Neutrosophic Sets and Systems, Vol. 97, 2026
University of New Mexico

CLASS OF ANALYTIC FUNCTION DEFINED BY NEUTROSOPHIC POISSON DISTRIBUTION

[1]Olushola ADEYEMO * [2]Sayo Abidemi GBANGBALA [3] Adeniyi Abimbola AYENI [4]Folorunso Isola AKINWALE [5]Abel Onaolapo AREMU
[1,2,4] Department of Pure and Applied Mathematics, Ladoke Akintola University of Technology Ogbomoso, P.M.B. 4000, Oyo State Nigeria.
oadeyemo28@lautech.edu.ng*,[2]sayoabidemi@gmail.com,[4]jimran2018@gmail.com, [3] Department of Mathematics, Federal College of Education (Special), Oyo Nigeria.
[3]ayeni.adeniyi1501@fces.edu.ng, [5]National University San Diego California, USA
[5]olaaremu6@gmail.com

Abstract. This article investigates the interplay between univalent functions, the Salagean operator, and Poisson distribution within the framework of neutrosophic distribution and neutrosophic Poisson distribution. We introduce new subclasses of analytic functions, leveraging the Ma-Minda class and utilizing the Salagean operator. Furthermore, we explore the application of Poisson distribution and neutrosophic Poisson distribution in defining these subclasses. The coefficient bounds and other significant properties of the defined class was also derived by using second form of Stirling numbers with decreasing factorial.

Keywords: Univalent function, Ma-Minda function, Stirling number of second kind, Neutrosophic Poisson Distribution

1. Introduction

In complex analysis, univalent is a holomorphic or an analytic function that is also injective in a domain. This implies that for any two distinct points, say, $\kappa_1$and κ_2 in a domain, the images of the two points are not the same and if the same, it implies that both points κ_1 and κ_2 are the same. Let A be collection of function expressed as

$$g(\kappa) = \kappa + \sum_{o=2}^{\infty} a_o \kappa^o \qquad (1)$$

which are analytic in the domain $U = \{\kappa : |\kappa| < 1\}$ with normalization condition $g(0) = 0$ and $g'(0) > 0$. Let S be the subclass of A which consists of the univalent function of the form

$$g(\kappa) = \kappa - \sum_{o=2}^{\infty} a_o \kappa^o \tag{2}$$

The subclasses of the class S are starlike functions and convex functions respectively denoted by S* and K. A function $g(\kappa)$ is said to be starlike if

$$Re\left(\frac{\kappa g'(\kappa)}{g(\kappa)}\right) > 0, \tag{3}$$

for all κ in the unit disk $|\kappa| < 1$. This condition ensures that the function maps the unit disk onto a star shaped region with respect to origin. An analytic function $g(\kappa)$ is said to be convex if

$$Re\left(1 + \frac{\kappa g''(\kappa)}{g'(\kappa)}\right) \forall\ \kappa \in \mathcal{U}. \tag{4}$$

This condition relates the geometric property of mapping to a convex domain to the analytical behavior of the function's first and second derivative. It implies that as the point transverses the unit circle, the tangent to the image curve rotates monotonically in one direction. Alexander [1] gave the relationship between the class of starlike function given by (3) and convex functions given by (4), by giving a necessary and sufficient condition for a function to be convex is $\kappa g'(\kappa)$ being starlike. This relationship allows the transition of results and techniques between these two subclasses of univalent functions. The extension of convex function is close to convex function which was introduced by Kaplan [9]. A function $g(\kappa)$ is said to be close to convex if there exists a starlike function $f(\kappa)$ such that

$$Re\left(\frac{\kappa g'(\kappa)}{f(\kappa)}\right) > 0\ \forall\ \kappa\ \in \mathcal{U}. \tag{5}$$

The condition for close to convexity implies that the image of the unit disk under $g(\kappa)$ is "close" to being starlike or, more precisely, its boundary does not turn back too much. The class of all close-to-convex functions is denoted by C. Let $g(\kappa) \equiv \kappa$, the class C reduced to the class C introduced and studied by Mahzoon and Kargar [13] and defined by

$$Re\left(e^{i\theta}\frac{g'(\kappa)}{\kappa}\right) > 0\ \forall\theta\ \in \mathcal{R} \text{ and } \kappa \in U. \tag{6}$$

In 1992, Ma and Minda [12] generalizes and unify the clases of starlike functions and convex functions. Broader classes of analytic function was defined by them by using the concept of subordination. It was unified and represented by

$$\psi(\kappa) = (1-\lambda)\frac{\kappa g'(\kappa)}{g(\kappa)} + \lambda\left(1 + \frac{g''(\kappa)}{g'(\kappa)}\right) \quad 0 \le \lambda \le 1, \kappa \in \mathcal{U}. \tag{7}$$

Instead of fixing the target region for expression like $\frac{\kappa g'(\kappa)}{g(\kappa)}$ or $1 + \frac{g''(\kappa)}{g'(\kappa)}$ to be the right half plane (as in the case of starlike and convex function of order zero), Ma Minda allowed this target region to be the image of the unit disk under a more general analytic function, $\psi(\kappa)$. This function $\psi(\kappa)$ is typically chosen to be univalent in the unit disk, with $\psi(0) = 0$, $\psi'(0) > 0$, and $\psi(\mathrm{U})$ being a starlike domain with respect to 1 and symmetric with respect to the real axis.

Smaradanche [16], developed Neutrosophy , a philosophical and mathematical framework that aims to provide a comprehensive approach to dealing with uncertainty, imprecision and inconsistency than existing theories like Fuzzy logic and intuitional logic. It achieved this by explicitly introducing the concept of indeterminacy alongside truth and falsity.

This research pioneers the application of neutrosophic crisp set theory to classical probability distributions (like Poisson, exponential, and uniform). This innovative approach opens up a new intrigues for analyzing problems where the underlying phenomena follow a classical distribution, yet the available data is indeterminate or imprecisely specified. A classical Poisson distribution is used to model the number of events which occurs in a fixed interval when the average rate of occurrence is a known precise value and constant values. For example, the number of acceptance of a project in a project managing company in a week must be modeled with a classical Poisson distribution if the average rate of acceptance is known to be exactly ten acceptance per week. In contrast, a neutrosophic Poisson distribution is a theoretical extension which added situation where data is imprecise or indeterminate. Instead of a single, crisp value,, its parameters such as the mean, are treated as indeterminate quantities-often represented by an interval or a set of values. this frame work use the neutrosophic crisp set theory to handle uncertainty and vague data. In essence, the neutrosophic approach provides a more flexible and realistic model for complex real world events where there is a lack of certainty about the exact values of parameters.

