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Eigen Neutrosophic Z- Set and Neutrosophic Z- Relation

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Abstract: This paper introduces an innovative framework for computing the Greatest Eigen Neutrosophic Z-set and the Least Eigen Neutrosophic Z-set using the composition operators, namely max-min-min and min-max-max. The proposed Eigen Neutrosophic Z-set, along with the Neutrosophic Z-relation, remains constant across different computational perspectives. This study addresses the limitation of existing neutrosophic and fuzzy models that fail to effectively capture eigen-based relationships under uncertainty by introducing the Eigen Neutrosophic Z-set framework for more consistent and interpretable decision analysis. Furthermore, Neutrosophic Z-matrices are developed, and their properties are examined in relation to Neutrosophic Z-relations. In this paper several similarity relations among Neutrosophic Z-matrices are presented, along with discussions on their permutations and the invertibility characteristics. Two distinct algorithms are formulated to establish the Greatest Eigen Neutrosophic Z-set and the Least Eigen Neutrosophic Z-set, accompanied by a numerical example. Additionally, a practical application is provided to demonstrate the enhancement of score value while addressing both effectiveness and uncertainty for future advancements of hotel management decision-making systems.

Keywords: Neutrosophic Z-set, Neutrosophic Z-relation, Neutrosophic Z-Matrices, Eigen Neutrosophic Z-set, Composition operators, Decision-making uncertainty modelling.

1. Introduction

Zadeh [1] proposed a notion namely Z-number, which is an ordered pair of fuzzy numbers $Z = (\breve{V}, \breve{R})$ in 2011.The reliability and restriction of fuzzy is mainly focused in Z-number [2]. Smarandache [3] introduced another concept of imprecise data called Neutrosophic data which deals with complicating aspects to process imprecision, vagueness, and uncertainty in data. Sanjib Mondal et.al., [4] developed similarity relations for Intuitionistic fuzzy matrices. Neutrosophic set was later developed to Quadri partitioned neutrosophic soft set, fuzzy neutrosophic soft matrices and fuzzy Quadri partitioned neutrosophic soft matrix [5, 6, 7] which was more useful in decision making.

Neutrosophic qualities and neutrosophic metrics to assess trustworthiness are united in the neutrosophic z-number set technique proposed by Shigui Du et al. [8] as a generalization of the z-numbers and the neutrosophic set. The three ordered pairs of neutrosophic numbers along with their reliability measures in indeterminate and inconsistent situations can be resolved by the suggested neutrosophic z-number set [9]. In z-numbers and their set, the multi criteria decision-making technique (MCDM) is readily embraced [10, 11, 12] later on MCDM developed to neutrosophic z-numbers. A fuzzy relations eigen fuzzy set was presented by Sanchez [13]. He provided three main algorithms to find the Greatest Eigen Fuzzy Set (GEFS) linked with fuzzy relations using max-min composition so that $R \circ A = A$. Eigen fuzzy sets have been successfully used in a number of real-world applications in decision-making, genetic algorithms, image analysis, and medicine. Guleria and Bajaj [14] later proposed eigen spherical fuzzy sets and applications. Using spherical fuzzy set, they have produced an astounding achievement by identifying two distinct techniques for finding eigen spherical fuzzy sets. Further T-spherical fuzzy set for similarity measure also found for decision-making [15].

Harikrishnan et al. [16] in their work min-max compositions for neutrosophic fuzzy matrices was demonstrated their application in diagnosing diseases. Their work shows that changing composition operators alters diagnostic outcomes, but they do not address eigen-based stability or Z-relations. But this shows the importance of composition operators but lacks information about eigen Z-set theory. Kamran et al. [17] examined the use of neutrosophic Z-numbers in AHP-based prioritization and Z-rough structures for ranking alternatives under uncertainty. Although Z-numbers provide richer representation of uncertainty, this work does not define eigen Z-sets or algorithms for stable relation evaluation. This work has a strong Z-number background but doesn't focus on similarity or eigen properties.

Mishra & Kumar [18] investigated algebraic properties of neutrosophic matrices, including invertibility and determinants for decision-oriented systems. Their theoretical work addresses classical neutrosophic matrices but does not extend these results to neutrosophic Z-matrices or eigen computations. This work doesn't involve the Z-matrix framework; only the foundation of matrix algebra is used for analysis.

Al-Faifi et al. [19] applied neutrosophic and plithogenic models to multi-criteria decision-making for uncertain preference structures. While they improve decision accuracy, they rely on distance and score measures and do not consider eigen-based consistency.

Saha & Abdel-Basset et al. [20] explored spectral measures such as the "energy" of neutrosophic matrices for network analysis and clustering. Their results demonstrate the usefulness of spectral neutrosophic properties, but they do not propose algorithms for greatest/least eigen Z-sets or Z-relations. The Z-set definition and the composition stability are missing.

2. Research Gap

The Eigen fuzzy set was introduced by Sanchez [13], along with the concept of fuzzy relations. This method established the Greatest Eigen Fuzzy Set using the max-min composition method. Numerous researchers have applied this max-min composition for image retrieval, genetic algorithms, and in the medicinal field. Subsequently, the Eigen Spherical Fuzzy Set was introduced by Guleria and Bajaj [14]. They offered two techniques for identifying the Greatest Eigen Spherical Fuzzy Set and the Least Eigen Spherical Fuzzy Set. The Neutrosophic Z-set is a novel method used to assess uncertainty in real-life scenarios. We have proposed a new composition operator for the Neutrosophic Z-set along with its Neutrosophic Z-relation. Many researchers have extensively explored Neutrosophic Fuzzy matrices, their relations, and similarity measures. Our study is significant because we extended similarity relations for Neutrosophic fuzzy matrices to Neutrosophic Z-matrices.

