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Linguistic Dual Hesitant Hypersoft Set and their Application with Decision Making in Medical Diagnosis and Treatment

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Abstract

This study presents Linguistic Dual Hesitant Hypersoft Set (LDHHS) for Analysing Medical Diagnose. In LDHHS, linguistic terms are combined with Dual Hesitant logic to handle uncertainty and vagueness in Decision-Making. The Hypersoft Set extend traditional soft sets by handling multi-attributes and interdependent parameters in Decision-Making. In the context of LDHHS can be used to categorize and assess different aspects of Aggregation and using Decision-Making in Medical Diagnose. In this study the development of this framework, its application and its potential impact using a Gastroesophageal reflux disease (GERD) case study to illustrate its effectiveness.

Keywords: Linguistic set, Dual Hesitant set, soft set, hypersoft set and multi-criteria decision-making (MCDM).

1.Introduction

Language, with its inherent ambiguity and subjectivity, often complicates medical diagnosis and treatment by introducing uncertainty and vagueness. Linguistic Dual Hesitant Hypersoft Sets (LDHHS) address these challenges by effectively managing linguistic uncertainty and modeling the complexities of medical data. By assigning Dual Hesitant values to descriptive terms like "minor" or "critical" for symptoms and "dissatisfied" or "very satisfied" for treatment effectiveness, LDHHS provides a precise framework for decision-making. This innovative approach has the potential to enhance diagnostic accuracy, optimize treatment strategies and improve healthcare outcomes, offering significant benefits for medical practitioners.

Motivation and Research Gap: In this study, medical diagnosis systems have increasingly required decision-making frameworks capable of processing vague linguistic information and multidimensional uncertainty. Traditional fuzzy and soft set-based models struggle to fully manage the nested and hierarchical nature of medical attributes such as symptom severity, disease progression, and treatment effectiveness. This creates a pressing need for an advanced model that can simultaneously incorporate

- linguistic subjectivity of medical descriptions,

- membership and non-membership in diagnostic evaluation,
- multi-level and multi-attribute clinical data,
- integration of decision-makers preferences.

A standardized framework for handling linguistic variables with further sub-attributes under Dual Hesitant environments. Aggregate operators, distance and similarity measures that support linguistic hypersoft information. Application of such a framework to medical diagnosis, where ambiguity and complexity are dominant. Thus, no comprehensive decision-making model currently exists that integrates linguistic uncertainty, attribute and Dual Hesitant non-membership simultaneously.

Research Questions

1. How can aggregation, distance, and similarity measures be constructed underthe LDHHS environment to support multi-criteria decision-making?
2. Does the proposed LDHHS based MCDM model provide more reliable and consistent diagnostic outcomes compared with existing approaches?

Novelty and Contributions: This study presents a novel decision-making framework based on Linguistic Dual Hesitant Hypersoft Sets (LDHHS) to address the persistent issues of linguistic ambiguity and uncertainty in medical diagnosis and treatment planning. The primary contribution lies in the ability of the proposed LDHHS model to incorporate multidimensional medical factors while simultaneously capturing the degrees of membership and non-membership associated with linguistically expressed symptoms and treatment effectiveness. In this study also contributes a practical application by demonstrating the effectiveness of N-LDHHS in a real-world medical decision-making scenario, providing new insights for healthcare practitioners and policymakers.

Literature review: Bin Zhu and Meimei Xia [10] introduced the concept of Dual Hesitant Fuzzy Sets (DHFSs), explores their properties and operations, and demonstrates their application in group forecasting. Dejian Yu, et al. [1] proposed new aggregation operators for dual hesitant fuzzy sets to better handle uncertainty. Their effectiveness is shown through a numerical example and applied to selecting HR outsourcing suppliers. Zhiliang Ren1 and Cuiping Wei [2] presents a prioritized multi-attribute decision-making method for dual hesitant fuzzy environments. A correctional score function and Dice similarity measure are introduced to better handle hesitant degrees and attribute priorities. The approach is demonstrated through a practical example. Baoquan Ning, et al. [3] proposed a MADM method using probabilistic dual hesitant fuzzy sets, introducing new distance and entropy measures and applying them to credit risk evaluation.

Muhammad Saqlain, et al. [4] introduced NHSS-TOPSIS, a decision-making method using neutrosophic hypersoft sets with new distance and similarity measures, applied to medical diagnosis and green security system selection. Hongjum Wang, et al. [5] proposed several aggregation operators for dual hesitant fuzzy MADM problems and demonstrates their effectiveness through a technology commercialization evaluation example. Jawad Ali and Muhammad Naeem [6] introduced new distance and similarity measures for normal wiggly dual hesitant fuzzy sets to better support decision-making, demonstrated through a disease detection example. Babitha and Sunil Jacob John [7] introduced a hybrid hesitant fuzzy soft set, combining soft sets and hesitant fuzzy sets, and explores its basic operations and application in decision-making. Sreelekshmi et al. [8] Proposed the Hesitant Fuzzy Hyper Soft Set to enhance decision-making accuracy, with defined operations and an application in robotics.

Glad Deschrijyer et al. [9] introduced aggregation operators on the lattice L∗, analyzes their properties via t-norms and implicators, and examines them under Smets–Magrez axioms. Ubaid Ur Rehman et al. [11] proposed complex dual hesitant fuzzy sets and their similarity measures, applying them to pattern recognition and medical diagnosis to demonstrate their effectiveness. Rana Muhammad Zulqarnain, et al. [12] developed algebraic operations and aggregation operators for interval-valued intuitionistic fuzzy hypersoft sets, applying them to material selection in cryogenic storage systems for improved decision-making. Sathiyapriya Bangarusamy, et al. [13] proposed approach enhances diagnostic accuracy and supports more tailored and reliable treatment strategies for improved patient care. Baoquan Ning et al. [14] proposed a novel correlation coefficient in the probabilistic dual hesitant fuzzy setting and applies it to a MADM method for evaluating project manager candidates. Florentin Smarandache [15] Introduced IndetermSoft and IndetermHyperSoft Sets to handle indeterminate data, extending soft set theory with applications in fuzzy and neutrosophic environments. Takkai Fujita and Shinjuku [16] explores advanced extensions of soft and rough sets, introducing Superhypersoft Hyperrough and Superhypersoft Superhyperrough sets to better handle uncertainty in decision-making. Saqlain, et. al [17] introduced Neutrosophic-linguistic valued hypersoft sets (N-LVHS) help manage linguistic uncertainty in medical diagnosis by assigning neutrosophic values to vague terms, improving accuracy in treatment and decision-making.

In this research paper explores Linguistic Dual Hesitant Hypersoft sets (LDHHS), starting with their fundamental principles and properties. It introduces operational laws and two mathematical Aggregated operators, LDHHSOWGAO and LDHHSWGAO, explaining their significance. A framework for Multi-Criteria Decision-Making (MCDM) is presented using an LDHHS algorithm, demonstrated through a case study. The paper concludes with findings and future research directions.

Acronyms

DHFS - Dual Hesitant Fuzzy Sets

LDHHS - Linguistic Dual Hesitant Hypersoft sets

ELDHHS - Empty Linguistic Dual Hesitant Hypersoft sets

LDHHSWGAO -Linguistic Dual Hesitant Hypersoft set Weighted geometric averaging Operator

LDHHSOWGAO -Linguistic Dual Hesitant Hypersoft Set Ordered Weighted Geometric Averaging Operator

GERD -Gastroesophageal Reflux Disease

2. Preliminary

2.1 Linguistic Set

Let w = $\{w_1, w_2, w_3, ... w_n\}$ where n = 2p + 1: p ≥ 1 and p ∈ $Ɽ^+$ (finite and real valued), be a finite strictly increasing set. For example, if p = 1 then,

w = $\{w_1, w_2, w_3\}$ = {dissatisfied, neutral, satisfied}

For Linguistic set, which is under consideration, the relationship to its elements w_n and the subscript n will be strictly increasing. To define the continuity this set is extended to w = $\{w_\omega: \omega \in Ɽ\}$ where ω is also strictly increasing.

