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A Retrospective Study on Neutrosophic Distributions and Their Applications

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Abstract: In recent years, Neutrosophic statistics has emerged as a powerful framework for handling uncertainty, indeterminacy, and imprecision in data. This review paper presents a comprehensive retrospective analysis of Neutrosophic distributions, tracing their theoretical foundations, historical evolution, and methodological advancements. The study systematically compares Neutrosophic distributions with classical probability distributions, emphasizing their unique ability to explicitly model indeterminacy through the incorporation of truth, indeterminacy, and falsity components. Key Neutrosophic distributions introduced in the literature are discussed alongside their related terms, with a tabular presentation to aid clarity. The methodologies employed in existing studies are examined in detail. Particular attention is given to their applications in biomedical research, where uncertainty is a critical factor in decision-making and data interpretation. The advantages, limitations, and challenges associated with Neutrosophic models are also analyzed. Finally, future research directions are proposed, including the development of new distributions, improved computational tools, and broader interdisciplinary applications. This review underscores the growing significance of Neutrosophic distributions as a generalization of classical models, offering a more realistic approach to data analysis in complex, uncertain environments.

Keywords: Neutrosophic Distributions, Uncertainty Modeling, Comparison with Classical Distributions

1 Introduction

The analysis of data in real-world scenarios often encounters the challenge of uncertainty. Traditional statistical methods, while powerful in many contexts, operate under the assumption of precise and well-defined data. However, real-world data is frequently fraught with vagueness, ambiguity, incompleteness, and even contradictions. This inherent uncertainty poses limitations for classical statistical approaches, necessitating the development of methodologies capable of effectively modeling and analyzing such complex information. Neutrosophic statistics has emerged as a valuable framework for addressing these limitations. It is rooted in neutrosophic logic, which extends fuzzy logic by incorporating the concept of indeterminacy alongside the traditional notions of truth and falsity [15]. This extension allows for a more nuanced representation of real-world systems where information might not be definitively true or false, but rather contain elements of the unknown or unclear. Neutrosophic statistics, therefore, serves as a generalization of classical statistics, specifically designed to handle indeterminate data and inference methods [4]. Its ability to accommodate varying degrees of truth, falsity, and indeterminacy makes it particularly suitable for analyzing the messy and complex nature of real-world phenomena [10]. Furthermore, Neutrosophic Statistics is built upon the foundation of Set Analysis, which positions Interval Statistics as a specific instance within the broader Neutrosophic framework. The introduction of indeterminacy as a fundamental component marks a significant departure from traditional statistical paradigms, offering a new perspective on how uncertainty is approached in data analysis. While classical statistics primarily models randomness using probability, and fuzzy statistics extends this to handle vagueness through degrees of membership, neutrosophic statistics broadens the scope further by explicitly addressing situations where information is not just vague but also genuinely unknown or contradictory. This three dimensional approach, encompassing truth, falsity, and indeterminacy, allows for a more comprehensive representation of the complexities inherent in real-world data.

This paper aims to provide a comprehensive retrospective analysis of the field of Neutrosophic distributions. The objective is to explore the historical development and evolution of these distributions, investigate their applications across various domains, compare them with other types of statistical distributions, identify their advantages and limitations, analyze the methodologies used in their study, and ultimately synthesize the findings to identify key insights, trends, and potential future research directions. This retrospective study seeks to offer a holistic view of Neutrosophic distributions, highlighting their impact and effectiveness in addressing the challenges posed by uncertainty in data analysis.

In Section 2 titled Fundamental Concepts of Neutrosophic Distributions we explain the key concepts of Neutrosophy, the structure of Neutrosophic sets, and how they extend classical and fuzzy logic systems. In Section 3, Historical Evolution of Neutrosophic Distributions provides a brief overview of the foundational concepts and key milestones in the development of Neutrosophic distributions. In Section 4, Overview of Key Neutrosophic Distributions, we present a comprehensive table listing various Neutrosophic distributions and the associated terminologies for ease of reference. Section 5 titled Comparison with Classical and Neutrosophic Distributions, highlights how Neutrosophic distributions extend classical frameworks by incorporating indeterminacy explicitly. In Section 6, Advantages and Challenges of Neutrosophic Distributions, explores the benefits of using Neutrosophic models, such as improved flexibility in modeling uncertainty, while also addressing their computational and theoretical challenges. Section 7 Methodologies used in Retrospective Neutrosophic Studies, outlines key methodologies used in retrospective studies, including derivations, simulations, real data analysis, and inference techniques. Section 8 titled Neutrosophic Distributions in Biomedical Research highlights the applications of Neutrosophic distributions in biomedical research. In Section 9, Conclusion and Future Directions, we summarize key findings and propose directions for future research, including new distribution development, software implementation, and expanded applications.

2 Fundamental Concepts of Neutrosophic Distributions

Neutrosophic distributions form the backbone of Neutrosophic statistics, offering a robust framework for analyzing data with uncertainty, vagueness, and contradiction. Unlike classical methods that assume precise values, Neutrosophic approaches explicitly incorporate indeterminacy into statistical modeling. This section introduces the essential concepts that define the structure and behavior of Neutrosophic distributions.

