NSS
Neutrosophic Sets and Systems, Vol. 98, 2026
University of New Mexico

Neutrosophic πgγ* closed sets in Neutrosophic Topological Spaces

Sakthivel K [1] and Gomathi A [2, *]

[1] Assistant professor, Department of Mathematics, Government Arts College, Udumalpet, Tamil Nadu, India; sakthivelaugust15@gmail.com

* Assistant professor, Department of Science and Humanities, Hindusthan College of Engineering and Technology, Coimbatore, Tamil Nadu, India; gomathimaths12@gmail.com

Abstract: This paper introduces and investigate the notion of Neutrosophic πgγ* closed sets within the framework of Neutrosophic topological spaces. The study begins defining Neutrosophic πgγ* closed sets and proceeds to investigate their fundamental properties and characterizations. Special attention is given to examining how these sets interrelate with and extend existing classes of Neutrosophic closed sets. By establishing inclusion relations and comparative hierarchies, the paper highlights the significance of Neutrosophic πgγ* closed sets in broadening the structural understanding of Neutrosophic topologies. Furthermore, several illustrative examples are provided to demonstrate the distinctive features of these sets and to clarify their role in the generalization process. The paper also derives various theorems that reveal the interplay between Neutrosophic πgγ* closed sets and other Neutrosophic closed families, thereby enriching the theoretical landscape of Neutrosophic topology. These results contribute to ongoing developments in generalized closed set theory, offering new insights and paths for further research in Neutrosophic mathematics.

Keywords: Neutrosophic topological spaces, Neutrosophic open sets, Neutrosophic closed sets, Neutrosophic πgγ* closed sets.

1. Introduction

The concept of fuzzy sets, introduced by Zadeh [13] in 1965, allow each element to have a degree of membership. This concept was expanded by K. Atanassov [1] in 1986 with the introduction of Intuitionistic Fuzzy sets, which assign both a degree of membership and a degree of non-membership to each element. Sakthivel K and Manikandan M [10] had studied the concept of πgγ* closed Sets in Intuitionistic Fuzzy Topological Spaces. Florentin Smarandache [2] introduced Neutrosophic Sets as a further generalization, which adds more flexibility. Later, Salama A A and Alblowi S A [11] extended the idea by developing Neutrosophic Topological Spaces.

In this article we define Neutrosophic πgγ* closed sets in Neutrosophic topological spaces and investigate their properties.

2. Preliminaries

This section reviews essential definitions, operations, and key results related to Neutrosophic sets.

Definition 2.1 [6] Let X be a non-empty fixed set. A Neutrosophic Set (NS) A is an object having the form A=$\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ where $\mu_A(x), \sigma_A(x)$ and $\nu_A(x)$ represent the degree of membership, degree of indeterminacy and the degree of non-membership respectively of each element x ∈ X the set A.

Definition 2.2 [6] Let X be a non-empty set and let A be a Neutrosophic Set
A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$.Then the complement of A is
$A^C = \{\langle x, \nu_A(x), 1 - \sigma_A(x), \mu_A(x) \rangle : x \in X\}$

Definition 2.3 [6] Let A and B be two Neutrosophic Sets, ∀ x ∈ X
A = $\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$
B = $\{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X\}$.
Then A ⊆ B ⇔ $\{\langle x, \mu_A(x) \le \mu_B(x), \sigma_A(x) \le \sigma_B(x), \nu_A(x) \ge \nu_B(x) \rangle : x \in X\}$

Definition 2.4 [6] Let X be a non-empty set and let A and B be two Neutrosophic Sets are
A=$\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$, B = $\{\langle x, \mu_B(x), \sigma_B(x), \nu_B(x) \rangle : x \in X\}$. Then

1. A ∩ B = $\{\langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X\}$
2. A ∪ B = $\{\langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(X), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X\}$

Definition 2.5 [6] Let X be a non-empty set and τ_N be the collection of Neutrosophic subsets of X satisfying the following properties:

1. $0_N, 1_N \in \tau_N$
2. $T_1 \cap T_2 \in \tau_N$ for any $T_1, T_2 \in \tau_N$
3. $\cup T_i \in \tau_N$ for every $\{T_i : i \in j\} \subseteq \tau_N$

Then the space (X, τ_N) is called a Neutrosophic Topological Space (NTS). The elements of τ_N are called Neutrosophic Open Set (NOS) and its complement is Neutrosophic Closed Set (NCS).