Definition 2.1 Dual Hesitant Fuzzy Set

Let Ụ be a fixed set, then a Dual Hesitant Fuzzy Set (DHFS) $\widehat{D}$ on Ụ is described as

$\widehat{D}$ = {< e, α(e), β(e) >| e ∈ Ụ},

in which α(e) and β(e) are two sets of some values in [0, 1], denoting the possible membership degrees and non-membership degrees of the element e ∈ Ụ to the set $\widehat{D}$ respectively, with the conditions

$0 \leq \gamma, \eta \leq 1, 0 \leq \gamma^+ + \eta^+ \leq 1,$

Where γ ∈ α(e), η ∈ β(e), $\gamma^+ \in \alpha^+(e) = \bigcup_{\gamma\in\alpha} \max\{\gamma\}$ and $\eta^+ \in \beta^+(e) = \bigcup_{\eta\in\beta} \max\{\eta\}$ for all e ∈ Ụ. For convenience, the pair $\widehat{D}$(e) = (α(e), β(e)) is called a dual hesitant fuzzy element denoted by

$\widehat{D}$ = (α, β), with the conditions: γ ∈ α, η ∈ β, $\gamma^+ \in \alpha^+ = \bigcup_{\gamma\in\alpha} \max\{\gamma\}$ and $\eta^+ \in \beta^+ = \bigcup_{\eta\in\beta} \max\{\eta\}$, $0 \leq \gamma, \eta \leq 1$ and $0 \leq \gamma^+ + \eta^+ \leq 1$.

Definition 2.2 Soft Set

Let Ụ be a universe set and let H = $\{h_1, h_2, h_3, ... h_n\}$ be a finite set of Parameters or Attributes. Let Ṕ (Ụ) denote the collection of all subsets of Ụ. For any E ⊆ H, a pair (D, H) is called soft Set over Ụ, where the mapping D is given by D: H → Ṕ (Ụ).

Definition 2.4 Hypersoft Set

Let Ụ be a universe of discourse, Ṕ (Ụ) be a power set of Ụ, Let H = $\{H_1, H_2, H_3, ... H_n\}$ for n ≥ 1 be n distinct attributes whose corresponding attributes values are respectively the sets $a_1, a_2, a_3, ... a_n$ with $a_p \cap a_q = \emptyset$, for p ≠ q and p, q ∈ {1, 2, 3, . . . n}. Then the pair (D, I) where I = $\{a_1 \times a_2 \times a_3 \times .... \times a_n$: n is finite and real valued} is known as Hypersoft set over Ụ with mapping

D: $a_1 \times a_2 \times a_3 \times .... \times a_n$ = I → Ṕ (Ụ).

Definition 2.5 Linguistic Hypersoft Set

Let $\lambda = (\lambda_1, \lambda_2, \lambda_3, ... \lambda_n)$ for n ≥1 be n distinct attributes, whose corresponding attribute values are respectively the sets $u_1, u_2, u_3, ... u_n$ with $u_p \cap u_q = \emptyset$, where p ≠ q for each n ≥ 1 and p, q ∈ {1, 2, 3, . . . n}. Then the pair $(\theta, \propto)$ where $\propto = \{u_1 \times u_2 \times u_3 \times .... \times u_n$: t is finite and real valued} is known as hypersoft set over $\underset{\wedge}{U}$ with mapping $\theta : (u_1 \times u_2 \times u_3 \times \ldots \times u_n) \rightarrow P(\underset{\wedge}{U})$.

Then the linguistic hypersoft set will be,

$\theta(\{\beta(\underset{\wedge}{U})(t)\})$: $\beta \subseteq \lambda$ & $t \in w_\omega = \{w_1, w_2, w_3, \ldots w_n\}$ where n = 2p + 1: p ≥ 1 and $m \in R^+\}$

3. Linguistic Dual Hesitant Hypersoft Set (LDHHS)

In this section, we propose LDHHS with its set structure properties.

Definition 3.1: Linguistic Dual Hesitant Hypersoft Set (LDHHS)

Let $\underset{\wedge}{U}$ be a universe of discourse $P(\underset{\wedge}{U})$ be a power set of $\underset{\wedge}{U}$. Take $\lambda = (\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n)$ for n ≥1. where $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$ are attributes, whose corresponding sub-attribute values are respectively the sets $u_1, u_2, u_3, \ldots u_n$ with $u_p \cap u_q = \emptyset$, where p ≠ q for each n ≥ 1 and p, q ∈ {1, 2, 3, . . . n}. Let $\propto = \{u_1 \times u_2 \times u_3 \times \ldots \times u_n$: where n is finite and real valued} and θ : $\propto = (u_1 \times u_2 \times u_3 \times \ldots \times u_n) \rightarrow P(\underset{\wedge}{U})$. Now the pair $(\theta, \propto)$ is known as the Linguistic Dual Hesitant Hypersoft Set (LDHHS) can be defined as

$\theta(\lambda(w)) = \{\beta(\lambda(\gamma^+, \eta^+)) \mid \beta \subseteq \lambda \;\&\; \gamma^+, \eta^+ \in w = \{w_1, w_2, w_3, \ldots w_n\}\}$

Where w is the set of Linguistic Parameters and γ^+, η^+ represent the Dual Hesitant maximum membership and maximum non-membership values in Linguistic Parameters with the condition

$0 \leq \gamma, \eta \leq 1$, $0 \leq \gamma^+ + \eta^+ \leq 1$.

Numerical Example 3.1.1

Let $\underset{\wedge}{U} = \{s_1, s_2, s_3\}$ be a universe of discourse, consisting of a set of three woods, describes the strength of wood $\theta(\lambda(w)) = \{s_1, s_2\}$. consider the attributes be λ^1 = Softwoods, λ^2 = Hardwoods and their corresponding sub-attributes values are

Softwoods = u_1 = {Cedar, Pine}

Hardwoods = u_2 = {Teak, Beech}

and set $\theta(\lambda(w)) = \{s_1, s_2\} \subset U$. Then the function $\theta: \propto = u_1 \times u_2 \rightarrow \acute{P}(\underset{\wedge}{U})$ and we have five Linguistic Parameters w = {w_1, w_2, w_3, w_4, w_5} = {very dissatisfied, dissatisfied, neutral, satisfied, very satisfied}, each linguistic Parameter corresponds a dual Hesitant value: w_1 = 0.01 for very dissatisfied, w_2 = 0.05 for dissatisfied, w_3 = 0.25 for neutral, w_4 = 0.35 for satisfied and w_5 = 0.52 for very satisfied.