2.1 Neutrosophic Sets and Logic

At the heart of Neutrosophic distributions lie the fundamental concepts of Neutrosophic sets, logic, and probability. A Neutrosophic set is characterized by the fact that each element within it possesses a degree of membership defined by three independent components: truth (T), indeterminacy (I), and falsity (F) [15]. These components represent the extent to which an element belongs to the set, the degree to which its membership is unknown or unclear, and the degree to which it does not belong to the set, respectively. Operating on these truth values is Neutrosophic logic, which serves as the underlying system for reasoning about propositions in the presence of indeterminacy. It allows for degrees of truth, falsehood, and indeterminacy, offering a more flexible framework than classical binary logic.

2.2 Neutrosophic Probability

Neutrosophic probability emerges as a generalization of classical probability. In this framework, the probability of an event is expressed not as a single value, but as a triplet (t, i, f), where 't' represents the chance of the event being true, 'i' represents the indeterminate chance (the probability of the event occurring or not occurring is unknown), and 'f' represents the chance of the event being false [18]. A key distinction from classical probability is that in the neutrosophic realm, the sum of all space probabilities equals 3, reflecting the inclusion of indeterminacy, whereas in classical probability, this sum is 1. The shift from a binary perspective of true/false or even the fuzzy perspective of a degree of truth to a trichotomy of truth, indeterminacy, and falsity represents a fundamental conceptual difference. This allows Neutrosophic methods to model a significantly wider range of real-world scenarios where uncertainty is not simply a matter of degree but can also involve genuine unknowns or contradictions.

2.3 Neutrosophic Random Variables

A Neutrosophic random variable is defined as a variable whose possible values are Neutrosophic numbers. These numbers comprise two parts: a determinate part, which represents the known or precise aspect, and an indeterminate part, which captures the uncertainty or vagueness [4]. A common representation of a Neutrosophic number is XN = a + bI, where 'a' is the determinate component and 'bI' is the indeterminate component. Consequently, a Neutrosophic distribution is a probability distribution that is defined for a Neutrosophic random variable. This distribution can be represented in various ways, such as by three separate functions or intervals corresponding to truth, indeterminacy, and falsity, or more commonly, by parameters of a classical distribution that are themselves Neutrosophic numbers. The function that models the Neutrosophic Probability of a random variable x is denoted as NP(x) = (T(x), I(x), F(x)). The use of Neutrosophic numbers to define both random variables and their associated distributions allows for the direct and inherent incorporation of uncertainty and vagueness into the statistical model itself, rather than treating uncertainty as an external factor to be addressed after data collection.

2.4 Representation of Indeterminancy

The representation of indeterminacy in Neutrosophic distributions is a crucial aspect. Indeterminacy is often expressed using an interval or by a parameter that is multiplied by an indeterminate factor 'I', where 'I' typically ranges within the interval or some other specified interval [4]. This indeterminate part serves to reflect the unknown, vague, or even contradictory information that is associated with the variable or the parameters of the distribution. It is important to note that Neutrosophic statistics possesses the capability to reduce indeterminacy through mathematical operations in certain cases, a feature that distinguishes it from interval statistics, where such operations might lead to an increase in indeterminacy. The interval-based representation of indeterminacy provides a practical and quantifiable way to work with uncertainty within statistical models. Instead of relying on a single precise value, this approach acknowledges the potential range of values that a parameter or observation might take, thus offering a more realistic and flexible framework for analysis in the face of imperfect information.

3 Historical Evolution of Neutrosophic Distributions

The genesis of Neutrosophic distributions can be traced to the introduction of Neutrosophy by Florentin Smarandache in 1995 [8]. Conceived as a generalization of fuzzy logic, Neutrosophy provided a philosophical framework for addressing neutrality and indeterminacy, both common in real-world situations. Following this foundation, Neutrosophic statistics emerged as a distinct discipline focused on analyzing data characterized by imprecision and indeterminacy [7]. Early research established fundamental Neutrosophic statistical concepts, paving the way for more advanced tools, including probability distributions. Key Contributors to Neutrosophic Distributions:

- Florentin Smarandache — Originator of Neutrosophy and a prolific contributor to Neutrosophic statistics [7].
- Muhammad Aslam — Developed numerous Neutrosophic statistical tests and distributions [15].
- Other notable researchers and their contributions include:
  - Patro Smarandache — Neutrosophic Binomial and Normal distributions [15].
  - Sherwani et al. — Contributions to the Neutrosophic Binomial distribution [15].
  - S. Al-Duais — Neutrosophic Log-Gamma distribution [12].
  - Khan et al. — Neutrosophic Negative Binomial distribution [15].
  - Musa — Neutrosophic Pareto distribution [11].