3. Contribution of this proposed work

- **Introduction of new composition operator for neutrosophic z-set:** Two distinct composition operators for neutrosophic z-sets,specifically max-min-min and min-max-max, have been developed alongside the concept of neutrosophic z-relation. These composition operators identify the Greatest Eigen Neutrosophic Z-sets (GENZS) and the Least Eigen Neutrosophic Z-sets (LENZS), which are tailored to yield suitable values in situations of uncertainty.
- **Neutrosophic Z-relation for Neutrosophic Z-matrices**: A Neutrosophic Z-matrix has been introduced together with the Neutrosophic Z-relation. Various properties of similarity relations

were examined, offering a foundational framework for Neutrosophic Z-matrices, with potential for further advancements through these matrices.

- **Algorithm for strategy finding:** This proposed work introduced two algorithms for each composition operator within the framework of Neutrosophic Z-sets. The primary objective of these algorithms is to determine the eigen neutrosophic z-set.
- **Numerical Example and Application**: The efficacy of the proposed method is illustrated using a numerical example. Effectiveness and uncertainty are assessed in a real-world setting. The decision-making scenario provides an in-depth comprehension regarding the methods feasibility and adaptability.

Table 1, depicts the comparison of the existing works in neutrosophic Z numbers and fuzzy matrices with the proposed work Eigen Neutrosophic Z set.

Table 1 Comparison of existing and proposed work

Dimension	Existing Works (Neutrosophic, Z-numbers, Fuzzy matrices)	Proposed Work
Handling of uncertainty	Uses truth, indeterminacy, falsity values; Z-numbers add reliability but no eigen characterization	Introduces Eigen Neutrosophic Z-set to measure stable relation values under uncertainty
Composition operators	Studies max–min / min–max families separately	Demonstrates both max–min–min and min–max–max operators and proves eigen-set consistency across compositions
Matrix framework	Classical neutrosophic matrices used for similarity or scoring	It defines Neutrosophic Z-matrices, explores invertibility, permutations, similarity relations
Eigen-based analysis	Mostly absent; spectral analysis exists but not for Z-relations	Provides algorithms for Greatest and Least Eigen Neutrosophic Z-sets with numerical examples
Practical decision-making	Score or distance-based ranking	Uses eigen Z-sets to enhance score and interpretability in hotel management decision-making
Reproducibility	Often conceptual or qualitative	Delivers two algorithms, operator consistency proof, and implementation steps

The paper is organized as follows: Key definitions and concepts are examined in Section 3. Neutrosophic Z-matrices, Neutrosophic Z-relations, invertibility requirements, and similarity relations are introduced in Section 4. NZM idempotent is also taken for consideration in this part. Numerous characteristics and findings pertaining to neutrosophic Z-matrices are examined. The composition operator and the Neutrosophic Z-relation are defined in Section 5. The notions of the

Least Eigen Neutrosophic Z-sets (LENZS) and the Greatest Eigen Neutrosophic Z-sets (GENZS) are presented in this study along with an algorithm. The principles of GENZS and LENZS are explained with the use of a numerical example. We show how the suggested technique can be used in practical situations in Section 6. In Section 7 the work is concluded and future research directions are discussed.

3 Preliminaries

3.1 Definition

Let X be a universe set then a Neutrosophic Z-number set (NZNs) in a universe set X is defined in the following as

$$S_Z = (< \mathrm{x}, \mathrm{T(V,R)(x)}, \mathrm{I(V,R)(x)}, \mathrm{F(V,R)(x)} > x \in \mathrm{X}) \quad (1)$$

here

$$\mathrm{T(V,R)(x)} = (T_V(\mathrm{x}), T_R(\mathrm{x})) \,, \mathrm{I(V,R)(x)} = (I_V(\mathrm{x}), I_R(\mathrm{x})), \mathrm{F\,(V,R)(x)} = (F_V(\mathrm{x}), F_R(\mathrm{x})) \colon \mathrm{X} \to [0,1]^2 \quad (2)$$

are the order pairs of neutrosophic values for truthfulness, indeterminacy, and falsehood; the first component consists of the neutrosophic values in a universe set X, and the second component consists of neutrosophic reliability measures, with the rule of

$$0 \le T_V(\mathrm{x}) + I_V(\mathrm{x}) + F_V(\mathrm{x}) \le 3 \text{ and } 0 \le T_R(\mathrm{x}) + I_R(\mathrm{x}) + F_R(\mathrm{x}) \le 3 \quad (3)$$

3.2 Definition

Let X be a universe set and F be a set of parameters. Consider a nonempty set $S_Z, S_Z \in F$. Let P(X) be the collection of all neutrosophic z- number sets of X. The set (E, S_Z) be termed as neutrosophic z- number sets (NZNs) over X, where $E : S_Z \to P(X)$. Consider S as neutrosophic z-matrices (NZMs) over X instead of (E, S_Z).

3.3 Definition

Let S_A be a $NZM_{m\times n}$ and S_B be a $NZM_{n\times p}$ then the composition of S_A and S_B is defined as

$$S_A \circ S_B = (< (\sum_{i=1}^{n} \left(T^A_{V_{ik}} \wedge T^B_{V_{kj}}\right), (\sum_{i=1}^{n} \left(T^A_{R_{ik}} \wedge T^B_{R_{kj}}\right)), (\prod_{i=1}^{n} (I^A_{V_{ik}} \vee I^B_{V_{kj}})), \\ (\prod_{i=1}^{n} (I^A_{R_{ik}} \vee I^B_{R_{kj}})), (\prod_{i=1}^{n} (F^A_{V_{ik}} \vee F^B_{V_{kj}})), (\prod_{i=1}^{n} (F^A_{R_{ik}} \vee F^B_{R_{kj}})) >) \quad (4)$$