Define the strength of woods in Linguistic Dual Hesitant Hypersoft Set (LDHHS)

$(\theta, \propto) = \theta(\lambda(w)) = \{\beta(\lambda(\gamma^+, \eta^+)) \mid \beta \subseteq \lambda \;\&\; \gamma^+, \eta^+ \in w = \{w_1, w_2, w_3, \ldots w_n\}\}$

θ({Cedar, Beech}) = {s_1, s_2} = {(s_1((dissatisfied, neutral), (very dissatisfied, dissatisfied))), (s_2 (neutral, satisfied), (very dissatisfied, dissatisfied))} = G

Similarly,

$\theta_1(\{\text{Pine, Teak}\}) = \{s_1, s_2\} = \{(s_1$((neutral, satisfied), (very dissatisfied, dissatisfied))), (s_2(dissatisfied, neutral), (very dissatisfied, neutral))} = G_1

$\theta_2(\{\text{Cedar, Teak}\}) = \{s_2, s_3\} = \{(s_2$((very dissatisfied, neutral), (very dissatisfied, dissatisfied)), s_3((neutral, satisfied), (very dissatisfied, dissatisfied))} = G_2.

Definition 3.2: Let $(\theta_1, \propto_1) = G_1$ be a LDHHS, then the subset G_b can be defined as. $\theta(\lambda\,(w)) = \{\beta(\lambda\,(\gamma^+, \eta^+)) \mid \beta \subseteq \lambda \;\&\; \gamma^+, \eta^+ \in w = \{w_1, w_2, w_3, \ldots w_n\}\}$

1. $G_b \subseteq G_1$
2. $\forall\, w \in G_b, \theta_2(w) \subseteq \theta_1(w)$.

This holds only when linguistic variables w_ω satisfy the property i.e., each w_ω of $(\theta_b, \propto_b) \leq w_\omega$ of $(\theta_1, \propto_1)$. Where w_ω represents Linguistic variables associated with Dual Hesitant evaluation.

Example 3.2.1

Recall Example 3.1.1. The function $\theta_1 : \propto_b = u_1 \times u_2 \rightarrow \acute{P}\,(\underset{\wedge}{U})$ and assume the hypersoft set, $\theta_1(\{\text{Pine, Teak}\}) = \{(s_1$(neutral, satisfied), (very dissatisfied, dissatisfied), (s_2 (dissatisfied, neutral), (very dissatisfied, neutral)} = G_b. Where $\propto_b \subseteq \propto$ and $G_b \subseteq G_1$.

Where G_b is a subset of the original set G_1. Linguistic variables are associated with Dual Hesitant evaluation (maximum of membership and non-membership).

Definition 3.3

Empty Linguistic Dual Hesitant Hypersoft Set (ELDHHS) can be defined as. $\theta_1 : \propto_E = u_1 \times u_2 \times \ldots\ldots \times u_n \rightarrow \acute{P}(\underset{\wedge}{U})$

Such that each $u_p (p \leq n)$ is empty. $\theta_1(\{G_E(\underset{\wedge}{U})\})$

1. $(\theta_1, \propto_E)(\emptyset) = G_E$ if $\forall \theta_1(w) = \emptyset$: $\forall\, w \in \propto_E$.

Example 3.3.1

Recall Example 3.1.1. The function $\theta_1 : \propto_E = u_1 \times u_2 \rightarrow \acute{P}(\underset{\wedge}{U})$, where u_1 and u_2 are all empty sets ($u_1 = u_2 = \emptyset$) and assume the Hypersoft set, $\theta_1(\emptyset) = \emptyset = H_E$, where $\propto_E \subseteq \propto$.

Definition 3.4

The AND operation on two $(\theta_1, \propto_1) = G_1$ and $(\theta_2, \propto_2) = G_2$ Linguistic Dual Hesitant hypersoft set (LDHHS) can be defined by

1. $G_1 \wedge G_2 = (\theta_{1\wedge2}, \propto_{1\wedge2}) = G_{1\wedge2}$
2. $(w_p, w_q) = w_\omega = G_{1\vee2}$, where $w_p \in G_1$ and $w_q \in G_2$ with $p \neq q$
3. $\theta_{1\cup2}(w_p, w_q) = \theta_1(w_p) \cup \theta_2\,(w_q)$.

Definition 3.5

The OR operation on two $(\theta_1, \propto_1) = G_1$ and $(\theta_2, \propto_2) = G_2$ Linguistic Dual Hesitant hypersoft set (LDHHS) can be defined by

1. $G_1 \vee G_2 = (\theta_{1\vee 2}, \propto_{1\vee 2}) = G_{1\vee 2}$
2. $(w_p, w_q) = w_\omega = G_{1\vee 2}$, where $w_p \in G_1$ and $w_q \in G_2$ with $p \neq q$
3. $\theta_{1\cap 2}(w_p, w_q) = \theta_1(w_p) \cap \theta_2(w_q)$

Definition 3.6

The NOT operation on $(\theta, \propto)$ Linguistic Dual Hesitant hypersoft set (LDHHS) can be defined by.

1. $\sim G = \sim(\theta, \propto) = \sim u_1 \times \sim u_2 \times \ldots \times \sim u_n$
2. $\sim G = \sim \Pi w_p : p = 1,2,3,\ldots\ldots n$

Definition 3.7

The Complement on $(\theta, \propto) = G$ Linguistic Dual Hesitant hypersoft set (LDHHS) can be defined by

1. $(\theta, \propto)^{\sim} = (\theta^{\sim}, \sim \propto)$, $\theta^{\sim}: \sim \propto \to \acute{P}(\underset{\wedge}{U})$.
2. $\theta^{\sim}(\sim w) = \underset{\wedge}{U} \setminus \theta(w)$; $\forall w \in G$.

Proposition 3.8: Let $(\theta, \propto) = G$, $(\theta_1, \propto_1) = G_1$ and $(\theta_2, \propto_2) = G_2$ be Linguistic Dual Hesitant hypersoft set (LDHHS) then following holds.

1. $(\theta_1, \propto_1) \subseteq (\theta_1, \propto_1)$
2. $(\theta_2, \propto_E)(\emptyset) \subseteq (\theta_2, \propto_2)$
3. $\sim(\sim G) = G$
4. $\sim (\theta_2, \propto_E)(\emptyset) = \underset{\wedge}{U}$
5. If $(\theta_1, \propto_1) \subseteq (\theta_2, \propto_2)$ and $(\theta_2, \propto_2) \subseteq (\theta_1, \propto_1)$ then $(\theta_1, \propto_1) = (\theta_2, \propto_2)$ iff each w_ω of $(\theta_1, \propto_1) = w_\omega$ of $(\theta_2, \propto_2)$.

This property holds only when Dual Hesitant variables satisfy the property i.e., each w_ω of $(\theta_1, \propto_1) = w_\omega$ of $(\theta_2, \propto_2)$.

Proof: Recall G, G_1 and G_2 from example 3.1.1.

1. θ_1 contains Dual Hesitant Variables $G_1, G_2, G_3, \ldots G_n$

For each $w_\omega \in \propto_1$, we have a mapping $\theta_1(w_\omega) \subseteq \theta_1(w_\omega)$ each Linguistic Dual Hesitant variable map to itself. Thus, the set $(\theta_1, \propto_1)$ is a subset of itself by the Definition of 3.2, as the mappings are trivially reflexive.

$\therefore (\theta_1, \propto_1) \subseteq (\theta_1, \propto_1)$.

2. $\propto_E$ refers to an empty domain, (i.e., no Linguistic Dual Hesitant variable) the complement

operation on LDHHS with empty domain means there are no variables to map to Linguistic Dual Hesitant values.

$\theta_2(w_\omega) = \emptyset$ for all $w_\omega \in \propto_E$ the function maps nothing to the power set $\underset{\wedge}{U}$. Since the empty set is a subset of any set, it follows that $(\theta_2, \propto_E)(\emptyset) \subseteq (\theta_2, \propto_2)$ because the empty set maps to no Linguistic Dual Hesitant variables, making it trivially a subset of any non-empty LDHHS.

