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On Fermatean Neutrosophic e-continuous and e-irresolute Maps

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Abstract. The primary objective of this paper is to introduce and develop the concept of Fermatean neutrosophic e-continuous maps in the framework of Fermatean neutrosophic topological spaces. This new class of mappings extends the idea of continuity by incorporating the higher expressive power of Fermatean neutrosophic sets, which are capable of handling more complex degrees of truth, indeterminacy, and falsity than classical fuzzy or intuitionistic fuzzy settings. In addition to defining and studying the basic formulation of Fermatean neutrosophic e-continuity, we investigate its fundamental properties, structural behavior, and interrelations with other existing types of continuity. Furthermore, we examine the notion of Fermatean neutrosophic e-irresolute maps, which play an important role in the preservation of Fermatean neutrosophic topological structures under mappings. Several characterizations and properties of these maps are provided, establishing their significance in extending the theory of neutrosophic topology. The results presented not only generalize existing concepts from classical and fuzzy topology but also open new avenues for applications of Fermatean neutrosophic theory in decision-making, information systems, and uncertain data analysis.

Keywords: Fermatean neutrosophic e-closed sets, fermatean neutrosophic e-continuous maps and Fermatean neutrosophic e-irresolute maps.

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1. Introduction

In 1965, Zadeh [17] introduced the concept of fuzzy sets to model ambiguity and uncertainty in real-world scenarios. Later, in 1986, Atanassov [1] extended this idea by proposing the concept of intuitionistic fuzzy sets (ifs's), which incorporated both membership and non-membership functions addressing a limitation in Zadehs model that only considered the membership degree. However, in practical applications, the sum of membership and non-membership degrees can exceed one, indicating a need for more flexible models.

To address these challenges, Yager [16] introduced the Pythagorean fuzzy set (pfs), where the sum of the squares of the membership and non-membership degrees is constrained to be less than or equal to one. Building on this, Senapati and Yager [9] proposed the Fermatean fuzzy set ($\mathfrak{F}fs$), where the cube of the membership and non-membership degrees must sum to less than or equal to one. This extension provides greater flexibility and enhances the capability of managing uncertainty, making Fermatean fuzzy sets more efficient than ifs's and pfs's in decision-making and other real-life applications.

Smarandache [10] later introduced the concept of neutrosophic sets (NS's), representing a major advancement in the field of decision theory and beyond. Neutrosophic sets are built on the idea that any concept inherently involves degrees of truth (T), indeterminacy (I), and falsity (F). Unlike intuitionistic fuzzy sets where membership and non-membership are dependent neutrosophic sets allow these three components to be mutually independent, thereby offering greater modeling flexibility for incomplete, inconsistent, and indeterminate information.

This independence makes neutrosophic sets a powerful tool for representing uncertainty in a wide range of fields, including engineering, philosophy, economics, and information science. In particular, they support complex analyses involving both dependent and independent variables, contributing significantly to decision-making and strategic planning.

To leverage the strengths of both Fermatean fuzzy sets and neutrosophic sets, the Fermatean neutrosophic set was developed. This hybrid framework effectively handles uncertainty, imprecision, and indeterminacy in complex decision making contexts. For example, in evaluating a local restaurant, Fermatean fuzzy theory is applied to assess the importance of attributes such as quality, naturalness, freshness, taste, and presentation which are often subject to human hesitation and subjective judgment. By incorporating neutrosophic principles, the model better captures the vagueness and uncertainty in customer preferences.

The domain of local food service was chosen as a practical application due to the high degree of uncertainty in consumer decision making uncertainty that neutrosophic sets can represent more accurately than traditional methods.

From a topological perspective, Vadivel et al. [12] introduced the notion of δ-open sets within neutrosophic topological spaces. Prior to this, in 2008, Ekici [4] introduced e-open sets in general topology. Building on this foundation, Seenivasan et al. [8] in 2014 developed the concept of fuzzy e-open sets and their corresponding fuzzy e-continuity. Later, Vadivel et al. [3] extended these ideas into the realm of intuitionistic fuzzy topological spaces. More recently, Vadivel and collaborators [13–15] have made notable contributions by exploring various types of open sets in Fermatean fuzzy topological spaces, advancing the study of fuzzy and neutrosophic topology.

Research Gap: To the best of our knowledge, no prior investigation has been carried out on the concepts of continuity and irresoluteness with respect to Fermatean neutrosophic e-open sets within the framework of Fermatean fuzzy topological spaces. While various notions of continuity and irresolute mappings have been studied extensively in classical topology, fuzzy topology, and intuitionistic fuzzy topology, the specific treatment of these notions under the richer structure of Fermatean neutrosophic sets remains unexplored in the existing literature. This absence highlights a significant research gap, thereby motivating the present study to initiate and advance the theory of Fermatean neutrosophic e-continuous and e-irresolute maps.

In this paper, we put forward the novel concept of Fermatean neutrosophic e-continuity within the framework of Fermatean neutrosophic topological spaces. The study begins with a formal definition of this new type of continuity, followed by an investigation of its essential properties. To enhance clarity and demonstrate the practical significance of the proposed ideas, several illustrative examples are provided. These examples not only validate the theoretical framework but also highlight how Fermatean neutrosophic e-continuity extends and generalizes existing notions of continuity in fuzzy and neutrosophic topological settings. Furthermore, the paper delves into the study of Fermatean neutrosophic e-irresolute maps, presenting their fundamental properties and characterizations in a comprehensive manner. Special attention is given to their structural behavior and their role in preserving Fermatean neutrosophic e-open sets under mappings. By establishing these results, the paper contributes to the enrichment of Fermatean neutrosophic topology and provides a foundation for further theoretical advancements and practical applications in areas involving uncertainty, vagueness, and complex decision-making processes.

2. Preliminaries

Definition 2.1. [9] Let X be a universe of discourse. A Fermatean fuzzy set ($\mathfrak{F}\mathcal{F}s$) F in X is an object having the form $F = \{< x, \alpha_F(x), \beta_F(x) >: x \in X\}$ where $\alpha_F(x) : X \to [0,1]$ and $\beta_F(x) : X \to [0,1]$, including the condition $0 \leq (\alpha_F(x))^3 + (\beta_F(x))^3 \leq 1$, for all $x \in X$. The numbers $\alpha_F(x)$ and $\beta_F(x)$ denote, respectively, the degree of membership and the degree of non-membership of the element x in the set F. For any $\mathfrak{F}\mathcal{F}s$ F and $x \in X$, $\pi_F(x) = \sqrt[3]{1 - [(\alpha_F(x))^3 - (\beta_F(x))^3]}$ is identified as the degree of indeterminacy of x to F. In the interest of simplicity, we shall mention the symbol $F = (\alpha_F, \beta_F)$ for the $\mathfrak{F}\mathcal{F}s$ $F = \{< x, \alpha_F(x), \beta_F(x) >: x \in X\}$.

Definition 2.2. [7] Let X be a non-empty set. A neutrosophic set (briefly, Ns) L is an object having the form $L = \{\langle x, \mu_L(x), \nu_L(x), \sigma_L(x)\rangle : x \in X\}$ where $\mu_L \to [0,1]$ denote the degree of membership function, $\nu_L \to [0,1]$ denote the degree of indeterminacy function and

$\sigma_L \to [0,1]$ denote the degree of non-membership function respectively of each element $x \in X$ to the set L and $0 \leq \mu_L(x) + \nu_L(x) + \sigma_L(x) \leq 3$ for each $x \in X$.

Definition 2.3. [6] A neutrosophic topology (briefly, Nt) on a non-empty set X is a family τ_N of neutrosophic subsets of X satisfying

(i) 0_N, $1_N \in \tau_N$.
(ii) $L_1 \cap L_2 \in \tau_N$ for any $L_1, L_2 \in \tau_N$.
(iii) $\bigcup L_a \in \tau_N, \forall \ L_a : a \in A \subseteq \tau_N$.

Then (X, τ_N) is called a neutrosophic topological space (briefly, Nts) in X. The τ_N elements are called neutrosophic open sets (briefly, Nos) in X. A Ns C is called a neutrosophic closed sets (briefly, Ncs) iff its complement C^c is Nos.