Equivalently it can be written as

$$S_A \circ S_B = (< (\bigcup_{i=1}^{n} \left(T^A_{V_{ik}} \wedge T^B_{V_{kj}}\right)), (\bigcup_{i=1}^{n} \left(T^A_{R_{ik}} \wedge T^B_{R_{kj}}\right)), (\bigwedge_{i=1}^{n} (I^A_{V_{ik}} \vee I^B_{V_{kj}})), \\ (\bigwedge_{i=1}^{n} (I^A_{R_{ik}} \vee I^B_{R_{kj}})), (\bigwedge_{i=1}^{n} (F^A_{V_{ik}} \vee F^B_{V_{kj}})), (\bigwedge_{i=1}^{n} (F^A_{R_{ik}} \vee F^B_{R_{kj}})) >) \quad (5)$$

If the number of S_A columns equal the number of rows S_B, then the product is defined. This multiplication procedure is called as max-min composition operator. Consequently, $S_A \circ S_B$ and are considered conformable for multiplication, rather than using $S_A \circ S_B$ it is denoted as $S_A S_B$, where $\sum_{i=1}^{n}\left(T_{V_{ik}}^{A} \wedge T_{V_{kj}}^{B}\right)$ means max- min operation and $\prod_{i=1}^{n}(I_{V_{ik}}^{A} \vee I_{V_{kj}}^{B}))$ means min-max operation.

4 Neutrosophic Z-relation

4.1 Definition

Let $H(A,A)$ be an Neutrosophic Z-relation (NZR) on a set S_A. Let $T_{V,R}: S_A \longrightarrow [0,1]^2$, $I_{V,R}: S_A \longrightarrow [0,1]^2, and\ F_{V,R}: S_A \longrightarrow [0,1]^2$ are the three membership function and M_H be the corresponding Neutrosophic Z- Matrices (NZM) in relation H.

4.2 Definition

The relation $H(A,A)$ is reflexive if the diagonal entries of M_H is $[<(1,1),(0,0),(0,0)>]$ where $T_{(V,R)_H}(x,x) = (1,1)$, $I_{(V,R)_H}(x,x) = (0,0)$ and $F_{(V,R)_H}(x,x) = (0,0)$ for all $x \in S_A$.

4.3 Definition

The relation $H(A,A)$ is symmetric if $M_H = M_H^T$ where M_H^T is the transpose of M_H such that $T_{(V,R)_H}(x,y) = T_{(V,R)_H}(y,x)$, $I_{(V,R)_H}(x,y) = I_{(V,R)_H}(y,x)$ and $F_{(V,R)_H}(x,y) = F_{(V,R)_H}(y,x)$ for all $x,y \in S_A$.

4.4 Definition

The relation $H(A,A)$ is transitive if $M_H \geq M_H^2$ i.e.,

$T_{(V,R)_H}(x,z) \geq$ max (min $(\left(T_{(V,R)_H}(y,x), T_{(V,R)_H}(y,z)\right))$,

$I_{(V,R)_H}(x,z) \leq$ min (max $(\left(I_{(V,R)_H}(y,x), I_{(V,R)_H}(y,z)\right))$ and

$F_{(V,R)_H}(x,z) \leq$ min (max$\left(\left(F_{(V,R)_H}(y,x), F_{(V,R)_H}(y,z)\right)\right)$ for all pair$(x,z) \in S_A \times S_A$.

4.5 Definition

Let $H(A,A)$ relation is reflexive, symmetric and transitive then $H(A,A)$ relation is called as similarity relation.

4.6 Proposition

For any $S_A \in NZM_{n\times n}$, S_A is reflexive if $S_A \geq I_n$.

proof

Sinc $S_A \geq I_n$, then matrix entries which is diagonal of S_A are $[<(1,1),(0,0),(0,0)>]$.

$\therefore$ S_A is a reflexive matrix.

Hence the proof.

4.7 Definition

For an $S_A \in NZM_{n\times n}$, we define

- S_A is Reflexive if $S_A \geq I_n$
- S_A is Weekly reflexive if $S_A \geq S_A$
- S_A is Symmetric $S_A = S_A^T$
- S_A is Idempotent $S_A = S_A^2$
- S_A is Transitive $S_A^2 \leq S_A$

4.8 Proposition

Let $S_A \in NZM_{n\times n}$ be a reflexive of NZM. Then

I. S_A^T is reflexive NZM, whereS_A^T is transpose of S_A.
II. S_A^K is reflexive NZM for positive integer k.
III. $S_A S_B \geq S_B$ for $S_B \in NZM_{n\times n}$
IV. $S_B S_A \geq S_B$ for $S_B \in NZM_{n\times n}$
V. $S_A S_B$ and $S_B S_A$ are reflexive NZMs if S_B is reflexive
VI. $S_A S_A^T$ and $S_A^T S_A$ are reflexive NZMs.

Proof:

I. Since S_A has reflexive properties only when its diagonal entries are $[<(1,1),(0,0),(0,0)>]$. Hence S_A^T is reflexive.

II. Since S_A is reflexive, $S_A \geq I_n$ then $S_A^2 \geq S_A \geq I_n$ (multiplying on both sides). Proceeding for (k-1) times we get $S_A^k \geq S_A^{k-1} \geq \cdots \ldots . . \geq S_A^2 \geq S_A \geq I_n$. The result holds for any scalar k. then S_A^k is reflexive.

III. $S_A \geq I_n$ then, $S_A S_B \geq I_n S_B \Rightarrow S_A S_B \geq S_B$

IV. $S_A \geq I_n$ then, $S_B S_A \geq I_n S_B \Rightarrow S_B S_A \geq S_B$

V. Since S_B is reflexive $S_B \geq I_n$ then $S_A S_B \geq S_B \geq I_n$ and $S_B S_A \geq S_B \geq I_n$ from (III) and (IV). Hence $S_A S_B$ and $S_B S_A$ are reflexive.

VI. Using (I) in (V) replace S_A^T in the place of S_B we derive the desire result.

Hence the proof

4.9 Proposition

If $S_A \in NZM_{n\times n}$ be transitive and also it is reflexive then S_A is idempotent.