Definition 2.6 [6] Let (X, τ_N) be a NTS and A=$\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ be a NS in X. Then
Neutrosophic closure of A is N_Cl(A) = ∩ { M : M is a NCS in X and A ⊆ M}
Neutrosophic interior of A is N_Int(A) = ∪ { H : H is a NOS in X and H ⊆A}

Definition 2.7 Let (X, τ_N) be a NTS and A=$\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ be a NS in X. Then A is said to be

- Neutrosophic Semi closed set [4] (NSCS) if N_Int (N_Cl (A)) ⊆A.
- Neutrosophic Semi open set [4] (NSOS) if A ⊆ N_Cl (N_Int(A)).
- Neutrosophic Pre closed set [12] (NPCS) if N_Cl(N_Int(A)) ⊆A.
- Neutrosophic Pre open set [12] (NPOS) if A⊆ N_Int(N_Cl(A)).
- Neutrosophic Regular closed set [7] (NRCS) if A= N_Cl(N_IntA)).
- Neutrosophic Regular open set [7] (NROS if A = N_Int(N_Cl(A)).
- Neutrosophic α closed set [5] (NαCS) if N_Cl (N_Int (N_Cl (A))) ⊆A.
- Neutrosophic α open set [5] (NαOS) if A ⊆ N_Int (N_Cl (N_Int (A))).
- Neutrosophic β closed set [8] (NβCS) if N_Int (N_Cl (N_Int (A))) ⊆A.
- Neutrosophic β open set [8] (NβOS in short) if A⊆ N_Cl (N_Int (N_Cl (A))).
- Neutrosophic b closed set [6] (NbCS) if N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆ A

- Neutrosophic b open set [6] (NbOS) if A ⊆ N_Int (N_Cl (A)) ∪ N_Cl (N_Int(A))
- Neutrosophic π closed set [9] if A is the union of Neutrosophic Regular closed sets.
- Neutrosophic π open set [9] if A is the union of Neutrosophic Regular open sets.

Definition 2.8 Let (X, τ_N) be a NTS and A=$\{\langle x, \mu_A(x), \sigma_A(x), \nu_A(x) \rangle : x \in X\}$ be a NS in X. Then A is said to be

- Neutrosophic Generalized closed set [3] (NGCS) if N_Cl(A)⊆ U whenever A ⊆ U and U is a NOS in X.
- Neutrosophic Generalized semi closed set [3] (NGSCS) if N_sCl(A) ⊆ U whenever A ⊆ U andU is a NOS in X.
- Neutrosophic α Generalized closed set [5] (NαGCS) if N_αCl(A) ⊆ U whenever A ⊆ U and U is a NOS in X

Remark 2.9 For any Neutrosophic Set A,

- N_Cl (A) ^{C} = (N_Int (A)) ^{C}
- N_Int (A)^C = (N_Cl (A)) ^{C}
- N_sCl (A) ^{C} = (N_sInt (A)) ^{C}
- N_sInt (A)^C = (N_sCl (A)) ^{C}
- N_sCl (A) = A ∪ N_Int (N_Cl (A))
- N_sInt (A) = A ∩ N_Cl (N_Int (A))
- N_αCl(A) = A ∪ N_Cl (N_Int (N_Cl A)))
- N_αInt(A) = A ∩ N_Int (N_Cl (N_Int (A)))

Remark 2.10

1. Each NOS is NPOS in NTS.
2. Each NRCS is NCS in NTS.
3. Each N πOS is NOS in NTS.

3. Neutrosophic πgγ* closed sets in Neutrosophic Topological Spaces

We have introduced Neutrosophic πgγ* closed sets and explored some of their properties.