Of recent, several scientists had developed neutrosophic Probaility distribution. [19] introduced Alhassan and Smarandache distribution, in which different neutrosophic distribution, like Rayleigh distributio, Wellbull distribution of five parameters in the sense of neutrosophic, the three parameters Wellbull distribution, neutrosophic continuous distribution and neutrosophic exponential distribution was studied by Alhabib et al [20]. In addition, Khan et al. [21] investigated distributions that apply to environmental data, such as the neutrosophic gamma distribution and the neutrosophic log normal distribution; Partro and Smarandache [21] investigated the neutrosophic normal and binomial distributions; Alhassan–ul-Haya and Zafar [22] investigated the neutrosophic discrete Rama Louza distribution; and Khan et al. [23] investigated the neutrosophic generalized Pareto model with application in data modeling and quality control, wherein neutrosophic data were used to construct a S control chart. [24–28] recently developed neutrosophic distributions, such as the neutrosophic Laplace distribution,

the neutrosophic negative binomial distribution, the neutrosophic logistic distribution, which is utilized in fuzzy data modeling, and the neutrosophic log gamma distribution, which can be utilized to analyze industrial growth

Specifically, we introduce the Neutrosophic Poisson Distribution. Unlike the classical Poisson distribution where the parameter (mean, m) is a precise value, in our model, m is treated as an imprecise quantity, often represented as an interval or a set of multiple elements. This allows incorporation and management of data uncertainties. The core distinction and novelty of this investigation lies in its ability to address real-world physical problems characterized by inherent fluctuations and indeterminate data by defining the analytic function in terms of Ma Minda. While previous works in geometric functions theory has primarily focused on classical distributions with perfectly specified data, our proposed work uniquely handles both classical Poisson distributions with precise data and neutrosophic Poisson distributions with indeterminate data using Ma-Minda concept. This dual capacity provides a more robust and realistic framework for modeling complex situations encountered in daily life.

A parameter y is said to be neutrosophic Poisson distribution if it takes the value in the set N ∪ {0} where N is the set of natural numbers with the probability

$$P(y=o) = m_N^o \frac{e^{-m_N}}{o!} \qquad o \in \mathrm{N} \cup \{0\} \tag{8}$$

and $NE[Y] = NV[Y] = m_N$ where$N = d + I$ a neutrosophic statistical number [17]. Al-Habib et al [2] introduced and investigated the power of Neutrosophic Poisson distribution which was further studied by Oladipo [15],through coeffiecient inequality expressed as

$$G(m_N, \kappa) = \kappa + \sum_{o=2}^{\infty} \frac{m_N^{o-1}}{(o-1)!} e^{-m_N} \kappa^o \qquad \kappa \in \mathrm{U} \tag{9}$$

where $m \in \mathrm{N}$ with the radius of convergence shown to be infinity using ratio test. Furthermore, a series R(m_N;κ) is defined by

$$\mathcal{R}(m_N;\kappa) = 2\kappa - G(m_N;\kappa) = \kappa - \sum_{o=2}^{\infty} \frac{m_N^{o-1}}{(o-1)!} e^{-m_N} \kappa^o \quad . \tag{10}$$

The Hadamand product of two power series given by $l(\kappa) = \kappa + \sum_{o=0}^{\infty} b_o \kappa^o$ and $j(\kappa) = \kappa + \sum_{o=0}^{\infty} t_o \kappa^o$ is given by

$$s(\kappa) = (l * j)(\kappa) = \kappa + \sum_{o=0}^{\infty} b_o t_o \kappa^o \quad . \tag{11}$$

comparing equation (2) and (10) implies that

$$a_o = \frac{m_N^{o-1}}{(o-1)!} e^{-m_N}$$

and taking the Hadamand product of (2) and (10) as in (11) gives

$$\Psi(m_N, \kappa) = \kappa + \sum_{o=2}^{\infty} \frac{m_N^{o-1}}{(o-1)!} e^{-m_N} a_o \kappa^o \quad . \tag{12}$$

Research on analytic univalent functions has led to significant discoveries, particularly in relation to Poisson distribution. A study introduced a class of functions connected to the Poisson distribution, establishing necessary and sufficient conditions for membership in specific function classes. Further research defined new subclasses of analytic univalent functions and investigated series conditions for these classes, revealing connection between various subclasses. in a related area, investigations into neutrosophic Poisson distribution series have yielded necessary and sufficient conditions for membership in certain function classes using coefficient inequalities. Building on this, a new subclass of analytic and bi-univalent functions associated with q-Gengebauer polynomials has been examined within the open unit disk, see see Porwal[15], Awolere and Oladio[10],Al-Habib .al [2] and Alsoboh et.al [3].

Meanwhile, Stirling numbers of the second kid have been extensively studied for their applications in combinatorial mathematics and statistics. These numbers count the numbers of ways to partition a set of distinct objects into non-empty , unlabeled subsets. Researchers have utilized Stirling numbers to calculate moments in statistics, derive formulas for moments of specific distributions, ad develop recursive formula for moment of binomial distributions. Recent studies have also explored the use of Stirling numbers in expressing powers of variables using falling facorials and deriving closed form formulas for sums of powers of integers. the connection between Stirling numbers and statistical distribution continues to be an area of active research, [6,8, 10, 11].

A common method to express a power of a variable, d^n, using falling factorials is through the use of Stirling numbers of the second kind.

$$(d)_j = d(d-1)\dots(d-j$$

$$d^n = \sum_{j=0}^{n} S(n;j)(d)_j$$

+ 1) is

where the Stirling numbers of the second kind, denoted as $S(n,j)(d)_j$ are used to express the power of a variable, as a linear combination of falling factorials,So, expanding $(d+1)^n$ in terms of $(d)_j = d(d-1)...(d-j+1)$. Thus,

$$o^n = \sum_{j-0}^{n} C(n,j)(o-1)_j$$

$$For\ o^1 : C(1,1) = 1, C(1,0) = 1$$

$$o^2 = C(2,2) = 1, C(2,1) = 3, C(2,0) = 1$$

$$o^3 = C(3,3) = 1, C(3,2) = 6, C(3,1) = 7, C(3,0) = 1$$

$$o^4 = C(4,4) = 1, C(4,3) = 10, C(4,2) = 25, C(4,1) = 15, C(4,0) = 1$$

$$\vdots$$

$$o^n = \sum_{j=0}^{n} C(n+1,j+1)(o-1)(o-2)...(o-j). \quad (13)$$

The present work uniquely addressed both classical Poisson distribution with precise data and Neutrosophic Poisson distribution with indeterminate data by using the Ma-Minda concept, providing a more realistic frame work for modeling complex, real world situation with inherent fluctuations and uncertain data. In this study, Stirling numbers of the second kind were used in conjunction with a decreasing factorial to determine coefficient bounds and other key properties of these new subclasses of functions.