Proof:

It is known that S_A is reflexive, $S_A \geq I_n$

$$S_A^2 \geq S_A \geq I_n \tag{6}$$

Also, S_A is transitive

$$S_A^2 \leq S_A \tag{7}$$

Combining (6) & (7) $\Rightarrow S_A^2 = S_A$

Hence S_A is idempotent

Note: Converse is not true.

Example

Let $S_A = \begin{bmatrix} <(0.8,0.3),(0.3,0.4),(0.4,0.4)> & <(0.8,0.3),(0.3,0.4),(0.4,0.4)> \\ <(0.5,0.3),(0.3,0.4),(0.4,0.4)> & <(0.8,0.3),(0.3,0.4),(0.4,0.4)> \end{bmatrix} \ngeq I_2$

Hence S_A is not reflexive, But

$S_A^2 = S_A S_A$ (max-min)

$$= \begin{bmatrix} <(0.8,0.3),(0.3,0.4),(0.4,0.4)> & <(0.8,0.3),(0.3,0.4),(0.4,0.4)> \\ <(0.5,0.3),(0.3,0.4),(0.4,0.4)> & <(0.8,0.3),(0.3,0.4),(0.4,0.4)> \end{bmatrix} = S_A$$

S_A is idempotent but not reflexive

4.10 Proposition

If S_A and S_B are two symmetric NZMs of order n x n such that $S_A S_B = S_B S_A$, then $S_A S_B$ is symmetric NZM.
It can be proved easily

Note:

If S_A is symmetric in $NZM_{n\times n}$ then S_A^K is also symmetric for any scalar k.

4.11 Proposition

Let S_A , $S_B \in NZM_{n\times n}$ is transitive, such that $S_A S_B = S_B S_A$, then S_A S_B will be transitive.

Proof

We know S_A and S_B both transitive $S_A^2 \leq S_A$ and $S_B^2 \leq S_B$. Now

$(S_A S_B)^2 = (S_A S_B)(S_A S_B)$

$= S_A(S_B S_A)S_B$

$= S_A(S_A S_B)S_B$

$= (S_A S_A)(S_B S_B)$

$= S_A^2 S_B^2$

$\Rightarrow (S_A S_B)^2 \leq S_A S_B$

hence $S_A S_B$ is transitive.

Note:

If S_A is transitive in $NZM_{n\times n}$ then S_A^k is also transitive for any scalar k.

4.12 Proposition:

If $S_A = [S_{A_{pq}}] = \left[< \left(T^A_{V_{pq}}, T^A_{R_{pq}}\right), \left(I^A_{V_{pq}}, I^A_{R_{pq}}\right), \left(F^A_{V_{pq}}, F^A_{R_{pq}}\right) >\right] \in NZM_{n\times n}$ is symmetric and transitive then $S_{A_{pq}} \leq S_{A_{pp}}$ for p,q ∈ { 1,2, 3………n}.

Proof

Let S_A is symmetric, $S_{A_{pq}} = S_{A_{qp}}$ for all p, q ∈ {1,2, 3………n}

Also, since S_A is transitive

$$S_A^2 \leq S_A \Longrightarrow S_A \geq S_A^2$$

Thus

$S_{A_{pq}} \geq \underbrace{max}_{r} [\min(S_{A_{pr}}, S_{A_{rj}})]$ for p= q and r ∈ {1,2, 3………n}

$S_{A_{pq}} \geq \underbrace{max}_{r} [\min(S_{A_{pr}}, S_{A_{rq}})]$ for p= q and r ∈ {1,2, 3………n}

$\geq \min(S_{A_{pr}}, S_{A_{rq}})$ for r=q and each p

$S_{A_{pp}} \geq S_{A_{pq}}$ (since $S_{A_{pq}} = S_{A_{qp}}$)

Hence proved.

4.13 Definition

Let $S_A \in NZM_n$ and S_B is said to be invertible if and only if there exist $S_B \in NZM_n$ such that $S_A S_B = S_B S_A = I_n$.

4.14 Definition

An $S_A \in NZM_n$ is called Neutrosophic Z-Permutation matrix (NZPM) if both row and column contains exactly one entry I and all other entries are ϕ .

4.15 Proposition

If S_A be a NZM_n of an NZPM then $S_A S_A^T = S_A^T S_A = I_n$

Proof:

$S_A = (< \left(T^A_{V_{ij}}, T^A_{R_{ij}}\right), \left(I^A_{V_{ij}}, I^A_{R_{ij}}\right), \left(F^A_{V_{ij}}, F^A_{R_{ij}}\right) >)$

Then $S_A^T = (< \left(T^A_{V_{ji}}, T^A_{R_{ji}}\right), \left(I^A_{V_{ji}}, I^A_{R_{ji}}\right), \left(F^A_{V_{ji}}, F^A_{R_{ji}}\right) >)$

now, i, jth entries of $S_A S_A^T$ is

$$\sum_{k=1}^{n} S_{A_{ik}} S_{B_{kj}} = \sum_{k=1}^{n} S_{A_{ik}} S_{A_{kj}} = \begin{cases} \phi & if\ i \neq j \\ I & if\ i = j \end{cases}$$

(since $S_A S_B$ is NZPM, $\sum_{k=1}^{n} S_{A_{ik}} S_{A_{ik}} = I$)

Hence $S_A S_A^T$ is an I_n.

converse can be proved easily.

Hence the proof.

4.19 Proposition

Let S_A be a NZMn, S_A is invertible if and only if S_A is an NZPM.