3. Let's assume $\sim(\theta_1, \propto_1)$ represents the complement of the LDHHS, which is $(\theta_1, \propto_1)$

with each Linguistic Dual Hesitant variables w_ω replaced by its complement. Now, apply the complement again would return the original Dual Hesitant values, as: $\sim(\sim(w_\omega)) = w_\omega$ for each w_ω thus $\sim(\sim G) = G$, Confirming the double complementation property.

4. When the LDHHS is complemented and the domain is the empty set, the result is

the complement of the empty set, which is the entire universal set $\underset{\wedge}{U}$.

Therefore, $\sim(\theta_2, \propto_E)(\emptyset) = \underset{\wedge}{U}$.

5. Let's assume that $(\theta_1, \propto_1) \subseteq (\theta_2, \propto_2)$ and $(\theta_2, \propto_2) \subseteq (\theta_1, \propto_1)$. This means that

for all, $w_\omega \in \theta_1$, there exists a corresponding $w_\omega \in \theta_2$ such that $\theta_1(w_\omega) \subseteq \theta_2(w_\omega)$

Similarly, $(\theta_2, \propto_2) \subseteq (\theta_1, \propto_1)$ implies: $\theta_2(w_\omega) \subseteq \theta_1(w_\omega)$

Since $\theta_1(w_\omega) \subseteq \theta_2(w_\omega)$ and $\theta_2(w_\omega) \subseteq \theta_1(w_\omega)$,

we conclude: $\theta_1(w_\omega) = \theta_2(w_\omega)$

Thus, $(\theta_1, \propto_1) = (\theta_2, \propto_2)$, provided that the Linguistic Dual Hesitant variables w_ω from both sets match exactly.

4. Operational Laws on LDHHSS

In this section, we discuss the importance of operational laws, theorems and propose for LDHHS. Let $(\theta_1, \propto_1) = G_1$ and $(\theta_2, \propto_2) = G_2$ be two LDHHS, where $\propto_1 = \{u_1 \times u_2 \times u_3 \times \ldots . \times u_p$: p is finite and real valued} over $\underset{\wedge}{U}$ with mapping $\theta: \propto_1 = u_1 \times u_2 \times u_3 \times \ldots . \times u_p \rightarrow \acute{P}(\underset{\wedge}{U})$ and $\propto_2 = u_1 \times u_2 \times u_3 \times \ldots . \times u_q$: y is finite and real valued} over Ă with mapping $\theta: \propto_2 = u_1 \times u_2 \times u_3 \times \ldots . \times u_q \rightarrow \acute{P}(\underset{\wedge}{U})$.

Such that

$$(\theta, \propto) = \theta(\lambda(w)) = \{\beta(\lambda(\gamma^+, \eta^+)) \mid \beta \subseteq \lambda \;\&\; \gamma^+, \eta^+ \in w = \{w_1, w_2, w_3, \ldots w_n\}\}$$

where w is the set of Linguistic terms and γ^+, η^+ represents the Dual Hesitant Maximum of Membership and non-membership values in Linguistic terms in ascending order i. e. very dissatisfied to very satisfied.

Then the operational laws on LDHHS can be defined with some necessary conditions.

Definition 4.1 Union of LDHHS

The union of two LDHHS, $(\theta_1, \propto_1) = G_1$ and $(\theta_2, \propto_2) = G_2$, can be represented as $G_1 \cup G_2$. Depending on the relationship between their Linguistic Dual Hesitant variables and their domains, the union is defined in two cases.

Case 1: Let $(\theta_1, \propto_1) = G_1$ and $(\theta_2, \propto_2) = G_2$ be two LDHHS, then the union can be defined as

$$G_1 \cup G_2 = \{\Pi\, \lambda_p(w_p) \times \Pi\, \lambda_q(w_q) \in \prod_{p=1}^{n} u_p \times \prod_{q=1}^{n} u_q\}$$

where, $\lambda_p(w_p) \in \prod_{p=1}^{n} u_p$ and $\lambda_q(w_q) \in \prod_{q=1}^{n} u_q$ should be distinct with $u_p \cup u_q = \emptyset$, for $p \neq q$ and $p, q \in \{1, 2, \ldots. n\}$and $w = \{w_1, w_2, w_3, \ldots w_n\}$.

Case 2: $G_1 \cup G_2 = \{\lambda_p(w_p) \in \prod_{p=1}^{n} u_p \times \prod_{q=1}^{n} u_q\}$

With p = q and Linguistic Dual Hesitant variable w_p of u_p should be same.

Example: Consider 3.1.1

Case 1: $\theta_1(\{\text{Pine, Teak}\}) = \{s_1, s_2\}$ = {s_1(satisfied, neutral), (very dissatisfied, dissatisfied), s_2(very dissatisfied, dissatisfied), (neutral, very dissatisfied)} = G_1

$\theta_2(\{\text{Cedar, Beech}\}) = \{s_1, s_2\}$ = {s_1(very dissatisfied, dissatisfied), (satisfied, neutral), s_2(neutral, dissatisfied), (very dissatisfied, very dissatisfied)} = G_2.

$\therefore u_p \cup u_q = \emptyset$ with $p \neq q$

$G_1 \cup G_2 = \{s_1(\text{satisfied, dissatisfied}),\ s_2(\text{dissatisfied, neutral}), s_1(\text{disatisfied, satisfied}), s_2(\text{neutral, very dissatisfied})\}$.

Case 2: $\theta_1(\{\text{Pine, Beach}\}) = \{s_1, s_2\}$ = {s_1(satisfied, neutral), (dissatisfied, very dissatisfied), s_2(neutral, dissatisfied), (very dissatisfied, dissatisfied)} = G_1

$\theta_2(\{\text{Pine, Teak}\}) = \{s_1, s_2\}$ = {s_1(satisfied, neutral), (dissatisfied, very dissatisfied), s_2(neutral, dissatisfied), (very dissatisfied, dissatisfied)} = G_2.

$\therefore u_p \cup u_q \neq \emptyset$ with p = q

$G_1 \cup G_2 = \{s_1(\text{satisfied, dissatisfied}), s_2(\text{neutral, dissatisfied})\}$.

Case 3: (counter example)

$\theta_1(\{\text{Pine, Beach}\}) = \{s_1, s_2\}$ = {s_1(satisfied, neutral), (dissatisfied, very dissatisfied), s_2(neutral, dissatisfied), (very dissatisfied, dissatisfied)} = G_1

$\theta_2(\{\text{Pine}, \text{Teak}\}) = \{s_1, s_2\} = \{s_1$(very satisfied, neutral), (dissatisfied, very dissatisfied), s_2(neutral, satisfied), (very dissatisfied, dissatisfied)$\} = G_2$.

$\therefore u_p \cup u_q \neq \emptyset$ with p = q

Each Linguistic Dual Hesitant variables w_p of G_1 is less then Linguistic Dual Hesitant variables w_p of G_2 then this implies $G_1 \cup G_2$ can be defined with some restriction i.e., consider highest Linguistic Dual Hesitant variables w_p of each Parameters.

Example

$G_1 = \{s_1$(satisfied, neutral), (dissatisfied, very dissatisfied), s_2(neutral, dissatisfied), (very dissatisfied, dissatisfied)$\}$

$G_2 = \{s_1$(very satisfied, neutral), (dissatisfied, very dissatisfied), s_2(neutral, satisfied), (very dissatisfied, dissatisfied)$\}$

As, s_1(satisfied, dissatisfied) < s_1(very satisfied, dissatisfied) and s_2(neutral, dissatisfied) < s_2(satisfied, dissatisfied)

Then $G_1 \cup G_2 = \{s_1$(very satisfied, dissatisfied), s_2(satisfied, dissatisfied)$\}$.

