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Nonagonal Neutrosophic Number: Analytical Aspects and its Role in Optimization technique for transportation problem

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Abstract: Neutrosophic numbers have received increasing attention from researchers and industrialists to address the indeterminacy and uncertainty inherent in real-life decision-making. This study aims to solve the transportation problem where supply, demand and transportation costs are Nonagonal Neutrosophic Numbers (NNNs). In the existing literature, various methods have been introduced to solve transportation problems (TPs) involving neutrosophic parameters. The application of Nonagonal Neutrosophic Numbers to transportation problems is a relatively recent development. NNNs offer a more detailed and adaptable representation of uncertainty by utilizing a nine-parameter structure that captures the degrees of truth, indeterminacy, and falsity. Therefore, in this paper, we solve the transportation problem using Nonagonal Neutrosophic Numbers for the first time. To facilitate this, we introduce two novel score functions for converting NNNs into crisp values. Based on these, we propose an algorithmic framework to obtain the optimal solution effectively. To exemplify the effectiveness of the proposed method, we solved a numerical example, and the obtained results are presented and compared with those in the existing literature. Finally, the significance of this study and potential directions for future research are mentioned.

Keywords: Transportation problem, Nonagonal neutrosophic numbers, Defuzzification, Crisp data, Optimization

1. Introduction

The transportation problem (TP) is one kind of Linear programming problem that aims to minimize the total cost involved in transporting goods from multiple sources to multiple destinations. George Dantzig is attributed with the initial formulation of the concept of linear programming in 1947.Hitchcock formulated classical transportation problem for the first time. Although the classical transportation problem cannot handle the uncertainties in real-world scenarios, it only narrates the rigorous data of supply and demand.

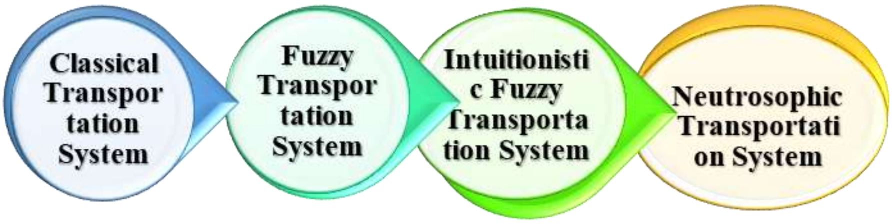
To address this gap, researchers have introduced classical transportation models into fuzzy systems. Fuzzy set theory, formulated by Lotfi A. Zadeh [3] in 1965, enables the modeling of vagueness and imprecision by incorporating degrees of membership. Building upon this concept, Atanassov proposed Intuitionistic Fuzzy Sets (IFS) in 1986, which consider both membership and non-membership functions. However, IFS falls short of explicitly representing indeterminacy which is a crucial factor in many real-world applications.

In response to the inability of existing models to address this shortcoming, the concept of neutrosophy was introduced by Florentin Smarandache in 1999. Neutrosophic sets extend classical and fuzzy theories by incorporating truth, indeterminacy, and falsity components, thereby offering a more comprehensive framework for modeling incomplete, inconsistent, and indeterminate information. Since then, neutrosophic theory has been successfully applied in various optimization domains, including transportation problems. Notably, Thamaraiselvi and Santhi [11] proposed optimization techniques for transportation problems under a neutrosophic environment. Singh Kumar, and Appadoo [6] further contributed to the field by presenting a modified approach for optimizing real-life transportation problems using neutrosophic sets in 2017.

Subsequent studies have explored various forms of neutrosophic numbers, such as triangular, trapezoidal, pentagonal, hexagonal, heptagonal, and octagonal, to enhance uncertainty modeling. Srinivas. S & Prabakaran. K [8] developed a method for solving transportation problems using triangular fuzzy neutrosophic numbers in 2023. Broumi et al. [12] introduced an algorithm for trapezoidal interval-valued neutrosophic network analysis, demonstrating the applicability of neutrosophic logic in complex decision-making systems. In 2024, Kalaivani Kaspar and Palanivel Kaliyaperumal [17] employed single-valued trapezoidal neutrosophic numbers to solve transportation problems with mixed constraints, highlighting the versatility of neutrosophic numbers in handling various types of uncertainty.

Moreover, Chakraborty, Broumi, and Singh [18] explored the use of pentagonal neutrosophic numbers, contributing to the expanding literature on neutrosophic-based transportation models. Nagalakshmi, T., Sudharani, R., & Ambika, G. [23] presented a comparative analysis of fuzzy transportation problems using hexagonal fuzzy numbers and neutrosophic triangular fuzzy numbers in 2022. Recent developments have also considered heptagonal and octagonal neutrosophic numbers, along with the formulation of appropriate score functions, further expanding the applicability of neutrosophic models in uncertain environments, have been discussed in [24]-[27].