Definition 1.1: A function $g \in U$ having the power series (10) is said to be in the family $J_{\lambda,\gamma}^{n}(\theta)$ of if

$$(2-\lambda)(D^n g(\kappa))' + \left(\frac{1+e^{i\theta}}{2}\lambda z(D^n g(\kappa))''\right) > \gamma$$

$\leq \gamma < 1,\ 0 \leq \lambda \leq 1,\ -\pi < \theta < \pi.$

(14)

where 0

Remarks: The defined class in (13) generalized the classes studied by the following authors

1 when $\lambda = 1$, the class reduced to class defined by Awolere and Oladipo [4]

2 When $\lambda = 1$ and $n = 0$, the cl;ass defined in (14) reduced to the class of analytic function defined by Mahmoon [13]

2. Main Results

Lemma 2.1: The class $J_{\lambda,\gamma}^{n}(\theta)$ includes a function g of (2) if and only if

$$\sum_{o=2}^{\infty} o^n[2(2-\lambda)o + \lambda o(o-1)(1+\cos\theta)]|a_o| \leq 2(2-(\lambda+\gamma)) \qquad (15)$$

where $0 \leq \gamma \leq 1, 0 \leq \lambda \leq 1, -\pi \leq \theta \leq \pi$ and $n \in \mathrm{N} \cup \{0\}$

Proof. Suppose that the function $g(\kappa) \in J_{\lambda,\gamma}^{n}$. Then by (14) we have

$$(2-\lambda)(D^n f(\kappa))' + \frac{1+e^{\theta}}{2}\lambda\kappa(D^{\kappa} f(\kappa))'' > \gamma$$

$$(2-\lambda)[1 - \sum_{o=2}^{\infty} o^{n+1}|a_o|z^{o-1}] + \frac{1+e^{i\theta}}{2}\lambda \sum_{o=2}^{\infty} o^{n+1}(o-1)|a_o|\kappa^{o-1} > \gamma$$

$$(2-\lambda) - \sum_{o=2}^{\infty} o^{n+1}(2-\lambda) + \frac{(1+\cos\theta)}{2}(o-1)\lambda)|a_o| > \gamma$$

$$\sum_{o=2}^{\infty} o^{n+1}(2-\lambda) + \lambda\frac{(1+\cos\theta)}{2}(o-1))|a_o| \leq 2-(\lambda+\gamma)$$

$$\sum_{o=2}^{\infty} o^n[2(2-\lambda)o + \lambda o(o-1)(1+\cos\theta)]|a_o| \leq 2(2-(\lambda+\gamma))$$

Conversely, suppose that (15) is true then, we have

$$|(2-\lambda)(D^n f(\kappa))' + \frac{1+e^{i\theta}}{2}\lambda\kappa(D^n f(\kappa))'' - (2-\lambda)|$$

$$\sum_{o=2}^{\infty}[o(2-\lambda) + \frac{\lambda o(o-1)}{2}(1+\cos\theta)]$$

since $o \geq 2$ and

$1 + \cos\theta \geq 0$, and $\lambda \in [0,1]$ the term $(2-\lambda) + \lambda o(o-1)(1+\cos\theta)$ is > 0

$$\leq \sum_{o=2}^{\infty} o^n[(2-\lambda) + o\lambda(o-1)(1+\cos\theta)]\frac{|a_o|}{2}\kappa^{o-1}$$

$$< \sum_{o=2}^{\infty} o^n[(2-\lambda) + o\lambda(o-1)(1+\cos\theta)]\frac{|a_o|}{2}$$

From assumption (14) the entire expression is

$$\leq \frac{2(2-(\lambda+\gamma)))}{2} \leq 2-(\lambda+\gamma)$$

Lemma 2.2: If g of the form (12) belongs to P(A,B,λ) then □

$$a_o \leq \frac{(A-B)T}{(2-\lambda)k} \tag{16}$$

where $T \in$ C\{0}, $-1 \leq B < A \leq 1$, $0 < \lambda \leq 1$.

Proof. A function $g \in$ A is in the class $G_T(A,B,\lambda)$ if

$$\left|\frac{(2-\lambda)g'(\kappa)}{(A-B)T - B(g'(\kappa)-(2-\lambda)}\right| < (2-\lambda)$$

$T \in \mathcal{C}\backslash\{0\},\ -1 \leq B < A \leq 1,\ 0 < \lambda \leq 1.$

Let $\omega(\kappa) = \frac{(2-\lambda)g'(\kappa)-(2-\lambda)}{(A-B)T-B(2-\lambda)g'(\kappa)-(2-\lambda)}$

By definition of Schwarsz function

$$\omega(0) = \frac{(2-\lambda)g'(0)-(2-\lambda)}{(A-B)T - B((2-\lambda)g'(0)-(2-\lambda)} = 0$$

which implies that from Schwarsz lemma $\omega(\kappa) \leq |\kappa|$ and from the definition of $\omega(\kappa)$, we can solve for $g'(\kappa)$

$$(2-\lambda)g'(\kappa) - (2-\lambda) = \omega(\kappa)[(A-B)T - B((2-\lambda)g'(\kappa)-(2-\lambda))]$$

$$(2-\lambda)(g'(\kappa)-1) + B\omega(\kappa)[(2-\lambda)g'(\kappa)-(2-\lambda)] = (A-B)T\omega(\kappa)$$

$$(2-\lambda)(g'(\kappa)-1) + (2-\lambda)(g'(\kappa)-1)B\omega(\kappa) = (A-B)T\omega(\kappa)$$

$$(2-\lambda-1)(g'(\kappa)-1)(1+B\omega(\kappa)) = (A-B)T\omega(\kappa)$$

$$(2-\lambda)(g'(\kappa)-1) = \frac{(A-B)T\omega(\kappa)}{1+B\omega(\kappa)}$$

$$(2-\lambda)(\sum a_o\kappa^{o-1}) = \frac{(A-B)T\omega(\kappa)}{1+B\omega(\kappa)}$$

But $\omega(\kappa) = c_1\kappa + c_2\kappa^2 + c +_3 \kappa^3 + \ldots \omega(0) = 0$

$= (A-B)T(c_1\kappa + c_2\kappa^2 + c +_3 \kappa^3 + \ldots)[1 + B(c_1\kappa + c_2\kappa^2 + c +_3 \kappa^3 + \ldots)]^{-1}$

$= (A - B)T(c_1\kappa + c_2\kappa^2 + c +_3 \kappa^3 + \dots)[(1 - B(c_1\kappa + c_2\kappa^2 + c +_3 \kappa^3 + \dots)$

$+ B^2(c_1\kappa + c_2\kappa^2 + c_3\kappa^3 + \dots)^2)]$

$= (A - B)Tc_1$

By Schwarsz Lemma $|c_i| \leq 1$

$$a_o \leq \frac{(A-B)T}{(2-\lambda)o} \tag{17}$$

□

Theorem 2.1: Suppose $m \in$ N,$0 \leq \gamma < 1, 0 \leq \lambda \leq 1and-\pi \leq \theta \leq \pi$.