Definition 2.4. [11] Let X be a non-empty set. A Fermatean neutrosophic set (briefly, $\mathfrak{FN}s$) L is an object having the form $L = \{\langle x, \mu_L(x), \nu_L(x), \sigma_L(x)\rangle : x \in X\}$ where $\mu_L \to [0,1]$ denote the degree of membership function, $\nu_L \to [0,1]$ denote the degree of indeterminacy function and $\sigma_L \to [0,1]$ denote the degree of non-membership function respectively of each element $x \in X$ to the set L such that $0 \leq (\mu_L(x))^3 + (\sigma_L(x))^3 \leq 1$ and $0 \leq (\nu_L(x))^3 \leq 1$. Then $0 \leq (\mu_L(x))^3 + (\nu_L(x))^3 + (\sigma_L(x))^3 \leq 2$ for all $x \in X$. Here $\mu_L(x)$ and $\sigma_L(x)$ are dependent components and $\nu_L(x)$ is an independent component.

The definitions of $1_{\mathfrak{FN}}$ and $0_{\mathfrak{FN}}$ that will be needed before proceeding to set operations will be given. In [6], possible definitions of $1_{\mathfrak{FN}}$ and $0_{\mathfrak{FN}}$ neutrosophic sets are given. In this paper, the theory will be constructed by defining $0_{\mathfrak{FN}}$ and $1_{\mathfrak{FN}}$ Fermatean neutrosophic sets in a single way. $0_{\mathfrak{FN}}$ and $1_{\mathfrak{FN}}$ are defined as $0_{\mathfrak{FN}} = \{(x, 0, 0, 1) : x \in X\}$ and $1_{\mathfrak{FN}} = \{(x, 1, 1, 0) : x \in X\}$ Now, the union, intersection and complement definitions necessary for the definition of the topological space will be given. These definitions are given in several different ways in classical neutrosophic spaces in [2]; to avoid confusion here, only one method will be given for sets with Fermatean structure, and this method is different from the method chosen in [6].

Definition 2.5. [11] Let X be a non-empty set & the $\mathfrak{FNs}$'s L & M in the form $L = \{\langle x, \mu_L(x), \nu_L(x), \sigma_L(x)\rangle : x \in X\}$, $M = \{\langle x, \mu_M(x), \nu_M(x), \sigma_M(x)\rangle : x \in X\}$, then

(i) $0_{\mathfrak{FN}} = \langle x, 0, 0, 1\rangle$ and $1_{\mathfrak{FN}} = \langle x, 1, 1, 0\rangle$,
(ii) $L \subseteq M$ iff $\mu_L(x) \leq \mu_M(x)$, $\nu_L(x) \leq \nu_M(x)$ & $\sigma_L(x) \geq \sigma_M(x) : x \in X$,
(iii) $L = M$ iff $L \subseteq M$ and $M \subseteq L$,
(iv) $1_{\mathfrak{FN}} - L = \{\langle x, \sigma_L(x), 1_{\mathfrak{FN}} - \nu_L(x), \mu_L(x)\rangle : x \in X\} = L^c$ or $C(L)$,
(v) $L \cup M = \{\langle x, \max(\mu_L(x), \mu_M(x)), \max(\nu_L(x), \nu_M(x)), \min(\sigma_L(x), \sigma_M(x))\rangle : x \in X\}$,
(vi) $L \cap M = \{\langle x, \min(\mu_L(x), \mu_M(x)), \min(\nu_L(x), \nu_M(x)), \max(\sigma_L(x), \sigma_M(x))\rangle : x \in X\}$.

Definition 2.6. [5] A Fermatean neutrosophic topology (briefly, $\mathfrak{FN}t$) on a non-empty set X is a family $\tau_{\mathfrak{FN}}$ of Fermatean neutrosophic subsets of X satisfying

(i) $0_{\mathfrak{FN}}$, $1_{\mathfrak{FN}} \in \tau_{\mathfrak{FN}}$,

(ii) $L_1 \cap L_2 \in \tau_{\mathfrak{FN}}$ for any $L_1, L_2 \in \tau_{\mathfrak{FN}}$,

(iii) $\bigcup L_a \in \tau_{\mathfrak{FN}}, \forall\ L_a : a \in A \subseteq \tau_{\mathfrak{FN}}$.

Then $(X, \tau_{\mathfrak{FN}})$ is called a Fermatean neutrosophic topological space (briefly, $\mathfrak{FN}ts$) in X. The $\tau_{\mathfrak{FN}}$ elements are called Fermatean neutrosophic open sets (briefly, $\mathfrak{FN}os$) in X. A $\mathfrak{FN}s$ C is called a Fermatean neutrosophic closed sets (briefly, $\mathfrak{FN}cs$) iff its complement C^c is $\mathfrak{FN}os$.

Definition 2.7. [5] Let $(X, \tau_{\mathfrak{FN}})$ be $\mathfrak{FN}ts$ on X and L be an $\mathfrak{FN}s$ on X, then the Fermatean neutrosophic interior of L (briefly, $\mathfrak{FN}int(L)$) and the Fermatean neutrosophic closure of L (briefly, $\mathfrak{FN}cl(L)$) are defined as

$$\mathfrak{FN}int(L) = \bigcup\{I : I \subseteq L \ \&\ I \text{ is a } \mathfrak{FN}os \text{ in } X\}$$

$$\mathfrak{FN}cl(L) = \bigcap\{I : L \subseteq I \ \&\ I \text{ is a } \mathfrak{FN}cs \text{ in } X\}.$$

Theorem 2.8. [5] Let L be an $\mathfrak{FN}s$ on X. In this case, the following four properties hold:

(i) $\mathfrak{FN}cl(L)$ is a closed Fermatean neutrosophic set,

(ii) $\mathfrak{FN}cl(1_{\mathfrak{FN}}) = 1_{\mathfrak{FN}}$, $\mathfrak{FN}cl(0_{\mathfrak{FN}}) = 0_{\mathfrak{FN}}$,

(iii) $\mathfrak{FN}int(L)$ is an open Fermatean neutrosophic set.

(iv) $\mathfrak{FN}int(1_{\mathfrak{FN}}) = 1_{\mathfrak{FN}}$, $\mathfrak{FN}int(0_{\mathfrak{FN}}) = 0_{\mathfrak{FN}}$.

Lemma 2.9. [5] For any Fermatean neutrosophic set A in $(X, \tau_{\mathfrak{FN}})$, we have $C(\mathfrak{FN}int(A)) = \mathfrak{FN}cl(C(A))$ and $C(\mathfrak{FN}cl(A)) = \mathfrak{FN}int(C(A))$. Here $C(A)$ or $\overline{A}$ denotes complement of A.

3. **Fermatean neutrosophic e-continuous maps**

In this section we introduce Fermatean neutrosophic e-continuous maps and study some of its properties.

Definition 3.1. Let $(X, \tau_{\mathfrak{FN}})$ be an $\mathfrak{FN}ts$ and A be an $\mathfrak{FN}s$. Then A is said to be an Fermatean neutrosophic (i) regular open set ($\mathfrak{FN}ros$ in short) if $A = \mathfrak{FN}int(\mathfrak{FN}cl(A))$. (ii) regular closed set ($\mathfrak{FN}rcs$ in short) if $A = \mathfrak{FN}cl(\mathfrak{FN}int(A))$. By Lemma 2.9, it follows that A is an $\mathfrak{FN}ros$ iff $\bar{A}$ is an $\mathfrak{FN}rcs$.

Definition 3.2. Let $(X, \tau_{\mathfrak{FN}})$ be an $\mathfrak{FN}ts$ and $A = \{< a, \mu_A(a), \nu_A(a), \sigma_A(a) > | a \in X\}$ be an $\mathfrak{FN}s$ in X. Then the δ-interior and the δ-closure of A are denoted by $\mathfrak{FN}\delta int(A)$ and $\mathfrak{FN}\delta cl(A)$ and are defined as follows. $\mathfrak{FN}\delta int(A) = \cup\{G | G$ is an $\mathfrak{FN}ros$ and $G \subseteq A\}$, $\mathfrak{FN}\delta cl(A) = \cap\{K | K$ is an $\mathfrak{FN}rcs$ and $A \subseteq K\}$.