Definition 4.2 Intersection of LDHHS

Case 1: Let $(\theta_1, \propto_1) = G_1$ and $(\theta_2, \propto_2) = G_2$ be two LDHHS, then the intersection can be defined as

$$G_1 \cap G_2 = \{\Pi\, \lambda_p(w_p) \times \Pi\, \lambda_q(w_q) \in \prod_{p=1}^{n} u_p \times \prod_{q=1}^{n} u_q\} = \emptyset$$

where, $\lambda_p(w_p) \in \prod_{p=1}^{n} u_p$ and $\lambda_q(w_q) \in \prod_{q=1}^{n} u_q$ should be distinct with $u_p \cup u_q = \emptyset$, for p = q and p, q ∈ {1, 2, …. , n}and $w = \{w_1, w_2, w_3, \ldots w_n\}$.

Case 2: $G_1 \cap G_2 = \{\lambda_p(w_p) \in \prod_{p=1}^{n} u_p \times \prod_{q=1}^{n} u_q\}$

With p = q and Dual Hesitant variable w_p of s_p Then $G_1 \cap G_2 = G_1$ OR G_2.

Example: Consider,

Case 1: $\theta_1(\{\text{Pine}, \text{Teak}\}) = \{s_1, s_2\} = \{s_1$(very satisfied, satisfied), (very dissatisfied, very dissatisfied), s_2(dissatisfied, very dissatisfied), (neutral, dissatisfied)$\} = G_1$

$\theta_2(\{\text{Cedar}, \text{Beech}\}) = \{s_1, s_2\} = \{s_1$(very dissatisfied, very dissatisfied), (very satisfied, neutral), s_2(dissatisfied, very dissatisfied), (satisfied, neutral)$\} = G_2$.

$\therefore u_p \cap u_q = \emptyset$

$G_1 \cap G_2 = \{\emptyset\}$.

Case 2: $\theta_1(\{\text{Pine}, \text{Beach}\}) = \{s_1, s_2\} = \{s_1$(satisfied, neutral), (very dissatisfied, dissatisfied), s_2(neutral, dissatisfied), (dissatisfied, very dissatisfied)$\} = G_1$

$\theta_2(\{\text{Pine}, \text{Teak}\}) = \{s_1, s_2\}$ = $\{s_1$(satisfied, neutral), (very dissatisfied, dissatisfied), s_2(neutral, dissatisfied), (dissatisfied, very dissatisfied)} = G_2.

$\therefore u_p \cap u_q \neq \emptyset$ with p = q

$G_1 \cap G_2 = \{s_1(\text{satisfied}, \text{dissatisfied}),\ s_2(\text{neutral}, \text{dissatisfied})\}$.

Case 3: (counter example)

$\theta_1(\{\text{Pine}, \text{ Beach}\}) = \{s_1, s_2\}$ = $\{s_1$(neutral, satisfied), (dissatisfied, very dissatisfied), s_2(neutral, dissatisfied), (very dissatisfied, neutral)} = G_1

$\theta_2(\{\text{Pine}, \text{Teak}\}) = \{s_1, s_2\}$ = $\{s_1$(very satisfied, neutral), (very dissatisfied, dissatisfied), s_2(satisfied, neutral), (very dissatisfied, dissatisfied)} = G_2.

$\therefore u_p \cap u_q \neq \emptyset$ with p = q

Each Linguistic Dual Hesitant variable w_p of G_1 is less then Linguistic Dual Hesitant variable w_p of G_2 then this implies $G_1 \cup G_2$ can be defined with some restriction i.e., consider highest Linguistic Dual Hesitant variable w_p of each Parameters.

Example

G_1 = $\{s_1$(neutral, satisfied), (dissatisfied, very dissatisfied), s_2(neutral, dissatisfied), (very dissatisfied, neutral)}

G_2 = $\{s_1$(very satisfied, neutral), (very dissatisfied, dissatisfied), s_2(satisfied, neutral), (very dissatisfied, dissatisfied)}

As, s_1(satisfied, dissatisfied) < s_1(very satisfied, dissatisfied) and s_2(neutral, neutral) < s_2(satisfied, dissatisfied)

Then $G_1 \cup G_2 = \emptyset$.

Theorem 4.3: If G_1 and G_2 be two LDHHS then the following holds

1. $G_1 \cup G_1 = G_1$
2. $G_1 \cup \emptyset = G_1$
3. $G_1 \cap G_1 = G_1$
4. $G_1 \cap \emptyset = \emptyset$
5. $G_1 \cup G_2 = G_2 \cup G_1$
6. $G_1 \cap G_2 = G_2 \cap G_1$
7. If $G_1 \subset G_2$ and $G_2 \subset G_1$ then $G_1 = G_2$.
8. $\zeta\,(G_1) = \zeta G_1;\ \zeta \geq 0$.
9. $\zeta\,(G_1 \cup g_2) = \zeta\,(G_2 \cup G_1)$

Proof

Straight Forward.

Theorem 4.4

If G_1, G_2 be two LDHHS then the operations are given as follows

1. $\zeta \times G_1 = G_{\zeta \times 1}$; ζ (Linguistic Dual Hesitant variable)
2. $G_1 \oplus G_2 = G_{1 \oplus 2}$
3. $G_1 \otimes G_2 = G_{1 \otimes 2}$
4. $(G_1)^{\zeta} = G_{1^{\zeta}}$.

Proof

Straight Forward.

5.Some Aggregation operators

Aggregate operators are essential in decision-making processes, combining and aggregating Linguistic Dual Hesitant quantifiers or numerical values to assess factors. They enable informed analysis and evaluation of complex information, handling multiple criteria simultaneously, such as language, quality, reliability and customer satisfaction allowing for comprehensive evaluation and comparison.

Definition 5.1 LDHHSWGAO

Consider $\lambda_1, \lambda_2, \lambda_3, \ldots. \lambda_n$ for $n \geq 1$, where $(\lambda_1, \lambda_2, \lambda_3, \ldots. \lambda_n)$ are attributes, whose corresponding sub-attribute values are respectively the sets $\propto = (u_1, u_2, u_3, \ldots. u_n)$ with $u_p \cap u_q = \emptyset$, for $p \neq q$ for each $n \geq 1$ and $p, q \in \{1,2,3,\ldots.t\}$.

$\mu^{\tau} : \propto = u_1, u_2, u_3, \ldots. u_n \rightarrow P(\underset{\hat{}}{U}) = (\theta, \propto) = \theta(\lambda (w)) = \{\beta(\lambda (\gamma^{+}, \eta^{+})) \mid \beta \subseteq \lambda \;\&\; \gamma^{+}, \eta^{+} \in w = \{w_1, w_2, w_3, \ldots w_n\}\}$ (1)

Where w is the set of Linguistic Parameters and γ^{+}, η^{+} represent the Dual Hesitant maximum membership and maximum non-membership values in Linguistic Parameters.

if $\mu^{\tau} (\lambda_1, \lambda_2, \lambda_3, \ldots. \lambda_n) = \prod_{n=1}^{t} \lambda_n(\gamma^{+}, \eta^{+})^{\tau_n}$

such that

$\mu^{\tau} (\lambda_1, \lambda_2, \lambda_3, \ldots. \lambda_n) = {\lambda_1}^{\tau_1} \otimes {\lambda_2}^{\tau_2} \otimes {\lambda_3}^{\tau_3} \otimes \ldots.. \otimes {\lambda_n}^{\tau_n} = \{\beta(\lambda (\gamma^{+}, \eta^{+})\}$

where $\tau = (\tau_1, \tau_2, \tau_3, \ldots.. \tau_n)$ is the exponential weighting vector of the $\lambda_n(\gamma^{+}, \eta^{+})^{\tau_n} \in \{\beta(\lambda (\gamma^{+}, \eta^{+})\}$ and $\tau_n \in [0,1]$ with $\sum_{n=1}^{t} \tau_n = 1$, then μ^{τ} is called Linguistic Dual Hesitant weighted geometric averaging operator (LDHHSWGAO).

