University of New Mexico

Filters in Hoops based on Lukasiewicz Neutrosophic set

N. Abirami[1] and M. Mary Jansirani[2,*]

[1]Research scholar, School of Sciences, Division of Mathematics, SRM- Institute of Science and Technology, Tiruchirappalli campus, SRM Nagar, Trichy-Chennai highway, near Samayapuram, Tiruchirappalli-621105, Tamil Nadu, India; abirami.n.2001@gmail.com,an0942@srmist.edu.in

[2]School of Sciences, Division of Mathematics, SRM- Institute of Science and Technology, Tiruchirappalli campus, SRM Nagar, Trichy-Chennai highway, near Samayapuram, Tiruchirappalli-621105, Tamil Nadu, India; anthuvanjansi@gmail.com

*Correspondence: anthuvanjansi@gmail.com

Abstract. The Lukasiewicz neutrosophic set ($\mathcal{LNS}$) is developed using the ideas of Lukasiewicz t-norm and s-norm. This set is then applied to the hoop structure and introduces the Lukasiewicz neutrosophic filter ($\mathcal{LNF}$). The properties of this filter, as well as its interconnection with the Lukasiewicz fuzzy filter ($\mathcal{LFF}$), are subsequently investigated.

Keywords: Lukasiewicz neutrosophic set; Lukasiewicz neutrosophic filter; Lukasiewicz fuzzy filter; Neutrosophic point ; Hoop

---

1. **Introduction**

Professor Lofti A. Zadeh, the father of fuzzy systems theory, proposed the idea of fuzzy logic in 1965. Extending this, Atanassov introduced intuitionistic fuzzy sets by adding the degree of belongingness and non-belongingness. As a generalization of the Zadeh and Atanassov system, Florentin Smarandache introduced the neutrosophic set, which deals with belongingness, in determinacy, and non-belongingness, in 1998 [15]. The concept of neutrosophic point and its properties was introduced by Gautam Chandra Ray and Sudeep Dey [13]. Neutrosophic algebraic structures were studied by Kandasamy and Smarandache [10, 17]. Many researchers have since explored neutrosophic filters in different algebraic frameworks [5–7, 12, 14, 16, 18, 19]. Hoops is an algebraic structure introduced by Bosbach [3]. Lukasiewicz logic is the many-valued logic, which is the extension of classical binary logic. Y.B. Jun introduced the Lukasiewicz fuzzy set using Lukasiewicz t-norm and the Lukasiewicz intuitionistic fuzzy set using Lukasiewicz t-norm and the dual of Lukasiewicz t-norm (Lukasiewicz t-conorm) and applied them in BCK algebras [9, 11], while Mohseni Takallo et al. studied the Lukasiewicz fuzzy filters in hoops [11] and Jun et al. extended these ideas to BE-algebras [8]. However, these filters cannot adequately capture indeterminacy, which plays a crucial role in uncertainty modeling.

In this paper, we address this gap by introducing the Lukasiewicz neutrosophic set, which extends Lukasiewicz fuzzy and intuitionistic fuzzy sets by capturing the indeterminacy component, which has remained unexplored in hoop structures. We introduced the Lukasiewicz neutrosophic filter and studied its characteristics using the neutrosophic point. We provide a relationship between the Lukasiewicz fuzzy filter and the Lukasiewicz neutrosophic filter.

1.1. *Comparative Analysis*

Lukasiewicz intuitionistic fuzzy filter handles uncertainty better than the Lukasiewicz fuzzy filter. However, both of these filters fail to capture indeterminancy. Our proposed Lukasiewicz neutrosophic filter plays a significant role where indeterminancy occurs, and is an advancement of existing theories.

Term	Notation
Lukasiewicz fuzzy set	$\mathcal{LFS}$
Lukasiewicz fuzzy filter	$\mathcal{LFF}$
Lukasiewicz neutrosophic set	$\mathcal{LNS}$
Lukasiewicz neutrosophic filter	$\mathcal{LNF}$

2. **Preliminaries**

Definition 2.1. [11]

If an algebra $(\mathcal{H}, \circledast, \rightsquigarrow, 1)$ satisfies the axioms $(\mathcal{H}1, \mathcal{H}2, \mathcal{H}3, \mathcal{H}4)$, then it is said to be a hoop.

$\mathcal{H}1 : n \rightsquigarrow n = 1 \; for \; all \; n \in \mathcal{H}$

$\mathcal{H}2 : n \circledast (n \rightsquigarrow s) = s \circledast (s \rightsquigarrow n) \; for \; all \; n, s \in \mathcal{H}$

$\mathcal{H}3 : n \rightsquigarrow (s \rightsquigarrow p) = (n \circledast s) \rightsquigarrow p \; for \; all \; n, s, p \in \mathcal{H}$

$\mathcal{H}4 : (\mathcal{H}, \circledast, 1)$ is a commutative monoid.

In a hoop $\mathcal{H}$, we say that an element n is less than or equal to s , that is $n \leq s$ if and only if $n \rightsquigarrow s = 1$.

Proposition 2.2. *[3]*

All of the following claims are met by every hoop $\mathcal{H}$.

$$n \circledast s \leq p \Longleftrightarrow n \leq s \rightsquigarrow p \tag{1}$$

$$n \circledast s \leq n, s \tag{2}$$

$$n \leq s \rightsquigarrow n \tag{3}$$

$$n \rightsquigarrow 1 = 1, 1 \rightsquigarrow n = n \tag{4}$$

$$(n \rightsquigarrow s) \circledast (s \rightsquigarrow p) \leq n \rightsquigarrow p \tag{5}$$

$$n \leq s \Longrightarrow n \circledast p \leq s \circledast p \tag{6}$$

$$n \circledast (n \rightsquigarrow s) \leq s \tag{7}$$

$\forall\, n, s, p \in \mathcal{H}$.

Definition 2.3. [13]

Let X be the universe of discourse. A neutrosophic set $\mathcal{N}$ over X is defined by

$$\mathcal{N} = \{(x, T_{\mathcal{N}}(x), I_{\mathcal{N}}(x), F_{\mathcal{N}}(x))|x \in X\}$$

where $T_{\mathcal{N}}, I_{\mathcal{N}}, F_{\mathcal{N}} : X \to [0,1]$ and $0 \leq T_{\mathcal{N}}(x) + I_{\mathcal{N}}(x) + F_{\mathcal{N}}(x) \leq 3$.