The field of Neutrosophic distributions, though relatively young, shows active development, with growing recognition of its potential to address uncertainty in data analysis. Milestones in the Evolution of Neutrosophic Distributions:

- Early developments focused on extending classical distributions into the Neutrosophic framework:
  - Neutrosophic Binomial, Normal, and Multinomial distributions [15].
- Subsequent expansions included:
  - Neutrosophic Negative Binomial distribution [15].
  - Neutrosophic Log-Gamma distribution [12].
  - Neutrosophic Inverse Exponential distribution [10].
  - Neutrosophic Maxwell distribution [13].
  - Neutrosophic Lindley distribution [8].
  - Neutrosophic Pareto distribution [11].
  - Neutrosophic q-Poisson distribution [3].

Beyond distributions, Neutrosophic concepts have been applied to generalize statistical tests and methodologies for hypothesis testing and inference under uncertainty [7]. This generalization strategy reflects a systematic effort to expand classical statistical tools to better accommodate real-world data affected by indeterminacy and imprecision.Over time, researchers steadily built on the early ideas, introducing new Neutrosophic distributions to better handle the complex and uncertain nature of real-world data. This steady growth shows how the field has evolved in response to the need for more flexible and realistic statistical tools

4 Overview of Key Neutrosophic Distributions

The field of Neutrosophic statistics has witnessed the development of several key probability distributions that extend their classical counterparts to handle uncertainty and indeterminacy. A closer look at some of these distributions reveals their unique properties and potential applications.
The following table summarizes the neutrosophic distributions and their corresponding probability functions.

Table 1: Neutrosophic Distributions and Probability Functions

SI no	Neutrosophic Distribution	Probability Function
1	Binomial	$f(x) = \begin{cases} \binom{n}{x} p_N^x (1-p_N)^{n-x}, & \text{if } x = 0,1,\ldots,n \\ 0, & \text{otherwise} \end{cases}$
2	Poisson	$f(x) = \begin{cases} e^{-\lambda_N} \frac{\lambda_N^x}{(x)!}, & \text{if } x = 0,1,\ldots,n \\ 0, & \text{otherwise} \end{cases}$
3	Exponential	$f(x) = \begin{cases} \lambda_N e^{-x\lambda_N}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$
4	Uniform	$f(x) = \begin{cases} \frac{1}{b^N - a^N} & \text{if } a \leq x \leq b, \\ 0 & \text{otherwise.} \end{cases}$
5	Normal	$f(x) = \begin{cases} \frac{1}{\sigma_N\sqrt{2\pi}} \exp\left(-\frac{(x-\mu_N)^2}{2\sigma_N^2}\right), & \text{if} \quad -\infty < x < \infty \\ 0, & \text{otherwise} \end{cases}$
6	Weibull	$f(x) = \begin{cases} \frac{\beta_N}{\alpha_N^{\beta_N}} x^{\beta_N - 1} e^{-(x/\alpha_N)^{\beta_N}}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$
7	Beta	$f(x) = \begin{cases} \frac{1}{B(\alpha_N,\beta_N)} x^{\alpha_N - 1}(1-x)^{\beta_N - 1}, & \text{if } 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$ where $B(\alpha_N, \beta_N) = \frac{\Gamma(\alpha_N)\Gamma(\beta_N)}{\Gamma(\alpha_N+\beta_N)}$

8	Gamma	$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha_N)\lambda_N^{\alpha_N}} x^{\alpha_N - 1} e^{-(\frac{x}{\lambda_N})}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$
9	Lomax	$f(x) = \begin{cases} \dfrac{\alpha_N}{\beta_N}\left(1 + \dfrac{x}{\beta_N}\right)^{-(\alpha_N+1)}, & \text{if } x \geq 0 \\ 0, & \text{otherwise} \end{cases}$
10	Laplace	$f(x) = \begin{cases} \frac{1}{2\beta_N} \exp\left(-\frac{\lvert x-\theta_N \rvert}{\beta_N}\right), & \text{for } -\infty < x < \infty \\ 0, & \text{otherwise} \end{cases}$
11	Geometric	$f(x) = \begin{cases} p_N(1-p_N)^x, & \text{if } x = 0, 1, \ldots, n \\ 0, & \text{otherwise} \end{cases}$
12	Burr-III	$f(x) = \begin{cases} ckx^{-c-1}\left(1 + x^{-c}\right)^{-k-1}, & \text{for } x > 0,\ c > 0,\ k > 0 \\ 0, & \text{otherwise} \end{cases}$
13	Generalized Pareto (NGPD)	$f(x) = \begin{cases} \dfrac{1}{\beta_N}\left(1 + \dfrac{\alpha_N x_N}{\beta_N}\right)^{-\frac{1}{\alpha_N}-1}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$
14	Negative Binomial	$f(x) = \begin{cases} \binom{r_N + x - 1}{} p_N^{r_N}(1-p_N)^x, & \text{if } x = 0, 1, \ldots, n \\ 0, & \text{otherwise} \end{cases}$
15	Kumaraswamy	$f(x) = \begin{cases} \alpha_N \beta_N x^{\alpha_N - 1}(1 - x^{\alpha_N})^{\beta_N - 1}, & \text{if } x \in (0, 1) \\ 0, & \text{otherwise} \end{cases}$
16	Rayleigh	$f(x) = \begin{cases} \frac{x}{\theta_N^2} e^{-\frac{x^2}{2\theta_N^2}}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$
17	Log Gamma (NLGD)	$f(x) = \begin{cases} \frac{b^p}{\Gamma(p)} x^{b-1}(\log(x))^{p-1}, & \text{if } x \geq 1,\ p, b > 0 \\ 0, & \text{otherwise} \end{cases}$
18	Inverse Exponential (NIE)	$f(x) = \begin{cases} \frac{\theta_N}{x^2} e^{-\frac{\theta_N}{x}}, & \text{if } x > 0 \\ 0, & \text{otherwise} \end{cases}$