Then $\Psi(m_N,\kappa) \in {}^{J}{}_{\lambda,\gamma}{}^{0}$ if and only if $\lambda(1+\cos\theta)m^2{}_N+[4+2\lambda\cos\theta]m_N+2(2-\lambda)(1-e^{-mN}) \leq$

$2(2-(\lambda+\gamma)) \leq 2(2-(\lambda+\gamma))$

(18)

Proof. From Lemma (2.1), let $n = 0$,

$$[2o(2-\lambda) + \lambda o(o-1)(1+cos\theta)]\frac{m_N^{o-1}}{(o-1)!}e^{-m_N}$$

$$= [2o(2-\lambda) + \lambda o^2(1+\cos\theta) - \lambda o(1+\cos\theta)]\frac{m_N^{o-1}}{(o-1)!}e^{-m_N}$$

$$= o[2(2-\lambda) - \lambda - \lambda\cos\theta] + \lambda o^2(1+\cos\theta)$$

From (13)

$$\Big\{((o-1)+1)[4-\lambda(3+\cos\theta)] + \lambda(1+\cos\theta)[(o-1)(o-2)$$

$$+ 3(o-1)+1]\Big\}\frac{m_N^{o-1}}{(o-1)!}e^{-m_N}$$

$$= \Big\{\lambda(1+\cos\theta)(o-1)(o-2) + (o-1)(4+2\lambda\cos\theta)2(2-\lambda)\Big\}\frac{m_N^{o-1}}{(o-1)!}e^{-m_N}$$

For $o \geq 2, \frac{(o-1)(o-2)}{(o-3)!} = \frac{1}{(o-3)!}$, let $j = o-3, o-1 = j+2$

$$\Big\{\lambda(1+\cos\theta)(o-1)(o-2) + (o-1)(4+2\lambda\cos\theta) + 2(2-\lambda)\Big\}\frac{m_N^{j+2}}{j!}e^{-m_N}$$

$$\lambda(1+\cos\theta)m_N^2 + (4+2\lambda\cos\theta)m_N + 2(2-\lambda)(1-e^{-m_N})$$

□

Theorem 2.2: Suppose $m \in$ N,$0 \leq \gamma < 1, 0 \leq \lambda \leq 1and-\pi \leq \theta \leq \pi$.
Then $\Psi(m_N,\kappa) \in {}^{J}{}_{\lambda,\gamma}{}^{1}$ if and only if $\lambda(1 + \cos\theta)m^3{}_N + [4 + \lambda(3 +$

$5\cos\theta)]m^2{}_N$

$$+ [2\lambda(2\cos\theta - 1) + 12]m_N + 2(2 - \lambda)(1 - e^{-mN}) \leq 2(2 - (\lambda + \gamma)) \quad (19)$$

Proof. From Lemma (2.1), let $n = 1$, and follow the same procedure as in theorem (2.1) and applying the Stirling formula of reduced factorial of (13) gives the result. □

Theorem 2.3: Suppose $m \in$ N,$0 \leq \gamma < 1, 0 \leq \lambda \leq 1and-\pi \leq \theta \leq \pi$.
Then $\Psi(m_N,\kappa) \in {}^{J}{}_{\lambda,\gamma}{}^{2}$ if and only if $\lambda(1 + \cos\theta)m^4{}_N + [4 + \lambda(7 + 9\cos\theta)]m^3{}_N + [\lambda(19\cos\theta + 7)$

$+ 24]m^2{}_N$

$$+ [28 + \lambda(8\cos\theta - 6)]m_N + 2(2 - \lambda)(1 - e^{-mN}) \leq 2(2 - (\lambda + \gamma)) \quad (20)$$

Proof. From Lemma (2.1), let $n = 2$, and follow the same procedure as in theorem (2.1) and applying the Stirling formula of reduced factorial of (13) gives the result. □

Theorem 2.4: Suppose $m \in$ N,$0 \leq \gamma < 1, 0 \leq \lambda \leq 1and-\pi \leq \theta \leq \pi$.
Then $\Psi(m_N,\kappa) \in {}^{J}{}_{\lambda,\gamma}{}^{3}$ if and only if $\lambda(1 + \cos\theta)m^5{}_N + [4 + 3\lambda(9 +$

$10\cos\theta)]m^4{}_N + [20\lambda(4\cos\theta + 3) + 40]m^3{}_N$

$$+ [100 + 5\lambda(8\cos\theta - 3)]m^2{}_N + 30(2 - \lambda)m_N + 2(2 - \lambda)(1 - e^{-mN}) \leq 2(2 - (\lambda + \gamma)) \quad (21)$$

Proof. From Lemma (2.1), let $n = 3$, and follow the same procedure as in theorem (2.1) and applying the Stirling formula of reduced factorial of (13) gives the result. □

Theorem 2.5: Given $m \in$ N,$0 \leq \gamma < 1, 0 \leq \lambda \leq 1$and $-\pi \leq \theta \leq \pi$.
Then $\Psi(m_N,\kappa) \in {}^{J}{}_{\lambda,\gamma}{}^{4}$ if and only if $\lambda(1 + \cos\theta)m^6{}_N + [4 + 2\lambda(30 + 31\cos\theta)]m^4{}_N5$

$+ [2\lambda(104\cos\theta + 135) + 124]m^4{}_N$

$$+ [360 + 20\lambda(13\cos\theta + 4)]m^3{}_N + 5\lambda(15\cos\theta - 11)m^2{}_N$$

$$+ [6\lambda(\cos\theta - 4) + 60] + 2(2 - \lambda)(1 - e^{-mN}) \leq 2(2 - (\lambda + \gamma)) \quad (22)$$

Proof. From Lemma (2.1), let $n = 4$, and follow the same procedure as in theorem (2.1) and applying the Stirling formula of reduced factorial of (13) gives the result. □

The effects of Neutrosophic on the class $J_{\lambda,\gamma}^n$ for $n = 0,1,2,3,4$ will be looked into the subsequent theorems using Lemma 2.2.