Definition 2.4. [13]

A neutrosophic set $\mathcal{P} = \{(x, T_{\mathcal{P}}(x), I_{\mathcal{P}}(x), F_{\mathcal{P}}(x))|x \in X\}$ is called a neutrosophic point if it is of the form

$$P(y) = \begin{cases} (\alpha, \beta, \gamma), & \text{if } y = x \\ (0, 1, 1), & \text{for } y \neq x \end{cases}$$

where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$.

For the neutrsophic point $\mathcal{P} = \{(x, T_{\mathcal{P}}(x), I_{\mathcal{P}}(x), F_{\mathcal{P}}(x))|x \in X\}$ with support x will be denoted by $P^{x}_{\alpha,\beta,\gamma}$ or $p < x, \alpha, \beta, \gamma >$ or $x_{\alpha,\beta,\gamma}$.

Definition 2.5. [13]

Let $\mathcal{N}$ be a neutrosophic set over X and $x_{\alpha,\beta,\gamma}$ be the neutrosophic point in X.

Then $x_{\alpha,\beta,\gamma}$ is said to belong to $\mathcal{N}$, denoted by $x_{\alpha,\beta,\gamma} \in \mathcal{N}$ if and only if

$$\alpha \leq T_{\mathcal{N}}(x), \beta \geq I_{\mathcal{N}}(x), \gamma \geq F_{\mathcal{N}(x)}.$$

Definition 2.6. [11]

Consider a fuzzy set $\mathcal{F}$ in $\mathcal{H}$ and let $\varepsilon \in (0,1)$. A function from $\mathcal{H}$ to [0,1] defined by

$$L^{\varepsilon}_{\mathcal{F}}(x) = max\{0, \mathcal{F}(x) + \varepsilon - 1\}$$

is called the Lukasiewicz fuzzy set ($\mathcal{LFS}$) of $\mathcal{F}$ in $\mathcal{H}$.

Definition 2.7. [11]

A $\mathcal{LFS}$ $L^{\varepsilon}_{\mathcal{F}}$ is called a Lukasiewicz fuzzy filter ($\mathcal{LFF}$) of $\mathcal{H}$ if it satisfies:

$$L^{\varepsilon}_{\mathcal{F}}(1) \textit{ is an upper bound of } \{L^{\varepsilon}_{\mathcal{F}}(x)|x \in \mathcal{H}\} \tag{8}$$

$$L^{\varepsilon}_{\mathcal{F}}(y) \geq \{L^{\varepsilon}_{\mathcal{F}}(x), L^{\varepsilon}_{\mathcal{F}}(x \rightsquigarrow y)\} \quad (9)$$

3. Lukasiewicz Neutrosophic Filters

The concepts of Lukasiewicz neutrosophic set($\mathcal{LNS}$) and Lukasiewicz neutrosophic filter($\mathcal{LNF}$) in the hoop structure were presented in this section, and their properties were examined.

Definition 3.1.
Let $\mathcal{N}$ be a neutrosophic set in a hoop $\mathcal{H}$ and let $\varepsilon, \delta, \theta \in [0,1]$ be such that $0 \leq \varepsilon + \delta + \theta \leq 3$. A mapping defined as an object in the form below

$$L_{\mathcal{N}} = \{(x, L^{\varepsilon}_{T_{\mathcal{N}}}, L^{\delta}_{I_{\mathcal{N}}}, L^{\theta}_{F_{\mathcal{N}}}) | x \in \mathcal{H}\}$$

where

$$L^{\varepsilon}_{T_{\mathcal{N}}} : \mathcal{H} \to [0,1], L^{\varepsilon}_{T_{\mathcal{N}}}(x) = max\{0, T_{\mathcal{N}}(x) + \varepsilon - 1\}$$
$$L^{\delta}_{I_{\mathcal{N}}} : \mathcal{H} \to [0,1], L^{\delta}_{I_{\mathcal{N}}}(x) = min\{1, I_{\mathcal{N}}(x) + \delta\}$$
$$L^{\theta}_{F_{\mathcal{N}}} : \mathcal{H} \to [0,1], L^{\theta}_{F_{\mathcal{N}}}(x) = min\{1, F_{\mathcal{N}}(x) + \theta\}$$

such that $0 \leq L^{\varepsilon}_{T_{\mathcal{N}}} + L^{\delta}_{I_{\mathcal{N}}} + L^{\theta}_{F_{\mathcal{N}}} \leq 3 \ \forall x \in \mathcal{H}$
is called the $\mathcal{LNS}$ of $\mathcal{N}$ in $\mathcal{H}$ denoted by $L_{\mathcal{N}} = (L^{\varepsilon}_{T_{\mathcal{N}}}, L^{\delta}_{I_{\mathcal{N}}}, L^{\theta}_{F_{\mathcal{N}}})$.

Let $L_{\mathcal{N}} = (L^{\varepsilon}_{T_{\mathcal{N}}}, L^{\delta}_{I_{\mathcal{N}}}, L^{\theta}_{F_{\mathcal{N}}})$ be the $\mathcal{LNS}$ of $\mathcal{N}$ in $\mathcal{H}$. If $(\varepsilon, \delta, \theta) = (1,0,0)$, then $max\{0, T_{\mathcal{N}}(x) + \varepsilon - 1\} = T_{\mathcal{N}}(x)$ and $min\{1, I_{\mathcal{N}}(x) + \delta\} = I_{\mathcal{N}}(x)$, $min\{1, F_{\mathcal{N}}(x) + \theta\} = F_{\mathcal{N}}(x)$. This shows that if $(\varepsilon, \delta, \theta) = (1,0,0)$, then the $\mathcal{LNS}$, $L_{\mathcal{N}} = (L^{\varepsilon}_{T_{\mathcal{N}}}, L^{\delta}_{I_{\mathcal{N}}}, L^{\theta}_{F_{\mathcal{N}}})$ of $\mathcal{N}$ is the classical neutrosophic set $\mathcal{N}$ in $\mathcal{H}$.
If $(\varepsilon, \delta, \theta) = (0,1,1)$, then $max\{0, T_{\mathcal{N}}(x) + \varepsilon - 1\} = 0$ and $min\{1, I_{\mathcal{N}}(x) + \delta\} = 1$, $min\{1, F_{\mathcal{N}}(x) + \theta\} = 1$.Thus if $(\varepsilon, \delta, \theta) = (0,1,1)$, then the $\mathcal{LNS}$, $L_{\mathcal{N}} = (L^{\varepsilon}_{T_{\mathcal{N}}}, L^{\delta}_{I_{\mathcal{N}}}, L^{\theta}_{F_{\mathcal{N}}})$ of $\mathcal{N}$ is the constant function with the value(0,1,1).Therefore, in handling the $\mathcal{LNS}$, the value of $(\varepsilon, \delta, \theta)$ can always be considered to be in (0,1) x (0,1) x (0,1).