Proof:

First part:

$S_A S_A^T = S_A^T S_A = I_n$ (by previous proposition)

hence S_A is invertible and S_A^T is the inverse of S_A (i.e) $S_A^- = S_A^T$

Second part:

Let S_A be invertible and S_B be the inverse of S_A. Thus $S_A S_B = S_B S_A = I_n$ follows that

$\sum_{k=1}^{n} S_{A_{pr}} S_{B_{rq}} = \sum_{k=1}^{n} S_{B_{pr}} S_{A_{rq}} = \phi$ for p≠q

$$\sum_{k=1}^{n} S_{A_{pr}} S_{B_{rq}} = \sum_{k=1}^{n} S_{B_{pr}} S_{A_{rp}} = I$$

$S_{A_{pq}} S_{B_{qr}} = S_{B_{pr}} S_{A_{rq}} = \phi$ for p≠q and r ∈{1,2,…..n} (8)

$S_{A_{pr}} S_{B_{rp}} = S_{B_{pr}} S_{A_{rp}} = I$ for atleast one r ∈{1,2,…..n} and for each p ∈{1,2,…..n} (9)

From (8)

$S_{A_{pr}} = \phi$ or $S_{B_{rq}} = \phi$ or both $S_{A_{pr}} = S_{B_{rq}} = \phi$ for p≠q and r ∈{1,2,…..n} (10)

and

$S_{B_{pr}} = \phi$ or $S_{A_{rq}} = \phi$ or both $S_{B_{pr}} = S_{A_{rq}} = \phi$ for p≠q and r ∈{1,2,…..n} (11)

Also, using (11)

$S_{A_{pr}} = S_{B_{rp}} = \mathrm{I}$ and $S_{A_{rp}} = S_{B_{pr}} = \mathrm{I}$
for atleast one r ∈{1,2,…..n} and for each p ∈{1,2,…..n} (12)

Let the results of (12) equation exists for k=p (say) that is

$S_{A_{pk}} = S_{B_{kp}} = \mathrm{I} = [< (1,1), (0,0), (0,0) >]$

Then from (10), we get $S_{B_{pj}} = \phi = [< (0,0), (1,1), (1,1) >]$ for all i≠ j and

$S_{A_{qk}} = \phi = [< (0,0), (1,1), (1,1) >]$ for all p≠ q.

There is exactly one I in the kth row of S_B, and all of the other entries are all ϕ. Similarly, there is exactly one I in the kth column of S_A. By applying (4), the kth row of S_A contains precisely one I, and all of the other entries are all$\phi$and vice versa.

Then S_A and S_B are NZPM.

Hence the proof.

Note:

A neutrosophic z permutation matrix S_A be an NZM_n is invertible and S_A^T will be the inverse of S_A.

The NZPM will be the only invertible matrices in NZM_n.

4.20 Example

$$\text{Let} \quad S_A = \begin{bmatrix} < (0,0), (1,1), (1,1) > & < (1,1), (0,0), (0,0) > & < (0,0), (1,1), (1,1) > \\ < (0,0), (1,1), (1,1) > & < (0,0), (1,1), (1,1) > & < (1,1), (0,0), (0,0) > \\ < (1,1), (0,0), (0,0) > & < (0,0), (1,1), (1,1) > & < (0,0), (1,1), (1,1) > \end{bmatrix}$$

$$S_A^T = \begin{bmatrix} < (0,0), (1,1), (1,1) > & < (0,0), (1,1), (1,1) > & < (1,1), (0,0), (0,0) > \\ < (1,1), (0,0), (0,0) > & < (0,0), (1,1), (1,1) > & < (0,0), (1,1), (1,1) > \\ < (0,0), (1,1), (1,1) > & < (1,1), (0,0), (0,0) > & < (0,0), (1,1), (1,1) > \end{bmatrix}$$

and then

$$S_A S_A^T = \begin{bmatrix} < (1,1), (0,0), (0,0) > & < (0,0), (1,1), (1,1) > & < (0,0), (1,1), (1,1) > \\ < (0,0), (1,1), (1,1) > & < (1,1), (0,0), (0,0) > & < (0,0), (1,1), (1,1) > \\ < (0,0), (1,1), (1,1) > & < (0,0), (1,1), (1,1) > & < (1,1), (0,0), (0,0) > \end{bmatrix}$$

$=I_3$

5. Eigen Neutrosophic Z-set

A special case of eigen neutrosophic z-set for determining two set of Greatest Eigen Neutrosophic Z-set (GENZS) and Smallest Eigen Neutrosophic Z-set (LENZS) were mentioned.

5.1 Definition

Let H be a neutrosophic z-relation on the elements of a neutrosophic z-set X, i.e., $H \in NZNH(X \times X)$. Consider $S_{Z_A} \subseteq X$. Then S_{Z_A} is an eigen neutrosophic z-set associated with relation H if $S_{Z_A} = [< (T_{V_i}^x, T_{R_i}^x), (I_{V_i}^x, I_{R_i}^x), (F_{V_i}^x, F_{R_i}^x) >]$ satisfies the condition $S_{Z_A} * \text{H} = S_{Z_A}$ with $T_{V_i}^x, T_{R_i}^x \in [0,1]^2$,

$I_{V_i}^x, I_{R_i}^x \in [0,1]^2$ and $F_{V_i}^x, F_{R_i}^x \in [0,1]^2$ where $*$ is any composition operator.

5.2 Algorithm for finding Greatest Eigen Neutrosophic Z-set (GENZS)

The Greatest Eigen Neutrosophic Z- set from the neutrosophic Z -relation H, we apply the max-min composition operator. LetS_{Z_A} be a neutrosophic Z set,$S_{Z_A} \in NZNs(U)$ in which the truth membership values is the greatest of all the elements of the column relation H, the other two values are the smallest of all the elements of the columns of H.

$$
\begin{aligned}
T_V^A(x') &= max_{x \in X}\, T_V^A(x, x') && \text{for all } x' \in X_2 \\
T_R^A(x') &= max_{x \in X}\, T_R^A(x, x') && \text{for all } x' \in X_2 \\
I_V^A(x') &= min_{x \in X}\, I_V^A(x, x') && \text{for all } x' \in X_2 \\
I_R^A(x') &= min_{x \in X}\, I_R^A(x, x') && \text{for all } x' \in X_2 \qquad (\star) \\
F_V^A(x') &= min_{x \in X}\, F_V^A(x, x') && \text{for all } x' \in X_2 \\
F_R^A(x') &= min_{x \in X}\, F_R^A(x, x') && \text{for all } x' \in X_2
\end{aligned}
$$

First, consider Q_0 as a constant neutrosophic Z-set with a value equal to the minimum element of the set Q_1 . Q_0 is an eigen neutrosophic z-set but not greatest always. To find sequence of Q_n.

$$
\begin{aligned}
Q_1 \circ H &= Q_2 \\
Q_2 \circ H &= Q_1 \circ H^2 = Q_3 \\
&\vdots \qquad \vdots \\
&\vdots \qquad \vdots \\
Q_n \circ H &= Q_n \circ H^n = Q_{n+1}
\end{aligned}
$$

We observe the sequence Q_n is a decreasing sequence and bounded by Q_0 and Q_1.