Theorem 2.6: Let $m \in [1,\infty], 0 \le \gamma < 1, 0 < \lambda \le 1, -\pi \le \theta \le \pi$ and $g \in G_T(A,B,\lambda)$. Then $\Psi(m_N,\lambda,z) \in J_{\lambda,\gamma}^0$ if and only if

$$\frac{(A-B)}{(2-\lambda)}|T|[\lambda(1+\cos\theta)m_N+2(2-\lambda)(1-e^{-m_N})] \le 2(2-(\lambda+\gamma)) \quad (23)$$

Proof. In view of theorem (2.1), it suffices to prove that

$$[2(2-\lambda)o+\lambda o(o-1)(1+\cos\theta)]\left(\frac{m_N^{o-1}}{(o-1)!}e^{-}m_N\right)|a_o| \le 2(2-(\lambda+\gamma))$$

from Lemma (2.2) for $g \in G_T(A,B,\lambda)$ we have

$$|a_o| \le \frac{(A-B)}{(2-\lambda)o}|T|$$

Substitute this upper bound for $|a_o|$ into the sum to get

$$\sum[2(2-\lambda)o+o\lambda(o-1)(1+\cos\theta)]\left(\frac{m_N^{o-1}}{(o-1)!}e^{-m_N}\right)\frac{(A-B)T}{(2-\lambda)o}$$

Factoring out the constant $\frac{(A-B)T}{(2-\lambda)k}$ and simplifying the summation inside the bracket

$$\sum[2(2-\lambda)o+o\lambda(o-1)(1+\cos\theta)]\left(\frac{m_N^{o-1}}{(o-1)!}\right.$$

$2(2-\lambda)\sum \frac{m_N^{o-1}}{(o-1)!}$ad let$j = o - 1$

$$2(2-\lambda)\sum \frac{m_N^{j}}{j!} = 2(2-\lambda)(e^{m_N}-1)$$.

Coefficient of

$$(1+\cos\theta) = \lambda(1+\cos\theta)\sum(o-1)\frac{m_N^{o-1}}{(o-1)!}$$

So, (1 + cos$\theta) \sum \frac{m_N^{o-1}}{(o-2)!}$; let$j = o-2, o-1 = j+1$

$$\lambda(1+\cos\theta)\sum \frac{m_N^{j}+1}{j!} = \lambda(1+\cos\theta)m_N \sum j = 0^{\infty}\frac{m_N^{j}}{j!} = \lambda(1+\cos\theta)m_N e^{m_N}$$

Then, we have

$$\frac{(A-B)|T|}{(2-\lambda)}e^{-m_N}[\lambda(1+\cos\theta)m_N e^{m_N} + 2(2-\lambda)(e^{m_N} - 1)] \quad (24)$$

$$\frac{(A-B)|T|}{(2-\lambda)}[\lambda(1+\cos\theta)m_N + 2(2-\lambda)(1-e^{-m_N})] \quad (25)$$ □

Theorem 2.7: Suppose $m \in$ N,$0 \le \gamma < 1, 0 < \lambda \le 1, -\pi \le \theta \le \pi$ and $g \in K_T(A,B,\lambda)$. Then $\Psi(m_N,\lambda,\kappa) \in J_{\lambda,\gamma}^1$ if and only if

$$\frac{(A-B)|T|}{(2-\lambda)}[\lambda(1+\cos\theta)m_N^2 + [4+\lambda(1+\cos\theta)]m_N + [4-\lambda(1-\cos\theta)](1-e^{-m_N})] \le 2(2-(\lambda+\gamma)) \quad (26)$$

Proof. In view of theorem (2.3),it suffices to prove that

$$[k[(2-\lambda)2k + \lambda(o-1)(1+\cos\theta)]]\frac{m_N^{o-1}}{(o-1)!}e^{-m_N}|a_o|$$

From Lemma (2.2) for $g \in K_T A,B,\lambda$ we have $|a_o| \le \frac{(A-B)}{(2-\lambda)o}|T|$. Substitute this for upper bound to $|a_o|$ into the sum to get an upper bound.

Thus,

$$\frac{(A-B)}{(2-\lambda)}|T|e^{-m_N}\sum[(2-\lambda)2o+\lambda o(o-1)(1+\cos\theta)]\frac{m_N^{o-1}}{(o-1)!}$$

$$\leq \frac{(A-B)}{(2-\lambda)}|T|e^{-m_N}\sum[2(2-\lambda)(o-1)+2(2-\lambda)+\lambda(1+\cos\theta)(o-1)(o-2)$$

$$+3\lambda(1+\cos\theta)(o-1)+\lambda(1+\cos\theta)]\frac{m_N^{o-1}}{(o-1)!}$$

$$\frac{(A-B)}{(2-\lambda)}|T|e^{-m_N}\Bigg[2(2-\lambda)\sum\frac{m_N(o-1)}{(o-3)!}$$

$$+2(2-\lambda)\sum\frac{m_N^{(o-1)}}{(o-1)!}+\lambda(1+\cos\theta)\sum\frac{m_N^{-(o-1)}}{(o-1)!}$$

$$+3\lambda(1+\cos\theta)\sum\frac{m_N^{o-1}}{(o-2)!}+\lambda(1+\cos\theta)\sum\frac{m_N^{(o-1)}}{(o-1)!}\Bigg]$$

Let $j = o-2$, $o-1 = j+1$

$$\leq \frac{(A-B)}{(2-\lambda)}|T|e^{-m_N}\Big[2(2-\lambda)m_Ne^{m_N}+2(2-\lambda)(e^{m_N}-1)$$

$$+\lambda(1\cos\theta)m_N^2e^{m_N}+3\lambda(1+\cos\theta)m_ne^{m_N}+\lambda(1+\cos\theta)(e^{m_N}-1)\Big]$$

$$\frac{(A-B)}{(2-\lambda)}|T|[\lambda(1+\cos\theta)m_N^2+[4+\lambda(3+\cos\theta)]m_N+(4-\lambda(1-\cos\theta))(1-e^{-m_N})] \quad (27)$$

□

Theorem 2.8: Given $m \in \mathrm{N}, 0 \leq \gamma < 1, 0 < \lambda \leq 1, -\pi \leq \theta \leq \pi$ and $g \in \mathrm{K}_T(A,B,\lambda)$. Then $\Psi(m_N,\lambda,\kappa) \in J_{\lambda,\gamma}{}^2$ if and only if

$$\frac{(A-B)}{(2-\lambda)}|T|[\lambda(1+\cos\theta)m_N^3+[4+3\lambda(1+\cos\theta)]m_N^2$$
$$+[12+2\lambda(\cos\theta-1)]m_N+(4-2\lambda)(1-e^{-m_N}) \quad (28)$$