Definition 3.3. Let $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ be an $\mathfrak{F}\mathfrak{N}ts$ and $A = \{< a, \mu_A(a), \nu_A(a), \sigma_A(a) > |a \in X\}$ be an $\mathfrak{F}\mathfrak{N}s$ in X. A set A is said to be $\mathfrak{F}\mathfrak{N}$

(i) δ-open set (briefly, $\mathfrak{F}\mathfrak{N}\delta os$) if $A = \mathfrak{F}\mathfrak{N}\delta int(A)$,

(ii) δ-pre open set (briefly, $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}os$) if $A \subseteq \mathfrak{F}\mathfrak{N}int(\mathfrak{F}\mathfrak{N}\delta cl(A))$.

(iii) δ-semi open set (briefly, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os$) if $A \subseteq \mathfrak{F}\mathfrak{N}cl(\mathfrak{F}\mathfrak{N}\delta int(A))$.

(iv) $\delta\alpha$ open set or a-open set (briefly, $\mathfrak{F}\mathfrak{N}\delta\alpha os$ or $\mathfrak{F}\mathfrak{N}aos$) if $A \subseteq \mathfrak{F}\mathfrak{N}int(\mathfrak{F}\mathfrak{N}cl(\mathfrak{F}\mathfrak{N}\delta int(A)))$.

(v) $\delta\beta$ open set or e^*-open set (briefly, $\mathfrak{F}\mathfrak{N}\delta\beta os$ or $\mathfrak{F}\mathfrak{N}e^*os$) if $A \subseteq \mathfrak{F}\mathfrak{N}cl(\mathfrak{F}\mathfrak{N}int(\mathfrak{F}\mathfrak{N}\delta cl(A)))$.

(vi) e open set (briefly, $\mathfrak{F}\mathfrak{N}eos$) if $A \subseteq \mathfrak{F}\mathfrak{N}cl(\mathfrak{F}\mathfrak{N}\delta int(A)) \cup \mathfrak{F}\mathfrak{N}int(\mathfrak{F}\mathfrak{N}\delta cl(A))$.

(vii) δ (resp. δ-pre, δ-semi, $\delta\alpha$, e and $\delta\beta$) dense if $\mathfrak{F}\mathfrak{N}\delta cl(A)$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}cl(A)$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}cl(A)$, $\mathfrak{F}\mathfrak{N}\delta\alpha cl(A)$, $\mathfrak{F}\mathfrak{N}ecl(A)$ and $\mathfrak{F}\mathfrak{N}\delta\beta cl(A)) = 1_{\mathfrak{F}\mathfrak{N}}$.

The complement of an $\mathfrak{F}\mathfrak{N}\delta os$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}os$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os, \mathfrak{F}\mathfrak{N}\delta\alpha os$, $\mathfrak{F}\mathfrak{N}eos$ and $\mathfrak{F}\mathfrak{N}\delta\beta os$) is called an $\mathfrak{F}\mathfrak{N}\delta$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}, \mathfrak{F}\mathfrak{N}\delta\alpha$, $\mathfrak{F}\mathfrak{N}e$ and $\mathfrak{F}\mathfrak{N}\delta\beta$) closed set (briefly, $\mathfrak{F}\mathfrak{N}\delta cs$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}cs$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}cs, \mathfrak{F}\mathfrak{N}\delta\alpha cs$, $\mathfrak{F}\mathfrak{N}ecs$ and $\mathfrak{F}\mathfrak{N}\delta\beta cs$)) in X.

The family of all $\mathfrak{F}\mathfrak{N}\delta os$ (resp. $\mathfrak{F}\mathfrak{N}\delta cs$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}os$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}cs$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}cs$, $\mathfrak{F}\mathfrak{N}\delta\alpha os$, $\mathfrak{F}\mathfrak{N}\delta\alpha cs$, $\mathfrak{F}\mathfrak{N}eos$, $\mathfrak{F}\mathfrak{N}ecs$, $\mathfrak{F}\mathfrak{N}\delta\beta os$ and $\mathfrak{F}\mathfrak{N}\delta\beta cs$) of X is denoted by $\mathfrak{F}\mathfrak{N}\delta OS(X)$, (resp. $\mathfrak{F}\mathfrak{N}\delta CS(X)$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}OS(X)$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}CS(X), \mathfrak{F}\mathfrak{N}\delta\mathcal{S}OS(X), \mathfrak{F}\mathfrak{N}\delta\mathcal{S}CS(X)$, $\mathfrak{F}\mathfrak{N}\delta\alpha OS(X), \mathfrak{F}\mathfrak{N}\delta\alpha CS(X)$, $\mathfrak{F}\mathfrak{N}eOS(X)$, $\mathfrak{F}\mathfrak{N}eCS(X)$, $\mathfrak{F}\mathfrak{N}\delta\beta OS(X)$ and $\mathfrak{F}\mathfrak{N}\delta\beta CS(X)$).

Definition 3.4. Let $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ be an $\mathfrak{F}\mathfrak{N}ts$ and $A = \{< a, \mu_A(a), \nu_A(a), \sigma_A(a) > |a \in X\}$ be an $\mathfrak{F}\mathfrak{N}s$ in X. Then the $\mathfrak{F}\mathfrak{N}\delta$ (resp. $\mathfrak{F}\mathfrak{N}\delta$-pre, $\mathfrak{F}\mathfrak{N}\delta$-semi, $\mathfrak{F}\mathfrak{N}\delta\alpha$, $\mathfrak{F}\mathfrak{N}e$ and $\mathfrak{F}\mathfrak{N}\delta\beta$)-interior and the $\mathfrak{F}\mathfrak{N}\delta$ (resp. $\mathfrak{F}\mathfrak{N}\delta$-pre, $\mathfrak{F}\mathfrak{N}\delta$-semi, $\mathfrak{F}\mathfrak{N}\delta\alpha$, $\mathfrak{F}\mathfrak{N}e$ and $\mathfrak{F}\mathfrak{N}\delta\beta$)-closure of A are denoted by $\mathfrak{F}\mathfrak{N}\delta int(A)$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}int(A)$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}int(A)$, $\mathfrak{F}\mathfrak{N}\delta\alpha int(A)$, $\mathfrak{F}\mathfrak{N}eint(A)$ and $\mathfrak{F}\mathfrak{N}\delta\beta int(A)$) and the $\mathfrak{F}\mathfrak{N}\delta cl(A)$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}cl(A)$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}cl(A), \mathfrak{F}\mathfrak{N}\delta\alpha cl(A)$, $\mathfrak{F}\mathfrak{N}ecl(A)$ and $\mathfrak{F}\mathfrak{N}\delta\beta cl(A)$) and are defined as follows:

$\mathfrak{F}\mathfrak{N}\delta int(A)$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}int(A), \mathfrak{F}\mathfrak{N}\delta\mathcal{S}int(A), \mathfrak{F}\mathfrak{N}\delta\alpha int(A)$, $\mathfrak{F}\mathfrak{N}eint(A)$ and $\mathfrak{F}\mathfrak{N}\delta\beta int(A)$) $=$ $\cup\{G|G$ in a $\mathfrak{F}\mathfrak{N}\delta os$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}os$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os, \mathfrak{F}\mathfrak{N}\delta\alpha os, \mathfrak{F}\mathfrak{N}eos$, and $\mathfrak{F}\mathfrak{N}\delta\beta os$) and $G \subseteq A\}$ and $\mathfrak{F}\mathfrak{N}\delta cl(A)$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}cl(A), \mathfrak{F}\mathfrak{N}\delta\mathcal{S}cl(A), \mathfrak{F}\mathfrak{N}\delta\alpha cl(A)$, $\mathfrak{F}\mathfrak{N}ecl(A)$ and $\mathfrak{F}\mathfrak{N}\delta\beta cl(A)$) $= \cap\{K|K$ is an $\mathfrak{F}\mathfrak{N}\delta cs$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}cs, \mathfrak{F}\mathfrak{N}\delta\mathcal{S}cs$, $\mathfrak{F}\mathfrak{N}\delta\alpha cs, \mathfrak{F}\mathfrak{N}ecs$ $\mathfrak{F}\mathfrak{N}\delta\beta cs$) and $A \subseteq K\}$.