Definition 3.1. A Neutrosophic Set A in (X, τ_N) is said to be a Neutrosophic πgγ* closed sets (Nπgγ*CS in short) if N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆ U whenever A⊆ U and U is NπOS in (X, τ_N).

Example 3.2. Let X = {p, q} with τ_N = {0_N, B, 1_N} be a NTS on X, where B = <x, (0.4,0.5,0.7), (0.4,0.4,0.6)>. Let us consider the NS, A = <x, (0.3,0.2,0.7), (0.3,0.2,0.8)>. Here NπOS is U = {1_N, B}. Clearly A ⊆ U . Now N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) = 0_N ⊆ U . Therefore NS, A is a Nπgγ*CS in (X, τ_N).

Theorem 3.3. Every NCS in (X, τ_N) is a N πgγ*CS (X, τ_N) but not conversely in general.
Proof: Consider 'A' is a NCS in (X, τ_N). Assume that A⊆ U and U is a NπOS in (X, τ_N). Since A is a NCS in (X, τ_N), N_Cl(A) = A . Now N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) = N_Int(A) ∩ N_Int(A) = N_Int(A) ⊆ A ⊆ U. Thus N_Cl (N_Int(A)) ∩ N_Int(N_Cl (A)) ⊆ U , whenever A⊆ U and U is NπOS in (X, τ_N) Therefore, A is a N πgγ*CS in (X, τ_N). However, the reverse implication is not true.

Example 3.4. Let X = {p, q} with τ_N = { 0_N , B, 1_N } be a NTS on X , where B = <x, (0.2,0.4,0.6), (0.3,0.5,0.7)>. Let us consider the NS, A = <x, (0.1,0.3,0.8), (0.2,0.4,0.7)>. Here NπOS is Ʊ = {1_N , B}. Clearly A ⊆ Ʊ. Now N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) = 0_N ⊆ Ʊ. But N_Cl(A) = B^c ⊈ A. Therefore NS, A is a N πgγ*CS but not NCS in (X, τ_N).

Theorem 3.5. Every NSCS in (X, τ_N) is a Nπgγ*CS (X, τ_N) but not conversely in general.
Proof: Consider A is a NSCS in (X, τ_N). Suppose that A ⊆ Ʊ and Ʊ is a NπOS in (X, τ_N). Since A is a NSCS in (X, τ_N), N_Int (N_Cl (A)) ⊆ A. Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ N_Cl (N_Int (A)) ∩ A ⊆ N_Cl (A) ∩ A = A ⊆ Ʊ. Thus N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ Ʊ , whenever A⊆ Ʊ and Ʊ is NπOS in (X, τ_N). Therefore, A is a Nπgγ*CS in (X, τ_N). However, the reverse implication is not true.

Example 3.6. Let X = {p, q} with τ_N = { 0_N ,B, 1_N } be a NTS on X , where B = <x, (0.2,0.4,0.6), (0.3,0.5,0.7)>. Let us consider the NS, A = <x, (0.1,0.3,0.8), (0.2,0.4,0.7)> . Here NπOS is Ʊ = {1_N , B}. Clearly A ⊆ Ʊ . Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) = 0_N ⊆ Ʊ . But N_Int (N_Cl (A)) = B ⊈ A. Therefore NS, A is a Nπgγ*CS but not NSCS in (X, τ_N).