In this context, the present study focuses on advancing transportation problem modeling using Nonagonal Neutrosophic Numbers (NNNs). These provide a finer and more detailed representation of uncertainty through nine parameters that reflect varying levels of truth, indeterminacy, and falsity. This novel approach aims to provide enhanced accuracy in decision-making for complex and uncertain transportation systems.



Motivation of the study

Real-world transportation systems are often fraught with uncertainty, imprecision and indeterminacy caused by fluctuating costs, unpredictable weather conditions, and incomplete information. To tackle these challenges, neutrosophic numbers have been increasingly utilized. Recent developments, such as Triangular, Trapezoidal, Hexagonal, and Octagonal neutrosophic numbers have offered improved modeling adaptability, but may still be limited in representing highly complex and layered uncertainties. This motivates the use of Nonagonal Neutrosophic Numbers**,** a more advanced representation that employs nine parameters to capture deeper nuances in uncertainty. NNNs allow for finer granularity in modeling expert opinions and fluctuating data, offering enhanced decision-making capabilities in environments characterized by high ambiguity and incomplete knowledge.

Research gaps

Based on the literature review, it is evident that numerous researchers have developed various methods to solve TPs in certain and uncertain environments. However, several research gaps still remain in the existing approaches, as listed below:

- The majority of existing studies focuses on triangular, trapezoidal, or hexagonal neutrosophic numbers, which, while effective, offer limited capacity to model highly complex or multi-layered uncertainty. The potential of NNNs-with their nine-parameter structure, for capturing more intricate and uncertainties remains underexplored.

- While the conceptual basis of NNNs is emerging, there is a lack of structured frameworks or algorithms specifically designed to apply NNNs in transportation problem contexts. This limits their practical utility and acceptance.
- There is limited literature comparing the performance of NNN-based solutions with other fuzzy and neutrosophic approaches in terms of accuracy, robustness, and computational efficiency. Such comparisons are essential to validate the effectiveness of NNNs in real-world scenario.
- Few studies have implemented NNN-based transportation models in practical domains such as disaster management, logistics planning and supply chain distribution. This highlights the gap between theoretical development and practical implementation.

Research highlights

This paper presents a comprehensive overview of transportation problem in a neutrosophic manner using NNNs.It explores the characteristics of NNNs, identifies gaps in existing approaches to various types of transportation problems, and introduces double defuzzification for NNNs. A step-by-step algorithm is introduced to solve the transportation problem in a neutrosophic nonagonal context. The supply, demand, and transportation cost entries in the transportation table are expressed using NNNs, allowing for a more refined modeling of uncertainty. The outcomes were compared with those obtained from existing methods to demonstrate their effectiveness and applicability.

This paper is structured as follows: Section 1 presents the abstract and introduction. Section 2 focuses on fundamental definitions. Sections 3 and 4 introduce NNNs and discuss their mathematical properties, respectively. The score function for defuzzification of NNNs is expressed in section 5.In section 6-9, include the proposed algorithm, illustrative diagrams, mathematical formulation, and a numerical example to demonstrate the methodology. Section 10-12 discuss the findings in the form of major conclusions, limitations and future scope.

2.Preliminaries

Definition 1 A fuzzy set $\mathfrak{F}$ in a non-empty set X is defined as $\mathfrak{F} = \{(x, \Theta_{\mathfrak{F}}(x)/x \in X)\}$

where $\Theta_{\mathfrak{F}}$, the degree of membership of x in fuzzy set $\mathfrak{F}$ and $\Theta_{\mathfrak{F}} : X \to [0,1]$.

Definition 2 A set $\mathfrak{I}$ in a non-empty set X is said to be an intuitionistic fuzzy set as