19	Maxwell	$f(x) = \begin{cases} \frac{2}{\pi\lambda_N^3} x^2 e^{-\frac{x^2}{2\lambda_N^2}} & \text{for } x > 0, \\ 0 & \text{otherwise.} \end{cases}$
20	Lindley	$f(x) = \begin{cases} \frac{\vartheta^2}{(1+\vartheta)}(1+x)e^{-\vartheta x}, & \text{for } x \geq 0, \\ 0, & \text{otherwise.} \end{cases}$
21	Pareto	$f(x) = \begin{cases} \dfrac{\alpha_N \theta_N^{\alpha_N}}{x^{\alpha_N+1}}, & \text{if } x > \theta_N,\ \alpha_N > 0,\ \theta_N > 0 \\ 0, & \text{otherwise} \end{cases}$

The following are some commonly used Neutrosophic probability distributions along with their key areas of application in real-world uncertain enivronments:

- **Neutrosophic Binomial**: Used in statistics and quality control to analyze defect rates when success probabilities are uncertain.
- **Neutrosophic Poisson**: Applied in engineering and healthcare to count events with unpredictable or variable rates.
- **Neutrosophic Exponential**: Useful in reliability and operations research to model time until failure under uncertain conditions.
- **Neutrosophic Uniform**: Helps simulate data within a known but imprecise range.
- **Neutrosophic Normal**: Used in finance and engineering when data has an uncertain mean or variance.
- **Neutrosophic Weibull**: Ideal for modeling product lifespans and reliability in vague or uncertain environments.
- **Neutrosophic Beta**: Commonly used in Bayesian inference and statistics to model uncertain success probabilities.
- **Neutrosophic Gamma**: Applied in healthcare and engineering to model treatment durations with varying effects.
- **Neutrosophic Lomax**: Useful in finance and actuarial science for modeling extreme losses with imprecise risks.
- **Neutrosophic Laplace**: Handles data with sharp peaks and uncertain parameters, often used in economics and signal processing.
- **Neutrosophic Geometric**: Models the number of trials until the first success, especially in quality control and computer science.
- **Neutrosophic Burr-III**: Used in finance and reliability engineering for failure time analysis under uncertain stress levels.
- **Neutrosophic Generalized Pareto**: Suitable for environmental science and hydrology to model rare extreme events with imprecise data.

- **Neutrosophic Negative Binomial**: Applied in epidemiology to model disease spread with uncertain transmission rates.
- **Neutrosophic Kumaraswamy**: Helps model bounded data like river flows with imprecise measurements.
- **Neutrosophic Rayleigh**: Used in telecommunications and electronics to model device lifespans under fluctuating conditions.
- **Neutrosophic Log-Gamma**: Models industrial growth or economic change with uncertain influencing factors.
- **Neutrosophic Inverse Exponential**: Suitable for components that improve over time but still have uncertain failure times.
- **Neutrosophic Maxwell**: Applied in physics and chemistry to study molecular speeds under uncertain measurements.
- **Neutrosophic Lindley**: Models survival times in medical studies where data is vague or imprecise.
- **Neutrosophic Pareto**: Useful in economics and sociology to study income inequality when high-income data is uncertain.
- **Neutrosophic Diagnosis Tests**: Designed for medical science to interpret unclear or vague symptoms and test results.

5 Comparison with Classical and Neutrosophic Distributions

Neutrosophic distributions extend classical probability models by incorporating indeterminacy, making them better suited for analyzing uncertain, vague, or incomplete data. When the indeterminacy component is set to zero, Neutrosophic distributions reduce to their classical counterparts [15]. Classical models assume precise data and parameters, whereas Neutrosophic models explicitly handle uncertainty, offering more flexibility in real-world applications [10]. The key difference lies in their ability to address various forms of uncertainty:

Table 2: Comparison of Classical and Neutrosophic Distributions

Feature	**Classical**	**Neutrosophic**
Handles Determinacy	Yes	Yes
Models Randomness	Yes (Primary Focus)	Yes
Models Vagueness	Limited	Yes (Through Indeterminacy)
Models Indeterminacy	No	Yes (Explicit Component)
Models Contradiction	No	Yes (Through Truth and Falsity)
Generalization of Classical	No	Yes
Based on Logic	Boolean Logic	Neutrosophic Logic

To highlight the practical significance of neutrosophic probability distributions, several case studies are presented below. These examples showcase real-world datasets from diverse fields such as healthcare, reliability engineering, environmental science, finance, and manufacturing. Each study includes a comparison between classical and neutrosophic models, demonstrating how the incorporation of indeterminacy improves model fit and interpretability.