Definition 3.5. A map $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \to (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is called Fermatean neutrosophic

(i) continuous (briefly, $\mathfrak{F}\mathfrak{N}Cts$) if the inverse image of every $\mathfrak{F}\mathfrak{N}os$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$,

(ii) δ-continuous (briefly, $\mathfrak{F}\mathfrak{N}\delta Cts$) if the inverse image of every $\mathfrak{F}\mathfrak{N}os$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}\delta os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$,

(iii) $\delta\mathcal{S}$-continuous (briefly, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}Cts$) if the inverse image of every $\mathfrak{F}\mathfrak{N}os$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$,

(iv) $\delta\mathcal{P}$-continuous (briefly, $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Cts$) if the inverse image of every $\mathfrak{F}\mathfrak{N}os$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$,

(v) $\delta\alpha$-continuous (briefly, $\mathfrak{F}\mathfrak{N}\delta\alpha Cts$) if the inverse image of every $\mathfrak{F}\mathfrak{N}os$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}\delta\alpha os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$,

(vi) e-continuous (briefly, $\mathfrak{F}\mathfrak{N}eCts$) if the inverse image of every $\mathfrak{F}\mathfrak{N}os$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}eos$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$,

(vii) $\delta\beta$-continuous (briefly, $\mathfrak{F}\mathfrak{N}\delta\beta Cts$) if the inverse image of every $\mathfrak{F}\mathfrak{N}os$ in $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}\delta\beta os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$.

Proposition 3.6. The statements are hold but the converse does not true.

(i) Every $\mathfrak{F}\mathfrak{N}\delta Cts$ is a $\mathfrak{F}\mathfrak{N}Cts$.
(ii) Every $\mathfrak{F}\mathfrak{N}\delta Cts$ is a $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}Cts$.
(iii) Every $\mathfrak{F}\mathfrak{N}\delta Cts$ is a $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Cts$.
(iv) Every $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Cts$ is a $\mathfrak{F}\mathfrak{N}eCts$.
(v) Every $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}Cts$ is a $\mathfrak{F}\mathfrak{N}eCts$.
(vi) Every $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Cts$ is a $\mathfrak{F}\mathfrak{N}\delta\beta Cts$.
(vii) Every $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}Cts$ is a $\mathfrak{F}\mathfrak{N}\delta\beta Cts$.
(viii) Every $\mathfrak{F}\mathfrak{N}\delta\alpha Cts$ is a $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}Cts$.
(ix) Every $\mathfrak{F}\mathfrak{N}\delta\alpha Cts$ is a $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Cts$.

Proof. We prove only (iv) and (v), the others are similar.

(iv) Let λ be a $\mathfrak{F}\mathfrak{N}os$ in Y. Since f is $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Cts$, $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}os$ in X. Since every $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}os$ is a $\mathfrak{F}\mathfrak{N}eos$, $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}eos$ in X. Hence f is a $\mathfrak{F}\mathfrak{N}eCts$.

(v) Let λ be a $\mathfrak{F}\mathfrak{N}os$ in Y. Since f is $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}Cts$, $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os$ in X. Since every $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os$ is a $\mathfrak{F}\mathfrak{N}eos$, $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}eos$ in X. Hence f is a $\mathfrak{F}\mathfrak{N}eCts$.

Example 3.7. Let $X = Y = \{a, b\}$ and the $\mathfrak{F}\mathfrak{N}s$'s A_1, A_2 and A_3 are defined as
$\mu_{A_1}(a) = 0.8,\ \nu_{A_1}(a) = 0.8,\ \sigma_{A_1}(a) = 0.1,$
$\mu_{A_1}(b) = 0.9,\ \nu_{A_1}(b) = 0.8,\ \sigma_{A_1}(b) = 0.2;$
$\mu_{A_2}(a) = 0.6,\ \nu_{A_2}(a) = 0.7,\ \sigma_{A_2}(a) = 0.2,$
$\mu_{A_2}(b) = 0.5,\ \nu_{A_2}(b) = 0.7,\ \sigma_{A_2}(b) = 0.6;$
$\mu_{A_3}(a) = 0.1,\ \nu_{A_3}(a) = 0.2,\ \sigma_{A_3}(a) = 0.8,$
$\mu_{A_3}(b) = 0.2,\ \nu_{A_3}(b) = 0.2,\ \sigma_{A_3}(b) = 0.9.$

Let $\tau_{\mathfrak{F}\mathfrak{N}} = \sigma_{\mathfrak{F}\mathfrak{N}} = \{0_{\mathfrak{F}\mathfrak{N}},\ 1_{\mathfrak{F}\mathfrak{N}},\ A_1,\ A_2,\ A_3\}$ be a $\mathfrak{F}\mathfrak{N}ts$ on X and let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \to (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be an identity mapping, then f is $\mathfrak{F}\mathfrak{N}Cts$ (resp. $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Cts$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Cts$, $\mathfrak{F}\mathfrak{N}eCts$ and $\mathfrak{F}\mathfrak{N}\delta\beta Cts$) but not $\mathfrak{F}\mathfrak{N}\delta Cts$ (resp. $\mathfrak{F}\mathfrak{N}\delta\alpha Cts$, $\mathfrak{F}\mathfrak{N}\delta Cts$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}Cts$ and $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}Cts$), the set A_2 is a $\mathfrak{F}\mathfrak{N}os$ in Y but $f^{-1}(A_2) = A_2$ is not $\mathfrak{F}\mathfrak{N}\delta os$ (resp. $\mathfrak{F}\mathfrak{N}\delta\alpha os$, $\mathfrak{F}\mathfrak{N}\delta os$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os$ and $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os$) in X.

Example 3.8. Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2 and A_3 are defined as
$\mu_{A_1}(a) = 0.2,\ \nu_{A_1}(a) = 0.5,\ \sigma_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.3,\ \nu_{A_1}(b) = 0.5,\ \sigma_{A_1}(b) = 0.7;$
$\mu_{A_2}(a) = 0.1,\ \nu_{A_2}(a) = 0.5,\ \sigma_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.1,\ \nu_{A_2}(b) = 0.5,\ \sigma_{A_2}(b) = 0.9;$
$\mu_{A_3}(a) = 0.2,\ \nu_{A_3}(a) = 0.5,\ \sigma_{A_3}(a) = 0.8,$
$\mu_{A_3}(b) = 0.4,\ \nu_{A_3}(b) = 0.5,\ \sigma_{A_3}(b) = 0.6.$

Let $\tau_{\mathfrak{FN}} = \sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}},\ 1_{\mathfrak{FN}},\ A_1,\ A_2,\ A_3\}$ be a $\mathfrak{FN}ts$ on X and let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is $\mathfrak{FN}\delta\mathcal{S}Cts$ but not $\mathfrak{FN}\delta Cts$, the set A_1 is a $\mathfrak{FN}os$ in Y but $f^{-1}(A_1) = A_1$ is not $\mathfrak{FN}\delta os$ in X.

