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Reliability and sensitivity analysis of consecutive k-out-of-r-from-n:F systems using trapezoidal neutrosophic numbers and UGF approach

Supriya[1], Deepak Kumar[2*]

[1]Department of Mathematics ; D.S.B. Campus, Kumaun University, Nainital–263002, India; Email: tiruwasupriya15792@gmail.com;

[2*]Department of Mathematics ; D.S.B. Campus, Kumaun University Nainital–263002, India ; Email: deepakdev16@gmail.com

[*]Correspondence: Email: deepakdev16@gmail.com

Abstract. Reliability analysis under uncertain, imprecise, and time-dependent conditions remains a critical challenge in system engineering, especially for complex configurations such as linear consecutive (LC) k-out-of-r-from-n:F systems. Existing probabilistic and fuzzy models often fail to capture the combined effects of vagueness, indeterminacy, and dynamic failure behavior. To address this gap, the present study proposes an integrated neutrosophic fuzzy reliability framework that employs trapezoidal neutrosophic fuzzy numbers (TrNFNs) with the universal generating function (UGF) technique for life-cycle reliability and sensitivity evaluation. The methodology incorporates the α, β, γ-cut approach to quantify uncertainty and applies Weibull and Pareto lifetime distributions to model time-dependent failure rates. Comparative results reveal that system reliability decreases over time but remains consistently higher under Pareto than Weibull distribution, indicating greater robustness for rare yet severe failure scenarios. A novel sensitivity of fuzzy reliability (SOFR) index is also developed to identify degradation intervals and optimize maintenance scheduling. The proposed TrNFN–UGF framework enhances neutrosophic reliability theory by integrating time-varying distributions with multidimensional uncertainty representation. Its applicability to mission-critical domains such as aerospace, healthcare, and smart manufacturing demonstrates its potential as a practical decision-support tool for reliability prediction and maintenance planning under complex uncertainty environments.

Keywords: Trapezoidal neutrosophic fuzzy numbers, Universal generating function, LC k-out-of-r-from-n system, Pareto distribution, Sensitivity of fuzzy reliability (SOFR)

1. Introduction

Reliability modeling plays a vital role in the design and performance evaluation of intelligent and safety-critical systems, particularly in domains such as smart manufacturing, mission-driven operations, wireless sensor networks, and industrial control infrastructures. Among various system configurations, the linear consecutive (LC) k-out-of-r-from-n: F system has gained significant attention because system failure occurs whenever k consecutive components among r active ones (out of n) become non-operational. Such configurations are extensively applied in assembly lines, power transmission networks, cyber-physical systems, and modular robotic structures. The concept of LC systems was first proposed by Griffith [11], extended to linear and circular configurations by Derman et al. [8], refined through combinatorial approaches by Levitin [17, 18], and analyzed via change-point methods by Triantafyllou [24].

Conventional probabilistic reliability models often assume precise and complete data, which is rarely available in real-world engineering applications. Consequently, fuzzy set-based reliability approaches have been introduced to handle imprecision and partial knowledge. For example, Elghamry et al. [9] applied rayleigh-based fuzzy methods, while Velmurugan et al. [26] employed fermatean fuzzy logic for LC systems. However, classical fuzzy frameworks are limited when the data exhibits ambiguity, contradiction, or indeterminacy. To overcome these challenges, advanced mathematical tools such as intuitionistic fuzzy sets (IFSs) [3] and neutrosophic sets (NSs) [22] have been developed, enabling more flexible modeling through the simultaneous consideration of truth, indeterminacy, and falsity degrees.

Among various neutrosophic structures, trapezoidal neutrosophic fuzzy numbers (TrNFNs), introduced by Deli and Subas [7] and later enhanced by Garai et al. [10], provide an effective means to represent multi-dimensional uncertainty. Recent developments in neutrosophic probability distributions, such as the neutrosophic weibull and neutrosophic family weibull distributions [13], have extended reliability analysis to include parametric families like weibull, inverse-weibull, and rayleigh under indeterminate conditions. Similarly, Abbas et al. [1] examined reliability in a neutrosophic manifold framework to geometrically capture indeterminacy, while Ahmad et al. [2] proposed the neutrosophic kumaraswamy distribution for bounded lifetime modeling under uncertainty.

Despite these advances, the integration of the universal generating function (UGF) technique with neutrosophic fuzzy reliability frameworks remains largely unexplored. Existing studies rarely combine time-dependent lifetime distributions such as Weibull, Pareto, and Rayleigh with neutrosophic uncertainty in LC k-out-of-r system modeling. Moreover, no prior work has introduced a sensitivity-based index for evaluating time-dependent degradation in fuzzy reliability.

Motivation and Objectives: To address the existing research gaps, this study proposes a comprehensive reliability framework that integrates trapezoidal neutrosophic fuzzy numbers (TrNFNs) with the universal generating function (UGF) approach for evaluating time-dependent and uncertainty-aware system reliability in LC k-out-of-r-from-n:F systems. The framework utilizes (α, β, γ)-cuts to quantify uncertainty propagation through the system's lifetime distribution, compares reliability trends under Weibull and Pareto distributions to identify distribution-specific degradation patterns, and introduces a novel Sensitivity of Fuzzy Reliability (SOFR) index to capture system degradation behavior over time. This study presents the first integration of TrNFNs with the UGF technique under dynamic lifetime distributions, including Weibull, Pareto, and Rayleigh, thus providing a unified and flexible mathematical structure for reliability analysis. The proposed SOFR index effectively evaluates time-dependent degradation, demonstrating that the Pareto distribution yields tighter and more realistic reliability bounds than the Weibull distribution. The framework successfully captures all reliability phases early life, steady state, and wear-out-offering a computationally efficient and uncertainty-inclusive model suitable for real-world reliability assessment. Earlier works, such as those by Kumar, Singh, and Ram (2019), analyzed consecutive-k-out-of-n:F systems using the intuitionistic fuzzy set (IFS) approach [14], which addressed partial uncertainty but lacked the capacity to represent higher-order indeterminacy. Building upon this foundation, the present research extends the reliability modeling paradigm from IFS-based systems to the neutrosophic environment, enabling richer representation of truth, indeterminacy, and falsity through TrNFNs and dynamic lifetime distributions. Moreover, this work draws on foundational contributions by Deepak Kumar and collaborators, including system reliability analysis using Pythagorean fuzzy sets [15], fuzzy lifetime modeling with triskaidecagonal fuzzy numbers [5], and neutrosophic reliability evaluation of high-pass filter-based non-series-parallel systems [16], thereby reinforcing the novelty, depth, and practical relevance of the proposed study.

1.1. *Contribution of the Study*

The present study proposes a novel reliability modeling framework that integrates trapezoidal neutrosophic fuzzy numbers (TrNFNs) with the universal generating function (UGF) approach to evaluate the reliability and sensitivity of linear consecutive (LC) k-out-of-r-from-n:F systems. The proposed framework effectively captures uncertainty, imprecision, and indeterminacy in the system parameters while maintaining computational efficiency. The study employs the α, β, γ-cut technique to quantify the propagation of neutrosophic uncertainty throughout the reliability evaluation process. Furthermore, two time-dependent lifetime distributions, namely Weibull and Pareto, are incorporated to analyze dynamic system degradation and failure behavior under uncertain environments. A novel Sensitivity of Fuzzy Reliability (SOFR) index is introduced to identify degradation trends and optimize maintenance intervals, thereby enhancing system performance prediction. Comparative results under Weibull and Pareto lifetime distributions confirm the robustness and adaptability of the proposed TrNFN-UGF model. Overall, this work extends the existing reliability theory by merging neutrosophic fuzzy modeling with the UGF technique, offering a general and flexible mechanism for the analysis of complex reliability structures.

1.2. *Importance of Neutrosophic Framework*

The incorporation of the neutrosophic framework plays a vital role in this study as it provides a powerful mathematical tool to handle multi-dimensional uncertainty in reliability systems. Unlike classical probabilistic or fuzzy approaches, which address only a single aspect of uncertainty, the neutrosophic theory simultaneously represents the degrees of truth (T), indeterminacy (I), and falsity (F), thereby offering a more realistic and comprehensive modeling of vague, incomplete, or contradictory data. In practical reliability engineering, component lifetimes and failure rates are often uncertain, imprecise, or time-dependent. The trapezoidal neutrosophic fuzzy representation efficiently accommodates these complexities by incorporating α, β, γ-cut, which provide a refined interpretation of uncertainty propagation. Consequently, the proposed neutrosophic framework enhances the accuracy, interpretability, and stability of the LC k-out-of-r-from-n:F system analysis. It also facilitates improved decision-making in critical domains such as aerospace systems, healthcare equipment, and intelligent manufacturing, where high reliability under uncertainty is of paramount importance.