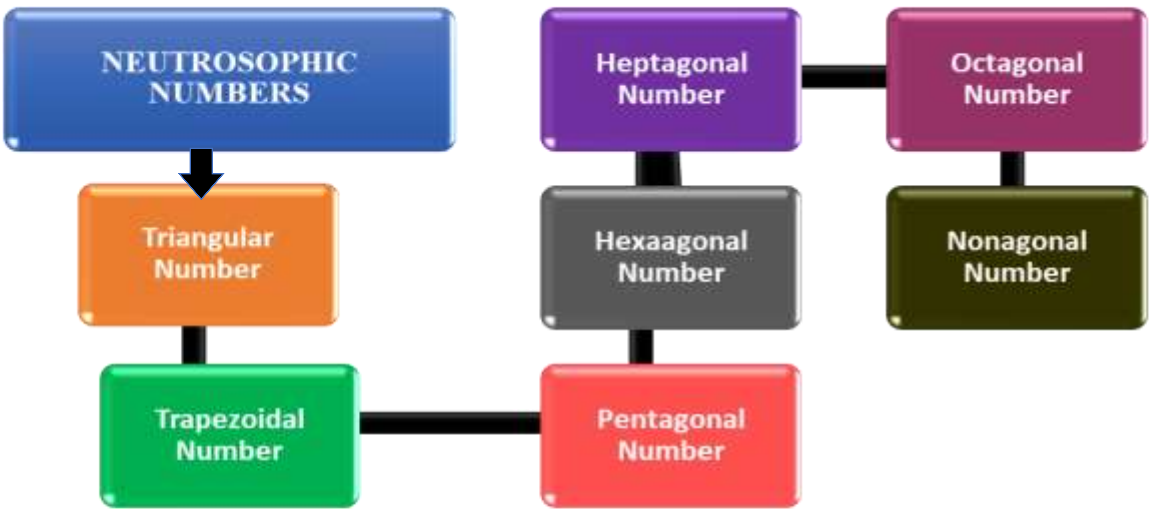
$\mathfrak{I} = \{(x, \Theta_{\mathfrak{I}}(x), \xi_{\mathfrak{I}}(x))/x \in X\}$ where $\Theta_{\mathfrak{I}}, \xi_{\mathfrak{I}} : X \to [0,1]$ and $\Theta_{\mathfrak{I}}(x)$, $\delta_M(x)$ indicates the degree of membership and non-membership of x to the set M such that $0 \le \Theta_{\mathfrak{I}}(x) + \xi_{\mathfrak{I}}(x) \le 1$.

Definition 3 Let X be a non-empty set. A neutrosophic set $\mathcal{N}$ in X is defined as $\mathcal{N} = \{(x, \Theta_{\mathcal{N}}(x), \xi_{\mathcal{N}}(x), \varpi_{\mathcal{N}}(x))/x \in X\}$ where $\Theta_{\mathcal{N}}$ is a truth membership function, $\xi_{\mathcal{N}}$ is an indeterminacy -membership function and $\varpi_{\mathcal{N}}$ is a falsity-membership function in which $\Theta_{\mathcal{N}}, \xi_{\mathcal{N}}, \varpi_{\mathcal{N}} : X \to -]0, 1[^{+}$ and $-0 \le \Theta_{\mathcal{N}} + \xi_{\mathcal{N}} + \le 3^{+}$.

Definition 4 A Neutrosophic set $\mathcal{N}$ in a set of real numbers R, called the neutrosophic number then it meets:

1) If there exist x0 $\in R$, such that $\Theta_{\mathcal{N}}(x_0) = 1$, $\xi_{\mathcal{N}}(x_0) = 0$ *and* $\varpi_{\mathcal{N}}(x_0) = 0$
2) $\Theta_{\mathcal{N}}(\mu x_1 + (1-\mu)x_2) \ge \min(\Theta_{\mathcal{N}}(x_1), \Theta_{\mathcal{N}}(x_2))$, $\forall\ x_1, x_2 \in R$ & $\mu \in [0,1]$.
3) $\xi_{\mathcal{N}}(\mu x_1 + (1-\mu)x_2) \ge \max(\xi_{\mathcal{N}}(x_1), \xi_{\mathcal{N}}(x_2))$, $\forall\ x_1, x_2 \in R$ & $\mu \in [0,1]$.
4) $\varpi_{\mathcal{N}}(\mu x_1 + (1-\mu)x_2) \ge \max(\varpi_{\mathcal{N}}(x_1), \varpi_{\mathcal{N}}(x_2))$, $\forall\ x_1, x_2 \in R$ & $\mu \in [0,1]$.

Definition 5: Classification of Neutrosophic numbers



3. Nonagonal Neutrosophic Number

A single valued Nonagonal neutrosophic number is represented as

$$\overline{\aleph\aleph\aleph} = \begin{pmatrix} [\,(\ddot{p}^{11}, \ddot{q}^{11}, \ddot{r}^{11}, \ddot{s}^{11}, \ddot{t}^{11}, \ddot{u}^{11}, \ddot{v}^{11}, \ddot{w}^{11}, \ddot{z}^{11}) : \Theta_{\mathcal{N}}\,], \\ [\,(\ddot{p}^{12}, \ddot{q}^{12}, \ddot{r}^{12}, \ddot{s}^{12}, \ddot{t}^{12}, \ddot{u}^{12}, \ddot{v}^{12}, \ddot{w}^{12}, \ddot{z}^{12}) : \xi_{\mathcal{N}}\,], \\ [\,(\ddot{p}^{13}, \ddot{q}^{13}, \ddot{r}^{13}, \ddot{s}^{13}, \ddot{t}^{13}, \ddot{u}^{13}, \ddot{v}^{13}, \ddot{w}^{13}, \ddot{z}^{13}) : \varpi_{\mathcal{N}}\,] \end{pmatrix}$$