Example 3.9. Let $X = Y = \{a, b, c, d\}$ and the $\mathfrak{FN}s$'s A_1, A_2, A_3, A_4 and A_5 are defined as
$\mu_{A_1}(a) = 1,\ \nu_{A_1}(a) = 0.5,\ \sigma_{A_1}(a) = 0,$
$\mu_{A_1}(b) = 1,\ \nu_{A_1}(b) = 0.5,\ \sigma_{A_1}(b) = 0,$
$\mu_{A_1}(c) = 1,\ \nu_{A_1}(c) = 0.5,\ \sigma_{A_1}(c) = 0,$
$\mu_{A_1}(d) = 1,\ \nu_{A_1}(d) = 0.5,\ \sigma_{A_1}(d) = 0;$
$\mu_{A_2}(a) = 1,\ \nu_{A_2}(a) = 0.5,\ \sigma_{A_2}(a) = 0,$
$\mu_{A_2}(b) = 0,\ \nu_{A_2}(b) = 0.5,\ \sigma_{A_2}(b) = 1,$
$\mu_{A_2}(c) = 0.2,\ \nu_{A_2}(c) = 0.5,\ \sigma_{A_2}(c) = 0.7,$
$\mu_{A_2}(d) = 0,\ \nu_{A_2}(d) = 0.5,\ \sigma_{A_2}(d) = 1;$
$\mu_{A_3}(a) = 0,\ \nu_{A_3}(a) = 0.5,\ \sigma_{A_3}(a) = 1,$
$\mu_{A_3}(b) = 1,\ \nu_{A_3}(b) = 0.5,\ \sigma_{A_3}(b) = 0,$
$\mu_{A_3}(c) = 0,\ \nu_{A_3}(c) = 0.5,\ \sigma_{A_3}(c) = 0,$
$\mu_{A_3}(d) = 0,\ \nu_{A_3}(d) = 0.5,\ \sigma_{A_3}(d) = 1;$
$\mu_{A_4}(a) = 1,\ \nu_{A_4}(a) = 0.5,\ \sigma_{A_4}(a) = 0,$
$\mu_{A_4}(b) = 1,\ \nu_{A_4}(b) = 0.5,\ \sigma_{A_4}(b) = 0,$
$\mu_{A_4}(c) = 0.2,\ \nu_{A_4}(c) = 0.5,\ \sigma_{A_4}(c) = 0.7,$
$\mu_{A_4}(d) = 0,\ \nu_{A_4}(d) = 0.5,\ \sigma_{A_4}(d) = 0.1;$
$\mu_{A_5}(a) = 0,\ \nu_{A_5}(a) = 0.5,\ \sigma_{A_5}(a) = 1,$
$\mu_{A_5}(b) = 0,\ \nu_{A_5}(b) = 0.5,\ \sigma_{A_5}(b) = 1,$
$\mu_{A_5}(c) = 0,\ \nu_{A_5}(c) = 0.5,\ \sigma_{A_5}(c) = 1,$
$\mu_{A_5}(d) = 0,\ \nu_{A_5}(d) = 0.5,\ \sigma_{A_5}(d) = 1.$
Let $\tau_{\mathfrak{FN}} = \sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}},\ 1_{\mathfrak{FN}},\ A_1,\ A_2,\ A_3,\ A_4,\ A_5\}$ be a $\mathfrak{FN}ts$ on X and let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is $\mathfrak{FN}\delta\mathcal{S}Cts$ (resp. $\mathfrak{FN}eCts$ and $\mathfrak{FN}\delta\beta Cts$) but not $\mathfrak{FN}\delta\alpha Cts$ (resp. $\mathfrak{FN}\delta\mathcal{P}Cts$ and $\mathfrak{FN}\delta\mathcal{P}Cts$), the set A_2 is a $\mathfrak{FN}os$ in Y but $f^{-1}(A_2) = A_2$ is not $\mathfrak{FN}\delta\alpha os$ (resp. $\mathfrak{FN}\delta\mathcal{P}os$ and $\mathfrak{FN}\delta\mathcal{P}os$) in X.

From the above Proposition 3.6, and the following Example, the implications are hold.

$$\begin{tikzcd}
 & & \mathfrak{FN}Cts & & \\
 & & \mathfrak{FN}\delta Cts \arrow[u] \arrow[ddll] \arrow[ddrr] & & \\
 & & \mathfrak{FN}\delta\alpha Cts \arrow[dll] \arrow[drr] & & \\
\mathfrak{FN}\delta\mathcal{S}Cts \arrow[rr] \arrow[drr] & & \mathfrak{FN}eCts & & \mathfrak{FN}\delta\mathcal{P}Cts \arrow[ll] \arrow[dll] \\
 & & \mathfrak{FN}\delta\beta Cts & &
\end{tikzcd}$$

Note: $A \rightarrow B$ denotes A implies B, but not conversely.

Theorem 3.10. A map $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ is $\mathfrak{FN}eCts$ iff the inverse image of each $\mathfrak{FN}cs$ in Y is $\mathfrak{FN}ecs$ in X.

Proof. Let λ be a $\mathfrak{FN}cs$ in Y. This implies λ^c is $\mathfrak{FN}os$ in Y. Since f is $\mathfrak{FN}eCts$, $f^{-1}(\lambda^c)$ is $\mathfrak{FN}eos$ in X. Since $f^{-1}(\lambda^c) = (f^{-1}(\lambda))^c$, $f^{-1}(\lambda)$ is a $\mathfrak{FN}ecs$ in X.

Conversely, let λ be a $\mathfrak{FN}cs$ in Y. Then λ^c is a $\mathfrak{FN}os$ in Y. By hypothesis $f^{-1}(\lambda^c)$ is $\mathfrak{FN}eos$ in X. Since $f^{-1}(\lambda^c) = \left(f^{-1}(\lambda)\right)^c$, $\left(f^{-1}(\lambda)\right)^c$ is a $\mathfrak{FN}eos$ in X. Therefore $f^{-1}(\lambda)$ is a $\mathfrak{FN}ecs$ in X. Hence f is $\mathfrak{FN}eCts$.

Definition 3.11. *A $\mathfrak{FN}t$ $(X, \tau_{\mathfrak{FN}})$ is said to be an Fermatean neutrosophic $eU_{\frac{1}{2}}$ (in short $\mathfrak{FN}eU_{\frac{1}{2}}$)-space, if every $\mathfrak{FN}eos$ in X is a $\mathfrak{FN}os$ in X.*

Theorem 3.12. Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be a $\mathfrak{FN}eCts$, then f is a $\mathfrak{FN}Cts$ if X is a $\mathfrak{FN}eU_{\frac{1}{2}}$-space.

Proof. Let λ be a $\mathfrak{FN}os$ in Y. Then $f^{-1}(\lambda)$ is a $\mathfrak{FN}eos$ in X, by hypothesis. Since X is a $\mathfrak{FN}eU_{\frac{1}{2}}$-space, $f^{-1}(\lambda)$ is a $\mathfrak{FN}os$ in X. Hence f is a $\mathfrak{FN}eCts$.

Theorem 3.13. Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be a $\mathfrak{FN}eCts$ map and $g : (Y, \sigma_{\mathfrak{FN}}) \to (Z, \rho_{\mathfrak{FN}})$ be an $\mathfrak{FN}eCts$, then $g \circ f : (X, \tau_{\mathfrak{FN}}) \to (Z, \rho_{\mathfrak{FN}})$ is a $\mathfrak{FN}eCts$.

Proof. Let λ be a $\mathfrak{FN}eos$ in Z. Then $g^{-1}(\lambda)$ is a $\mathfrak{FN}os$ in Y, by hypothesis. Since f is a $\mathfrak{FN}eCts$ map, $f^{-1}\left(g^{-1}(\lambda)\right)$ is a $\mathfrak{FN}eos$ in X. Hence $g \circ f$ is a $\mathfrak{FN}eCts$ map.

Theorem 3.14. Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be a $\mathfrak{FN}eCts$ map. Then the following conditions are hold.

(i) $f(\mathfrak{FN}ecl(\lambda)) \subseteq \mathfrak{FN}cl(f(\lambda))$, for all $\mathfrak{FN}cs$ λ in X.

(ii) $\mathfrak{FN}ecl(f^{-1}(\mu)) \subseteq f^{-1}(\mathfrak{FN}cl(\mu))$, for all $\mathfrak{FN}cs$ μ in Y.

Proof. (i) Since $\mathfrak{FN}ecl(f(\lambda))$ is a $\mathfrak{FN}ecs$ in Y and f is $\mathfrak{FN}eCts$, then $f^{-1}(\mathfrak{FN}ecl(f(\lambda)))$ is $\mathfrak{FN}ec$ in Y. Now, since $\lambda \subseteq f^{-1}(\mathfrak{FN}cl(f(\lambda)))$, $\mathfrak{FN}ecl(\lambda) \subseteq f^{-1}(\mathfrak{FN}ecl(f(\lambda)))$. Therefore, $f(\mathfrak{FN}ecl(\lambda)) \subseteq \mathfrak{FN}cl(f(\lambda))$.