Proof. The proof follows as above. □

Theorem 2.9: Suppose $m \in \mathrm{N}, 0 \leq \gamma < 1, 0 < \lambda \leq 1, -\pi \leq \theta \leq \pi$ and $g \in \mathrm{K}_T(A,B,\lambda)$. Then $\Psi(m_N,\lambda,\kappa) \in J_{\lambda,\gamma}{}^3$ if and only if

$$\frac{(A-B)}{(2-\lambda)}|T|[\lambda(1+\cos\theta)m_N^4+[4+7\lambda(1+\cos\theta)]m_N^3$$
$$+[24+\lambda(19\cos\theta+7)]m_N^2+[28+2\lambda(\cos\theta-1)]m_N+(4-2\lambda)(1-e^{-m_N}) \quad (29)$$

Proof. The proof follows as above. □

The graphical illustration of the results are presented below which is follow by an example to illustrate the concept along with the decision rule.

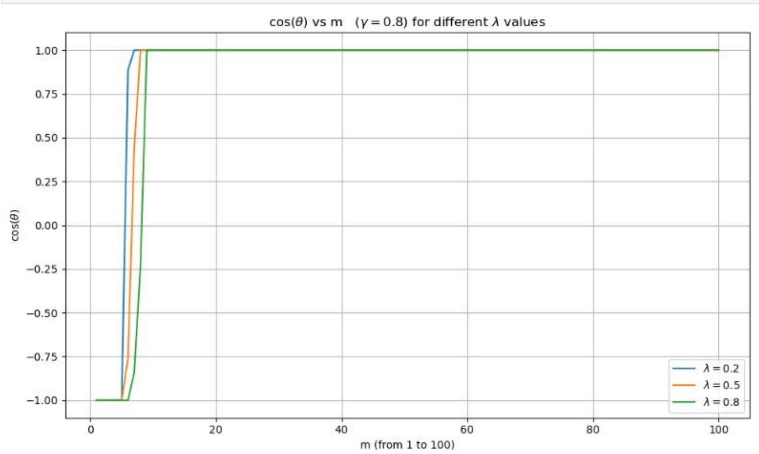


Figure 1. graph of cosθ against m_N a range of values of λ ans γ

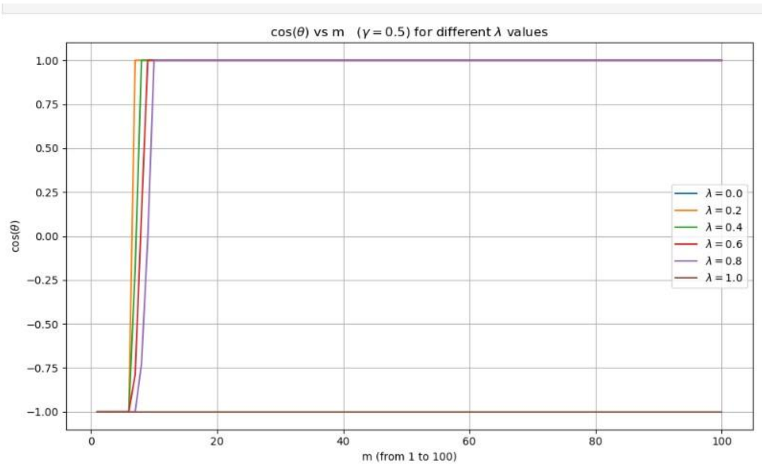


Figure 2.

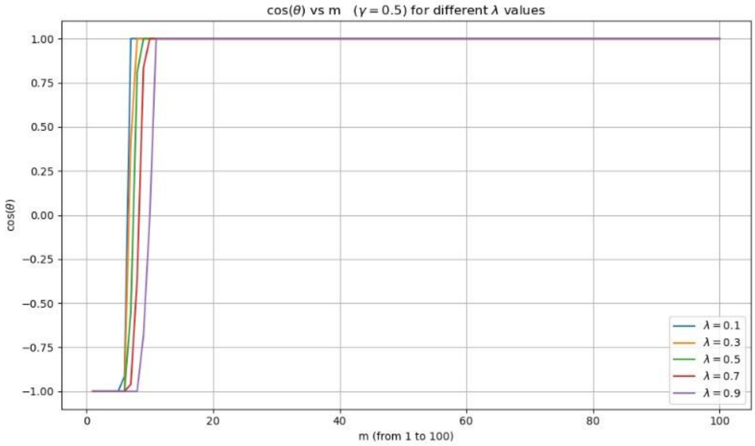
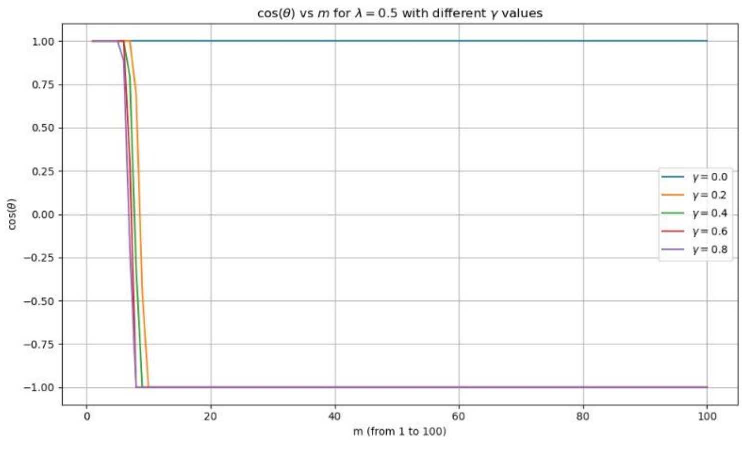


Figure 3. graph of cosθ against m_N for a range of values of λ ans γ Figure 4.



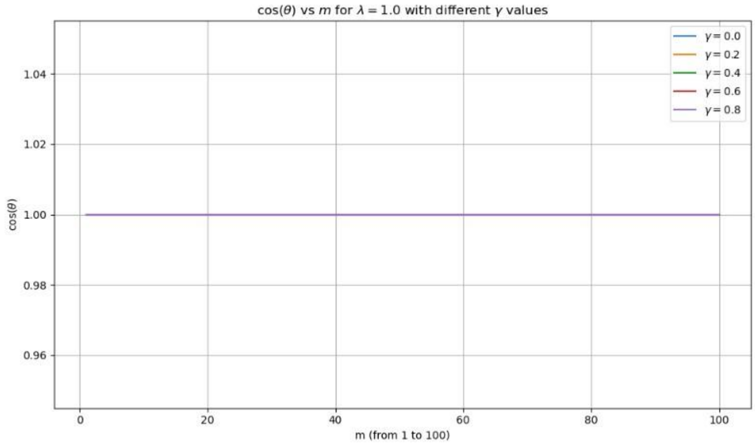


Figure 6.