In summary, this paper develops a time-sensitive, uncertainty-aware, and computationally efficient neutrosophic reliability framework for LC k-out-of-r-from-n: F systems. The proposed model enriches the theoretical foundations of neutrosophic reliability theory and demonstrates practical applicability for mission-critical domains such as aerospace safety, healthcare devices, and industrial automation. The paper is organized as follows: Section 1 provides the introduction; Section 2 presents the preliminaries and basic definitions; Section 3 describes the model description of the proposed TrNFN-based UGF framework; Section 4 illustrates the methodology through numerical examples; Section 5 offers a comparative analysis of weibull and pareto lifetime distributions; Section 6 introduces the Sensitivity of Fuzzy reliability assessment (SOFR); Section 7 discusses the results and their Implications; Section 8 draws the conclusion; and finally, Section 9 highlights the limitations and future work of the present study.

2. Preliminaries and Basic Definitions

2.1. *Fuzzy set [27]*

"Let $X = \{x_1, x_2, x_3, ..., x_n\}$ be a universe of discourse, then a fuzzy set A in X is defined as follows:

$$A = \{\langle x, \mu_A(x)\rangle | x \in X\}$$

where $\mu_A(x) : X \to [0, 1]$ is the membership degree."

2.2. *Intuitionistic fuzzy set [4]*

"An Atanassov's intuitionistic fuzzy set A in x can be written as:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

where $\mu_A(x) : X \to [0, 1]$ & $\nu_A(x) : X \to [0, 1]$ are membership degree and non-membership degree, respectively, with the condition: $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ $\pi_A(x)$ determined by the following expression:

$$\pi_A(x) = 1 - \mu_A(x) + \nu_A(x)$$

$\pi_A(x)$ is called the hesitancy degree of the element to the $x \in X$ to the set A, and $\pi_A(x) \in [0, 1], \forall x \in X$"

2.3. *Neutrosophic sets* [22]

"Let X be the universal set and $x \in X$.A NS U in X is characterized by a truth membership function ϕ_U , an indeterminacy membership function ν_U and a falsity membership function χ_U where ϕ_U , ν_U and χ_U are real standard elements of [0,1]. It can be written as

$A = \{\langle x, (\phi_U(x), \nu_U(x), \chi_U(x))\rangle : x \in X, \phi_U, \nu_U, \chi_U \in (0, 1)$ There is no restriction on the sum of $\phi_U(x)$, $\nu_U(x)$ and $\chi_U(x)$

and so $$0 \leqslant \phi_U(x) + \nu_U(x) + \chi_U(x) \leqslant 3."$$

2.4. *Trapezoidal neutrosophic fuzzy number [7]*

" A trapezoidal neutrosophic fuzzy number(TrNFN) $\hat{A}$ with parameters $a \leq b \leq c \leq d \leq$ is denoted by

$$\hat{A} =< a, b, c, d; \phi_N, \nu_N, \chi_N >$$

In this case , its truth-membership function $T(x)$, indeterminacy-membership function $I(x)$ and falsity-membership function $F(x)$ are defined as follows:

$$T(x) = \begin{cases} \left(\dfrac{x-a}{b-a}\right)\phi_N, & a \le x \le b \\ \phi_x, & b \le x \le c \\ \left(\dfrac{d-x}{d-c}\right)\phi_N, & c \le x \le d \\ 0, & \text{otherwise} \end{cases}$$

$$I(x) = \begin{cases} \left(\dfrac{b-x+\nu_x(x-a)}{b-a}\right), & a \le x \le b \\ \nu_x, & b \le x \le c \\ \left(\dfrac{x-c+\nu_x(d-x)}{d-c}\right), & c \le x \le d \\ 1, & \text{otherwise} \end{cases}$$

$$F(x) = \begin{cases} \left(\dfrac{b-x+\chi_x(x-a)}{b-a}\right), & a \le x \le b \\ \chi_x, & b \le x \le c \\ \left(\dfrac{x-c+\chi_x(d-x)}{d-c}\right), & c \le x \le d \\ 1, & \text{otherwise”} \end{cases}$$

2.5. *Trapezoidal neutrosophic fuzzy number (α, β, γ) -cut [10]*

“The (α, β, γ)-cut for a trapezoidal neutrosophic number $TrNN$=$\langle a, b, c, d, \phi_N, \nu_N, \chi_N\rangle$, can be defined as

$\hat{T_N}^{(\alpha,\beta,\gamma)} = \{x : \phi_N \ge \alpha, \mu_N \le \beta, \chi_N \le \gamma\}$, where $0 \le \alpha \le \phi_N, \mu_N\beta \le 1$ *and* $\chi_N \le \gamma \le 1$

The upper bounds $U(\alpha), U(\beta)$ *and* $U(\gamma)$, *and* the lower bounds $L(\alpha), L(\beta)$ *and* $L(\gamma)$ for(α, β, γ) -cut of the TrNN

$$\begin{aligned} \hat{T}_\alpha &= [L(\alpha), U(\alpha)] \\ &= \left[a + \alpha\left(\frac{b-a}{\phi_N}\right),\ d - \alpha\left(\frac{d-c}{\phi_N}\right)\right] \end{aligned}$$

$$\begin{aligned} \hat{T}_\beta &= [L(\beta), U(\beta)] \\ &= \left[\frac{(\beta-\nu_N)a + (1-\beta)b}{1-\nu_N},\ \frac{(\beta-\nu_N)d + (1-\beta)c}{1-\nu_N}\right] \end{aligned}$$

$$\hat{T}_\gamma = [L(\gamma), U(\gamma)]$$
$$= \left[\frac{(\gamma - \chi_N)a + (1-\gamma)b}{1-\chi_N},\ \frac{(\gamma - \chi_N)d + (1-\gamma)c}{1-\chi_N}\right],$$

2.6. *Universal generating function*

"There are multiple methods to evaluating reliability. However, the complexities of multiple states might make it a difficult and very difficult operation. [25] introduced the UGF approach, which has been used in various industries, including reliability, due to its simplicity and efficiency. The system states are presented in a structured way using logical and predictable approaches. [18] proposes that they can replace complex combinatorial algorithms commonly used in networks and sequential systems. The UGF approach evaluates reliability by constructing u-functions for discrete random variables and using composition operations. [18]analyses the use of complex combinatorial algorithms in networks for sequential systems. For evaluating accuracy using the UGF. This method involves defining u-functions for a variable that is discrete in nature, then using composition operators.

The u-function for an independent and discrete random variable with N possible values and probability $\tilde{P}_N$ is defined as follows:

$$u(z) = \sum_{n=1}^{N} \tilde{P}_N z^{\rho_n}$$

If α and β are two components with their respective u-functions, $u_\alpha(z)$ and $u_\beta(z)$ the combined u-functions of α and β can be determined as

$$u(z) = u_\alpha(z) \otimes_\rho \phi u_\beta(z) = \sum_{n_1=1}^{N_\alpha} \tilde{p}_{\alpha n_1} z^{m_{\alpha n_1}} \otimes_\rho \sum_{n_2=1}^{N_\beta} \tilde{p}_{\beta n_2} z^{m_{\beta n_2}}$$

$$= \sum_{n_1=1}^{N_\beta} \sum_{n_1=1}^{N_\beta} \tilde{P}_{\alpha n_1} \tilde{P}_{\beta n_2} z^{\rho\{m_{\alpha n_1}, m_{\beta n_2}\}}$$

where,

$$\rho\{m_{\alpha n_1}, m_{\beta n_2}\} = \begin{cases} min\{m_{\alpha n_1}, m_{\beta n_2}\},\ if\ \alpha\ and\ \beta\ are\ in\ series. \\ max\{m_{\alpha n_1}, m_{\beta n_2}\},\ if\ \alpha\ and\ \beta\ are\ in\ parallel." \end{cases} \quad (1)$$