Definition 3.2.
A $\mathcal{LNS}$ $L_{\mathcal{N}}$ in $\mathcal{H}$ is called a $\mathcal{LNF}$ of $\mathcal{H}$ if it satisfies:

$$n_{\alpha_1,\beta_1,\gamma_1} \in L_{\mathcal{N}}, s_{\alpha_2,\beta_2,\gamma_2} \in L_{\mathcal{N}} \Rightarrow (n \circledast s)_{\alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2, \gamma_1 \vee \gamma_2} \in L_{\mathcal{N}} \quad (10)$$

$$n \leq s, n_{\alpha_1,\beta_1,\gamma_1} \in L_{\mathcal{N}} \Rightarrow s_{\alpha_1,\beta_1,\gamma_1} \in L_{\mathcal{N}} \quad (11)$$

$for\ all\ n, s \in \mathcal{H}, 0 < \alpha_1, \alpha_2 \leq 1, 0 \leq \beta_1, \beta_2 < 1, 0 \leq \gamma_1, \gamma_2 < 1.$

Example 3.3.
Consider a Hoop structure $\mathcal{H}$ with the binary operations $''\circledast''\ and\ ''\rightsquigarrow''$ as below.

Table 3.1

$\circledast$	n	s	p	1
n	n	n	n	n
s	n	n	s	s
p	n	s	p	p
1	n	s	p	1

Table 3.2

$\rightsquigarrow$	n	s	p	1
n	1	1	1	1
s	s	1	1	1
p	n	s	1	1
1	n	s	p	1

Define the neutrosophic set $\mathcal{N}$ in $\mathcal{H}$ as follows,

$$\mathcal{N} = \begin{cases} (0.63, 0.23, 0.35), & \text{if } x = n \\ (0.63, 0.05, 0.30), & \text{if } x = s \\ (0.70, 0.01, 0.27), & \text{if } x = p \\ (0.91, 0, 0.2), & \text{if } x = 1 \end{cases}$$

For (ε, δ, θ)=(1,0.5,0.6), the $\mathcal{LNS}$ is given as follows

$$L_{\mathcal{N}} = \begin{cases} (0.63, 0.73, 0.95), & \text{if } x = n \\ (0.63, 0.55, 0.90), & \text{if } x = s \\ (0.70, 0.51, 0.87), & \text{if } x = p \\ (0.91, 0.5, 0.8), & \text{if } x = 1 \end{cases}$$

Now we can check that it is a $\mathcal{LNF}$ of $\mathcal{H}$.

Theorem 3.4.

A $\mathcal{LNS}$ $L_{\mathcal{N}}$ in $\mathcal{H}$ is a $\mathcal{LNF}$ of $\mathcal{H}$ if and only if the following conditions are valid.

$$\begin{gathered} L_{T^{\varepsilon}_{\mathcal{N}}}(1) \text{ is an upper bound of } \{L_{T^{\varepsilon}_{\mathcal{N}}}(n) | n \in \mathcal{H}\} \\ L^{\delta}_{I_{\mathcal{N}}}(1) \text{ is a lower bound of } \{L^{\delta}_{I_{\mathcal{N}}}(n) | n \in \mathcal{H}\} \\ L^{\theta}_{F_{\mathcal{N}}}(1) \text{ is a lower bound of } \{L^{\theta}_{F_{\mathcal{N}}}(n) | n \in \mathcal{H}\} \end{gathered} \tag{12}$$

$$n_{(\alpha_1,\beta_1,\gamma_1)} \in L_{\mathcal{N}}, (n \rightsquigarrow s)_{\alpha_2,\beta_2,\gamma_2} \in L_{\mathcal{N}} \Rightarrow s_{\alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2, \gamma_1 \vee \gamma_2} \in L_{\mathcal{N}} \tag{13}$$

for all $n, s \in \mathcal{H}, \forall\, 0 < \alpha_1, \alpha_2 \leq 1, 0 \leq \beta_1, \beta_2 < 1, 0 \leq \gamma_1, \gamma_2 < 1.$

Proof.

Suppose that $L_{\mathcal{N}}$ is a $\mathcal{LNF}$ of $\mathcal{H}$. If $L_{T^{\varepsilon}_{\mathcal{N}}}(1)$ *is not an upper bound of* $\{L_{T^{\varepsilon}_{\mathcal{N}}}(n) | n \in \mathcal{H}\}$,then $L_{T^{\varepsilon}_{\mathcal{N}}}(1) < L_{T^{\varepsilon}_{\mathcal{N}}}(m)$ for some $m \in \mathcal{H}$. since $m \leq 1$ and $m_{L_{T^{\varepsilon}_{\mathcal{N}}}(m), L^{\delta}_{I_{\mathcal{N}}}(m), L^{\theta}_{F_{\mathcal{N}}}(m)} \in L_{\mathcal{N}}$, it follows from (11) that $1_{L_{T^{\varepsilon}_{\mathcal{N}}}(m), L^{\delta}_{I_{\mathcal{N}}}(m), L^{\theta}_{F_{\mathcal{N}}}(m)} \in L_{\mathcal{N}}$. That is, $L_{T^{\varepsilon}_{\mathcal{N}}}(1) \geq L_{T^{\varepsilon}_{\mathcal{N}}}(m)$, $L^{\delta}_{I_{\mathcal{N}}}(1) \leq L^{\delta}_{I_{\mathcal{N}}}(m)$, $L^{\theta}_{F_{\mathcal{N}}}(1) \leq L^{\theta}_{F_{\mathcal{N}}}(m)$.

Thus $L_{T^{\varepsilon}_{\mathcal{N}}}(1)$ *is an upper bound of* $\{L_{T^{\varepsilon}_{\mathcal{N}}}(n) | n \in \mathcal{H}\}$,
$L^{\delta}_{I_{\mathcal{N}}}(1)$ *is a lower bound of* $\{L^{\delta}_{I_{\mathcal{N}}}(n) | n \in \mathcal{H}\}$, and
$L^{\theta}_{F_{\mathcal{N}}}(1)$ *is a lower bound of* $\{L^{\theta}_{F_{\mathcal{N}}}(n) | n \in \mathcal{H}\}$.