Example

Assume $\tau = (0.6,0.2)^{n}$ then LDHHSWGAO $\{s_1$(Pine, Beech), s_2(Cedar, Teak)$\}$ = $\{s_1$(Pine (neutral, satisfied), (dissatisfied, very dissatisfied), (Beech (satisfied, dissatisfied), (neutral, dissatisfied))$\}$

$\therefore \mu^{\tau} (\lambda_1, \lambda_2, \lambda_3, \ldots. \lambda_n) = \prod_{n=1}^{t} (\lambda_n(\gamma^{+}, \eta^{+})^{\tau_n}) = {\lambda_1}^{\tau_1} \otimes {\lambda_2}^{\tau_2} \otimes {\lambda_3}^{\tau_3} \otimes \ldots.. \otimes {\lambda_n}^{\tau_n}$

$= \{\beta(\lambda (\gamma^{+}, \eta^{+})\}$

= $\{s_1$(Pine (neutral, satisfied)$^{0.6}$, (dissatisfied, very dissatisfied)$^{0.2}$), (Beech (satisfied, neutral)$^{0.6}$, (neutral, dissatisfied)$^{0.2}$)$\}$

= $\{s_1$((neutral, satisfied)$^{0.6}$ + (dissatisfied, very dissatisfied)$^{0.2}$), ((satisfied, neutral)$^{0.6}$ + (neutral, dissatisfied)$^{0.2}$)$\}$

= $\{s_1$(neutral, dissatisfied), (satisfied, neutral)$\}$

Similarly, s_2(none, none).

Definition 5.2 LDHHSOWGAO

Consider $\lambda_1, \lambda_2, \lambda_3, \ldots. \lambda_n$ for $n \geq 1$, where $(\lambda_1, \lambda_2, \lambda_3, \ldots. \lambda_n)$ are attributes, whose corresponding sub-attribute values are respectively the sets $\propto = (u_1, u_2, u_3, \ldots. u_n)$ with $u_p \cap u_q = \emptyset$, for $p \neq q$ for each $n \geq 1$ and $p, q \in \{1,2,3, \ldots. t\}$.

$\mu' : \propto = u_1, u_2, u_3, \ldots. u_n \rightarrow$ Ᵽ (Ụ) = $(\theta, \propto) = \theta(\lambda\ (w)) = \{\beta(\lambda\ (\gamma^+, \eta^+)) \mid \beta \subseteq \lambda\ \&\ \gamma^+, \eta^+ \in w = \{w_1, w_2, w_3, \ldots w_n\}\}$ (2)

if $\mu'^{\tau}\ (\lambda_1, \lambda_2, \lambda_3, \ldots. \lambda_n) = \prod_{n=1}^{t} \lambda_n (\gamma^+, \eta^+)^{\tau_n}$

such that

$\mu'^{\tau}\ (\lambda_1, \lambda_2, \lambda_3, \ldots. \lambda_n) = {\lambda_1}^{\tau_1} \otimes {\lambda_2}^{\tau_2} \otimes {\lambda_3}^{\tau_3} \otimes \ldots.. \otimes {\lambda_n}^{\tau_n} = \{\beta(\lambda\ (\gamma^+, \eta^+)\}$

where $\tau = (\tau_1, \tau_2, \tau_3, \ldots.. \tau_n)$ is the exponential weighting vector of the $\lambda_n (\gamma^+, \eta^+)^{\tau_n} \in \{\beta(\lambda\ (\gamma^+, \eta^+)\}$ and $\tau_n \in [0,1]$ with $\sum_{n=1}^{t} \tau_n = 1$, then μ'^{τ} is called Linguistic Dual Hesitant Hypersoft Set Ordered Weighted Geometric Averaging Operator (LDHHSOWGAO).

Example

Assume $\tau = (0.5, 0.4)^n$ then LDHHSOWGAO $\{s_1$(Cedar, Beech), s_2(Pine, Teak)$\}$ = $\{s_1$(Pine (neutral, satisfied), (dissatisfied, very dissatisfied), (Beech (satisfied, dissatisfied), (neutral, dissatisfied))$\}$

$\therefore \mu^{\tau}\ (\lambda_1, \lambda_2, \lambda_3, \ldots. \lambda_n) = \prod_{n=1}^{t} (\lambda_n (\gamma^+, \eta^+)^{\tau_n}) = {\lambda_1}^{\tau_1} \otimes {\lambda_2}^{\tau_2} \otimes {\lambda_3}^{\tau_3} \otimes \ldots.. \otimes {\lambda_n}^{\tau_n}$

$= \{\beta(\lambda\ (\gamma^+, \eta^+)\}$

= $\{s_1$(Cedar (satisfied, dissatisfied)$^{0.5}$, (neutral, dissatisfied)$^{0.4}$), (Beech (neutral, very dissatisfied)$^{0.5}$, (dissatisfied, very dissatisfied)$^{0.4}$)$\}$

= $\{s_1$((satisfied, dissatisfied)$^{0.5}$ + (neutral, dissatisfied)$^{0.4}$), ((neutral, very dissatisfied)$^{0.5}$ + (dissatisfied, very dissatisfied)$^{0.4}$)$\}$

= $\{s_1$(satisfied, neutral), (neutral, dissatisfied)$\}$

Similarly, s_2(none, none).

Theorem 5.1

1. $\min_{p}(\lambda_n(\gamma^+,\eta^+)) \leq \mu^{\tau}\,(\lambda_1,\lambda_2,\lambda_3,\ldots.\lambda_n) \leq \max_{p}(\lambda_n(\gamma^+,\eta^+))$
2. $\min_{p}(\lambda_n(\gamma^+,\eta^+)) \leq \mu'^{\tau}\,(\lambda_1,\lambda_2,\lambda_3,\ldots.\lambda_n) \leq \max_{p}(\lambda_n(\gamma^+,\eta^+))$

Proof

Straight forward

Theorem 5.2

1. $\mu'^{\tau}(\lambda_p(\gamma^+,\eta^+)) = \mu'^{\tau}(\lambda_p(\gamma^+,\eta^+))$, where $(\lambda_p(\gamma^+,\eta^+))$ is any permutation of $(\lambda_p(\gamma^+,\eta^+))$
2. If $\forall(\lambda_p(\gamma^+,\eta^+)) = (\lambda_p(\gamma^+,\eta^+))\ \forall p$, then $\mu'^{\tau}\,(\lambda_p(\gamma^+,\eta^+)) = w_{\omega}(\lambda_p(\gamma^+,\eta^+))$

Proof: Straight forward.

6. LDHHS Algorithm to solve MCDM problem

A decision-making technique based on Linguistic Dual Hesitant Hypersoft Set Weighted Geometric Averaging Operator (LDHHSWGAO) has been used to construct an algorithm known as Linguistic Dual Hesitant Hypersoft Set based multi criteria group decision-making (LDHHS) algorithm. The graphical representation of the proposed LDHHS algorithm is presented in figure 2.