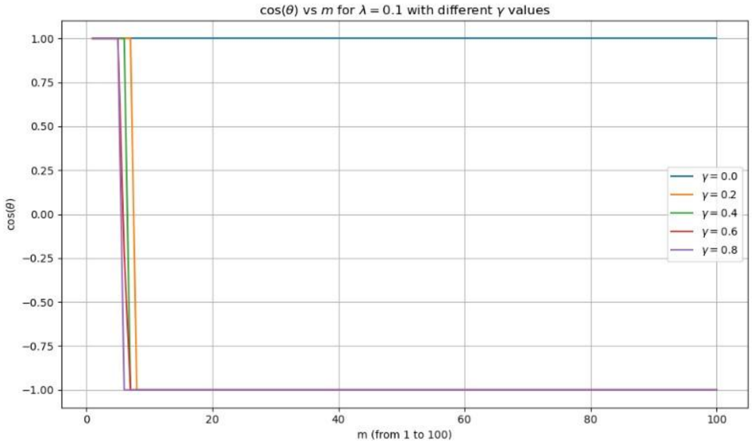


Figure 7. graph of $\cos\theta$ against m_N for a range of values of λ ans γ

3. Examples

An example that illustrates how a neutrosophic inspired decision rule can lead to the sharp step like behavior of the graph above. Let $\cos\theta$ represents the acceptance degree which depends on the performance score, m_N and λ represent the tolerance level.

Assume the evaluation of a project based in its performance score, m_N, and its acceptance degree is to be evaluated which is map to $\cos\theta$. in the case of Neutrosophic set or Neutrosophic logic, the following are members of the set: a truth or support for acceptance, an indeterminancy or ambiguity and a falsity or rejection and λ is a factor that influences how strictly the evaluation m_N is made.

Consider a simplified neutrosophic evaluation that outputs a score $S(m_N,\lambda)$ which is assigned to $\cos\theta$. Let

$$S(m_N,\lambda) = m_N - \lambda \cdot (uncertainty/Penalty)$$

where uncertainty/penalty can a constant function or variable function but for a sharp transition, a comparison to a fixed threshold. Let τ be an underlying acceptance threshold

1 If m_N is well below τ, the project is not accepted ($\cos\theta = -1$)

2 If m_N is well above τ, the project is accepted ($\cos\theta = +1$)

3 The ambiguity parameter λ influence how much m_N needs to exceed or fall below a base threshold to trigger a clear decision.

As an illustration, let $\cos\theta$ be determined by the following piecewise function which allows a base threshold T_0 and a λ adjusting the critical point. For a given parameter score $m_N \geq 1$ and a parameter $\lambda \in [0,1]$.

Define a critical threshold

$C(\lambda) = T_0 + l \cdot \lambda$

where T_0 is a base threshold, say, 5 and l a positive constant that scale the effect of λ, say, l = 10. define $\cos\theta$ based on this critical threshold,

$$\cos\theta = \begin{cases} -1 & \text{If } m_N < C(\lambda) - \delta \\ -1 < \cos\theta < +1 & \text{if } C(\lambda) - \delta \leq m_N \leq C(\lambda) + \delta \\ 1 & \text{if } m_N > C(\lambda) + \delta \end{cases} \quad (30)$$

where δ is a small positive value representing the transistion width say, $\delta = 0.5$. Choosing a specific value $T_0 = 5$, $l = 10$, $\delta = 0.5$. Now, let see $\cos\theta$ behaves for different values of λ and increase m_N. This simple model clearly demonstrates the step like behavior as seen in the graphs. For low m_N values, $\cos\theta$ is fixed. As m_N increases, it hits a λ dependent threshold $C(\lambda)$. Around this threshold, there is a quick sharp transition defined by δ where $\cos\theta$ jumps from −1 to +1. after the transition, $\cos\theta$ is fixed at +1 for higher m_N values. Crucially, increasing λ shifts this transition point to higher values of m_N just as seen in the figures where the red line (higher λ)shifts the jump to the right. This examples illustrates how a decision rule, often found in system dealing with uncertainties or degree of belonginess like neutrosophic sets, can translate a continuous input m_N into a seemingly crisp or threshold based output for $\cos\theta$.

Data and Results

level of decision	
λ	$C(\lambda)$
0.2	$C(\lambda) = 5 + 10(0.2) = 7$
0.5	$C(\lambda) = 5 + 0(0.5) = 10$
0.8	$C(\lambda) = 5 + 10(0.8) = 13$

As shown in this data, an increase in the degree of indeterminacy shifts the performance score m_N required for a jump in acceptance. The higher the tolerance for ambiguity (λ),the higher the performance score a project needs to be accepted. this illustrates the core function of the degree of indeterminacy within the neutrosophic framework,

2. In financial risk assessment, neutrosophic numbers can be applied to model like Capital Asset Pricing Method (CAPM). Instead of a single value to β which measures the stock volatility, neutrosophic β_N ca represented as an interval. This accounts for market fluctuations and data uncertainty. The traditional CAPM formula for expected return
is given as

$$E[R_i] = R_f + \beta_i(E[R_m] - R_f) \quad (31)$$

where $E[R_i]$ is the expected return of the investment R_f is the risk free rate β_i is the rate of volatility of the investment.

$E[R_m]$ is the expected market return. In a neutrosophic framework, the volatility rate, $\beta_N = [1.2, 15]$ due to market uncertainty. If the risk free rate is 3% and the expected market return is 10%. The range of the expected return is

$$E[R_i]_L = 0.03 + 1.2(0.10 - 0.03) = 0.114$$

which is 11.4% rate of return.

$$E[R_i]_U = 0.03 + 1.5(0.10 - 0.03) = 0.135$$

which is 13.5% rate of return. The neutrosophic expected return would be the interval [11.4%, 13.5%], providing a more realistic and flexible risk assessment than a single value result.

Conclusion This paper introduces and studies novel subclasses of analytic functions using the Neutrosophic Poisson distribution. The core achievement of this study is a framework that can handle both the classical Poisson distribution for exact data and the Neutrosophic Poisson distribution for uncertain data. This dual approach offers a more resilient and authentic method for modeling complex scenarios with inherent variability and data imprecision.The study utilized Stirling numbers of the second kind to derive bounds for coefficients and their properties within these new function classes. The findings, which include the necessary and sufficient conditions for functions to be members of these classes, establish a solid basis for further investigation. Future work could focus on applying the Neutrosophic Probability distribution to other fields of mathematics and science. This would be particularly useful in areas that require a comprehensive approach to managing uncertainty, imprecision, and inconsistency.