5.1 Engineering Data Modeling Using the Neutrosophic Burr-XII Distribution

To demonstrate the effectiveness of the Neutrosophic Burr-XII (NeS-BrXII) distribution, it was applied to a real-world dataset (DS) involving time-to-failure data for 20 electronic components [1]. The failure times were reported as intervals (e.g., (0.001, 0.06), (0.011, 0.15)) due to measurement limitations, making them ideal for neutrosophic modeling.

Table 3: Descriptive neutrosophic summary of DS

Data	**N**	**Mean**	**Median**	**Variance**	**Skewness**	**Kurtosis**	**Min**	**Max**
DS1	20	2.53	1.87	6.86	2.29	5.77	0.03	12

Table 4: ML estimates and information criteria for DS

Distribution	**MLEs (SE)**		**Information Criterion**			
	$\hat{\eta}_{\mathbf{NeS}}$	$\hat{\gamma}_{\mathbf{NeS}}$	**AIC**	**CAIC**	**BIC**	**HQIC**
NeS-BrXII	[0.72,1.57] ([0.16, 0.13])	[1.32,0.81] ([0.13, 0.13])	[69.01, 77.43]	[69.72, 78.13]	[71.00, 79.42]	[69.40, 77.82]
BuXII	1.60 (0.36)	0.70 (0.19)	86.11	86.82	88.10	86.50
Wb	0.83 (0.21)	0.64 (0.08)	68.49	69.19	70.48	68.88
BrIII	1.27 (0.23)	1.40 (0.32)	85.94	86.65	87.93	86.33
NH	1.08 (0.51)	0.35 (0.27)	81.09	81.79	83.08	81.48

A summary of the dataset (Table 3) shows moderate variability and positive skewness, with key statistics like mean, variance, and kurtosis expressed as intervals—reflecting uncertainty in the data. The NeS-BrXII model was compared with four classical models (Burr-XII, Burr-III, Weibull, Nadarajah–Haghighi) using maximum likelihood estimation. Goodness-of-fit was assessed via AIC, BIC, CAIC, and HQIC (Table 4).

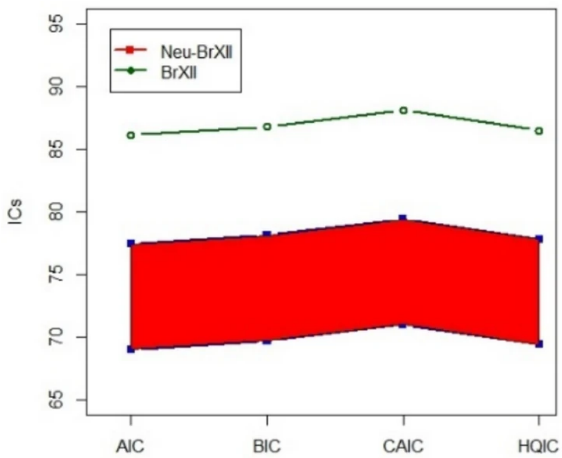

Figure 1: Comparison of information criteria for DS across five models

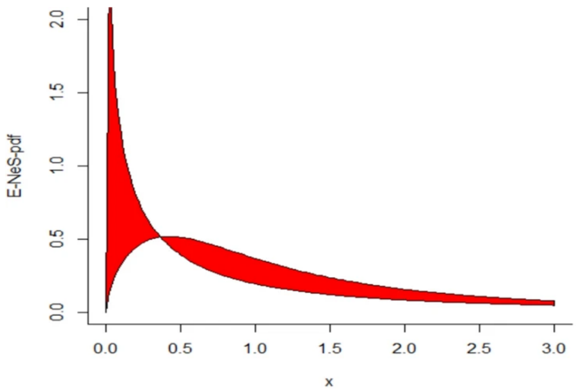

Figure 2: Fitted PDF curves for DS using NeS-BrXII

- The NeS-BrXII model achieved the lowest information criteria values.
- Its fitted curve closely matched the empirical data (Figures 1 and 2).
- It outperformed classical models, particularly in capturing measurement imprecision.

The Neutrosophic Burr-XII distribution proved highly effective for modeling real-world engineering data with uncertainty, highlighting the advantage of neutrosophic methods in reliability analysis.

5.2 Modeling Environmental Health Data Using the Neutrosophic Lindley Distribution

The Neutrosophic Lindley Distribution (NLiD) was applied to a real-world dataset to assess its ability to handle uncertain and imprecise data [8]. The dataset includes average daily dioxin ingestion from typical diets between 1998 and 2015, reported as intervals, making classical models unsuitable.