Theorem 3.7. Every NPCS in (X, τ_N) is a Nπgγ*CS (X, τ_N) but not conversely in general.
Proof: Consider A is a NPCS in (X, τ_N). Suppose that A ⊆ Ʊ and Ʊ is a NπOS in (X, τ_N). Since A is a NPCS in (X, τ_N), N_Cl (N_Int (A)) ⊆ A. Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ A ∩ N_Int (N_Cl (A)) ⊆ A ∩ N_Cl (A) = A ⊆ Ʊ. Thus N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆ Ʊ , whenever A ⊆ Ʊ and Ʊ is NπOS in (X, τ_N). Therefore, A is a Nπgγ*CS in (X, τ_N). However, the reverse implication is not true.

Example 3.8. Let X = { p, q } with τ_N = { 0_N , B_1, B_2, 1_N } be a NTS on X , where B_1 = <x, (0.5,0.5,0.5), (0.6,0.5,0.4)> and B_2 = <x, (0.4,0.5,0.6), (0.3,0.5,0.7)>. Let us consider the NS, A = <x, (0.4,0.5,0.6), (0.4,0.5,0.6)> . Here NπOS is Ʊ = {$1_N, B_1, B_2$}. Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) = B_2 ⊆ Ʊ . But N_Cl (N_Int (A) = B_1^C ⊈ A. Therefore NS, A is a Nπgγ*CS but not NPCS in (X, τ_N).

Theorem 3.9. Every NRCS in (X, τ_N) is a Nπgγ*CS in (X, τ_N) but not conversely in general.
Proof: Let A be a NRCS in (X, τ_N). Since every NRCS is a NCS, by Theorem 3.3 A is a Nπgγ*CS in (X, τ_N). However, the reverse implication is not true.

Example 3.10. Let X = { p, q } with τ_N ={ 0_N ,B, 1_N } be a NTS on X , where B= <x, (0.5,0.5,0.5), (0.4,0.5,0.6)>. Let us consider the NS, A=<x, (0.5,0.5,0.5), (0.5,0.5,0.4)>. Here NπOS is Ʊ = {1_N , B}. Clearly A ⊆ Ʊ. Now N_Cl (N_Int (A))∩N_Int (N_Cl (A)) = B ⊆ Ʊ . But N_Cl (N_Int (A) = $B^C \neq A$ .Therefore NS, A is a N$\pi g\gamma^*$CS but not NRCS in (X, τ_N).

Theorem 3.11. Every NROS in (X, τ_N) is a Nπgγ*CS in (X, τ_N) but not conversely in general.
Proof: Let A be a NROS in (X, τ_N). A is a Nπgγ*CS in (X, τ_N). Then N_Int(N_Cl(A)) = A and N_Int(A) = A. Suppose that A ⊆ Ʊ and Ʊ is a NπOS in (X, τ_N). Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ N_Cl (A) ∩ A = A ⊆ Ʊ . Thus N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A))⊆Ʊ , whenever A⊆ Ʊ and Ʊ is NπOS in (X, τ_N). Therefore, A is a Nπgγ*CS in (X, τ_N). However, the reverse implication is not true.

Example 3.12. Let X = { p, q } with τ_N = { 0_N ,B, 1_N } be a NTS on X , where B = <x, (0.5,0.5,0.5), (0.4,0.5,0.6)>. Let us consider the NS, A = <x, (0.4,0.5,0.6), (0.4,0.5,0.6)>. Here

NπOS is $\cup = \{1_N, B\}$. Clearly A ⊆ ∪. Now N_Cl (N_Int(A)) ∩ N_Int(N_Cl (A)) = 0_N ⊆ ∪ . But N_Int (N_Cl (A)) = B ≠ A. Therefore NS, A is a Nπgγ*CS but not NROS in (X, τ_N).

Theorem 3.13. Every NαCS in (X, τ_N) is a Nπgγ*CS (X, τ_N) but not conversely in general.

Proof: Consider A is a NαCS in (X, τ_N). Suppose that A⊆ ∪ and ∪ is a NπOS in (X, τ_N). Since A is a NαCS in (X, τ_N), N_Cl (N_Int (N_Cl (A))) ⊆ A. Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ N_Cl (N_Int (N_Cl (A))) ∩ N_Cl (N_Int (N_Cl (A))) ⊆ A ∩ A = A ⊆ ∪ . Thus N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ ∪ , whenever A ⊆ ∪ and ∪ is NπOS in (X, τ_N). Therefore, A is a Nπgγ*CS in (X, τ_N). However, the reverse implication is not true.