Ethical Approval and Consent to Participate: We confirm that the data and results included in this manuscript are original and have not been submitted to or published by another journal. The authors hold all rights to the material and no permissions are necessary.

Consent for Publication: No studies involving human participants or animals were conducted by the authors of this article.

Availability of Data and Material: The data that support this study's findings are available from the corresponding author upon a reasonable request.

Conflict of Interest : The authors have no competing interests to disclose regarding this research.

Funding : No outside funding was used for this research.

Author Contribution : All authors contributed equally to the manuscript's creation, and they have all read and approved the final version

Acknowledgement : The authors are grateful for the assistance and useful feedback provided by the editorial team, reviewers, and the assigned editor.

References

[1] Alexander, J.W., (1915) Functions which maps the interior of the unit circle upon simple regions, Annals of Mathematics 17 91) 12-22

[2] Al-Habib, R., Ranna, M.,M., Farch, H., and Salamy, A.A., (2018), Some Neutrosophic Probability distributions, Neutrosophic sets and System 22 30-33

[3] Alsboh, A., Amourah, A., Darus, M., and Sharefen, R.I., (2023) Application of Neutrosophic q-Poisson distribution series for subclass of analytic functions and bi-univalent function. Mathematics 11, 868 https://doi.org/10.3390/math 11040868

[4] Awolere, I.,T. and Oladipo, A., T., (2023): Applications of Neutrosophic Poisson Probability distribution series for certain subclass of analytic univalent functions. TWMS J.App and Eng.Math.V. 13,N.3 1042-1052

[5] Awolere, I.,T., Oladipo, A., T., and Altinkaya, S., (2024): Application of Neutrosophic Poisson Distribution series on harmonic classes of analytic functions defined by q-derivative operator and Sigmoid function. Dergipark

[6] Benyl, A., and Manago, S., M. (2005) A recursive formula for moments of a binomial distribution. The college Mathematics journal 36(1) 68-72

[7] Gbolagade, A., M., Awolere, I., T.,Adeyemo,O., and Oladipo, A.,T., (2024) Application of the Neutrosophic Poisson Distribution series on the harmonic subclass of Analytic Functions using the Salagean Derivative Operator. Neutrosophic Systems with Application 23 https://10.64356/j.nswa.2024.23390

[8] Harris, B (1966) Theory of Probability Wesley Reading, Massachuchets

[9] Kaplan, W., 91952) Close to convex Schlicht Functions, Michigan Math.J., , (2) 169-185

[10] Kochler, A., (2008) Closed form expression for the moments of the binomial probability distribution SIAM Journal of Applied Mathematics 69 (1) 197-204

[11] Joardam, A.H and Mahmood, M., (1997) An introduction derivation of Stirling numbers of the second kind and their applications. Journal of Applied Mathematics and Decision Sciences. 1 92) 151-157

[12] Ma, W.C. and Minda, D. 91992) A unified treatment of some special class of univalent functions. In proceedings of the conference on Complex Analysis tainjin 157-169, conf.Proc.lecvture notes, Anal, loint press, Cambridge, M

[13] Mahzoon, H., and Kargar, R., (2018) On certain subclass of close to convex functions related with the second order differential subordination, arXiv. 1901.0267002(math cv) 17 1-9

[14] Nguyen, D. (2021) A probabilistic approach to the moments of binomial random variables and application. The American Statistician 75 (1) 101-103

[15] Oladipo, A., T., (2023) Bounds for Poisson neutrosophic Poisson distribution associated with Chebyschev Polynomial, Palestine Journal of mathematics 10 (10) 169-174.

[16] Porwal, S., (2014) An Application of a Poisson distribution series on certain analytic functions. J. Complex Analysis Art ID 984135 1-3

[17] Sinh, M., Singh, G.P., and Singh, I (2025) Neutrosophic Rama Distribution with an application . Neutrosophic sets and system 86

[18] Smarandanche, F., (1998) Neutrosophy, Neutrosophic Probability, set and logic, Pro-quest information learning, Ann Arbor, Michigan USA 105 118-123

[19] Alhasan, K. F. H., & Smarandache, F. (2019). Neutrosophic Weibull distribution and neutrosophic family Weibull distribution. Infinite Study.

[20] Alhabib, R., Ranna, M. M., Farah, H., & Salama, A. A. (2018). Some neutrosophic probability distributions. Neutrosophic Sets and Systems, 22, 30-38.

[21] Khan, Z., Al-Bossly, A., Almazah, M. M.,& Alduais, F. S. (2021). On statistical development of neutrosophic gamma distribution with applications to complex data analysis. Complexity, 2021(1), 3701236.

[22] 18. Khan, Z., Amin, A., Khan, S. A., & Gulistan, M. (2021). Statistical development of the neutrosophic lognormal model with application to environmental data. Neutrosophic Sets and Systems, 47(1), 1.

[23] Patro, S. K., & Smarandache, F. (2016). The neutrosophic statistical distribution, more problems, more solutions. Infinite Study.

[24] Ahsan-ul-Haq, M., & Zafar, J. (2023). A new one-parameter discrete probability distribution with its neutrosophic extension: mathematical properties and applications. International Journal of Data Science and Analytics, 1-11.

[25] 1. Khan, Z., Almazah, M. M., Hamood Odhah, O., & Alshanbari, H. M. (2022). Generalized Pareto model: properties and applications in neutrosophic data modeling. Mathematical Problems in Engineering, 2022(1), 3686968.

[26] Ibrahim, A. M. M., & Khan, Z. (2024). Neutrosophic Laplace Distribution with Properties and Applications in Decision Making. International Journal of Neutrosophic Science (IJNS), 23(1).

Aslam, M. (2024). The neutrosophic negative binomial distribution: algorithms and practical application: accepted-August 2024. REVSTAT-Statistica Journal.

[28] Al-Essa, L. A., Khan, Z.,& Alduais, F. S. (2024). Neutrosophic logistic model with applications in fuzzy data modeling. Journal of Intelligent & Fuzzy Systems, (Preprint), 1-14

[29] S Al-Duais, F. (2024). Neutrosophic Log-Gamma Distribution and its Applications to Industrial Growth. Neutrosophic Sets and Systems, 72(1),

Received: May 30, 2025. Accepted: Nov 12, 2025