2.7. *Weibull distribution*

"The weibull distribution is used to assess system reliability when the failure rate is non-linear. Changing the shape parameter value can simplify this to an exponential and rayleigh distribution. Fuzzy parameters are used to address imprecision due to lack of data or human mistake. The distribution is converted into a fuzzy Weibull distribution. If a parameter that represents the lifespan is replaced with a $\tilde{\varpi}$. The fuzzy parameter $\tilde{\varpi}$ affects the distribution function [12]

$$k(\varepsilon,\tilde{\varpi}) = \{k(\varepsilon)[\alpha], \mu_{k(\varepsilon)} : [k(\varepsilon)[\alpha] = [k_{min}(\varepsilon)[\alpha], k_{max}(\varepsilon)[\alpha], \mu_{k(\varepsilon)} = \alpha\},$$

$$k_{min}(\varepsilon)[\alpha] = inf\{k(\varepsilon,\varpi) : \varpi \in \tilde{\varpi}[\alpha]\}$$

$$k_{max}(\varepsilon)[\alpha] = sup\{k(\varepsilon,\varpi) : \varpi \in \tilde{\varpi}[\alpha]\}$$

Hence, the fuzzy reliability $R(\tilde{t})[\alpha]$ is now defined as,

$$R(\tilde{t})[\alpha] = \left\{\int_0^{\infty} \frac{\beta}{\varpi}\left(\frac{\varepsilon}{\varpi}\right)^{\beta-1} e^{\left(-\left(\frac{\varepsilon}{\varpi}\right)^{\beta}\right)} d\varepsilon : \varpi \in \tilde{\varpi}[\alpha]\right\}$$

$$R(\tilde{t})[\alpha] = \left\{exp\left[\left(\frac{t}{\varpi}\right)^{\beta}\right] d\varepsilon : \varpi \in \tilde{\varpi}[\alpha]\right\}\text{''}$$

Therefore, if

Let us consider that (a, b, c, d) are any neutrosophic trapezoidal fuzzy number and ϕ_N, ν_N, χ_N are three membership functions for the truth, indeterminacy, and falsity then if the $\hat{T}_\alpha, \hat{T}_\beta, \hat{T}_\gamma$ be represent the (α, β, γ) cut of the neutrosophic trapezoidal fuzzy number then the reliability function is,

$$\hat{R}(t)[\alpha] = \left\langle \exp\left[-\left(\frac{t}{a+\alpha\left(\frac{b-a}{\phi_N}\right)}\right)^{\theta}\right]\right\rangle, \quad \left\langle \exp\left[-\left(\frac{t}{d-\alpha\left(\frac{d-c}{\phi_N}\right)}\right)^{\theta}\right]\right\rangle,$$
$$\left\langle \exp\left[-\left(\frac{t}{\frac{(\beta-\nu_N)a+(1-\beta)b}{1-\nu_N}}\right)^{\theta}\right]\right\rangle, \quad \left\langle \exp\left[-\left(\frac{t}{\frac{(\beta-\nu_N)d+(1-\beta)c}{1-\nu_N}}\right)^{\theta}\right]\right\rangle,$$
$$\left\langle \exp\left[-\left(\frac{t}{\frac{(\gamma-\chi_N)a+(1-\gamma)b}{1-\chi_N}}\right)^{\theta}\right]\right\rangle, \quad \left\langle \exp\left[-\left(\frac{t}{\frac{(\gamma-\chi_N)d+(1-\gamma)c}{1-\chi_N}}\right)^{\theta}\right]\right\rangle \quad (2)$$

2.8. *Pareto Distribution*

"Compared to the Weibull distribution, the pareto distribution can be useful in life time analysis. [21] used the three-parameter Pareto distribution to assess reliability, whereas [20] used the two-parameter Pareto distribution. The two-parameter pareto distribution is used in this study to simplify the analysis. Using the two-parameter Pareto distribution, the reliability function $\tilde{R}[\alpha]$ may be represented as follows:

$$R(\tilde{t})[\alpha] = \left\{ \int_0^{\infty} \frac{\beta \varpi^{\beta}}{\varepsilon^{\beta+1}} d\varepsilon : \varpi \in \tilde{\varpi}[\alpha] \right\}",$$

Thus, the reliability function is,
$R(\tilde{t})[\alpha] = \{\varpi^{\beta}\varepsilon^{-\beta} : \varpi \in \tilde{\varpi}[\alpha]\}$

Let us consider that (a, b, c, d) are any neutrosophic trapezoidal fuzzy number and ϕ_N, ν_N, χ_N are three membership functions for the truth, indeterminacy, and falsity then if the $\hat{T}_{\alpha}, \hat{T}_{\beta}, \hat{T}_{\gamma}$ be represent the (α, β, γ) cut of the neutrosophic trapezoidal fuzzy number then the reliability function is,

$$\tilde{R}(t)[\alpha] = \left\{ \left(\frac{\delta}{t+\delta} \right)^{\theta} \right\}$$

where θ is the shape parameters and δ is the fuzzy scale parameters.

$$"R(\hat{t})[\alpha] = \left\langle \left(\left[\left(\frac{a + \alpha\left(\frac{b-a}{\phi_N}\right)}{t + a + \alpha\left(\frac{b-a}{\phi_N}\right)} \right)^{\theta} \right] \right) \right\rangle, \left\langle \left(\left[\left(\frac{d - \alpha\left(\frac{d-c}{\phi_N}\right)}{t + d - \alpha\left(\frac{d-c}{\phi_N}\right)} \right)^{\theta} \right] \right) \right\rangle$$

$$\left\langle \left(\left[\left(\frac{\frac{(\beta-\nu_N)a+(1-\beta)b}{1-\nu_N}}{\frac{t+(\beta-\nu_N)a+(1-\beta)b}{1-\nu_N}} \right)^{\theta} \right] \right) \right\rangle, \left\langle \left(\left[\left(\frac{\frac{(\beta-\nu_N)d+(1-\beta)c}{1-\nu_N}}{\frac{t+(\beta-\nu_N)d+(1-\beta)c}{1-\nu_N}} \right)^{\theta} \right] \right) \right\rangle$$

$$\left\langle \left(\left[\left(\frac{\frac{(\gamma-\chi_N)a+(1-\gamma)b}{1-\chi_N}}{\frac{t+(\gamma-\chi_N)a-(1-\gamma)b}{1-\chi_N}} \right)^{\theta} \right] \right) \right\rangle, \left\langle \left(\left[\left(\frac{\frac{(\gamma-\chi_N)d+(1-\gamma)c}{1-\chi_N}}{\frac{t+(\gamma-\chi_N)d-(1-\gamma)c}{1-\chi_N}} \right)^{\theta} \right] \right) \right\rangle" \quad (3)$$

3. Model description

In this study, we explore the reliability of a linear consecutive LC k-out-of-r-from-n: F system. Such systems are commonly found in applications like conveyor belts in manufacturing plants, wireless communication modules, and network security frameworks. The system is considered failed if at least 3 consecutive out of any 4 components become non-operational. The foundational work in this area began with Nelson [19] and Tong [23], who studied reliability models involving ordered sequences of component failures. These studies helped establish the groundwork for understanding how sequence-based failures can critically impact overall system performance. Griffith (1986) further refined this concept by introducing models tailored to systems with consecutive component dependencies. Later, [17] emphasized the applicability of LC k-out-of-r-from-n systems in phased-mission reliability evaluations, where systems pass through multiple stages of operation. In continuation, [18] developed an efficient mathematical formulation using the Universal Generating Function (UGF) approach, which has since become a standard tool in multi-state system reliability analysis.

In our proposed model, each of the five components is assumed to be non-identical in performance and reliability. Their behaviors are described using trapezoidal neutrosophic fuzzy numbers (TrNFNs), which offer an advanced method to incorporate uncertainties via truth-membership, indeterminacy, and falsity functions. The system's total reliability is evaluated using the UGF technique. Initially, individual component u-functions are constructed based on their fuzzy probabilities. These u-functions are combined sequentially, applying shifting and composition operations. Unreliable configurations (i.e., those with less than 3 consecutive working components within any subset of 4) are filtered out. To express varying levels of confidence in the data, reliability is modeled with (α, β, γ)-cuts. The final system reliability is then calculated using the inclusion-exclusion principle, yielding a comprehensive fuzzy reliability profile. By integrating the contributions of Nelson, Tong, Griffith, and Levitin with modern fuzzy modeling, this approach ensures a humanized, computationally efficient, and uncertainty-aware reliability analysis.

3.1. *Reliability function evaluation*

To evaluate the reliability of the LC 3-out-of-4-from-5:F system, we apply a UGF-based procedure specifically designed for LC k-out-of-r-from-n:F configurations, as discussed by Levitin [18] .