Let $n, s \in \mathcal{H}$ and $0 < \alpha_1, \alpha_2 \leq 1, 0 \leq \beta_1, \beta_2 < 1, 0 \leq \gamma_1, \gamma_2 < 1, n_{(\alpha_1,\beta_1,\gamma_1)} \in L_{\mathcal{N}}$ *and* $(n \rightsquigarrow s)_{\alpha_2,\beta_2,\gamma_2} \in L_{\mathcal{N}}$. Then by (10) we have $(n \circledast (n \rightsquigarrow s))_{\alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2, \gamma_1 \vee \gamma_2} \in L_{\mathcal{N}}$. Since $(n \circledast (n \rightsquigarrow s)) \leq s$, by (11) we have that, $s_{\alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2, \gamma_1 \vee \gamma_2} \in L_{\mathcal{N}}$.

Assume that $L_{\mathcal{N}}$ satisfies (12) and (13).

Let $n, s \in \mathcal{H}, 0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$, be such that $n \leq s$ and $n_{\alpha,\beta,\gamma} \in L_{\mathcal{N}}$. Then it follows from (12) that

$$\begin{gathered} L_{T^{\varepsilon}_{\mathcal{N}}}(n \rightsquigarrow s) = L_{T^{\varepsilon}_{\mathcal{N}}}(1) \geq L_{T^{\varepsilon}_{\mathcal{N}}}(n) \geq \alpha \\ L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s) = L^{\delta}_{I_{\mathcal{N}}}(1) \leq L^{\delta}_{I_{\mathcal{N}}}(n) \leq \beta \\ L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s) = L^{\theta}_{F_{\mathcal{N}}}(1) \leq L^{\theta}_{F_{\mathcal{N}}}(n) \leq \gamma \\ \Rightarrow (n \rightsquigarrow s)_{\alpha,\beta,\gamma} \in L^{\varepsilon}_{\mathcal{N}}. \end{gathered}$$

The condition (13) leads to $s_{\alpha,\beta,\gamma} \in L^{\varepsilon}_{\mathcal{N}}$ which proves (11).

Let $n, s \in \mathcal{H},$ *and* $0 < \alpha_1, \alpha_2 \leq 1, 0 \leq \beta_1, \beta_2 < 1, 0 \leq \gamma_1, \gamma_2 <$ be such that $n_{(\alpha_1,\beta_1,\gamma_1)} \in L_{\mathcal{N}}$ and $s_{\alpha_2,\beta_2,\gamma_2} \in L_{\mathcal{N}}$.

Since,

$$\begin{aligned} n \rightsquigarrow (s \rightsquigarrow n \circledast s) &= (n \circledast s) \rightsquigarrow (n \circledast s)\, (by\ \mathcal{H}3) \\ &= 1\, (by\ \mathcal{H}1). \end{aligned}$$

we have

$$\begin{gathered} L_{T^{\varepsilon}_{\mathcal{N}}}(n \rightsquigarrow (s \rightsquigarrow n \circledast s)) = L_{T^{\varepsilon}_{\mathcal{N}}}(1) \geq L_{T^{\varepsilon}_{\mathcal{N}}}(s) \geq \alpha_2 \\ L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow (s \rightsquigarrow n \circledast s)) = L^{\delta}_{I_{\mathcal{N}}}(1) \leq L^{\delta}_{I_{\mathcal{N}}}(s) \leq \beta_2 \\ L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow (s \rightsquigarrow n \circledast s)) = L^{\theta}_{F_{\mathcal{N}}}(1) \leq L^{\theta}_{F_{\mathcal{N}}}(s) \leq \gamma_2 \\ \text{i.e., } (n \rightsquigarrow (s \rightsquigarrow n \circledast s))_{\alpha_2,\beta_2,\gamma_2} \in L_{\mathcal{N}} \end{gathered}$$

Hence by (13) we have, $(s \rightsquigarrow n \circledast s)_{\alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2, \gamma_1 \vee \gamma_2} \in L_{\mathcal{N}}$.

Again by (13) we have, $(n \circledast s)_{\alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2, \gamma_1 \vee \gamma_2} \in L_{\mathcal{N}}$ which shows (10).

$\therefore L_{\mathcal{N}}$ in $\mathcal{H}$ is a $\mathcal{LNF}$ of $\mathcal{H}$. □

Theorem 3.5. *A* $\mathcal{LNS}$ $L_{\mathcal{N}}$ *in* $\mathcal{H}$ *is a* $\mathcal{LNF}$ *of* $\mathcal{H}$ *if and only if it satisfies:*

$$n_{\alpha,\beta,\gamma} \in L_{\mathcal{N}} \Rightarrow 1_{\alpha,\beta,\gamma} \in L_{\mathcal{N}} \tag{14}$$

$\forall\ n \in \mathcal{H},\ 0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1.$

$$L_{T^{\varepsilon}_{\mathcal{N}}}(s) \geq min\{L_{T^{\varepsilon}_{\mathcal{N}}}(n), L_{T^{\varepsilon}_{\mathcal{N}}}(n \rightsquigarrow s)\}$$
$$L^{\delta}_{I_{\mathcal{N}}}(s) \leq max\{L^{\delta}_{I_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\} \tag{15}$$
$$L^{\theta}_{F_{\mathcal{N}}}(s) \leq max\{L^{\theta}_{F_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\}$$

$forall\ n, s \in \mathcal{H}.$

Proof.

Assume that $L_{\mathcal{N}}$ is a $\mathcal{LNF}$ of $\mathcal{H}$.

Let $n \in \mathcal{H}$ and $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$ be such that $n_{\alpha,\beta,\gamma} \in L_{\mathcal{N}}$.

The condition (12) leads to

$$L_{T^{\varepsilon}_{\mathcal{N}}}(1) \geq L_{T^{\varepsilon}_{\mathcal{N}}}(n) \geq \alpha$$
$$L^{\delta}_{I_{\mathcal{N}}}(1) \leq L^{\delta}_{I_{\mathcal{N}}}(n) \leq \beta$$
$$L^{\theta}_{F_{\mathcal{N}}}(1) \leq L^{\theta}_{F_{\mathcal{N}}}(n) \leq \gamma$$

which implies that $1_{\alpha,\beta,\gamma} \in L_{\mathcal{N}}$.

we know that $n_{L_{T^{\varepsilon}_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n)} \in L_{\mathcal{N}}$ and

$(n \rightsquigarrow s)_{L_{T^{\varepsilon}_{\mathcal{N}}}(n\rightsquigarrow s), L^{\delta}_{I_{\mathcal{N}}}(n\rightsquigarrow s), L^{\theta}_{F_{\mathcal{N}}}(n\rightsquigarrow s)} \in L_{\mathcal{N}}\ \forall n, s \in \mathcal{H}$. It follows from (13) that