Example 3.14. Let X = { p, q } with τ_N ={ 0_N , B, 1_N } be a NTS on X , where B = <x, (0.5,0.5,0.5), (0.4,0.5,0.6)>. Let us consider the NS, A = <x, (0.4,0.5,0.4), (0.5,0.5,0.6)>. Here NπOS is $\cup = \{1_N, B\}$. Clearly A ⊆ ∪. Now N_Cl (N_Int(A)) ∩ N_Int (N_Cl(A)) = 0_N ⊆ ∪ . But N_Cl (N_Int (N_Cl (A))) $= B^C \nsubseteq A$. Therefore NS, A is a Nπgγ*CS but not NαCS in (X, τ_N).

Theorem 3.15. Every NbCS in (X, τ_N) is a Nπgγ*CS (X, τ_N), but not conversely in general.

Proof: Consider A is a NbCS in (X, τ_N). Suppose that A ⊆ ∪ and ∪ is a NπOS in (X, τ_N). Since A is a NbCS in (X, τ_N), N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆ A. Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ A ⊆ ∪. Thus N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ ∪, whenever A ⊆ ∪ and ∪ is NπOS in (X, τ_N). Therefore, A is a Nπgγ*CS in (X, τ_N). However, the reverse implication is not true.

Example 3.16. Let X ={ p , q } with τ_N ={ 0_N , B_1, B_2, 1_N } be a NTS on X , where B_1 = <x, (0.5,0.5,0.5), (0.6,0.5,0.4)> and B_2 = <x, (0.4,0.5,0.6), (0.3,0.5,0.7)>. Let us consider the NS, A =<x, (0.4,0.5,0.6), (0.6,0.5,0.4)> . Here NπOS is $\cup = \{1_N, B_1, B_2\}$. Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ ∪ whenever A ⊆ ∪ . But N_Cl (N_Int(A)) ∩ N_Int (N_Cl(A)) = $B_1^C \nsubseteq A$. Therefore NS, A is a Nπgγ*CS but not NbCS in (X, τ_N).

Theorem 3.17. Every NGCS in (X, τ_N) is a Nπgγ*CS in (X, τ_N) but not conversely.

Proof: Consider A is a NGCS in (X, τ_N). Assume that A⊆ ∪ and ∪ is a NπOS in (X, τ_N). Since A is a NGCS in (X, τ_N), N_Cl(A) ⊆ ∪ . Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ N_Cl (A) ⊆ ∪ .Thus N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ ∪ , whenever A ⊆ ∪ and ∪ is NπOS in (X, τ_N). Therefore, A is a Nπgγ*CS in (X, τ_N). However, the reverse implication is not true.

Example 3.18. Let X ={ p, q } with τ_N ={ 0_N , B, 1_N } be a NTS on X , where B = <x, (0.3,0.5,0.4), (0.2,0.5,0.3)>. Consider NS, A = <x, (0.2,0.4,0.6), (0.2,0.4,0.4)>. Here NπOS $\cup = \{1_N, B\}$. Clearly A ⊆ ∪. Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) = 0_N ⊆ ∪. But N_Cl(A) = $B^C \nsubseteq B$, whenever A ⊆ B . Therefore NS, A is a Nπgγ*CS but not NGCS in (X, τ_N).

Theorem 3.19. Every NGSCS in (X, τ_N) is a Nπgγ*CS in (X, τ_N) but not conversely.