(ii) By replacing λ with μ in (i), we obtain $f(\mathfrak{FN}ecl(f^{-1}(\mu))) \subseteq \mathfrak{FN}cl(f(f^{-1}(\mu))) \subseteq \mathfrak{FN}cl(\mu)$. Hence, $\mathfrak{FN}ecl(f^{-1}(\mu)) \subseteq f^{-1}(\mathfrak{FN}cl(\mu))$.

Remark 3.15. If f is $\mathfrak{FN}eCts$, then

(i) $f(\mathfrak{FN}ecl(\lambda))$ is not necessarily equal to $\mathfrak{FN}cl(f(\lambda))$ where $\lambda \in X$.

(ii) $\mathfrak{FN}ecl(f^{-1}(\mu))$ is not necessarily equal to $f^{-1}(\mathfrak{FN}cl(\mu))$ where $\mu \in Y$.

Example 3.16. In Example 3.9, f is a $\mathfrak{FN}eCts$. Let the $\mathfrak{FN}s$'s A_6 and A_7 are defined as

$\mu_{A_6}(a) = 0.7,\ \nu_{A_6}(a) = 0.5,\ \sigma_{A_6}(a) = 0.2,$
$\mu_{A_6}(b) = 0.8,\ \nu_{A_6}(b) = 0.5,\ \sigma_{A_6}(b) = 0.1,$
$\mu_{A_6}(c) = 0.6,\ \nu_{A_6}(c) = 0.5,\ \sigma_{A_6}(c) = 0.1,$
$\mu_{A_6}(d) = 0.9,\ \nu_{A_6}(d) = 0.5,\ \sigma_{A_6}(d) = 0.2;$
$\mu_{A_7}(a) = 0.3,\ \nu_{A_7}(a) = 0.5,\ \sigma_{A_7}(a) = 0.6,$
$\mu_{A_7}(b) = 0.4,\ \nu_{A_7}(b) = 0.5,\ \sigma_{A_7}(b) = 0.8,$
$\mu_{A_7}(c) = 0.4,\ \nu_{A_7}(c) = 0.5,\ \sigma_{A_7}(c) = 0.2,$
$\mu_{A_7}(d) = 0.6,\ \nu_{A_7}(d) = 0.5,\ \sigma_{A_7}(d) = 0.3.$

(i) Let $\lambda = A_6$. Then $f(\mathfrak{FN}ecl(\lambda)) = A_6$. But $\mathfrak{FN}cl(f(\lambda)) = A_5^c$. Thus $f(\mathfrak{FN}ecl(\lambda)) \neq \mathfrak{FN}cl(f(\lambda))$.

(ii) Let $\lambda = A_7$. Then $f(\mathfrak{FN}ecl(\lambda)) = A_7$. But $\mathfrak{FN}cl(f(\lambda)) = A_5^c$. Thus $f^{-1}(\mathfrak{FN}cl(\lambda)) \neq \mathfrak{FN}ecl(f^{-1}(\lambda))$.

Theorem 3.17. *If f is $\mathfrak{FN}eCts$, then $f^{-1}(\mathfrak{FN}int(\mu)) \subseteq \mathfrak{FN}eint(f^{-1}(\mu))$, for all $\mathfrak{FN}cs$ μ in Y.*

Proof. If f is $\mathfrak{FN}eCts$ and $\mu \in \sigma_{\mathfrak{FN}}$. $\mathfrak{FN}int(\mu)$ is $\mathfrak{FN}o$ in Y and hence, $f^{-1}(\mathfrak{FN}int(\mu))$ is $\mathfrak{FN}eo$ in X. Therefore $\mathfrak{FN}eint\ (f^{-1}(\mathfrak{FN}eint(\mu))) = f^{-1}(\mathfrak{FN}int(\mu))$. Also, $\mathfrak{FN}int(\mu) \subseteq \mu$, implies that $f^{-1}(\mathfrak{FN}int(\mu)) \subseteq f^{-1}(\mu)$. Therefore $\mathfrak{FN}eint\ (f^{-1}(\mathfrak{FN}int(\mu))) \subseteq \mathfrak{FN}eint(f^{-1}(\mu))$. That is $f^{-1}(\mathfrak{FN}int(\mu)) \subseteq \mathfrak{FN}eint\ (f^{-1}(\mu))$.

Conversely, let $f^{-1}(\mathfrak{FN}int(\mu)) \subseteq \mathfrak{FN}eint(f^{-1}(\mu))$ for all subset μ of Y. If μ is $\mathfrak{FN}o$ in Y, then $\mathfrak{FN}int(\mu) = \mu$. By assumption, $f^{-1}(\mathfrak{FN}int(\mu)) \subseteq \mathfrak{FN}eint(f^{-1}(\mu))$. Thus $f^{-1}(\mu) \subseteq \mathfrak{FN}eint(f^{-1}(\mu))$. But $\mathfrak{FN}eint(f^{-1}(\mu)) \subseteq f^{-1}(\mu)$. Therefore $\mathfrak{FN}eint(f^{-1}(\mu)) = f^{-1}(\mu)$. That is, $f^{-1}(\mu)$ is $\mathfrak{FN}eo$ in X, for all $\mathfrak{FN}os$ μ in Y. Therefore f is $\mathfrak{FN}eCts$ on X.

Remark 3.18. If f is $\mathfrak{FN}eCts$, then $\mathfrak{FN}eint(f^{-1}(\mu))$ is not necessarily equal to $f^{-1}(\mathfrak{FN}int(\mu))$ where $\mu \in Y$.

Example 3.19. In Example 3.9, f is a $\mathfrak{F}\mathfrak{N}eCts$. Let $\eta = A_2^c$. Then $f^{-1}(\mathfrak{F}\mathfrak{N}int(\eta)) = A_3$. But $\mathfrak{F}\mathfrak{N}eint(f^{-1}(\eta)) = A_2^c$. Thus $f^{-1}(\mathfrak{F}\mathfrak{N}int(\eta)) \neq \mathfrak{F}\mathfrak{N}eint(f^{-1}(\eta))$.

4. Fermatean neutrosophic e-irresolute maps

In this section we introduce Fermatean neutrosophic e-irresolute maps and study some of its characterizations.

Definition 4.1. A map $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \to (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ is called a Fermatean neutrosophic

(i) irresolute (briefly, $\mathfrak{F}\mathfrak{N}Irr$) map if $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ for every $\mathfrak{F}\mathfrak{N}os$ λ of $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$,

(ii) δ-irresolute (briefly, $\mathfrak{F}\mathfrak{N}\delta Irr$) map if $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}\delta os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ for every $\mathfrak{F}\mathfrak{N}\delta os$ λ of $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$,

(iii) $\delta\mathcal{S}$-irresolute (briefly, $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}Irr$) map if $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ for every $\mathfrak{F}\mathfrak{N}\delta\mathcal{S}os$ λ of $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$,

(iv) $\delta\mathcal{P}$-irresolute (briefly, $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Irr$) map if $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ for every $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}os$ λ of $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$,

(v) $\delta\alpha$-irresolute (briefly, $\mathfrak{F}\mathfrak{N}\delta\alpha Irr$) map if $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}\delta\alpha os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ for every $\mathfrak{F}\mathfrak{N}\delta\alpha os$ λ of $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$,

(vi) e-irresolute (briefly, $\mathfrak{F}\mathfrak{N}eIrr$) map if $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}eos$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ for every $\mathfrak{F}\mathfrak{N}eos$ λ of $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$,

(vii) $\delta\beta$-irresolute (briefly, $\mathfrak{F}\mathfrak{N}\delta\beta Irr$) map if $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}\delta\beta os$ in $(X, \tau_{\mathfrak{F}\mathfrak{N}})$ for every $\mathfrak{F}\mathfrak{N}\delta\beta os$ λ of $(Y, \sigma_{\mathfrak{F}\mathfrak{N}})$.