Table 5: Descriptives statistics for the NLiD with dataset

Descriptives	Ingestion of dioxins
Mean	[0.6887, 0.7115]
Variance	[0.3243, 0.3448]
Median	[0.2823, 0.2973]
First Quartile	[0.1192, 0.1257]
Third Quartile	[0.5517, 0.5802]
Skewness	[-0.0548, -0.0500]
Kurtosis	[-0.8476, -0.7604]

Table 6: ML estimates and standard errors of the NLiD for ingestion of dioxins dataset

Distribution	$\boldsymbol{\theta}$	**Standard Error**
N Lindley	[3.0168,2.8884]	[0.5554,0.5297]
Lindely	3.0168	0.5554

Table 7: Model selection criteria for ingestion of dioxins dataset

Models	**LL**	**AIC**	**BIC**	**CAIC**	**HQIC**
N Lindley IN=[0.05]	[-1.0638, -2.1032]	[4.1277, 6.2063]	[4.9609, 7.0395]	[4.3943, 6.473]	[4.2105, 6.2891]
Lindely	-1.0638	4.1277	4.9609	4.3943	4.2105

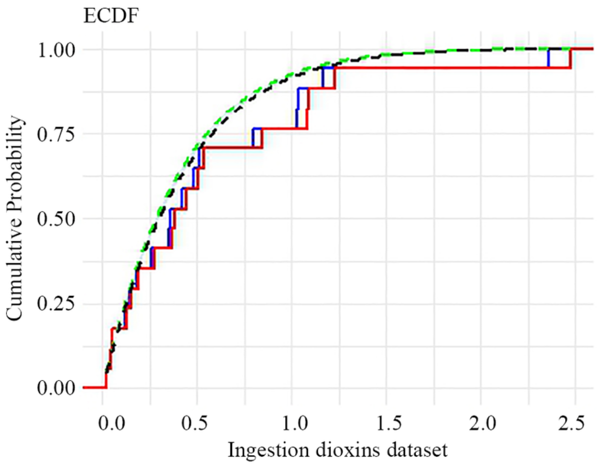

Figure 3: Empirical cumulative distribution function with real life dataset

- Descriptive statistics (Table 5) show variability in variance, stable mean values, near-zero skewness (approximate symmetry), and negative kurtosis (flatter distribution), supporting the model's suitability.
- Parameter estimates obtained via Maximum Likelihood Estimation (Table 6) indicate a better fit compared to classical models.
- The empirical cumulative distribution function (Figure 3) aligns closely with the NLiD fit, confirming its effectiveness.

The Neutrosophic Lindley Distribution outperforms classical models by effectively handling data uncertainty, making it ideal for environmental and biomedical studies where measurements are imprecise.

5.3 Wind Speed Modeling Using the Neutrosophic Weibull Distribution

The Neutrosophic Weibull Distribution (NWD) was applied to wind speed data from Bahawalpur station to assess its effectiveness in handling atmospheric uncertainty [2].

Table 8: Some descriptive measures for the dataset.

Min.	**Q1**	**Median**	**Q3**	**Mean**	**Var**	**Skewness**	**Kurtosis**	**Max.**
0.0004	2.4118	3.3181	4.3006	3.5049	2.8053	1.1346	6.0846	20.073

Table 9: MLEs and goodness-of-it measures for the dataset.

Model	$\hat{\alpha}$ **(SE)**	$\hat{\beta}$ **(SE)**	**IN**	**LogLik**	**AIC**	**BIC**
WD	3.943394	2.155532	0	-130683.9	261371.8	261390.1
	4.337733	2.371085	0.1	-125756.0	251518.1	251545.5
NWD	4.732073	2.586638	0.2	-124587.8	249181.7	249209.1
	5.126412	2.802192	0.3	-126932.5	253871.0	253898.4

- The data (Table 8) shows positive skewness and heavy tails (kurtosis > 6), making it ideal for a flexible model like NWD.
- Parameters for both classical Weibull (WD) and NWD were estimated using Maximum Likelihood, with model fit compared via log-likelihood, AIC, and BIC (Table 9).
- As the indeterminacy parameter (IN) increased, log-likelihood improved, and AIC/BIC values decreased, confirming a better fit for NWD.
- For example, AIC dropped from 261,371.8 (WD) to 249,181.7 (NWD, IN $= 0.2$), highlighting substantial improvement.

The NWD offers a more robust, adaptable approach for modeling uncertain and skewed wind speed data, making it highly suitable for energy forecasting and climate studies

5.4 Modeling Uncertain Automotive Data with the Neutrosophic Beta Distribution

The Neutrosophic Beta (N-Beta) distribution was applied to real-world industrial data from Korean automobile manufacturing, containing 28 uncertain observations [19].

Table 10: Parameter estimation and goodness of fit for automobile data using N-Beta (α, β)

Distribution	$\hat{\alpha}$	$\hat{\beta}$	**AIC**	**BIC**
N-Beta	1.6386	[2.8437, 0.8561]	[728.068, 96.1830]	[658.7123, 49.1298]
Classical Beta	3.6587	9.5462	898.3621	846.8352

- The α parameter was treated classically, while β was modeled neutrosophically to capture data uncertainty.
- Parameters were estimated using Maximum Likelihood Estimation (MLE).