$s_{L_{T^{\varepsilon}_{\mathcal{N}}}(n)\wedge L_{T^{\varepsilon}_{\mathcal{N}}}(n\rightsquigarrow s), L^{\delta}_{I_{\mathcal{N}}}(n)\vee L^{\delta}_{I_{\mathcal{N}}}(n\rightsquigarrow s), L^{\theta}_{F_{\mathcal{N}}}(n)\vee L^{\theta}_{F_{\mathcal{N}}}(n\rightsquigarrow s)} \in L_{\mathcal{N}}$ and hence

$$L_{T^{\varepsilon}_{\mathcal{N}}}(s) \geq min\{L_{T^{\varepsilon}_{\mathcal{N}}}(n), L_{T^{\varepsilon}_{\mathcal{N}}}(n \rightsquigarrow s)\}$$
$$L^{\delta}_{I_{\mathcal{N}}}(s) \leq max\{L^{\delta}_{I_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\}$$
$$L^{\theta}_{F_{\mathcal{N}}}(s) \leq max\{L^{\theta}_{F_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\}$$

for all n,s $\in \mathcal{H}$.

Conversely suppose that $L_{\mathcal{N}}$ satisfies (14) and (15).

Since $n_{L_{T^{\varepsilon}_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n)} \in L_{\mathcal{N}}$ for all $n \in \mathcal{H}$, we have by (14) that

$1_{L_{T^{\varepsilon}_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n)} \in L_{\mathcal{N}}$ and so $L_{T^{\varepsilon}_{\mathcal{N}}}(1) \geq L_{T^{\varepsilon}_{\mathcal{N}}}(n)$, $L^{\delta}_{I_{\mathcal{N}}}(1) \leq L^{\delta}_{I_{\mathcal{N}}}(n)$, $L^{\theta}_{F_{\mathcal{N}}}(1) \leq L^{\theta}_{F_{\mathcal{N}}}(n)$ for all $n \in \mathcal{H}$. Hence

$$L_{T^{\varepsilon}_{\mathcal{N}}}(1)\ is\ an\ upper\ bound\ of\ \{L_{T^{\varepsilon}_{\mathcal{N}}}(n) | n \in \mathcal{H}\}$$
$$L^{\delta}_{I_{\mathcal{N}}}(1)\ is\ a\ lower\ bound\ of\ \{L^{\delta}_{I_{\mathcal{N}}}(n) | n \in \mathcal{H}\}$$
$$L^{\theta}_{F_{\mathcal{N}}}(1)\ is\ a\ lower\ bound\ of\ \{L^{\theta}_{F_{\mathcal{N}}}(n) | n \in \mathcal{H}\}$$

Let $n, s \in \mathcal{H}$ and $0 < \alpha_1, \alpha_2 \leq 1, 0 \leq \beta_1, \beta_2 < 1, 0 \leq \gamma_1, \gamma_2 < 1$ be such that $n_{(\alpha_1,\beta_1,\gamma_1)} \in L_{\mathcal{N}}$ and $(n \rightsquigarrow s)_{\alpha_2,\beta_2,\gamma_2} \in L_{\mathcal{N}}$. Then

$$L_{T^{\varepsilon}_{\mathcal{N}}}(n) \geq \alpha_1,\ L_{T^{\varepsilon}_{\mathcal{N}}}(n \rightsquigarrow s) \geq \alpha_2$$
$$L^{\delta}_{I_{\mathcal{N}}}(n) \leq \beta_1,\ L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s) \leq \beta_2$$
$$L^{\theta}_{F_{\mathcal{N}}}(n) \leq \gamma_1,\ L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s) \leq \gamma_2$$

which imply from (15) that,

$$L_{T^{\varepsilon}_{\mathcal{N}}}(s) \geq min\{L_{T^{\varepsilon}_{\mathcal{N}}}(n), L_{T^{\varepsilon}_{\mathcal{N}}}(n \rightsquigarrow s)\} \geq min\{\alpha_1, \alpha_2\}$$
$$L^{\delta}_{I_{\mathcal{N}}}(s) \leq max\{L^{\delta}_{I_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\} \leq max\{\beta_1, \beta_2\}$$
$$L^{\theta}_{F_{\mathcal{N}}}(s) \leq max\{L^{\theta}_{F_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\} \leq max\{\gamma_1, \gamma_2\}$$

Thus $s_{\alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2, \gamma_1 \vee \gamma_2} \in L_{\mathcal{N}}$.
Therefore $L_{\mathcal{N}}$ is a $\mathcal{LNF}$ of $\mathcal{H}$ by (3.4). □

Proposition 3.6.
Every $\mathcal{LNF}$ $L_{\mathcal{N}}$ of $\mathcal{H}$ satisfies the following condition.

$$p \leq n \rightsquigarrow s, n_{(\alpha_1,\beta_1,\gamma_1)} \in L_{\mathcal{N}}, p_{\alpha_2,\beta_2,\gamma_2} \in L_{\mathcal{N}} \Rightarrow s_{\alpha_1 \wedge \alpha_2, \beta_1 \vee \beta_2, \gamma_1 \vee \gamma_2} \in L_{\mathcal{N}} \quad (16)$$

$forall\ n, s, p \in \mathcal{H}, 0 < \alpha_1, \alpha_2 \leq 1, 0 \leq \beta_1, \beta_2 < 1, 0 \leq \gamma_1, \gamma_2 < 1.$

Proof.
Let $n, s, p \in \mathcal{H}$ and $0 < \alpha_1, \alpha_2 \leq 1, 0 \leq \beta_1, \beta_2 < 1, 0 \leq \gamma_1, \gamma_2 < 1$ be such that $p \leq n \rightsquigarrow s$, $n_{(\alpha_1,\beta_1,\gamma_1)} \in L_{\mathcal{N}}$ and $p_{\alpha_2,\beta_2,\gamma_2} \in L_{\mathcal{N}}$.
Then $p \rightsquigarrow (n \rightsquigarrow s) = 1$.
$L_{T^{\varepsilon}_{\mathcal{N}}}(n) \geq \alpha_1$, $L^{\delta}_{I_{\mathcal{N}}}(n) \leq \beta_1$, $L^{\theta}_{F_{\mathcal{N}}}(n) \leq \gamma_1$
$L_{T^{\varepsilon}_{\mathcal{N}}}(p) \geq \alpha_2$, $L^{\delta}_{I_{\mathcal{N}}}(p) \leq \beta_2$, $L^{\theta}_{F_{\mathcal{N}}}(p) \leq \gamma_2$
Hence