Theorem 4.2. *Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \to (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be a $\mathfrak{F}\mathfrak{N}eIrr$ (resp. $\mathfrak{F}\mathfrak{N}Irr$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Irr$ and $\mathfrak{F}\mathfrak{N}\delta\alpha Irr$), then f is a $\mathfrak{F}\mathfrak{N}eCts$ (resp. $\mathfrak{F}\mathfrak{N}Cts$, $\mathfrak{F}\mathfrak{N}\delta\mathcal{P}Cts$ and $\mathfrak{F}\mathfrak{N}\delta\alpha Cts$) map. But not conversely.*

Proof. Let f be a $\mathfrak{F}\mathfrak{N}eIrr$ map. Let λ be any $\mathfrak{F}\mathfrak{N}os$ in Y. Since every $\mathfrak{F}\mathfrak{N}os$ is a $\mathfrak{F}\mathfrak{N}eos$, λ is a $\mathfrak{F}\mathfrak{N}eos$ in Y. By hypothesis $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}eos$ in Y. Hence f is a $\mathfrak{F}\mathfrak{N}eCts$ map. The other cases are similar.

Example 4.3. Let $X = Y = \{a, b\}$ and the $\mathfrak{F}\mathfrak{N}s$'s A_1, A_2, A_3, B_1 and B_2 are defined as
$\mu_{A_1}(a) = 0.8,\ \nu_{A_1}(a) = 0.8,\ \sigma_{A_1}(a) = 0.1,$
$\mu_{A_1}(b) = 0.9,\ \nu_{A_1}(b) = 0.8,\ \sigma_{A_1}(b) = 0.2;$
$\mu_{A_2}(a) = 0.6,\ \nu_{A_2}(a) = 0.7,\ \sigma_{A_2}(a) = 0.2,$
$\mu_{A_2}(b) = 0.5,\ \nu_{A_2}(b) = 0.7,\ \sigma_{A_2}(b) = 0.6;$
$\mu_{A_3}(a) = 0.1,\ \nu_{A_3}(a) = 0.2,\ \sigma_{A_3}(a) = 0.8,$
$\mu_{A_3}(b) = 0.2,\ \nu_{A_3}(b) = 0.2,\ \sigma_{A_3}(b) = 0.9;$
$\mu_{B_1}(a) = 0.7,\ \nu_{B_1}(a) = 0.7,\ \sigma_{B_1}(a) = 0.2,$

$\mu_{B_1}(b) = 0.8,\ \nu_{B_1}(b) = 0.7,\ \sigma_{B_1}(b) = 0.1;$
$\mu_{B_2}(a) = 0.3,\ \nu_{B_2}(a) = 0.6,\ \sigma_{B_2}(a) = 0.6,$
$\mu_{B_2}(b) = 0.4,\ \nu_{B_2}(b) = 0.6,\ \sigma_{B_2}(b) = 0.5.$

Then we have $\tau_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, A_1, A_2, A_3, 1_{\mathfrak{FN}}\}$ and $\sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, B_1, B_2, 1_{\mathfrak{FN}}\}$. Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is $\mathfrak{FN}\mathcal{S}Cts$ (resp. $\mathfrak{FN}eCts$) but not $\mathfrak{FN}Irr$ (resp. $\mathfrak{FN}eIrr$), the set (i) A_1 is a $\mathfrak{FN}\mathcal{S}os$ in Y but $f^{-1}(A_1)$ is not $\mathfrak{FN}\mathcal{S}os$ in X, (ii) B_1^c is a $\mathfrak{FN}eos$ in Y but $f^{-1}(B_1^c)$ is not $\mathfrak{FN}eos$ in X.

Example 4.4. Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2 and B_1 are defined as
$\mu_{A_1}(a) = 0.7,\ \nu_{A_1}(a) = 0.7,\ \sigma_{A_1}(a) = 0.2,$
$\mu_{A_1}(b) = 0.8,\ \nu_{A_1}(b) = 0.7,\ \sigma_{A_1}(b) = 0.1;$
$\mu_{A_2}(a) = 0.3,\ \nu_{A_2}(a) = 0.6,\ \sigma_{A_2}(a) = 0.6,$
$\mu_{A_2}(b) = 0.4,\ \nu_{A_2}(b) = 0.6,\ \sigma_{A_2}(b) = 0.5;$
$\mu_{B_1}(a) = 0.1,\ \nu_{B_1}(a) = 0.2,\ \sigma_{B_1}(a) = 0.7,$
$\mu_{B_1}(b) = 0.2,\ \nu_{B_1}(b) = 0.3,\ \sigma_{B_1}(b) = 0.8.$

Then we have $\tau_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, A_1, A_2, 1_{\mathfrak{FN}}\}$ and $\sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, B_1, 1_{\mathfrak{FN}}\}$. Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is $\mathfrak{FN}\delta\mathcal{P}Cts$ but not $\mathfrak{FN}\delta\mathcal{P}Irr$, the set A_1 is a $\mathfrak{FN}\delta\mathcal{P}os$ in Y but $f^{-1}(A_1)$ is not $\mathfrak{FN}\delta\mathcal{P}os$ in X.

Example 4.5. Let $X = Y = \{a, b\}$ and the $\mathfrak{FN}s$'s A_1, A_2, A_3, A_4, B_1, B_2 and B_3 are defined as
$\mu_{A_1}(a) = 0.2,\ \nu_{A_1}(a) = 0.5,\ \sigma_{A_1}(a) = 0.8,$
$\mu_{A_1}(b) = 0.4,\ \nu_{A_1}(b) = 0.5,\ \sigma_{A_1}(b) = 0.6;$
$\mu_{A_2}(a) = 0.1,\ \nu_{A_2}(a) = 0.5,\ \sigma_{A_2}(a) = 0.9,$
$\mu_{A_2}(b) = 0.3,\ \nu_{A_2}(b) = 0.5,\ \sigma_{A_2}(b) = 0.7;$
$\mu_{A_3}(a) = 0.9,\ \nu_{A_3}(a) = 0.5,\ \sigma_{A_3}(a) = 0.1,$
$\mu_{A_3}(b) = 0.7,\ \nu_{A_3}(b) = 0.5,\ \sigma_{A_3}(b) = 0.3;$
$\mu_{A_4}(a) = 0.2,\ \nu_{A_4}(a) = 0.5,\ \sigma_{A_4}(a) = 0.8,$
$\mu_{A_4}(b) = 0.3,\ \nu_{A_4}(b) = 0.5,\ \sigma_{A_4}(b) = 0.7;$
$\mu_{B_1}(a) = 0.9,\ \nu_{B_1}(a) = 0.5,\ \sigma_{B_1}(a) = 0.1,$
$\mu_{B_1}(b) = 0.7,\ \nu_{B_1}(b) = 0.5,\ \sigma_{B_1}(b) = 0.3;$
$\mu_{B_2}(a) = 0.1,\ \nu_{B_2}(a) = 0.5,\ \sigma_{B_2}(a) = 0.9,$
$\mu_{B_2}(b) = 0.3,\ \nu_{B_2}(b) = 0.5,\ \sigma_{B_2}(b) = 0.7;$
$\mu_{B_3}(a) = 0.2,\ \nu_{B_3}(a) = 0.5,\ \sigma_{B_3}(a) = 0.8,$
$\mu_{B_3}(b) = 0.4,\ \nu_{B_3}(b) = 0.5,\ \sigma_{B_3}(b) = 0.6.$ Then we have $\tau_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, A_1, A_2, A_3, A_4, 1_{\mathfrak{FN}}\}$ and $\sigma_{\mathfrak{FN}} = \{0_{\mathfrak{FN}}, B_1, B_2, B_3, 1_{\mathfrak{FN}}\}$. Let $f : (X, \tau_{\mathfrak{FN}}) \to (Y, \sigma_{\mathfrak{FN}})$ be an identity mapping, then f is $\mathfrak{FN}\delta\alpha Cts$ but not $\mathfrak{FN}\delta\alpha Irr$, the set A_4 is a $\mathfrak{FN}\delta\alpha os$ in Y but $f^{-1}(A_4)$ is not $\mathfrak{FN}\delta\alpha os$ in X.

Theorem 4.6. *Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \to (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be a $\mathfrak{F}\mathfrak{N}eIrr$, then f is a $\mathfrak{F}\mathfrak{N}Irr$ map if X is a $\mathfrak{F}\mathfrak{N}eU_{\frac{1}{2}}$-space.*

Proof. Let λ be a $\mathfrak{F}\mathfrak{N}os$ in Y. Then λ is a $\mathfrak{F}\mathfrak{N}eos$ in Y. Therefore $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}eos$ in X, by hypothesis. Since X is a $\mathfrak{F}\mathfrak{N}eU_{\frac{1}{2}}$-space, $f^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}os$ in X. Hence f is a $\mathfrak{F}\mathfrak{N}Irr$ map.