1. Initially assign F =0
2. u-function $U_{1-r}(z)$ is defined as $U_{1-r}(z) = z^{x_0}$ the vector x_0 consists of r zeroes.
3. A shift operator is defined as follows, Shifts all the elements of the vector to one place

left. $U_{i+1-r}(z) = U_{1-r}(z) \bigotimes u_i(z)$ for $i = 1, 2, ..., n$ such that the operators $\bigotimes$ shifts all the elements of the vector to one place left. 3.1 If $i \leq r$, then remove the terms having more than k zeroes from $U_{1-r}(z)$ and add to F.

4. The system reliability can be finally attained as $R = 1–F$. Alternately, it can also be obtained by adding the sum of the coefficients of the last u-function $U_i + 1 - r(z)$, where $i = n$. Using the above algorithm for the LC3-out-of-4-from-5: F system with non-identical components, let us assign $F = 0$ and $U\_3(z)$=$z^{(0,0,0,0)}$

Let$\tilde{N}_1^p, \tilde{N}_2^p, \tilde{N}_3^p, \tilde{N}_4^p, and \tilde{N}_5^p$ be the probabilities of the lifetimes of the individual components as form of (α, β, γ) of TrNFn .

Then, the u-functions of these individual components are,

$u_1(z) = \tilde{N}_1^p z^1 + \tilde{L}_1^p z^0, u_2(z) = \tilde{N}_2^p z^1 + \tilde{L}_2^p z^0, u_3(z) = \tilde{N}_3^p z^1 + \tilde{L}_3^p z^0, u_4(z) = \tilde{N}_4^p z^1 + \tilde{L}_4^p z^0, u_5(z) = \tilde{N}_5^p z^1 + \tilde{L}_5^p z^0$,where $\tilde{L}_i^p = 1 - \tilde{N}_i^p$ for $i = 1, 2, 3, 4, 5$

Using the previously described technique, the system reliability function of the LC 3-out-of-4-from-5: F are given below

If we have used this technique for the neutrosophic trapezoidal fuzzy number, which is explain with the help of truthness (T_i), indeterminacy (I_i) and falsity (F_i) where $(i = 1, 2, 3, 4, 5)$ then the system operates as follows:

$$\tilde{R}^p(t) = \Bigg\{ \Big(\tilde{T}_4^p(\tilde{T}_2^p + \tilde{T}_3^p) - \tilde{T}_2^p\tilde{T}_3^p\tilde{T}_4^p + \tilde{T}_3^p\tilde{T}_5^p(\tilde{T}_1^p + \tilde{T}_2^p) - \tilde{T}_2^p\tilde{T}_3^p\tilde{T}_4^p\tilde{T}_5^p + \tilde{T}_1^p\tilde{T}_5^p(\tilde{T}_2^p + \tilde{T}_4^p)$$

$$-2(\tilde{T}_1^p\tilde{T}_2^p\tilde{T}_4^p\tilde{T}_5^p) - 2(\tilde{T}_1^p\tilde{T}_3^p\tilde{T}_4^p\tilde{T}_5^p) - 2(\tilde{T}_1^p\tilde{T}_2^p\tilde{T}_3^p\tilde{T}_5^p) + 3(\tilde{T}_1^p\tilde{T}_2^p\tilde{T}_3^p\tilde{T}_4^p\tilde{T}_5^p) \Big),$$

$$\Big(\tilde{I}_4^p(\tilde{I}_2^p + \tilde{I}_3^p) - \tilde{I}_2^p\tilde{I}_3^p\tilde{I}_4^p + \tilde{I}_3^p\tilde{I}_5^p(\tilde{I}_1^p + \tilde{I}_2^p) - \tilde{I}_2^p\tilde{I}_3^p\tilde{I}_4^p\tilde{I}_5^p + \tilde{I}_1^p\tilde{I}_5^p(\tilde{I}_2^p + \tilde{I}_4^p)$$

$$-2(\tilde{I}_1^p\tilde{I}_2^p\tilde{I}_4^p\tilde{I}_5^p) - 2(\tilde{I}_1^p\tilde{I}_3^p\tilde{I}_4^p\tilde{I}_5^p) - 2(\tilde{I}_1^p\tilde{I}_2^p\tilde{I}_3^p\tilde{I}_5^p) + 3(\tilde{I}_1^p\tilde{I}_2^p\tilde{I}_3^p\tilde{I}_4^p\tilde{I}_5^p) \Big),$$

$$\Big(\tilde{F}_4^p(\tilde{F}_2^p + \tilde{F}_3^p) - \tilde{F}_2^p\tilde{F}_3^p\tilde{F}_4^p + \tilde{F}_3^p\tilde{F}_5^p(\tilde{F}_1^p + \tilde{F}_2^p) - \tilde{F}_2^p\tilde{F}_3^p\tilde{F}_4^p\tilde{F}_5^p + \tilde{F}_1^p\tilde{F}_5^p(\tilde{F}_2^p + \tilde{F}_4^p)$$

$$-2(\tilde{F}_1^p\tilde{F}_2^p\tilde{F}_4^p\tilde{F}_5^p) - 2(\tilde{F}_1^p\tilde{F}_3^p\tilde{F}_4^p\tilde{F}_5^p) - 2(\tilde{F}_1^p\tilde{F}_2^p\tilde{F}_3^p\tilde{F}_5^p) + 3(\tilde{F}_1^p\tilde{F}_2^p\tilde{F}_3^p\tilde{F}_4^p\tilde{F}_5^p) \Big) \Bigg\} \quad (4)$$

4. **Numerical examples**

4.1. *Fuzzy reliability assessment*

For the fuzzy reliability modelling, let us consider the example which are given below Here the example of Trepezoidal neutrosophic fuzzy number

$$N_1 = \{(3.25, 3.75, 4.25, 4.65), 0.8\}, \{(3.25, 3.75, 4.25, 4.65), 0.2\}, \{(3.25, 3.75, 4.25, 4.65), 0.4\}$$
$$N_2 = \{(4.45, 4.75, 5.25, 5.45), 0.9\}, \{(4.45, 4.75, 5.25, 5.65), 0.2\}, \{(4.45, 4.75, 5.25, 5.45), 0.3\}$$
$$N_3 = \{(4.65, 4.75, 5.45, 6.25), 0.7\}, \{(4.65, 4.75, 5.45, 6.25), 0.2\}, \{(4.65, 4.75, 5.45, 6.25), 0.4\}$$
$$N_4 = \{(5.25, 6.25, 6.45, 7.45), 0.8\}, \{(5.25, 6.25, 6.45, 7.45), 0.3\}, \{(5.25, 6.25, 6.45, 7.45), 0.4\}$$
$$N_5 = \{(5.75, 6.45, 6.75, 7.65), 0.8\}, \{(3.25, 3.75, 4.25, 4.65), 0.2\}, \{(3.25, 3.75, 4.25, 4.65), 0.5\}$$

let us consider the probabilities of the lifetimes of the individual components N_i^P for $i = 1, 2, 3, 4, 5$ as follows,

Table 1. Lifetime Probabilities for Components Using (α, β, γ)-cuts

Component	**Lifetime Probabilities (TrNFN)**
N_1^P	$(3.25 + 0.625\alpha,\ 4.65 - 0.5\alpha)$, $(3.875 - 0.625\beta,\ 4.15 + 0.5\beta)$, $(4.0834 - 0.8334\gamma,\ 3.9833 + 0.6667\gamma)$
N_2^P	$(4.45 + 0.3333\alpha,\ 5.45 - 0.2222\alpha)$, $(4.825 - 0.375\beta,\ 5.2 + 0.25\beta)$, $(4.8785 - 0.4285\gamma,\ 5.1642 + 0.2857\gamma)$
N_3^P	$(4.65 + 0.1428\alpha,\ 6.25 - 1.1428\alpha)$, $(4.775 - 0.125\beta,\ 5.25 + 1\beta)$, $(4.8166 - 0.1666\gamma,\ 4.9166 + 1.3333\gamma)$
N_4^P	$(5.25 + 1.25\alpha,\ 7.45 - 1.25\alpha)$, $(4.675 - 1.4285\beta,\ 6.0214 + 1.4285\beta)$, $(6.9166 - 1.6666\gamma,\ 5.7833 + 1.6666\gamma)$
N_5^P	$(5.75 + 0.875\alpha,\ 7.65 - 1.125\alpha)$, $(6.625 - 0.875\beta,\ 6.525 + 1.125\beta)$, $(7.15 - 1.4\gamma,\ 5.85 + 1.8\gamma)$