$$\begin{aligned} L_{T^{\varepsilon}_{\mathcal{N}}}(s) &\geq min\{L_{T^{\varepsilon}_{\mathcal{N}}}(n), L_{T^{\varepsilon}_{\mathcal{N}}}(n \rightsquigarrow s)\} \\ &\geq min\{L_{T^{\varepsilon}_{\mathcal{N}}}(n), min\{L_{T^{\varepsilon}_{\mathcal{N}}}(p \rightsquigarrow (n \rightsquigarrow s)), L_{T^{\varepsilon}_{\mathcal{N}}}(p)\}\} \\ &= min\{L_{T^{\varepsilon}_{\mathcal{N}}}(n), min\{L_{T^{\varepsilon}_{\mathcal{N}}}(1), L_{T^{\varepsilon}_{\mathcal{N}}}(p)\}\} \\ &= min\{L_{T^{\varepsilon}_{\mathcal{N}}}(n), L_{T^{\varepsilon}_{\mathcal{N}}}(p)\} \\ &\geq min\{\alpha_1, \alpha_2\} \end{aligned}$$

and

$$\begin{aligned} L^{\delta}_{I_{\mathcal{N}}}(s) &\leq max\{L^{\delta}_{I_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\} \\ &\leq max\{L^{\delta}_{I_{\mathcal{N}}}(n), max\{L^{\delta}_{I_{\mathcal{N}}}(p \rightsquigarrow (n \rightsquigarrow s)), L^{\delta}_{I_{\mathcal{N}}}(p)\}\} \\ &= max\{L^{\delta}_{I_{\mathcal{N}}}(n), max\{L^{\delta}_{I_{\mathcal{N}}}(1), L^{\delta}_{I_{\mathcal{N}}}(p)\}\} \\ &= max\{L^{\delta}_{I_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(p)\} \\ &\leq max\{\beta_1, \beta_2\} \end{aligned}$$

and

$$\begin{aligned}L^{\theta}_{F_{\mathcal{N}}}(s) &\leq max\{L^{\theta}_{F_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\}\\ &\leq max\{L^{\theta}_{F_{\mathcal{N}}}(n), max\{L^{\theta}_{F_{\mathcal{N}}}(p \rightsquigarrow (n \rightsquigarrow s)), L^{\theta}_{F_{\mathcal{N}}}(p)\}\}\\ &= max\{L^{\theta}_{F_{\mathcal{N}}}(n), max\{L^{\theta}_{F_{\mathcal{N}}}(1), L^{\theta}_{F_{\mathcal{N}}}(p)\}\}\\ &= max\{L^{\theta}_{F_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(p)\}\\ &\leq max\{\gamma_1, \gamma_2\}\end{aligned}$$

Thus we have that $s_{\alpha_1\wedge\alpha_2,\beta_1\vee\beta_2,\gamma_1\vee\gamma_2} \in L_{\mathcal{N}}$. □

4. Relation between Lukasiewicz Neutrosophic Filter and Lukasiewicz Fuzzy Filter

In this section, we discussed the relationship between $\mathcal{LNF}$ and $\mathcal{LFF}$.

Theorem 4.1.

A $\mathcal{LNS}$ $L_{\mathcal{N}}$ in a hoop is a $\mathcal{LNF}$ if and only if $L_{\mathcal{N}}$ satisfies the following three conditions.

(1) $L^{\varepsilon}_{T_{\mathcal{N}}}$ *is a* $\mathcal{LFF}$ *of* $\mathcal{H}$

(2) $1 - L^{\delta}_{I_{\mathcal{N}}}$ *is a* $\mathcal{LFF}$ *of* $\mathcal{H}$

(3) $1 - L^{\theta}_{F_{\mathcal{N}}}$ *is a* $\mathcal{LFF}$ *of* $\mathcal{H}$,
where $(1 - L^{\delta}_{I_{\mathcal{N}}})(n) = 1 - L^{\delta}_{I_{\mathcal{N}}}(n)$, $(1 - L^{\theta}_{F_{\mathcal{N}}})(n) = 1 - L^{\theta}_{F_{\mathcal{N}}}(n)$

Proof.

Suppose that $L_{\mathcal{N}}$ is a $\mathcal{LNF}$ in $\mathcal{H}$.

From (14) we have that $L^{\varepsilon}_{T_{\mathcal{N}}}(1)$ *is an upper bound of* $\{L^{\varepsilon}_{T_{\mathcal{N}}}(n)|n \in \mathcal{H}\}$ and by (15) we have $L^{\varepsilon}_{T_{\mathcal{N}}}(s) \geq min\{L^{\varepsilon}_{T_{\mathcal{N}}}(n), L^{\varepsilon}_{T_{\mathcal{N}}}(n \rightsquigarrow s)\}$. Hence $L^{\varepsilon}_{T_{\mathcal{N}}}$ is a $\mathcal{LFF}$ of $\mathcal{H}$ by (2.7).

Similarly from (14) we have that $L^{\delta}_{I_{\mathcal{N}}}(1)$ *is a lower bound of* $\{L^{\delta}_{I_{\mathcal{N}}}(n)|n \in \mathcal{H}\}$.

That is,

$$L^{\delta}_{I_{\mathcal{N}}}(1) \leq L^{\delta}_{I_{\mathcal{N}}}(n).$$
$$1 - L^{\delta}_{I_{\mathcal{N}}}(1) \geq 1 - L^{\delta}_{I_{\mathcal{N}}}(n)$$
$$(1 - L^{\delta}_{I_{\mathcal{N}}})(1) \geq (1 - L^{\delta}_{I_{\mathcal{N}}})(n)$$

$\therefore (1 - L^{\delta}_{I_{\mathcal{N}}})(1)$ is an upper bound of $\{(1 - L^{\delta}_{I_{\mathcal{N}}})(n)|n \in \mathcal{H}\}$.

By (15), we have

$$\begin{aligned}L^{\delta}_{I_{\mathcal{N}}}(s) &\leq max\{L^{\delta}_{I_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\}\\ 1 - L^{\delta}_{I_{\mathcal{N}}}(s) &\geq 1 - max\{L^{\delta}_{I_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\}\\ &= min\{1 - L^{\delta}_{I_{\mathcal{N}}}(n), 1 - L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\}.\end{aligned}$$

Thus $(1 - L^{\delta}_{I_{\mathcal{N}}})(s) \geq min\{(1 - L^{\delta}_{I_{\mathcal{N}}})(n), (1 - L^{\delta}_{I_{\mathcal{N}}})(n \rightsquigarrow s)\}$

$\therefore 1 - L^{\delta}_{I_{\mathcal{N}}}$ is a $\mathcal{LFF}$ of $\mathcal{H}$ by (2.7).