(i.e) $Q_0 \subseteq \ldots\ldots\ldots\ldots \subseteq Q_{n+1} \subseteq Q_n \subseteq \ldots\ldots\ldots \subseteq Q_3 \subseteq Q_2 \subseteq Q_1$

5.2.1 Type I Algorithm for GENZS

1. Find the set Q_1 from H as directed by $\star$ equation
2. Set the index n=1 and calculate $Q_{n+1} = Q_n \circ H$

3. If $Q_{n+1} \neq Q_n$ then return to step 2
4. If $Q_{n+1} = Q_n$ then Q_n is the greatest eigen neutrosophic Z -set relatively associated with H.

5.2.2 Example (Type I on -GENSZ)

Let S ={S_1, S_2, S_3} be a neutrosophic z-set and H be the neutrosophic z relation on S represented as follows.

$$H = \begin{bmatrix} <(0.8,0.7),(0.3,0.2),(0.5,0.3)> & <(0.7,0.8),(0.4,0.5),(0.3,0.4)> & <(0.7,0.8),(0.4,0.5),(0.3,0.4)> \\ <(0.5,0.2),(0.2,0.1),(0.3,0.1)> & <(0.6,0.8),(0.3,0.1),(0.2,0.4)> & <(0.8,0.7),(0.2,0.3),(0.2,0.1)> \\ <(0.8,0.6),(0.2,0.4),(0.3,0.2)> & <(0.8,0.7),(0.8,0.1),(0.2,0.1)> & <(0.3,0.6),(0.3,0.6),(0.1,0.2)> \end{bmatrix}$$

First, we calculate Q_1

$$Q_1 = [<(0.8,0.7),(0.2,0.1),(0.5,0.3)> \quad <(0.8,0.8),(0.3,0.1),(0.2,0.1)> \quad <(0.8,0.1),(0,0.1),(0.1,0.1)>]$$

Next step for n=1, $Q_2 = Q_1 \circ H$

We get

$$Q_2 = [<(0.8,0.7),(0.2,0.1),(0.5,0.3)> \quad <(0.8,0.8),(0.3,0.1),(0.2,0.1)> \quad <(0.8,0.1),(0,0.1),(0.1,0.1)>] \circ \begin{bmatrix} <(0.8,0.7),(0.3,0.2),(0.5,0.3)> & <(0.7,0.8),(0.4,0.5),(0.3,0.4)> & <(0.7,0.8),(0.4,0.5),(0.3,0.4)> \\ <(0.5,0.2),(0.2,0.1),(0.3,0.1)> & <(0.6,0.8),(0.3,0.1),(0.2,0.4)> & <(0.8,0.7),(0.2,0.3),(0.2,0.1)> \\ <(0.8,0.6),(0.2,0.4),(0.3,0.2)> & <(0.8,0.7),(0.8,0.1),(0.2,0.1)> & <(0.3,0.6),(0.3,0.6),(0.1,0.2)> \end{bmatrix}$$

$$Q_2 = [<(0.8,0.7),(0.2,0.1),(0.3,0.1)> \quad <(0.8,0.8),(0.3,0.1),(0.2,0.1)> \quad <(0.8,0.7),(0.2,0.1),(0.1,0.1)>]$$

Now, check $Q_2 \neq Q_1$ we set n=2 and find $Q_3 = Q_2 \circ H$

$$Q_3 = [<(0.8,0.7),(0.2,0.1),(0.3,0.1)> \quad <(0.8,0.8),(0.3,0.1),(0.2,0.1)> \quad <(0.8,0.7),(0.2,0.1),(0.1,0.1)>]$$

Now, $Q_3 = Q_2$ then Q_2 is the desired GENZS resulted with association of H.

5.2.3 Type II algorithm for GENZS

- Find the set Q_1 from H as directed by $\star$ equation
- Successive composition of H, we get $H^{n+1} = \underbrace{H \circ H \ldots\ldots . \circ H}_{n+1\ times}$ compute Q_{n+1} from H^{n+1} using $\star$ equation
- If $Q_{n+1} \neq Q_n$ then return to step 2
- If $Q_{n+1} = Q_n$ then Q_n is the greatest eigen neutrosophic Z -set associated with H.

5.2.3.1. Illustration (using example 5.2.2 on type II algorithm-GENSZ)

First calculated Q_1

$$Q_1 = [< (0.8, 0.7), (0.2,0.1), (0.5,0.3) > \quad < (0.8, 0.8), (0.3,0.1), (0.2,0.1) > \quad < (0.8, 0.1), (0\ ,0.1), (0.1,0.1) >]$$

Next step to find Q_2

$$H^2 = H \circ H$$

$$H^2 = \begin{bmatrix} < (0.8, 0.7), (0.2,0.2), (0.3,0.2) > & < (0.8,0.8), (0.4,0.1), (0.2,0.1) > & < (0.8,0.7), (0.3,0.2), (0.1,0.2) > \\ < (0.8,0.6), (0.2,0.1), (0.3,0.2) > & < (0.8,0.8), (0.3,0.1), (0.2,0.1) > & < (0.6,0.7), (0.2,0.1), (0.2,0.2) > \\ < (0.8,0.6), (0.3,0.1), (0.3,0.1) > & < (0.7,0.7), (0.4,0.1), (0.2,0.2) > & < (0.8,0.7), (0.2,0.3), (0.1,0.1) > \end{bmatrix}$$

$$Q_2 = [< (0.8, 0.7), (0.2,0.1), (0.3,0.1) > \quad < (0.8, 0.8), (0.3,0.1), (0.2,0.1) > \quad < (0.8, 0.7), (0.2\ ,0.1), (0.1,0.1) >]$$

