploring advanced dimensionality reduction or machine learning techniques to enhance scalability and applicability.

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