Proof: Consider 'A' is a NGSCS in (X, τ_N). Assume that A⊆ ∪ and ∪ is a NπOS in (X, τ_N). Since A is a NGSCS in (X, τ_N), N_sCl(A) = A ∪ N_Int(N_Cl(A)) ⊆ ∪ .This implies N_Int (N_Cl(A) ⊆ ∪. Now N_Cl (N_Int(A)) ∩ N_Int (N_Cl(A)) ⊆ N_Cl (N_Int(A)) ∩ ∪ ⊆ ∪ .Thus N_Cl (N_Int(A)) ∩ N_Int (N_Cl(A)) ⊆ ∪ , whenever A⊆ ∪ and ∪ is NπOS in (X, τ_N) Therefore, A is a Nπgγ*CS in (X, τ_N). However, the reverse implication is not true.

Example 3.20. Let X = { p , q } with τ_N ={ 0_N , B_1, B_2, 1_N } be a NTS on X , where B_1 = <x, (0.5,0.5,0.5), (0.3,0.5,0.7)> and B_2 = <x, (0.4,0.5,0.6), (0.3,0.5,0.7)>. Let us consider the NS, A = <x, (0.3,0.5,0.4), (0.2,0.5,0.8)>. Here NπOS is $\cup = \{1_N, B_1, B_2\}$. Now N_Cl (N_Int(A)) ∩ N_Int

(N_Cl(A)) = 0_N ⊆ Ʋ whenever A ⊆ Ʋ. But N_sCl (A) = A ∪ N_Int (N_Cl (A)) = A ∪ B_1 = B_1 ⊈ B and A ⊆ B_2. Therefore NS, A is a Nπgγ*CS but not NGSCS in (X, τ_N).

Theorem 3.21. Every NαGCS in (X, τ_N) is a Nπgγ*CS (X, τ_N) but not conversely in general.

Proof: Consider A is a NαGCS in (X, τ_N). Suppose that A ⊆ Ʋ and Ʋ is a NπOS in (X, τ_N). Since A is a NαGCS in (X, τ_N), N_αCl(A)⊆ Ʋ. Therefore A ∪ N_Cl (N_Int (N_Cl (A)))⊆ Ʋ, so N_Cl (N_Int (N_Cl A)))⊆ Ʋ and N_Int (N_Cl A)) ⊆ Ʋ. Now N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A))⊆ N_Cl (N_Int (A)) ∩ Ʋ ⊆ Ʋ. Thus N_Cl (N_Int (A)) ∩ N_Int (N_Cl (A)) ⊆ Ʋ, whenever A ⊆ Ʋ and Ʋ is NπOS in (X, τ_N) . Therefore, A is a Nπgγ*CS in (X, τ_N). However, the reverse implication is not true.

Example 3.22. In Example 3.20, the NS A =<x, (0.3,0.5,0.7), (0.2,0.5,0.8)> is Nπgγ*CS in (X, τ_N) but not a NαGCS in (X, τ_N) as N_αCl(A) = A ∪ N_Cl (N_Int (N_Cl (A)))= A ∪ B_1^C = B_1^C ⊈ B_1, B_2 and A ⊆ B_1, B_2 .

In the following figure (a) we have provided relation between Nπgγ*CS and other closed sets in Neutrosophic topological space.

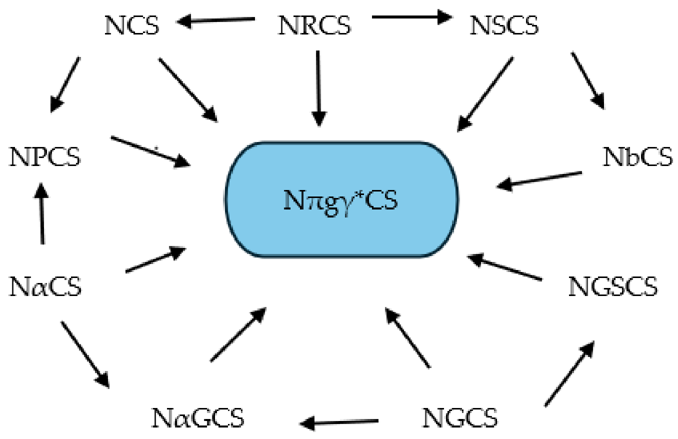

Figure (a)

Remark 3.23. The union of two Nπgγ*CSs need not be a Nπgγ*CS in (X, τ_N) in general.