Table 2. System Reliability with Weibull Distribution at $t = 7$

α	$\tilde{R}^P(T,\alpha)(t)$	$\tilde{R}^P(I,\beta)(t)$	$\tilde{R}^P(F,\gamma)(t)$
0	(0.208661, 0.288107)	(0.155796, 0.013489)	(0.114986, 0.077872)
0.1	(0.197970, 0.284282)	(0.152789, 0.014055)	(0.112028, 0.084181)
0.2	(0.190336, 0.280413)	(0.149755, 0.014630)	(0.109051, 0.090530)
0.3	(0.186117, 0.276501)	(0.146693, 0.015215)	(0.106053, 0.096904)
0.4	(0.185550, 0.272544)	(0.143602, 0.015800)	(0.103036, 0.103292)
0.5	(0.188730, 0.268542)	(0.140482, 0.016412)	(0.099980, 0.109684)
0.6	(0.195599, 0.264494)	(0.137332, 0.017024)	(0.096938, 0.116069)
0.7	(0.205944, 0.260399)	(0.134151, 0.017644)	(0.093858, 0.122441)
0.8	(0.219420, 0.256257)	(0.130938, 0.018274)	(0.090756, 0.128791)
0.9	(0.235580, 0.252067)	(0.127693, 0.018911)	(0.087632, 0.135114)
1	(0.253922, 0.247827)	(0.124414, 0.019558)	(0.084486, 0.141405)

Table 3. System Reliability with Weibull Distribution at $t = 10$

α	$\tilde{R}^P(T,\alpha)(t)$	$\tilde{R}^P(I,\beta)(t)$	$\tilde{R}^P(F,\gamma)(t)$
0	(0.130692, 0.194898)	(0.091033, 0.004455)	(0.063764, 0.040351)
0.1	(0.122400, 0.191664)	(0.088893, 0.004674)	(0.061813, 0.044214)
0.2	(0.116553, 0.188407)	(0.086744, 0.004899)	(0.059861, 0.048168)
0.3	(0.113342, 0.185126)	(0.084585, 0.005129)	(0.057906, 0.052202)
0.4	(0.112894, 0.181822)	(0.082417, 0.005365)	(0.055951, 0.056307)
0.5	(0.115271, 0.178494)	(0.080238, 0.005606)	(0.053994, 0.060473)
0.6	(0.120467, 0.175143)	(0.078050, 0.005852)	(0.052035, 0.064693)
0.7	(0.128397, 0.171767)	(0.075853, 0.006103)	(0.050076, 0.068958)
0.8	(0.138900, 0.168368)	(0.073645, 0.006359)	(0.006359, 0.073263)
0.9	(0.151744, 0.164946)	(0.071427, 0.006620)	(0.046156, 0.077600)
1	(0.166635, 0.161500)	(0.069199, 0.006887)	(0.044195, 0.081964)

Table 4. System Reliability with Weibull Distribution at $t = 15$

α	$\tilde{R}^P(T,\alpha)(t)$	$\tilde{R}^P(I,\beta)(t)$	$\tilde{R}^P(F,\gamma)(t)$
0	(0.067593, 0.111520)	(0.042941, 0.000989)	(0.028260, 0.016418)
0.1	(0.062200, 0.109177)	(0.041697, 0.001048)	(0.027219, 0.018300)
0.2	(0.058460, 0.106830)	(0.040550, 0.001108)	(0.026185, 0.020267)
0.3	(0.056422, 0.104418)	(0.039215, 0.001171)	(0.025156, 0.022314)
0.4	(0.056120, 0.102122)	(0.037977, 0.001235)	(0.024134, 0.038477)
0.5	(0.057587, 0.099762)	(0.036741, 0.001302)	(0.023119, 0.026626)
0.6	(0.060847, 0.097399)	(0.035507, 0.001371)	(0.022112, 0.028882)
0.7	(0.065912, 0.095032)	(0.034276, 0.001441)	(0.021111, 0.031199)
0.8	(0.072772, 0.092663)	(0.033048, 0.001514)	(0.020118, 0.033573)
0.9	(0.081378, 0.090292)	(0.031823, 0.001589)	(0.019133, 0.036001)
1	(0.091639, 0.087919)	(0.030601, 0.001666)	(0.018157, 0.038477)

Table 5. System Reliability with Pareto Distribution at $t = 7$

α	$\tilde{R}^P(T,\alpha)(t)$	$\tilde{R}^P(I,\beta)(t)$	$\tilde{R}^P(F,\gamma)(t)$
0	(0.750593, 0.825857)	(0.700270, 0.406083)	(0.614085, 0.583909)
0.1	(0.754579, 0.822760)	(0.695843, 0.425294)	(0.608325, 0.592155)
0.2	(0.758461, 0.819580)	(0.691560, 0.444542)	(0.602418, 0.600150)
0.3	(0.762242, 0.816315)	(0.687175, 0.463817)	(0.596357, 0.607905)
0.4	(0.765928, 0.812961)	(0.682683, 0.483109)	(0.590138, 0.615431)
0.5	(0.769521, 0.809515)	(0.678080, 0.502408)	(0.583751, 0.622737)
0.6	(0.773025, 0.805973)	(0.673361, 0.521706)	(0.577191, 0.629832)
0.7	(0.776443, 0.802333)	(0.668522, 0.540992)	(0.570448, 0.636725)
0.8	(0.779778, 0.798589)	(0.665359, 0.560262)	(0.563514, 0.643425)
0.9	(0.783034, 0.794738)	(0.658464, 0.579508)	(0.556380, 0.649939)
1	(0.786212, 0.790776)	(0.653234, 0.598723)	(0.549036, 0.656274)

Table 6. System Reliability with Pareto Distribution at $t = 10$

α	$\tilde{R}^P(T,\alpha)(t)$	$\tilde{R}^P(I,\beta)(t)$	$\tilde{R}^P(F,\gamma)(t)$
0	(0.645882, 0.735428)	(0.589510, 0.232060)	(0.501760, 0.472861)
0.1	(0.650395, 0.731563)	(0.585028, 0.244188)	(0.496081, 0.480838)
0.2	(0.654809, 0.727614)	(0.580460, 0.256427)	(0.490287, 0.488625)
0.3	(0.659130, 0.723577)	(0.575802, 0.268770)	(0.484372, 0.496228)
0.4	(0.663360, 0.719449)	(0.571052, 0.281209)	(0.478334, 0.503655)
0.5	(0.667502, 0.715229)	(0.566206, 0.293737)	(0.472166, 0.510912)
0.6	(0.671558, 0.710912)	(0.561262, 0.306348)	(0.465864, 0.518004)
0.7	(0.675533, 0.706495)	(0.556215, 0.319035)	(0.459422, 0.524938)
0.8	(0.679427, 0.701976)	(0.551061, 0.331793)	(0.452834, 0.531718)
0.9	(0.683245, 0.697351)	(0.545797, 0.344615)	(0.446094, 0.538351)
1	(0.686988, 0.692617)	(0.540418, 0.357496)	(0.439195, 0.544839)

Table 7. System Reliability with Pareto Distribution at $t = 15$

α	$\tilde{R}^P(T,\alpha)(t)$	$\tilde{R}^P(I,\beta)(t)$	$\tilde{R}^P(F,\gamma)(t)$
0	(0.517641, 0.613317)	(0.461297, 0.115724)	(0.381078, 0.356099)
0.1	(0.522223, 0.608976)	(0.456992, 0.122296)	(0.376038, 0.363062)
0.2	(0.526780, 0.604562)	(0.452622, 0.128972)	(0.370919, 0.369902)
0.3	(0.531157, 0.600071)	(0.448185, 0.135748)	(0.365719, 0.376623)
0.4	(0.535513, 0.595502)	(0.443679, 0.142622)	(0.360436, 0.383228)
0.5	(0.539797, 0.590853)	(0.439103, 0.149589)	(0.355065, 0.389720)
0.6	(0.544011, 0.586122)	(0.434453, 0.156646)	(0.349605, 0.396103)
0.7	(0.548158, 0.581307)	(0.429727, 0.163790)	(0.344051, 0.402380)
0.8	(0.552240, 0.576406)	(0.424924, 0.171018)	(0.338400, 0.408554)
0.9	(0.556258, 0.571416)	(0.420039, 0.178326)	(0.332647, 0.414628)
1	(0.560213, 0.566335)	(0.415071, 0.185711)	(0.326789, 0.420604)

TABLE 8. Sensitivity of fuzzy reliability (SOFR) with Weibull distribution at $t = 7$