Step 1: Let, $\lambda_1,\lambda_2,\lambda_3,\ldots,\lambda_n$ for $n \geq 1$, be n distinct attributes, whose corresponding sub-attribute values are respectively the sets $u_1,u_2,\ldots u_n$ with $u_p \cap u_q = \emptyset$, for $p \neq q$, and p, q $\in \{1,2,\ldots.n\}$. let $\tau = (\tau_1,\tau_2,\tau_3,\ldots..\tau_n)$ be the exponential weighting vector. Where $\tau_n \geq [0,1]$ with $\sum_{n=1}^{t}\tau_n = 1$.

Let $\theta{:}\propto = u_1 \times u_2 \times u_3 \times \ldots\ldots \times u_n \rightarrow Ᵽ(U̬) = \theta(\lambda\,(w)) = \{\beta(\lambda\,(\gamma^+,\eta^+)) \mid \beta \subseteq \lambda\ \&\ \gamma^+, \eta^+ \in w = \{w_1, w_2, w_3, \ldots w_n\}\}$ The decision-maker Ḏ assign the values with the Linguistic Dual Hesitant Parameters and assign Linguistic Dual Hesitant variable to each alternative as $G_p = \{(\lambda_n(\gamma^+, \eta^+)) \mid p = 1,2,\ldots n\}$, $w = \{w_1, w_2, w_3, \ldots w_n\}$ and construct a Linguistic Dual Hesitant preference table for $(\lambda_n(\gamma^+,\eta^+))^{(\tau_n)}$.

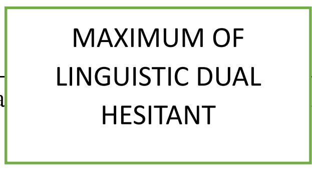
Step 2: Construct a matrix $[\lambda_{pq}]_{p\times q}$ for Ḏ using Linguistic Dual Hesitant Hypersoft weighted geometric averaging operator (LDHHSWGAO),

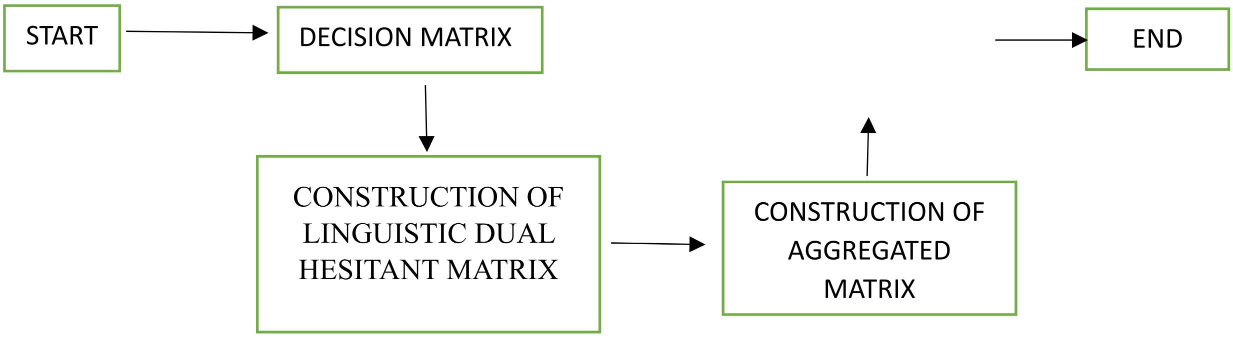
$\beta(\lambda\,(\gamma^+,\eta^+) = {\lambda_1}^{\tau_1} \otimes {\lambda_2}^{\tau_2} \otimes {\lambda_3}^{\tau_3} \otimes \ldots\ldots \otimes {\lambda_n}^{\tau_n}$

Step 3: List the aggregated values of all the alternatives $\beta(\lambda\,(\gamma^+, \eta^+)$.

Step 4: Finally, list the alternatives with maximum membership (γ^+) value. The maximum membership will represent the positive ideal alternative.

Figure 1. Graphical representation of proposed LDHHS algorithm





6.1 Illustrative Example

When ten patients visit a doctor with symptoms. Like Dry cough, Bitter taste, Upper Abdominal Discomfort, Epigastric pain, Belching. These symptoms are making a doubt to affect Gastroesophageal reflux disease (GERD) and also making the diagnosis questionable even if they are symptomatic of several medical diseases, including GERD. To evaluate their symptoms more precisely, the doctor uses LDHHS, and data presented in table 1.

Consider $P = \{P_1, P_2, P_3, \ldots .P_{10}\}$ be ten patients as alternatives and doctor want to diagnose them. The medical diagnose system should be to identify Gastroesophageal reflux disease (GERD) patients, while minimum of stomach issues and improve outcomes.

Consider the attributes λ_1= Dry cough, λ_2 = Bitter taste, λ_3= Upper Abdominal Discomfort, λ_4 = Epigastric pain and λ_5 = Belching.

Then the function $\mu: \propto = u_1 \times u_2 \times u_3 \times u_4 \times u_5 \to P(\underset{\sim}{U})$ and assume the hyper soft set $P = \{P_1, P_2, P_3, \ldots .P_{10}\} \subset \underset{\sim}{U}$ where $\underset{\sim}{U} = \{P_1, P_2, P_3, \ldots .P_{10}\}$ be the universal set.

Step1: Construction of Linguistic Dual Hesitant preference table for alternatives

This table organizes the system of each patient as fallows

Table 1: Doctor patient interaction and information gathering in Linguistic Dual Hesitant

Patient No./ Symptoms	Dry cough	Bitter taste	Upper Abdominal Discomfort	Epigastric pain	Belching
P_1	(very dissatisfied, satisfied), (very dissatisfied, dissatisfied)	(neutral, satisfied), (dissatisfied, neutral)	(dissatisfied, neutral), (satisfied, dissatisfied)	(satisfied, dissatisfied), (neutral, very dissatisfied)	(very dissatisfied, dissatisfied), (neutral, very dissatisfied)
P_2	(satisfied, neutral), (neutral, dissatisfied)	(satisfied, dissatisfied), (dissatisfied, very dissatisfied)	(neutral, very dissatisfied), (dissatisfied, very dissatisfied)	(dissatisfied, neutral), (very dissatisfied, dissatisfied)	(satisfied, very dissatisfied), (dissatisfied, very dissatisfied)
P_3	(very satisfied, very dissatisfied), (very dissatisfied, dissatisfied)	(satisfied, neutral), (very dissatisfied, dissatisfied)	(neutral, dissatisfied), (dissatisfied,	(satisfied, very dissatisfied),	(very satisfied, dissatisfied), (dissatisfied,