Example 3.24. Let X = {p , q} with τ_N = { 0_N ,B_1, B_2, 1_N} be a NTS on X, where B_1 = <x, (0.4,0.5,0.6), (0.2,0.5,0.8) > and B_2 = <x, (0.4,0.5,0.5), (0.4,0.5,0.5)>. Here A = <x, (0.4,0.5,0.5), (0.5,0.5,0.4) > and B = <x, (0.5,0.5,0.4), (0.2,0.5,0.6) > are Nπgγ*CS in (X, τ_N), but A ∪ B = <x, (0.5,0.5,0.4), (0.5,0.5,0.4)> is not a Nπgγ*CS in (X, τ_N).

Theorem 3.25. Let (X, τ_N) be a NTS. Then for every A ∈ Nπgγ*CS in (X, τ_N) and for every B ∈ NS in (X, τ_N), A ⊆ B ⊆ N_Cl (N_Int(A)) which implies B ∈ Nπgγ*CS in (X, τ_N).

Proof: Let B ⊆ Ʋ and Ʋ be an NπOS in X. Since A ⊆ B, A ⊆ Ʋ. Also, B ⊆ N_Cl (N_Int(A)) which implies N_Cl (N_Int(B)) ⊆ N_Cl (N_Int(A)). Now N_Int (N_Cl (B)) ⊆ N_Int (N_Cl (A)). Therefore N_Cl (N_Int(B)) ∩ N_Int (N_Cl (B)) ⊆ N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆ Ʋ, by hypothesis. Hence B ∈ Nπgγ*CS in (X, τ_N).

Theorem 3.26. If A is both NπOS and Nπgγ*CS in (X, τ_N) then A is a NbCS in (X, τ_N).

Proof: Let A be a NπOS and a Nπgγ*CS in (X, τ_N). Then N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆A, as A ⊆ A, by hypothesis. Therefore N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆ A. Hence A is a NbCS in (X, τ_N).

Theorem 3.27. If A is both a NπOS and a Nπgγ*CS in (X, τ_N) then A is a NβCS in (X, τ_N).
Proof: Let A be a NπOS and a Nπgγ*CS in (X, τ_N). Then N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆ A, as A ⊆ A, by hypothesis. Now N_Int (N_Cl (N_Int(A))) = N_Int (N_Cl (N_Int(A))) ∩ N_Cl (N_Int(A)) ⊆ N_Int (N_Cl (A)) ∩ N_Cl (N_Int(A)) = N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆ A. Therefore N_Int (N_Cl (N_Int(A))) ⊆ A . Hence A is a NβCS in (X, τ_N).

Theorem 3.28. If A is both a NπOS and a Nπgγ*CS in (X, τ_N) then A is a NSCS in (X, τ_N).
Proof: Let A be a NπOS and a Nπgγ*CS in (X, τ_N). That is A is a NOS in (X, τ_N). Then N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆ A, as A ⊆ A, by hypothesis. Clearly N_Int (N_Cl (A)) = N_Cl (A) ∩ N_Int (N_Cl (A)) = N_Cl (N_Int(A)) ∩ N_Int (N_Cl (A)) ⊆ A. Therefore N_Int (N_Cl (A)) ⊆ A. Hence A is a NSCS in (X, τ_N).

Theorem 3.29. If A is both a NπOS and a Nπgγ*CS in (X, τ_N), then A is a NROS in (X, τ_N).
Proof: Let A be a NπOS and a Nπgγ*CS in (X, τ_N). That is A is a NOS in (X, τ_N). Then N_Int (N_Cl (A)) = N_Int (N_Cl (A)) ∩ N_Cl (A) = N_Int (N_Cl (A)) ∩ N_Cl (N_Int(A)) ⊆ A. Since A is a NOS, it is a NPOS and A ⊆ N_Int (N_Cl (A)). Therefore A = N_Int (N_Cl (A)). Hence A is a NRCS in (X, τ_N).