α	$\tilde{R}^W(T,\alpha)(t)$	$\tilde{R}^W(I,\beta)(t)$	$\tilde{R}^W(F,\gamma)(t)$
0	(-0.21431, -0.33175)	(-0.31682, -0.00061)	(-0.42507, -0.29812)
0.1	(-0.21445, -0.33168)	(-0.31628, -0.00062)	(-0.42185, -0.30002)
0.2	(-0.21458, -0.33158)	(-0.31571, -0.00062)	(-0.41847, -0.30181)
0.3	(-0.21472, -0.33146)	(-0.31511, -0.00063)	(-0.41493, -0.30350)
0.4	(-0.21485, -0.33133)	(-0.31446, -0.00063)	(-0.41123, -0.30509)
0.5	(-0.21498, -0.33116)	(-0.31381, -0.00064)	(-0.40736, -0.30659)
0.6	(-0.21511, -0.33097)	(-0.31311, -0.00064)	(-0.40329, -0.30800)
0.7	(-0.21523, -0.33076)	(-0.31237, -0.00065)	(-0.39903, -0.30933)
0.8	(-0.21536, -0.33052)	(-0.31159, -0.00065)	(-0.39455, -0.31059)
0.9	(-0.21548, -0.33024)	(-0.31077, -0.00066)	(-0.38986, -0.31177)
1	(-0.21560, -0.32994)	(-0.30990, -0.00066)	(-0.38493, -0.31289)

TABLE 9. Sensitivity of fuzzy reliability (SOFR) with Weibull distribution at $t = 10$

α	$\tilde{R}^W(T,\alpha)(t)$	$\tilde{R}^W(I,\beta)(t)$	$\tilde{R}^W(F,\gamma)(t)$
0	(-0.22062, -0.35432)	(-0.31992, -0.00064)	(-0.34506, -0.28993)
0.1	(-0.22089, -0.35377)	(-0.31892, -0.00065)	(-0.34028, -0.29276)
0.2	(-0.22115, -0.35319)	(-0.31788, -0.00067)	(-0.33534, -0.29545)
0.3	(-0.22142, -0.35257)	(-0.31680, -0.00068)	(-0.33024, -0.29803)
0.4	(-0.22168, -0.35192)	(-0.31568, -0.00069)	(-0.32498, -0.30049)
0.5	(-0.22194, -0.35123)	(-0.31451, -0.00070)	(-0.31953, -0.30283)
0.6	(-0.22220, -0.35050)	(-0.31330, -0.00071)	(-0.31391, -0.30508)
0.7	(-0.22245, -0.34973)	(-0.31204, -0.00072)	(-0.30810, -0.30723)
0.8	(-0.22270, -0.34891)	(-0.31073, -0.00073)	(-0.30211, -0.30928)
0.9	(-0.22295, -0.34805)	(-0.30936, -0.00074)	(-0.29592, -0.31125)
1	(-0.22320, -0.34714)	(-0.30794, -0.00075)	(-0.28953, -0.31313)

Table 10. Sensitivity of fuzzy reliability (SOFR) with Weibull distribution at $t = 15$

α	$\tilde{R}^W(T,\alpha)(t)$	$\tilde{R}^W(I,\beta)(t)$	$\tilde{R}^W(F,\gamma)(t)$
0	(-0.20994, -0.35474)	(-0.29657, -0.00047)	(-0.21106, -0.25553)
0.1	(-0.21035, -0.35354)	(-0.29505, -0.00049)	(-0.20561, -0.25918)
0.2	(-0.21076, -0.35229)	(-0.29349, -0.00050)	(-0.20006, -0.26270)
0.3	(-0.21117, -0.35099)	(-0.29188, -0.00052)	(-0.19443, -0.26610)
0.4	(-0.21158, -0.34965)	(-0.29022, -0.00053)	(-0.18872, -0.26938)
0.5	(-0.21198, -0.34825)	(-0.28851, -0.00055)	(-0.18293, -0.27255)
0.6	(-0.21238, -0.34681)	(-0.28676, -0.00056)	(-0.17707, -0.27561)
0.7	(-0.21278, -0.34530)	(-0.28494, -0.00058)	(-0.17115, -0.27858)
0.8	(-0.21318, -0.34375)	(-0.28307, -0.00059)	(-0.16517, -0.28144)
0.9	(-0.21358, -0.34213)	(-0.28114, -0.00060)	(-0.15915, -0.28421)
1	(-0.21397, -0.34045)	(-0.27915, -0.00062)	(-0.15311, -0.28689)

Table 11. Sensitivity of fuzzy reliability (SOFR) with Pareto distribution at $t = 7$

α	$\tilde{R}^P(T,\alpha)(t)$	$\tilde{R}^P(I,\beta)(t)$	$\tilde{R}^P(F,\gamma)(t)$
0	(-0.21431, -0.33170)	(-0.22107, -0.31857)	(-0.43237, -0.29812)
0.1	(-0.21445, -0.33168)	(-0.31628, -0.00062)	(-0.42185, -0.30002)
0.2	(-0.21458, -0.33158)	(-0.31571, -0.00062)	(-0.41847, -0.30181)
0.3	(-0.21472, -0.33146)	(-0.31511, -0.00063)	(-0.41493, -0.30350)
0.4	(-0.21485, -0.33133)	(-0.31446, -0.00063)	(-0.41123, -0.30509)
0.5	(-0.21498, -0.33116)	(-0.31381, -0.00064)	(-0.40736, -0.30659)
0.6	(-0.21511, -0.33097)	(-0.31311, -0.00064)	(-0.40329, -0.30800)
0.7	(-0.21523, -0.33076)	(-0.31237, -0.00065)	(-0.39903, -0.30933)
0.8	(-0.21536, -0.33052)	(-0.31159, -0.00065)	(-0.39455, -0.31059)
0.9	(-0.21548, -0.33024)	(-0.31077, -0.00066)	(-0.38986, -0.31177)
1	(-0.21560, -0.32994)	(-0.30990, -0.00066)	(-0.38493, -0.31289)

Table 1 presents the α, β, and γ-cut values for the neutrosophic fuzzy probabilities P_i^N, where $i = 1$ to 5. To evaluate the system's neutrosophic fuzzy reliability, two types of component lifetime distributions are considered weibull and pareto. For the weibull distribution, the α, β, and γ-cut values from Table 1 are first used in equation (4), and the resulting values are then substituted into equation (6) to compute the system reliability. This evaluation is performed for three time instances: $t = 7$, 10, and 15, with the shape parameter $\beta = 0.5$. In the case of the pareto distribution, the same α, β, and γ-cut values are applied to equation (5). The final system reliability values are then obtained using equation (6) and are summarized in Table 4 for the time values $t = 7$, 10, and 15, as referenced in Table 3.

Table 12. Sensitivity of fuzzy reliability (SOFR) with Pareto distribution at $t = 10$

α	$\tilde{R}^P(T,\alpha)(t)$	$\tilde{R}^P(I,\beta)(t)$	$\tilde{R}^P(F,\gamma)(t)$
0	(-0.22084, -0.35402)	(-0.22438, -0.32277)	(-0.34725, -0.28993)
0.1	(-0.22107, -0.35349)	(-0.22338, -0.32431)	(-0.34227, -0.29276)
0.2	(-0.22130, -0.35292)	(-0.22233, -0.32579)	(-0.33714, -0.29545)
0.3	(-0.22153, -0.35232)	(-0.22124, -0.32721)	(-0.33184, -0.29803)
0.4	(-0.22176, -0.35168)	(-0.22010, -0.32857)	(-0.32639, -0.30049)
0.5	(-0.22198, -0.35101)	(-0.21891, -0.32987)	(-0.32076, -0.30283)
0.6	(-0.22221, -0.35030)	(-0.21767, -0.33112)	(-0.31497, -0.30508)
0.7	(-0.22243, -0.34954)	(-0.21638, -0.33232)	(-0.30899, -0.30723)
0.8	(-0.22265, -0.34875)	(-0.21504, -0.33347)	(-0.30283, -0.30928)
0.9	(-0.22287, -0.34791)	(-0.21363, -0.33458)	(-0.29649, -0.31125)
1	(-0.22308, -0.34702)	(-0.21216, -0.33564)	(-0.28996, -0.31313)

Table 13. Sensitivity of fuzzy reliability (SOFR) with Pareto distribution at $t = 15$