Now, check $Q_2 \neq Q_1$ find Q_3

$$H^3 = H^2 \circ H$$

$$H^3 = \begin{bmatrix} < (0.8, 0.7), (0.3,0.1), (0.3,0.1) > & < (0.7,0.8), (0.4,0.1), (0.2,0.2) > & < (0.8,0.7), (0.2,0.2), (0.1,0.1) > \\ < (0.8,0.7), (0.2,0.1), (0.3,0.1) > & < (0.7,0.8), (0.3,0.1), (0.2,0.1) > & < (0.8,0.7), (0.2,0.1), (0.2,0.1) > \\ < (0.8,0.6), (0.2,0.2), (0.3,0.2) > & < (0.8,0.7), (0.4,0.1), (0.2,0.1) > & < (0.8,0.7), (0.3,0.1), (0.1,0.2) > \end{bmatrix}$$

$$Q_2 = [< (0.8, 0.7), (0.2,0.1), (0.3,0.1) > \quad < (0.8, 0.8), (0.3,0.1), (0.2,0.1) > \quad < (0.8, 0.7), (0.2\ ,0.1), (0.1,0.1) >]$$

Now, $Q_3 = Q_2$ then Q_2 is the desired GENZS resulted with association of H.

5.3 Algorithm for finding Smallest Eigen Neutrosophic Z-set (LENZS)

The smallest eigen neutrosophic Z- set from the neutrosophic Z relation H_1, apply the min-max composition operator (★★). Let S_{Z_A} be a neutrosophic Z set, $S_{Z_A} \in NZNs(U)$ in which the truth membership values is the least of all the elements of the column relation H_1, the other membership values are the highest of all the elements of the columns of H_1.

$$\begin{aligned} T_V^A(x') &= min_{x \in X}\, T_V^A(x, x') && \text{for all } x' \in X_2 \\ T_R^A(x') &= min_{x \in X}\, T_R^A(x, x') && \text{for all } x' \in X_2 \\ I_V^A(x') &= max_{x \in X}\, I_V^A(x, x') && \text{for all } x' \in X_2 \\ I_R^A(x') &= max_{x \in X}\, I_R^A(x, x') && \text{for all } x' \in X_2 \\ F_V^A(x') &= max_{x \in X}\, F_V^A(x, x') && \text{for all } x' \in X_2 \\ F_R^A(x') &= max_{x \in X}\, F_R^A(x, x') && \text{for all } x' \in X_2 \end{aligned} \qquad (★★)$$

First consider Q'_0 as a constant neutrosophic Z-set with a value equal to the maximum element of the set Q'_1. Q'_0 is an eigen neutrosophic z- set but not smallest always. Find sequence of Q'_n.

$$Q'_1 \cdot H_1 = Q'_2$$

$$Q'_2 \cdot H_1 = Q'_1 \cdot H_1^2 = Q'_3$$

$$\vdots \qquad \vdots$$

$$\vdots \qquad \vdots$$

$$Q'_n \cdot H_1 = Q'_n \cdot H_1^n = Q'_{n+1}$$

5.3.1 Type I Algorithm for LENZS:

- Find the set Q'_1 from H_1 as directed by $\star\star$ equation
- Set the index n=1 and calculate $Q'_{n+1} = Q'_n \cdot H_1$
- If $Q'_{n+1} \neq Q'_n$ then return to step 2
- If $Q'_{n+1} = Q'_n$ then Q'_n is the lowest eigen neutrosophic Z -set relatively associated with H1.

5.3.1.1 Illustration (using example 4.2.2 on type I -LENSZ)

First, we calculate Q'_1

$$Q'_1 = [<(0.5, 0.2), (0.3,0.4), (0.5,0.3)> \quad <(0.6, 0.7), (0.8,0.5), (0.3,0.4)> \quad <(0.3, 0.6), (0.3, 0.6), (0.2,0.2)>]$$

Next step for n=1, $Q'_2 = Q'_1 \cdot H_1$

We get

$$Q'_2 = [<(0.6, 0.6), (0.3,0.4), (0.5,0.3)> \quad <(0.6, 0.7), (0.3,0.4), (0.3,0.4)> \quad <(0.3, 0.6), (0.3, 0.6), (0.2,0.2)>]$$

Now, check $Q'_2 \neq Q'_1$ we set n=2 and find $Q'_3 = Q'_2 \cdot H_1$

$$Q'_3 = [<(0.6, 0.6), (0.3,0.4), (0.5,0.3)> \quad <(0.6, 0.7), (0.3,0.4), (0.3,0.4)> \quad <(0.3, 0.6), (0.3, 0.6), (0.2,0.2)>]$$

Now, $Q'_3 = Q'_2$ then Q'_2 is the desired LENZS resulted with association of H1.

5.3.2 Type II Algorithm for LENZS:

Find the set Q'_1 from H_1 as directed by $\star\star$ equation

- Using min-max composition of H1, we get $H_1^{n+1} = \underbrace{H_1 \cdot H_1 \cdot \ldots\ldots \cdot H_1}_{n+1\ times}$ compute Q'_{n+1} from H_1^{n+1} using $\star\star$ equation
- If $Q'_{n+1} \neq Q'_n$ then return to step 2

- If $Q'_{n+1} = Q'_n$ then Q'_n is the lowest eigen neutrosophic Z -set relatively associated with H1.

Check the algorithm II for LENZS using above analogous method.