And from (14) we have that $L^{\theta}_{F_{\mathcal{N}}}(1)$ *is a lower bound of* $\{L^{\theta}_{F_{\mathcal{N}}}(n)|n \in \mathcal{H}\}$.

That is,

$$L^{\theta}_{F_{\mathcal{N}}}(1) \leq L^{\theta}_{F_{\mathcal{N}}}(n).$$
$$1 - L^{\theta}_{F_{\mathcal{N}}}(1) \geq 1 - L^{\theta}_{F_{\mathcal{N}}}(n)$$
$$(1 - L^{\theta}_{F_{\mathcal{N}}})(1) \geq (1 - L^{\theta}_{F_{\mathcal{N}}})(n)$$

$\therefore (1 - L^{\theta}_{F_{\mathcal{N}}})(1)$ is an upper bound of $\{(1 - L^{\theta}_{F_{\mathcal{N}}})(n)|n \in \mathcal{H}\}$.

By (15), we have

$$L^{\theta}_{F_{\mathcal{N}}}(s) \leq max\{L^{\theta}_{F_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\}$$
$$1 - L^{\theta}_{F_{\mathcal{N}}}(s) \geq 1 - max\{L^{\theta}_{F_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\}$$
$$= min\{1 - L^{\theta}_{F_{\mathcal{N}}}(n), 1 - L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\}$$

Thus $(1 - L^{\theta}_{F_{\mathcal{N}}})(s) \geq min\{(1 - L^{\theta}_{F_{\mathcal{N}}})(n), (1 - L^{\delta}_{I_{\mathcal{N}}})(n \rightsquigarrow s)\}$.

Therefore by (2.7), $1 - L^{\theta}_{F_{\mathcal{N}}}$ is a $\mathcal{LFF}$ of $\mathcal{H}$.

Conversely suppose that a $\mathcal{LNS}$ in $\mathcal{H}$ satisfies the conditions (1),(2),(3). Then by definition 2.7 we have,

$\forall n, s \in \mathcal{H}$, $L_{T^{\varepsilon}_{\mathcal{N}}}(1) \geq L_{T^{\varepsilon}_{\mathcal{N}}}(n)$ and $L_{T^{\varepsilon}_{\mathcal{N}}}(s) \geq min\{L_{T^{\varepsilon}_{\mathcal{N}}}(n), L_{T^{\varepsilon}_{\mathcal{N}}}(n \rightsquigarrow s)\}$

$\forall n, s \in \mathcal{H}$, $(1-L^{\delta}_{I_{\mathcal{N}}})(1) \geq (1-L^{\delta}_{I_{\mathcal{N}}})(n)$ and $(1-L^{\delta}_{I_{\mathcal{N}}})(s) \geq min\{(1-L^{\delta}_{I_{\mathcal{N}}})(n), (1-L^{\delta}_{I_{\mathcal{N}}})(n \rightsquigarrow s)\}$

$\forall n, s \in \mathcal{H}$, $(1 - L^{\theta}_{F_{\mathcal{N}}})(1) \geq (1 - L^{\theta}_{F_{\mathcal{N}}})(n)$ and

$(1 - L^{\theta}_{F_{\mathcal{N}}})(s) \geq min\{(1 - L^{\theta}_{F_{\mathcal{N}}})(n), (1 - L^{\theta}_{F_{\mathcal{N}}})(n \rightsquigarrow s)\}$

Let $n_{\alpha,\beta,\gamma} \in L^{\varepsilon}_{\mathcal{N}}$.

$\Rightarrow L_{T^{\varepsilon}_{\mathcal{N}}}(n) \geq \alpha, L^{\delta}_{I_{\mathcal{N}}}(n) \leq \beta, L^{\theta}_{F_{\mathcal{N}}}(n) \leq \gamma$

Hence $L_{T^{\varepsilon}_{\mathcal{N}}}(1) \geq L_{T^{\varepsilon}_{\mathcal{N}}}(n) \geq \alpha$.

$\therefore L_{T^{\varepsilon}_{\mathcal{N}}}(1) \geq \alpha$

$$\begin{aligned} L^{\delta}_{I_{\mathcal{N}}}(1) &= (1 - (1 - L^{\delta}_{I_{\mathcal{N}}}))(1) \\ &\leq (1 - (1 - L^{\delta}_{I_{\mathcal{N}}}))(n) \\ &= L^{\delta}_{I_{\mathcal{N}}}(n) \\ &\leq \beta \\ \therefore L^{\delta}_{I_{\mathcal{N}}}(1) &\leq \beta \end{aligned}$$

$$\begin{aligned} L^{\theta}_{F_{\mathcal{N}}}(1) &= (1 - (1 - L^{\theta}_{F_{\mathcal{N}}}))(1) \\ &\leq (1 - (1 - L^{\theta}_{F_{\mathcal{N}}}))(n) \\ &= L^{\theta}_{F_{\mathcal{N}}}(n) \\ &\leq \gamma \\ \therefore L^{\theta}_{F_{\mathcal{N}}}(1) &\leq \gamma \end{aligned}$$

Hence $1_{\alpha,\beta,\gamma} \in L_{\mathcal{N}}$. Therefore the condition 14 is proved.

Also we have

$$\begin{aligned}
L_{T^{\varepsilon}_{\mathcal{N}}}(s) &\geq min\{L_{T^{\varepsilon}_{\mathcal{N}}}(n), L_{T^{\varepsilon}_{\mathcal{N}}}(n \rightsquigarrow s)\}. \\
(1 - L^{\delta}_{I_{\mathcal{N}}})(s) &\geq min\{(1 - L^{\delta}_{I_{\mathcal{N}}})(n), (1 - L^{\delta}_{I_{\mathcal{N}}})(n \rightsquigarrow s)\} \\
&= min\{1 - L^{\delta}_{I_{\mathcal{N}}}(n), 1 - L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\} \\
&= 1 - max\{L^{\delta}_{I_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\} \\
1 - L^{\delta}_{I_{\mathcal{N}}}(s) &\geq 1 - max\{L^{\delta}_{I_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\} \\
L^{\delta}_{I_{\mathcal{N}}}(s) &\leq max\{L^{\delta}_{I_{\mathcal{N}}}(n), L^{\delta}_{I_{\mathcal{N}}}(n \rightsquigarrow s)\} \\
(1 - L^{\theta}_{F_{\mathcal{N}}})(s) &\geq min\{(1 - L^{\theta}_{F_{\mathcal{N}}})(n), (1 - L^{\theta}_{F_{\mathcal{N}}})(n \rightsquigarrow s)\} \\
&= min\{1 - L^{\theta}_{F_{\mathcal{N}}}(n), 1 - L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\} \\
&= 1 - max\{L^{\theta}_{F_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\} \\
1 - L^{\theta}_{F_{\mathcal{N}}}(s) &\geq 1 - max\{L^{\theta}_{F_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\} \\
L^{\theta}_{F_{\mathcal{N}}}(s) &\leq max\{L^{\theta}_{F_{\mathcal{N}}}(n), L^{\theta}_{F_{\mathcal{N}}}(n \rightsquigarrow s)\}
\end{aligned}$$

Therefore the condition 15 is proved.