4. Conclusion: In this paper we have introduced Neutrosophic πgγ* closed sets in Neutrosophic Topological Spaces and discussed some of its properties and some contradicting examples. This idea can be developed and extended in the area of continuous functions, homeomorphisms, compactness and connected and so on.

Funding: This research received no external funding.
Acknowledgements: The authors are highly grateful to the Referees for their consecutive suggestions.
Conflicts of Interest: The authors declare no conflict of interest.

REFERENCES:

[1] Atanassov, K. Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems*, 1986,*20*, 87–94.
[2] Florentin Smarandache, Neutrosophic Set, A Generalization of the Intuitionistic Fuzzy Set. *Journal of Defense Resources Management*, 2010, *1*(1), 107–116. https://www.researchgate.net/publication/354204356
[3] Iswarya, P; Bageerathi, K. A Study on Neutrosophic Generalized Semi-Closed Sets in Neutrosophic Topological Spaces. *Journal of Emerging Technologies and Innovative Research*, 2019, *6*(2), 452–457. www.jetir.org
[4] Iswarya, P; Bageerathi, K. On Neutrosophic Semi-Open sets in Neutrosophic Topological Spaces. *International Journal of Mathematics Trends and Technology*, 2019, *37*(3), 24–33. http://www.ijmttjournal.org

[5] Jayanthi, D. α Generalized Closed Sets in Neutrosophic Topological Spaces. *International Journal of Mathematics Trends and Technology (IJMTT), 2018, Special Issue*, 88–92. http://www.ijmttjournal.org

[6] Maheswari, C; Sathyabama, M; Chandrasekar, S. Neutrosophic generalized b-closed sets in Neutrosophic topological spaces. *Journal of Physics: Conference Series*, 2018, *1139*(1), 1–7. https://doi.org/10.1088/1742-6596/1139/1/012065

[7] Mohammed Ali Jaffer, I ; Ramesh, K. Neutrosophic Generalized Pre Regular Closed Sets. *Neutrosophic Sets and Systems*, 2019, *30*, 171–181. https://doi.org/10.5281/zenodo.3569681

[8] Narmatha, S; Glory Bebina, E; Vishnu Priya, R. On πgβ–Closed Sets and Mappings in Neutrosophic Topological Spaces. *International Journal of Innovative Technology and Exploring Engineering*, 2019, *8*(12S), 505–510. https://doi.org/10.35940/ijitee.l1126.10812s19

[9] Poornima, R; Girija, S; Gayathri, C; Gayathri, P. *A study of Neutrosophic Rg– closed sets,* IIP Series. e-ISBN: 978-93-6252-737-0, Book 2, Part 3, Chapter 1, 2024,3,67-94

[10] Sakthivel, K; Manikandan, M. πgγ* closed Sets in Intuitionistic Fuzzy Topological Spaces. *International Journal of Mathematics Trends and Technology*, 2019, *65*(3), 167–174. http://www.ijmttjournal.org

[11] Salama, A. A; Albowi, S. A. Neutrosophic Set and Neutrosophic Topological Spaces. *IOSR Journal of Mathematics*, 2012, *3*(4), 31–35. https://doi.org/10.9790/5728-0343135

[12] Venkateswara Rao, V; Srinivasa Rao, Y. Neutrosophic Pre-open Sets and Pre-closed Sets in Neutrosophic Topology. *International Journal of Chem Tech Research*, 2017, *10*(10), 449–458.

[13] Zadeh, L. A. Fuzzy Sets*. *Information and Control*, 1965, *8*, 338–353.

Received: July 16, 2025. Accepted: Dec 15, 2025