6. Applications in real life

The hotel management and the important features to look out for future enhancement of the hotel S_1 Ambiance, S_2 Taste of food, S_3 Monetary Value. Construct a NZMs using the relation $H(S_j, S_k)$which has true membership values (satisfied), the indeterminacy values (abstain), and the falsity value (not satisfied) which range from 0 to 1 for both neutrosophic (V) and reliability (R) values

$$H(S_j, S_k) = \left(\sum_{p=1,q=1}^{p=m,q=n} (\frac{T_{V_{pq}}}{m}, \frac{T_{R_{pq}}}{m}), \sum_{p=1,q=1}^{p=m,q=n} (\frac{I_{V_{pq}}}{m}, \frac{I_{R_{pq}}}{m}), \sum_{p=1,q=1}^{p=m,q=n} (\frac{F_{V_{pq}}}{m}, \frac{F_{R_{pq}}}{m}) \right) \tag{13}$$

and

$$H(S_i, S_i) = \frac{H(S_i, S_j) + H(S_i, S_k)}{m} \qquad \text{where j, k = 1,2,………..n} \tag{14}$$

$$\text{Score Value} = \frac{2 + T_{V_{ij}} T_{V_{ij}} - I_{V_{ij}} I_{R_{ij}} - F_{V_{ij}} F_{R_{ij}}}{3} \tag{15}$$

$$S_A max = \frac{GENZs(S_i) + LENZs(S_i)}{2} \tag{16}$$

$$S_A min = \frac{GENZs(S_i) - LENZs(S_i)}{2} \tag{17}$$

The effectiveness (E) and Uncertainty(U) can be found using average of max, min value and difference of max, min value divided by 2.

Using (13) and (14) equation we can find the normalized values and a NZM set is found.

$$H = \begin{bmatrix} <(0.6, 0.7), (0.4,0.3), (0.2,0.1)> & <(0.7,0.8), (0.4,0.3), (0.1,0.1)> & <(0.9,0.8), (0.6,0.7), (0.3,0.4)> \\ <(0.8,0.6), (0.6,0.4), (0.3,0.1)> & <(0.8,0.6), (0.5,0.6), (0,0.2)> & <(0.6,0.7), (0.5,0.3), (0.2,0.1)> \\ <(0.7,0.5), (0.4,0.3), (0.2,0.1)> & <(0.5,0.6), (0.4,0.3), (0.4,0.3)> & <(0.5,0.6), (0.4,0.3), (0.3,0.1)> \end{bmatrix}$$

First, we calculate Q_1

$$Q_1 = [<(0.8, 0.7), (0.4,0.3), (0.2,0.1)> \quad <(0.8, 0.9), (0.4,0.3), (0.1,0.1)> \quad <(0.9, 0.8), (0.4,0.3), (0.2,0.1)>]$$

Next step for n=1, $Q_2 = Q_1 \circ H$

$$Q_2 = [<(0.8, 0.7), (0.4,0.3), (0.2,0.1)> \quad <(0.8, 0.7), (0.4,0.3), (0.2,0.1)> \quad <(0.8, 0.7), (0.4,0.3), (0.2,0.1)>]$$

Now, find $Q_3 = Q_2 \circ H$

Q_3
$= [< (0.8, 0.7), (0.4,0.3), (0.2,0.1) > \quad < (0.8, 0.7), (0.4,0.3), (0.2,0.1) > \quad < (0.8, 0.7), (0.4,0.3), (0.2,0.1) >]$

Now, $Q_3 = Q_2$ then Q_2 is the desired GENZ set

Next, algorithm I to Calculate LENZS

calculate Q_1'

Q_1'
$= [< (0.6, 0.5), (0.6,0.4), (0.3,0.1) > \quad < (0.5, 0.6), (0.5,0.3), (0.4,0.3) > \quad < (0.5, 0.6), (0.6 ,0.7), (0.3,0.4) >]$

Next step for n=1, $Q_2' = Q_1' \cdot H_1$

Q_2'
$= [< (0.6, 0.6), (0.5,0.3), (0.3,0.1) > \quad < (0.5, 0.6), (0.5,0.3), (0.3,0.3) > \quad < (0.5, 0.6), (0.6 ,0.4), (0.3,0.1) >]$

Now, find $Q_3' = Q_2' \cdot H_1$

Q_3'
$= [< (0.6, 0.6), (0.5,0.3), (0.3,0.1) > \quad < (0.5, 0.6), (0.5,0.3), (0.3,0.3) > \quad < (0.5, 0.6), (0.6 ,0.4), (0.3,0.1) >]$

Now, $Q_3' = Q_2'$ then Q_2' is the acquired LENZS.

Then using (15), (16) and (17) find $S_A max$ and $S_A min$.

The effectiveness (E) and Uncertainty(U) can be found using average of max, min value and difference of max, min value divided by 2.

Table 2: Result of Effectiveness and Uncertainty

Parameters	S_1	S_2	S_3
$S_A max$	0.8066	0.8066	0.8066
$S_A min$	0.7266	0.6866	0.6766
E	0.7666	0.7463	0.7416
U	0.04	0.06	0.065

Table 2 depicts effectiveness E_1 is higher and uncertainty U_1 is lower which makes the ambiance is good. The highest uncertainty of U_3 gives the feedback of monetary value can be considered in future.

7. Conclusion

In this work, neutrosophic z-relation along with neutrosophic z- matrices and their certain connected properties and models are introduced. The algorithms for calculating two types of eigen neutrosophic z-set with analogous were shown. At last, a utilization of neutrosophic z- set in choice strategy problem using eigen neutrosophic z-set were found. As, an extension of this work in future, Neutrosophic Z-relations and Z-matrices may be generalized to higher-order structures such as Neutrosophic Z-tensors, enabling the modeling of multi-dimensional and highly uncertain data.

Eigen neutrosophic Z-set integration with machine learning, deep learning, and hybrid intelligent systems could open up new opportunities for uncertain data-driven decision making.

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