Hence $L_{\mathcal{N}}$ is a $\mathcal{LNF}$ in $\mathcal{H}$ □

Example 4.2. consider the example of $\mathcal{LNF}$ given in 3.3.

$$L_{T^{\varepsilon}_{\mathcal{N}}} = \begin{cases} 0.63, & \text{if } x = n \\ 0.63, & \text{if } x = s \\ 0.70, & \text{if } x = p \\ 0.91, & \text{if } x = 1 \end{cases}$$

$$1 - L_{I^{\delta}_{\mathcal{N}}} = \begin{cases} 0.27, & \text{if } x = n \\ 0.45, & \text{if } x = s \\ 0.49, & \text{if } x = p \\ 0.5, & \text{if } x = 1 \end{cases}$$

$$1 - L_{F^{\theta}_{\mathcal{N}}} = \begin{cases} 0.05, & \text{if } x = n \\ 0.1, & \text{if } x = s \\ 0.13, & \text{if } x = p \\ 0.2, & \text{if } x = 1 \end{cases}$$

It can be easily verified that $L^{\varepsilon}_{T_{\mathcal{N}}}, 1 - L^{\delta}_{I_{\mathcal{N}}}, 1 - L^{\theta}_{F_{\mathcal{N}}}$ is a $\mathcal{LFF}$ of $\mathcal{H}$.

5. Limitations

This work focuses on theoretical development. And a comparison with real-life applications or a case study is not provided.

6. Conclusions

Using the Lukasiewicz t-norm and Lukasiewicz s-norm, the concept of Lukasiewicz neutrosophic set is introduced. Lukasiewicz neutrosophic filter is defined based on the neutrosophic point, and its properties have been studied.

7. Future work

In the future, we will extend this to other algebraic structures and provide a case study and real-world implementation of this theory. And also we examine the types of Lukasiewicz neutrosophic filter and the relation among them. **Conflicts of Interest:** We declare that there is no conflict of interest.

References

1. Abirami, N., Jansirani, M. M. (2025). £ ukasiewicz Intuitionistic Fuzzy Filters in Hoops and its Application in Medical Diagnosis. International Journal of Neutrosophic Science (IJNS), 26(2).
2. Borzooei, R. A., Aaly, M. (2020). Results on hoops. Journal of Algebraic Hyperstructures and Logical Algebras, 1(1), 61-77..
3. Bosbach, B. (1970). Komplementäre halbgruppen kongruenzen und quotienten. Fundamenta Mathematicae, 69(1), 1-14.
4. Gokila, T., Jansirani, M. M. (2025). Sv-Lukasiewicz Neutrosophic Fuzzy Matrix: A Robust Approach to Identifying an Ultimate Laptop Model. Neutrosophic Sets and Systems, 82, 335-351.
5. Malleswari, V. S., Luka, G., Kothuru, B., Rao, T. S., Babu, V. A. (2023). Application of Neutrosophic implicative filters and Neutrosophic positive implicative filters in Lattice implication algebra. International Journal of Neutrosophic Science (IJNS), 21(1).
6. Ibrahim, A., Gunaseeli, S. (2023). On Neutrosophic filter and fantastic filter of BL-algebras. International Journal of Neutrosophic Science (IJNS), 21(2).
7. Ibrahim, A., Gunaseeli, S. K. H., Broumi, S. (2023). Some Types of Neutrosophic Filters in Basic Logic Algebras. Neutrosophic Systems With Applications.
8. Jun, Y. B., Ahn, S. S. (2022). Lukasiewicz fuzzy BE-algebras and BE-filters. European Journal of Pure and Applied Mathematics, 15(3), 924-937.
9. Jun, Y. B. (2025). Ł ukasiewicz intuitionistic fuzzy sets and its application in BCK-algebras and BCI-algebras. ANNALS OF FUZZY MATHEMATICS AND INFORMATICS, 29(3), 291-305.
10. Kandasamy, W. B. V., Smarandache, F. (2006). Neutrosophic rings. https://doi.org/10.48550/ARXIV.MATH/0607765
11. Mohseni Takallo, M., Aaly Kologani, M., Jun, Y. B., Borzooei, R. A. (2024). Łukasiewicz fuzzy filters in hoops. Journal of Algebraic Systems, 12(1), 1-20.
12. Takallo, M. M., Kologani, M. A. (2020). MBJ-neutrosophic filters of equality algebras. Infinite Study.
13. Ray, G. C., Dey, S. (2021). Neutrosophic point and its neighbourhood structure. Neutrosophic sets and Systems, 43, 156-168.

14. Salama, A. A., Elagamy, H. (2013). Neutrosophic filters. International Journal of Computer Science Engineering and Information Technology Reseearch (IJCSEITR), 3(1), 307-312.
15. Smarandache, F. (2001). A unifying field in logics: Neutrosophic logic, neutrosophic set, neutrosophic probability and statistics. arXiv preprint math/0101228.
16. Topal, S., Sönmez, M. G. (2020). Ideals and Filters in Neutrosophic Lattices. Fundamentals of Contemporary Mathematical Sciences, 1(1), 30-36.
17. Kandasamy, W. V., Smarandache, F. (2006). Some neutrosophic algebraic structures and neutrosophic N-algebraic structures. Infinite Study.
18. Zhang, X. H., Yu, P., Smarandache, F., Park, C. (2018). Redefined neutrosophic filters in BE-algebras. Ital J Pure Appl Math.
19. Zhang, X., Mao, X., Wu, Y., Zhai, X. (2018). Neutrosophic filters in pseudo-BCI algebras. International Journal for Uncertainty Quantification, 8(6).

Received: June 1, 2025. Accepted: Nov 17, 2025