- Model performance was compared using AIC and BIC values (Table 10).
- The N-Beta model achieved significantly lower AIC and BIC values than the classical Beta distribution, confirming a better fit.
- The neutrosophic interval for β reflects the model's ability to incorporate indeterminacy present in industrial data.

The N-Beta distribution effectively handles vague, interval-based data, demonstrating superior flexibility and accuracy compared to classical models for real-world automotive applications

5.5 Financial Return Modeling with the Neutrosophic Laplace Distribution

The Neutrosophic Laplace Distribution (NLD) was applied to model daily return percentages of the NIFTY50 index from the Indian Stock Market, covering 827 observations from January 2020 to March 2023 [19]. Given the uncertainty in financial markets due to volatility and external factors, NLD provides a better alternative to classical models.

Table 11: The descriptive statistics of the daily returns of NIFTY50

Min	**1st Q**	**Median**	**Mean**	**3rd Q**	**MAD**	**Var**	**Skewness**	**Kurtosis**	**Max**
-6.818	-0.571	-0.0068	-0.0795	0.4508	0.718	1.1703	0.725	13.27992	9.306

Table 12: MLE, AIC and BIC for LD and NLD applied to NIFTY50 daily return data

Model	**IN**	**logL**	**AIC**	**BIC**
LD	0	-1122.67	2249.34	2258.78
NLD	0.1	-1043.85	2091.70	2101.13
	0.2	-971.89	1947.78	1957.22
	0.3	-905.69	1815.39	1824.82
	0.4	-844.41	1692.81	1702.25
	0.5	-787.35	1578.70	1588.14
	0.6	-733.98	1471.95	1481.39
	0.7	-683.84	1371.68	1381.12
	0.8	-636.57	1277.14	1286.58
	0.9	-591.86	1187.71	1197.15
	1.0	-549.44	1102.87	1112.31

- Descriptive statistics (Table 12) show the dataset contains ambiguity and heavy tails, supporting the need for neutrosophic modeling.
- Model comparison using Maximum Likelihood Estimation (Table 13) indicates:
  - As the indeterminacy parameter (IN) increases, log-likelihood improves.
  - AIC drops from 2249.34 (classical LD) to 1102.87 (NLD, IN = 1).
  - BIC reduces similarly, confirming better model fit.

The NLD outperforms the classical Laplace Distribution by effectively handling uncertainty, vagueness, and extreme variations in financial data. It is a robust tool for financial risk analysis and understanding market behavior

6 Advantages and Challenges of Neutrosophic Distributions

Neutrosophic distributions effectively model imprecise, vague, and indeterminate data, offering a more realistic approach to uncertainty in real-world applications [15]. As generalizations of classical distributions, they extend traditional models to uncertain environments and can improve accuracy in prediction and data analysis [10]. In some cases, mathematical operations within Neutrosophic models can help reduce indeterminacy—a unique benefit over other uncertainty-handling methods. Despite their advantages, Neutrosophic distributions come with limitations:

- Higher computational complexity compared to classical models, especially for advanced applications.
- Interpreting the indeterminacy component requires expertise and domain knowledge.
- The field is still developing, with limited software tools and theoretical resources.
- Defining and quantifying indeterminacy can be subjective and context-dependent.
- Parameter estimation is often more complex than for classical models.
- The retraction of studies (e.g., Neutrosophic Negative Binomial distribution by [14] highlights the need for rigorous mathematical validation.

While Neutrosophic distributions offer a powerful approach for modeling uncertainty, ongoing research, better computational tools, and standardized practices are essential for broader acceptance and reliable application.

7 Methodologies used in Retrospective Neutrosophic Studies

Research on Neutrosophic distributions uses methods similar to classical statistics but adapted to handle uncertainty and indeterminacy.
Key Approaches:

- New Neutrosophic distributions are mathematically derived, including probability functions, moments, survival functions, and hazard rates [10].
- Data is generated under different levels of indeterminacy to assess model performance, parameter estimation, and comparison with classical models [11].
- Neutrosophic models are applied to uncertain datasets in fields like industrial growth, oil exploration, healthcare, finance, and environmental science to demonstrate practical effectiveness [11].
- Classical tests (e.g., hypothesis tests) are extended to handle imprecise or vague data, enabling reliable inference under uncertainty [6].
- Maximum Likelihood Estimation (MLE) is commonly used to estimate parameters for Neutrosophic models based on real or simulated data [1].

These methodologies — combining theory, simulation, real-world applications, and extended statistical tools — are essential for developing and validating Neutrosophic distributions and ensuring their practical utility in uncertain environments.

8 Neutrosophic Distributions in Biomedical Research

Neutrosophic probability distributions have emerged as a powerful tool in biomedical research, where uncertainty, vagueness, and indeterminacy are often inherent in clinical and experimental data [17]. Traditional statistical models typically assume complete and precise information; however, real-world biomedical data frequently involve incomplete records, measurement errors, conflicting clinical findings, and imprecise diagnostic outcomes. Neutrosophic distributions address these challenges by incorporating truth, indeterminacy, and falsity components, offering a more flexible and realistic framework for data analysis.