α	$\tilde{R}^P(T,\alpha)(t)$	$\tilde{R}^P(I,\beta)(t)$	$\tilde{R}^P(F,\gamma)(t)$
0	(-0.21068, -0.35462)	(-0.20864, -0.30048)	(-0.20846, -0.25553)
0.1	(-0.21105, -0.35344)	(-0.20712, -0.30280)	(-0.20302, -0.25918)
0.2	(-0.21142, -0.35222)	(-0.20554, -0.30505)	(-0.19751, -0.26270)
0.3	(-0.21178, -0.35095)	(-0.20392, -0.30722)	(-0.19191, -0.26610)
0.4	(-0.21215, -0.34963)	(-0.20225, -0.30933)	(-0.18625, -0.26938)
0.5	(-0.21251, -0.34826)	(-0.20052, -0.31137)	(-0.18051, -0.27255)
0.6	(-0.21287, -0.34684)	(-0.19874, -0.31335)	(-0.17472, -0.27561)
0.7	(-0.21323, -0.34537)	(-0.19689, -0.31526)	(-0.16887, -0.27858)
0.8	(-0.21359, -0.34383)	(-0.19499, -0.31712)	(-0.16298, -0.28144)
0.9	(-0.21394, -0.34224)	(-0.19302, -0.31893)	(-0.15706, -0.28421)
1	(-0.21429, -0.34059)	(-0.19099, -0.32068)	(-0.15112, -0.28689)

5. Comparative Analysis of Weibull and Pareto Lifetime Distributions

Reliability analysis under different lifetime distributions is essential to understanding how varying failure behaviors influence the performance of linear consecutive (LC) k-out-of-r-from-n: F systems. In this section, the proposed trapezoidal neutrosophic fuzzy number (TrNFN) based universal generating function (UGF) model is applied using two distinct lifetime distributions Weibull and Pareto to demonstrate the adaptability and robustness of the neutrosophic reliability framework. The (α, β, γ)-cut approach is used to convert neutrosophic parameters into deterministic intervals for reliability computation under both distributions.

5.1. *Weibull Lifetime Distribution*

"The Weibull distribution is widely used for modeling life data with monotonic failure rates. Its probability density function (PDF) is given by:

$$f(t) = \frac{\beta}{\varpi}\left(\frac{t}{\varpi}\right)^{\beta-1} e^{-(t/\varpi)^{\beta}},$$

where η represents the scale parameter and β the shape parameter. A smaller β (< 1) corresponds to an early-life failure phase, $\beta = 1$ to a steady-state failure rate, and $\beta > 1$ to a wear-out phase. Within the neutrosophic fuzzy environment, these parameters are represented by TrNFNs, allowing the model to incorporate indeterminacy in lifetime estimation. The resulting reliability function is expressed as:

$$R_W(t) = e^{-(t/\varpi)^{\beta}}.$$

The Weibull-based neutrosophic reliability curves exhibit a smooth and gradual decline, indicating predictable system degradation. This distribution effectively models systems where aging and wear dominate failure behavior."

5.2. *Pareto Lifetime Distribution*

"The Pareto distribution captures heavy-tailed failure characteristics, representing systems prone to rare but catastrophic events. Its probability density function is expressed as: function $\tilde{R}[\alpha]$ may be represented as follows:

$$f(t) = \frac{\beta\varpi^{\beta}}{\varpi^{\beta+1}}, \quad \varpi \geq \beta\varpi,$$

where α denotes the shape parameter and t_m the minimum lifetime threshold. The corresponding reliability function is:

$$R_P(t) = \left(\frac{\beta\varpi}{\varpi}\right)^{\beta}$$

In the proposed TrNFN UGF framework, ϖ and $\beta\varpi$ are treated as trapezoidal neutrosophic fuzzy quantities, enabling the model to capture high uncertainty in tail behavior. The Pareto distribution demonstrates higher reliability in early operational stages but a sharper decline after critical degradation time. This behavior makes it suitable for systems exposed to external shocks, uncertain environments, or rare-event failures."

5.3. *Comparative Discussion*

In figure t=7 to t=15 Pareto distribution (illustrative) and corresponding numerical results indicate that the Pareto-based neutrosophic model yields slightly higher reliability values at early and mid-life stages compared to the Weibull-based model. This is because Pareto's heavy-tailed nature accommodates long-lifetime components even under uncertain failure rates. However, at later stages, the Weibull distribution provides smoother degradation trends, suitable for steady or predictable failure processes.

The comparison highlights that:

(1) The Pareto model better reflects systems dominated by infrequent but severe failure events.

(2) The Weibull model more effectively characterizes regular aging or wear-out processes.

(3) Under neutrosophic uncertainty, the Pareto distribution produces wider reliability intervals, capturing higher indeterminacy.

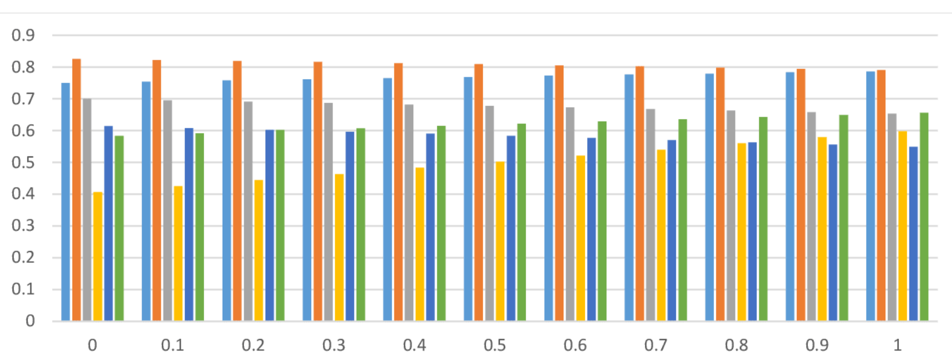
(4) The proposed TrNFN UGF framework remains consistent and stable under both distributions, confirming its generalizability and robustness.

Overall, the comparative analysis confirms that the proposed neutrosophic fuzzy framework provides a flexible and unified mechanism for evaluating reliability across multiple lifetime assumptions. This capability enhances its applicability to heterogeneous and dynamically evolving engineering systems.

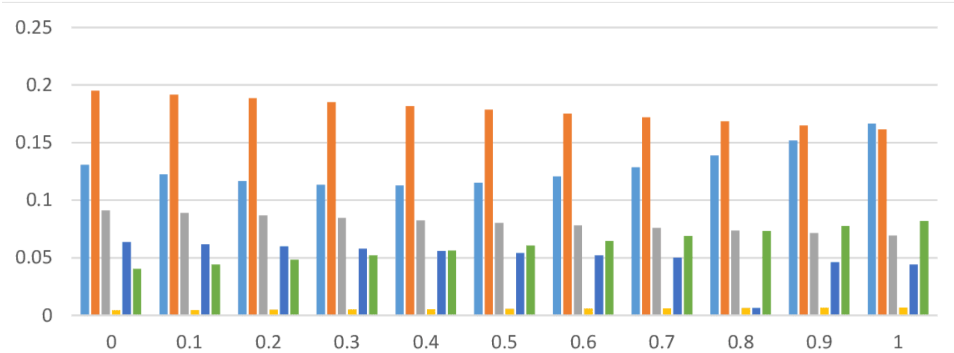
6. Sensitivity of fuzzy reliability assessment (SOFR assessment)

Sensitivity analysis analyses how changes in a parameter effect the dependent variable. Sensitivity refers to how quickly a change in a distribution parameter impacts the chance of failure or reliability [6]

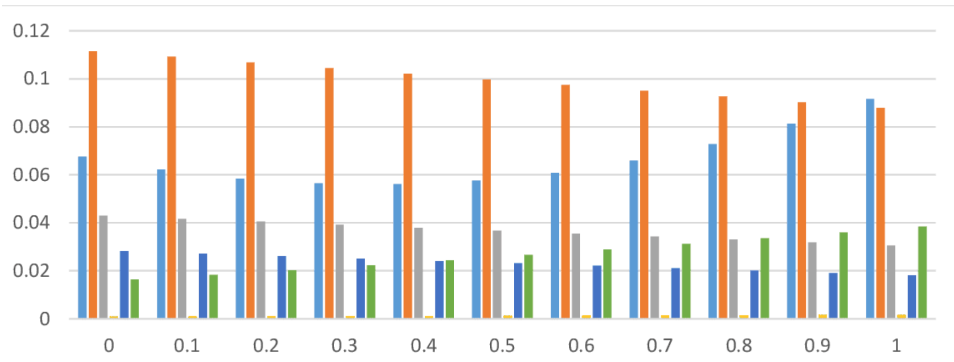
The sensitivity of fuzzy reliability (SOFR) with respect to time (t) is defined as $\frac{dR(t)}{dt}$. Tables 5 *and* 6 show the sensitivity of fuzzy reliability SOFR in terms of α, β, γ for Weibull and Pareto distributions, derived using Eq. (6) and the previously mentioned method for $\beta = 0.5, t = 7, 10,$ *and* 15