Theorem 4.7. *Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \to (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ and $g : (Y, \sigma_{\mathfrak{F}\mathfrak{N}}) \to (Z, \rho_{\mathfrak{F}\mathfrak{N}})$ be $\mathfrak{F}\mathfrak{N}eIrr$ maps, then $g \circ f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \to (Z, \rho_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}eIrr$ map.*

Proof. Let λ be a $\mathfrak{F}\mathfrak{N}eos$ in Z. Then $g^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}eos$ in Y. Since f is a $\mathfrak{F}\mathfrak{N}eIrr$ map. $f^{-1}(g^{-1}(\lambda))$ is a $\mathfrak{F}\mathfrak{N}eos$ in X. Hence $g \circ f$ is a $\mathfrak{F}\mathfrak{N}eIrr$ map.

Theorem 4.8. Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \to (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be $\mathfrak{F}\mathfrak{N}eIrr$ map and $g : (Y, \sigma_{\mathfrak{F}\mathfrak{N}}) \to (Z, \rho_{\mathfrak{F}\mathfrak{N}})$ be $\mathfrak{F}\mathfrak{N}eCts$ map, then $g \circ f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \to (Z, \rho_{\mathfrak{F}\mathfrak{N}})$ is a $\mathfrak{F}\mathfrak{N}eCts$ map.

Proof. Let λ be a $\mathfrak{F}\mathfrak{N}os$ in Z. Then $g^{-1}(\lambda)$ is a $\mathfrak{F}\mathfrak{N}eos$ in Y. Since f is a $\mathfrak{F}\mathfrak{N}eIrr$, $f^{-1}(g^{-1}(\lambda))$ is a $\mathfrak{F}\mathfrak{N}eos$ in X. Hence $g \circ f$ is a $\mathfrak{F}\mathfrak{N}eCts$ map.

Theorem 4.9. Let $f : (X, \tau_{\mathfrak{F}\mathfrak{N}}) \to (Y, \sigma_{\mathfrak{F}\mathfrak{N}})$ be a map from a $\mathfrak{F}\mathfrak{N}t$ X into a $\mathfrak{F}\mathfrak{N}t$ Y. Then the following conditions are equivalent if X and Y are $\mathfrak{F}\mathfrak{N}eU_{\frac{1}{2}}$-spaces.

(i) f is a $\mathfrak{F}\mathfrak{N}eIrr$ map.
(ii) $f^{-1}(\mu)$ is a $\mathfrak{F}\mathfrak{N}eos$ in X for each $\mathfrak{F}\mathfrak{N}eos$ μ in Y.
(iii) $\mathfrak{F}\mathfrak{N}cl(f^{-1}(\mu)) \subseteq f^{-1}(\mathfrak{F}\mathfrak{N}cl(\mu))$ for each $\mathfrak{F}\mathfrak{N}s$ μ of Y.

Proof. (i) $\to$ (ii): Let μ be any $\mathfrak{F}\mathfrak{N}eos$ in Y. Then μ^c is a $\mathfrak{F}\mathfrak{N}ecs$ in Y. Since f is $\mathfrak{F}\mathfrak{N}eIrr$, $f^{-1}(\mu^c)$ is a $\mathfrak{F}\mathfrak{N}ecs$ in X. But $f^{-1}(\mu^c) = \left(f^{-1}(\mu)\right)^c$. Therefore $f^{-1}(\mu)$ is a $\mathfrak{F}\mathfrak{N}eos$ in X.

(ii) $\to$ (iii): Let μ be a any $\mathfrak{F}\mathfrak{N}s$ in Y and $\mu \subseteq \mathfrak{F}\mathfrak{N}cl(\mu)$. Then $f^{-1}(\mu) \subseteq f^{-1}(\mathfrak{F}\mathfrak{N}cl(\mu))$. Since $\mathfrak{F}\mathfrak{N}cl(\mu)$ is a $\mathfrak{F}\mathfrak{N}cs$ in Y, $\mathfrak{F}\mathfrak{N}cl(\mu)$ is a $\mathfrak{F}\mathfrak{N}ecs$ in Y. Therefore $(\mathfrak{F}\mathfrak{N}cl(\mu))^c$ is a $\mathfrak{F}\mathfrak{N}eos$ in Y. By hypothesis, $f^{-1}((\mathfrak{F}\mathfrak{N}cl(\mu))^c)$ is a $\mathfrak{F}\mathfrak{N}eos$ in X. Since $f^{-1}((\mathfrak{F}\mathfrak{N}cl(\mu))^c) = \left(f^{-1}(\mathfrak{F}\mathfrak{N}cl(\mu))\right)^c$, $f^{-1}(\mathfrak{F}\mathfrak{N}cl(\mu))$ is a $\mathfrak{F}\mathfrak{N}ecs$ in X. Since X is $\mathfrak{F}\mathfrak{N}eU_{\frac{1}{2}}$-space, $f^{-1}(\mathfrak{F}\mathfrak{N}cl(\mu))$ is a $\mathfrak{F}\mathfrak{N}cs$ in X. Hence $\mathfrak{F}\mathfrak{N}cl\left(f^{-1}(\mu)\right) \subseteq \mathfrak{F}\mathfrak{N}cl(f^{-1}(\mathfrak{F}\mathfrak{N}cl(\mu))) = f^{-1}(\mathfrak{F}\mathfrak{N}cl(\mu))$. That is $\mathfrak{F}\mathfrak{N}cl\left(f^{-1}(\mu)\right) \subseteq f^{-1}(\mathfrak{F}\mathfrak{N}cl(\mu))$.

(iii) $\to$ (i): Let μ be any $\mathfrak{F}\mathfrak{N}ecs$ in Y. Since Y is $\mathfrak{F}\mathfrak{N}eU_{\frac{1}{2}}$-space, μ is a $\mathfrak{F}\mathfrak{N}cs$ in Y and $\mathfrak{F}\mathfrak{N}cl(\mu) = \mu$. Hence $f^{-1}(\mu) = f^{-1}(\mathfrak{F}\mathfrak{N}ecl(\mu)) \supseteq \mathfrak{F}\mathfrak{N}ecl(f^{-1}(\mu))$. But clearly $f^{-1}(\mu) \subseteq \mathfrak{F}\mathfrak{N}cl(f^{-1}(\mu))$. Therefore $\mathfrak{F}\mathfrak{N}cl(f^{-1}(\mu)) = f^{-1}(\mu)$. This implies $f^{-1}(\mu)$ is a $\mathfrak{F}\mathfrak{N}cs$ and hence it is a $\mathfrak{F}\mathfrak{N}ecs$ in X. Thus f is a $\mathfrak{F}\mathfrak{N}eIrr$ map.

5. Conclusions

In this research paper, we employ the notion of $\mathfrak{F}\mathfrak{N}e$-open sets ($\mathfrak{F}\mathfrak{N}eos$) as a foundation to introduce and formally define $\mathfrak{F}\mathfrak{N}e$-continuous maps ($\mathfrak{F}\mathfrak{N}eCts$) within Fermatean neutrosophic topological spaces. The study carefully investigates the fundamental properties of these mappings and provides a systematic analysis of their behavior. To highlight the novelty of the proposed framework, we present a comparative study between the newly defined $\mathfrak{FN}e$-continuous maps and the existing notions of Fermatean neutrosophic continuity, thereby establishing their distinctive role and scope in generalizing earlier results. Building upon this framework, the concept is further extended to introduce and examine $\mathfrak{FN}e$-irresolute maps, along with a detailed exploration of their properties and characterizations. These newly developed notions enrich the theory of Fermatean neutrosophic topology by providing deeper insights into the preservation of $\mathfrak{FN}e$-open structures under mappings. The contributions of this work not only fill a notable gap in the fuzzy and neutrosophic literature but also pave the way for future research directions. In particular, the results presented here may stimulate further investigations in related mathematical structures and hold potential applications in decision sciences, information systems, and other domains where uncertainty and vagueness play a critical role.

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