Several Neutrosophic distributions have been proposed for biomedical applications. The Neutrosophic Normal distribution has been utilized to model physiological parameters such as blood pressure and cholesterol levels, where readings may be influenced by measurement uncertainty or patient variability. Similarly, the Neutrosophic Exponential and Weibull distributions have found applications in survival analysis, particularly in modeling time-to-event data such as disease remission or patient survival times, which often suffer from censoring and incomplete follow-up [9].

The Neutrosophic Weibull distribution, in particular, has been highlighted as an effective model for survival data under uncertainty. By introducing indeterminate components into the scale and shape parameters, it offers a more adaptable fit for censored and incomplete survival datasets. Compared to the classical Weibull model, the Neutrosophic version provides improved flexibility in capturing the complex variability associated with patient prognoses and treatment outcomes.

In addition, the Neutrosophic Binomial distribution has been applied to model success and failure outcomes in clinical trials and treatment evaluation studies. For instance, when patient responses to a new drug are partially reported, indeterminacy can be effectively modeled using Neutrosophic parameters, leading to more reliable inferences about treatment efficacy [9].

The Neutrosophic Geometric distribution, although initially applied in industrial inspection settings, presents strong potential for biomedical research [5]. It can model the number of clinical visits required until a diagnosis is made or the number of treatments administered until a positive response is observed. In cases where diagnostic criteria are vague or patient compliance is inconsistent, incorporating indeterminacy into the model provides a more realistic representation of clinical processes.

Recent applications have also demonstrated the use of neutrosophic statistical tests, such as the Neutrosophic Kruskal-Wallis H test and the Neutrosophic sign test, in analyzing biomedical data. These tests have been employed in studies of COVID-19 patients to assess changes in vital signs, such as blood pressure, pulse rate, and temperature, under uncertain and rapidly evolving clinical conditions. The neutrosophic versions of these tests enable more robust statistical inference by explicitly accounting for vagueness and ambiguity in patient observations.

Beyond individual distributions and tests, the broader application of Neutrosophic statistics in biomedical research extends to the analysis of biomedical images, clinical decision support systems, and medical artificial intelligence [9]. Neutrosophic approaches have enhanced diagnostic imaging techniques by better handling ambiguous regions in medical scans, leading to improved segmentation and classification accuracy.

In clinical decision-making contexts, Neutrosophic models facilitate consensus-building among experts when faced with incomplete or contradictory information. For instance, the Neutrosophic Delphi method enables the integration of diverse expert opinions while managing the inherent indeterminacy in complex medical cases.

Overall, the integration of Neutrosophic distributions into biomedical research methodologies represents a significant advancement in statistical modeling under uncertainty. These approaches provide researchers and clinicians with tools that better mirror the realities of medical data, leading to more accurate analyses, improved predictive models, and ultimately better patient outcomes. As biomedical data continue to grow in complexity and variability, the role of

Neutrosophic distributions is expected to expand further, contributing to the advancement of evidence-based medicine in uncertain environments.

9 Conclusion and Future Directions

This retrospective study provides a structured overview of the evolution, theoretical foundations, and real-world applications of Neutrosophic probability distributions. By generalizing classical distributions to incorporate indeterminacy, these models offer a powerful framework for analyzing data characterized by uncertainty, vagueness, and imprecision—challenges often encountered in fields such as engineering, healthcare, environmental science, and decision-making. A unique contribution of this work is the systematic comparison between classical and Neutrosophic distributions, which highlights how Neutrosophic models extend beyond randomness to explicitly capture various forms of uncertainty. The study also synthesizes practical applications, demonstrating the superior performance of Neutrosophic models in handling imprecise and interval-valued data compared to classical methods. However, certain limitations remain. The theoretical development of Neutrosophic distributions is ongoing, with specific challenges in standardizing the interpretation and quantification of indeterminacy. Additionally, increased computational complexity and limited availability of user-friendly software currently restrict their widespread adoption.

Future research directions are crucial to address these gaps and unlock the full potential of Neutrosophic methods. Key areas for further exploration include:

- Advancing theoretical properties of existing and new Neutrosophic distributions.
- Developing efficient computational algorithms and accessible software tools.
- Extending Neutrosophic models to multivariate, time-series, and high-dimensional data.
- Creating robust goodness-of-fit tests and model selection criteria specific to Neutrosophic frameworks.
- Applying Neutrosophic methods in emerging fields such as artificial intelligence, big data analytics, biomedical research, and complex system modeling.
- Conducting comparative studies with other uncertainty modeling approaches, including fuzzy logic and Bayesian statistics.
- Investigating higher-order Neutrosophic statistics to capture more complex forms of uncertainty.

In conclusion, Neutrosophic distributions offer a promising advancement in statistical modeling for uncertain environments. Their ability to integrate indeterminacy provides a richer and more realistic analytical framework. Continued research is essential to strengthen their theoretical underpinnings, broaden their applications, and establish them as practical tools for real-world decision-making under uncertainty.

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