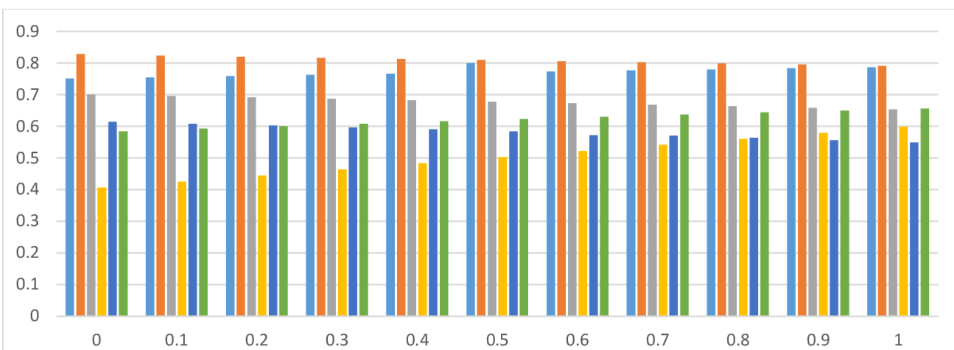
System reliability when t=7(Weibull Distribution)



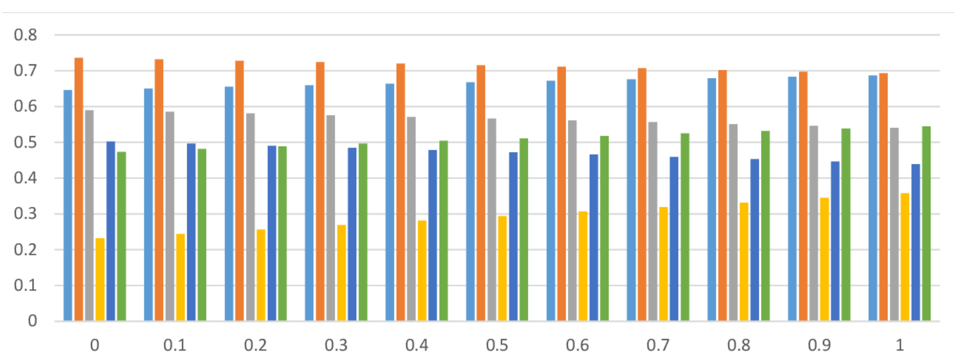
System reliability when t=10(Weibull Distribution)



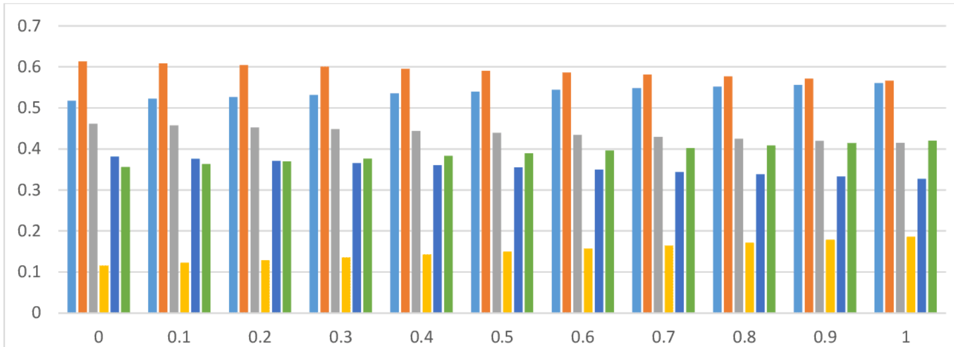
System reliability when t=15(Weibull Distribution)



System reliability when t=7(Pareto Distribution)



System reliability when t=10(Pareto Distribution)



System reliability when t=15(Pareto Distribution)

7. Results and discussion

This study applies the universal generating function (UGF) technique to evaluate the neutrosophic fuzzy reliability of a linear consecutive (LC) k-out-of-r-from n:F system that includes components with different characteristics. Two popular lifetime distributions Weibull and Pareto are used to illustrate the method, with the reliability of each component modeled using trapezoidal neutrosophic fuzzy numbers $(TrNFNs)$. The system's fuzzy reliability values at time points $t = 7$, 10, and 15 are shown in Table 1. To help visualize the results, Figures 1 to 3 display the reliability trends under the Weibull distribution. These clearly show that the system's reliability decreases as time increases, with the highest value observed at $t = 7$ and the lowest at $t = 15$. The use of $TrNFNs$ is important here because they effectively handle uncertainty and imprecision in system performance analysis. The system's fuzzy reliability under the Pareto distribution is also shown in Table 1 and Figures 4 to 6, which follow the same trend higher reliability at $t = 7$ and lower at $t = 15$. Interestingly, when comparing the two distributions, the Pareto distribution consistently yields higher system reliability than the Weibull. For instance, at $t = 7$, the highest $TrNFN$ based reliability under the Weibull distribution is around $(0.208661, 0.288107)$, while under the Pareto distribution it reaches up to $(0.750593, 0.825857)$. This clearly highlights the advantage of using Pareto for reliability modeling in this case.

A sensitivity analysis is also performed for both distributions to understand how reliability changes with varying parameters. The results (shown in Tables 5 and 6 reveal negative sensitivity for both Weibull and Pareto, meaning that as time progresses, the system's reliability declines. This matches the earlier findings and reflects the real-world behavior of systems that naturally degrade over time.

8. Conclusion

This paper evaluate reliability and senstivity of the linear consecutive (LC) k-out-of-r-from-n:F system by using universal generating function. Here we have used a new method for analysing reliability under fuzzy environment by taking the concept of trapezoidal neutrosophic fuzzy number. The results obtained by the graphs and tables reveal that system reliability reduces over time under uncertain environment . In this method we have considered Wiebull and Pareto distribution and got the result that the Pareto distribution is better then Weibull distribution.

This concept is also applicable to the Exponential and Rayliegh distributions, which are simply derived from the Wiebull distribution. It may also be extented to involve more then two parameter distributions. The proposed technique is very useful in real world sytems of the field of multicriteria decision making, engineering, management and medical etc. Similarly, a sensitivity analysis was executed to confirm that the results remain correct. The negative senstivity of fuzzy reliability (SOFR) implies that the system's reliability decreases over time, which is a basic concept of all systems.

9. Limitations and Future Work

The proposed trapezoidal neutrosophic fuzzy number (TrNFN) based universal generating function (UGF) framework provides a robust and flexible methodology for modeling uncertainty in life-cycle reliability assessment. However, several limitations remain that simultaneously highlight promising directions for future research.

First, the present model assumes statistically independent component lifetimes and uniform distributional behavior within each reliability phase. In practical engineering systems, dependencies often arise due to environmental coupling, structural interconnections, or shared operational stresses. Extending the TrNFN UGF model to account for inter-component dependency or common-cause failures possibly through copula-based or Markovian formulations would enhance its practical accuracy and realism.

Second, the current study focuses on three major lifetime distributions: Weibull, Pareto, and Rayleigh. Incorporating other probabilistic families such as exponential, lognormal, Burr, or generalized Pareto distributions could better represent specific reliability behaviors. Future work may explore these alternatives within the same neutrosophic framework to identify the most suitable models for diverse industrial contexts.

Third, the computational complexity of the proposed approach increases with system size and dimensionality of neutrosophic representation. To overcome this, advanced hybrid algorithms, pruning-based UGF composition, and Monte Carlo simulation can be integrated to improve scalability and efficiency. Moreover, Bayesian inference or data-driven learning techniques may be utilized to estimate neutrosophic parameters from real field data.

Finally, the present study primarily addresses static reliability analysis. Extending the framework to repairable systems by incorporating preventive and corrective maintenance, as well as dynamic degradation modeling, would substantially broaden its applicability. Further generalization toward higher-order neutrosophic environments such as hexagonal hesitant or dual-hesitant fuzzy numbers may provide deeper insights into uncertainty representation. Empirical validation on real-world applications including renewable energy networks, sensor-based control systems, and intelligent manufacturing setups can reinforce the framework's practical significance.

In conclusion, the integration of the neutrosophic fuzzy paradigm with probabilistic, statistical, and computational intelligence tools promises to advance next-generation reliability engineering by enabling more accurate, adaptive, and uncertainty-aware decision-making in complex systems.

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