			very dissatisfied)	(neutral, dissatisfied)	very dissatisfied)
P_4	(neutral, dissatisfied), (neutral, very dissatisfied)	(neutral, very dissatisfied), (very dissatisfied, dissatisfied)	(very dissatisfied, dissatisfied), (dissatisfied, neutral)	(satisfied, very dissatisfied), (dissatisfied, very dissatisfied)	(very dissatisfied, dissatisfied), (very dissatisfied, dissatisfied)
P_5	(dissatisfied, very dissatisfied), (very dissatisfied, neutral)	(neutral, very dissatisfied), (dissatisfied, very dissatisfied)	(very dissatisfied, dissatisfied), (dissatisfied, very dissatisfied)	(very dissatisfied, neutral), (neutral, dissatisfied)	(satisfied, very dissatisfied), (very dissatisfied, dissatisfied)
P_6	(satisfied, very dissatisfied), (very satisfied, very dissatisfied)	(dissatisfied, neutral), (very dissatisfied, neutral)	(satisfied, dissatisfied), (very dissatisfied, dissatisfied)	(neutral, very dissatisfied), (satisfied, very dissatisfied)	(dissatisfied, satisfied), (neutral, dissatisfied)
P_7	(neutral, dissatisfied), (very dissatisfied, dissatisfied)	(very dissatisfied, dissatisfied), (neutral, very dissatisfied)	(satisfied, neutral), (dissatisfied, very dissatisfied)	(neutral, dissatisfied), (neural, very dissatisfied)	(satisfied, very dissatisfied), (dissatisfied, very dissatisfied)
P_8	(dissatisfied, satisfied), (very dissatisfied, dissatisfied)	(neutral, very dissatisfied), (neutral, dissatisfied)	(satisfied, dissatisfied), (dissatisfied, very dissatisfied)	(very dissatisfied, neutral), (neutral, dissatisfied)	(neutral, very dissatisfied), (dissatisfied, very dissatisfied)
P_9	(satisfied, neutral), (very dissatisfied, neutral)	(satisfied, dissatisfied), (very dissatisfied, dissatisfied)	(dissatisfied, neutral), (very dissatisfied, neutral)	(dissatisfied, very dissatisfied), (neutral, dissatisfied)	(satisfied, dissatisfied), (neutral, dissatisfied)
P_{10}	(satisfied, very dissatisfied), (neutral, dissatisfied)	(satisfied, very dissatisfied), (neutral, dissatisfied)	(neutral, dissatisfied), (neutral, very dissatisfied)	(satisfied, neutral), (very dissatisfied, dissatisfied)	(very satisfied, dissatisfied), (very dissatisfied, dissatisfied)

Step 2: The LDHHSWGAO is designed to aggregate values (weights: for Dry cough = 0.2, for Bitter taste = 0.3, for Upper Abdominal Discomfort = 0.2, for Epigastric = 0.1, for Belching = 0.2). across different symptoms (attributes) for each and every single patient.

Patients **LDHHSWGAO Values**

$$\begin{array}{c} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \end{array} = \begin{bmatrix} \text{(dissatisfied, neutral)} \\ \text{(satisfied, dissatisfied)} \\ \text{(very satisfied, very dissatisfied)} \\ \text{(very dissatisfied, dissatisfied)} \\ \text{(very dissatisfied, dissatisfied)} \\ \text{(satisied, dissatisfied)} \\ \text{(satisfied, dissatisied)} \\ \text{(neutral, dissatisfied)} \\ \text{(satisfied, neutral)} \\ \text{(satisfied, neutral)} \end{bmatrix}$$

Step 3

Next, the doctor uses this operator to aggregate the Linguistic Dual Hesitant variables for each Patients. This aggregation takes into account maximum of Membership and Non membership of all symptoms to calculate an overall score for each Patients.

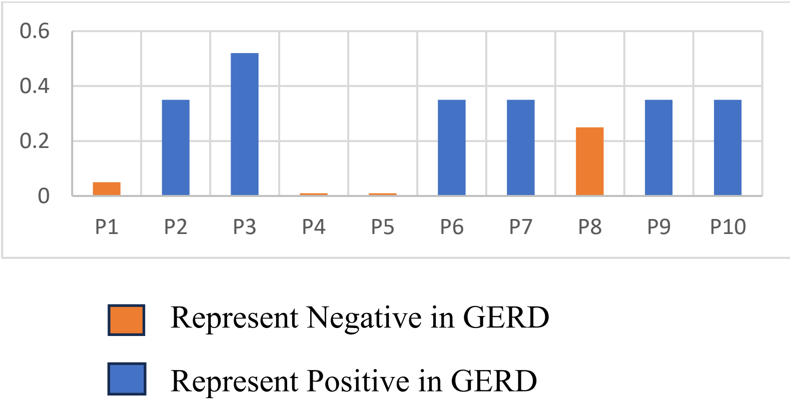
$$\begin{array}{ccc} \textbf{Patients} & & \textbf{Aggregated Values} \\ \begin{array}{c} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \end{array} & = & \begin{bmatrix} \text{(dissatisfied)} \\ \text{(satisfied)} \\ \text{(very satisfied)} \\ \text{(very dissatisfied)} \\ \text{(very dissatisfied)} \\ \text{(satisfied)} \\ \text{(satisfied)} \\ \text{(neutral)} \\ \text{(satisfied)} \\ \text{(satisfied)} \end{bmatrix} \end{array}$$

Step 4: Finally, list the alternatives with maximum Membership (γ^+) Values. The maximum (γ^+), will represent the positive ideal alternative.

$$\begin{array}{ccc} \textbf{Alternative} & & \textbf{Result} \\ \begin{array}{c} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \\ P_9 \\ P_{10} \end{array} & = & \begin{bmatrix} \text{(Negative)} \\ \text{(Positive)} \\ \text{(Positive)} \\ \text{(Negative)} \\ \text{(Negative)} \\ \text{(Positive)} \\ \text{(Positive)} \\ \text{(Negative)} \\ \text{(Positive)} \\ \text{(Positive)} \end{bmatrix} \end{array}$$

In this case study highlights how the LDHHS algorithm helped doctors overcome diagnostic challenges with patients showing common symptoms like Dry cough, Bitter taste, Upper Abdominal Discomfort, Epigastric pain and belching which overlap across multiple illness, including Gastroesophageal Reflux Disease (GERD). By leveraging advanced language models and analysing a wide range of medical data, the algorithm provided accurate, data-driven diagnoses, reducing uncertainty. Figure: 2 visually represents the relationship between Patients and GERD.

Figure 2: Negative and Positive Patients in GERD



6.3 Result Discussion Comparison and Future Direction

The comparison between proposed LDHHS algorithm and traditional diagnostic methods underscores its potential to revolutionize healthcare. Unlike conventional techniques reliant on clinical judgement, LDHHS leverages advanced language models and real-time medical data to deliver more accurate and adaptive diagnoses. Table 2 illustrates its comparable performance with existing methods, while its ability to incorporate new research and manage risk, particularly crises for GERD, highlights its superiority. By complementing, rather than replacing, the expertise of healthcare professionals LDHHS paves the way for more precise, efficient and patient-centered diagnostic solutions.

Table 2: Comparing Research Result with Existing Studies.

METHOD	POSITIVE	NEGATIVE
FDHS	P_1, P_3, P_4, P_6, P_7, P_{10}	P_2, P_5, P_8, P_9
LDHHS	P_2, P_5, P_6, P_8, P_{10}	P_1, P_3, P_4, P_7, P_9

7.Conclusion

In conclusion, this study underscores the critical role of language and the challenge it poses in medical diagnosis and treatment. By introducing Linguistic Dual Hesitant Hypersoft Sets (LDHHS), a robust framework for managing Linguistic Dual Hesitant uncertainty, the study presents a promising approach to enhancing healthcare decision-making. The proposed definition, concepts, aggregation operators and algorithms demonstrate the utility of LDHHS in addressing the complexity of modern medical practice, fostering more effective and patient-centered care. Looking ahead, expanding LDHHS to encompass a broader spectrum of medical conditions and fostering collaboration between data scientists and healthcare professionals will be key to advancing its potential. Beyond healthcare, the versatile framework of LDHHS holds promise for applications in diverse fields such as market research, environmental assessments and disaster preparedness, offering a powerful tool for navigating complex and uncertain environments.

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