where $\Theta_{\mathcal{N}}$ is a truth membership function, $\xi_{\mathcal{N}}$ is an indeterminacy -membership function and $\varpi_{\mathcal{N}}$ is a falsity-membership function is defined as

$$\Theta_{\mathcal{N}}(x) = \begin{cases} \Theta_{\mathcal{N}1}(x) & \ddot{p}^{11} \le x \le \ddot{q}^{11} \\ \Theta_{\mathcal{N}2}(x) & \ddot{q}^{11} \le x \le \ddot{r}^{11} \\ \Theta_{\mathcal{N}3}(x) & \ddot{r}^{11} \le x \le \ddot{s}^{11} \\ \Theta_{\mathcal{N}4}(x) & \ddot{s}^{11} \le x \le \ddot{t}^{11} \\ \Theta & x = \ddot{t}^{11} \\ \Theta_{\mathcal{N}4}(x) & \ddot{t}^{11} \le x \le \ddot{u}^{11} \\ \Theta_{\mathcal{N}3}(x) & \ddot{u}^{11} \le x \le \ddot{v}^{11} \\ \Theta_{\mathcal{N}2}(x) & \ddot{v}^{11} \le x \le \ddot{w}^{11} \\ \Theta_{\mathcal{N}1}(x) & \ddot{w}^{11} \le x \le \ddot{z}^{11} \\ 0 & otherwise \end{cases}, \quad \xi_{\mathcal{N}}(x) = \begin{cases} \xi_{\mathcal{N}1}(x) & \ddot{p}^{12} \le x \le \ddot{q}^{21} \\ \xi_{\mathcal{N}2}(x) & \ddot{q}^{12} \le x \le \ddot{r}^{12} \\ \xi_{\mathcal{N}3}(x) & \ddot{r}^{12} \le x \le \ddot{s}^{12} \\ \xi_{\mathcal{N}4}(x) & \ddot{s}^{12} \le x \le \ddot{t}^{12} \\ \xi & x = \ddot{t}^{12} \\ \xi_{\mathcal{N}4}(x) & \ddot{t}^{12} \le x \le \ddot{u}^{12} \\ \xi_{\mathcal{N}3}(x) & \ddot{u}^{12} \le x \le \ddot{v}^{12} \\ \xi_{\mathcal{N}2}(x)) & \ddot{v}^{12} \le x \le \ddot{w}^{12} \\ \xi_{\mathcal{N}1}(x) & \ddot{w}^{12} \le x \le \ddot{z}^{12} \\ 1 & otherwise \end{cases}$$

$$\varpi_{\mathcal{N}}(x) = \begin{cases} \varpi_{\mathcal{N}1}(x) & \ddot{p}^{13} \le x \le \ddot{q}^{13} \\ \varpi_{\mathcal{N}2}(x)) & \ddot{q}^{13} \le x \le \ddot{r}^{13} \\ \varpi_{\mathcal{N}3}(x)) & \ddot{r}^{13} \le x \le \ddot{s}^{13} \\ \varpi_{\mathcal{N}4}(x)) & \ddot{s}^{13} \le x \le \ddot{t}^{13} \\ \Theta & x = \ddot{t}^{13} \\ \varpi_{\mathcal{N}4}(x) & \ddot{t}^{13} \le x \le \ddot{u}^{13} \\ \varpi_{\mathcal{N}3}(x) & \ddot{u}^{13} \le x \le \ddot{v}^{13} \\ \varpi_{\mathcal{N}2}(x) & \ddot{v}^{13} \le x \le \ddot{w}^{13} \\ \varpi_{\mathcal{N}1}(x) & \ddot{w}^{13} \le x \le \ddot{z}^{13} \\ 1 & otherwise \end{cases}$$

4. Some operational laws of NNNs

If $\overline{\aleph\aleph\aleph_1}$ *and* $\overline{\aleph\aleph\aleph_2}$ are two nonagonal neutrosophic numbers having truth membership $\Theta_{\mathcal{N}}{}^1(x)$ *and* $\Theta_{\mathcal{N}}{}^2(x)$, indeterminancy membership $\xi_{\mathcal{N}}{}^1(x)$, $\xi_{\mathcal{N}}{}^2(x)$ and falsity membership $\varpi_{\mathcal{N}}{}^1(x), \varpi_{\mathcal{N}}{}^2(x)$ respectively.

$$\overline{\aleph\aleph\aleph}_1 = \begin{pmatrix} [(\ddot{\mathfrak{p}}^{11}, \ddot{\mathfrak{q}}^{11}, \ddot{\mathfrak{r}}^{11}, \ddot{\mathfrak{s}}^{11}, \ddot{\mathfrak{t}}^{11}, \ddot{\mathfrak{u}}^{11}, \ddot{\mathfrak{v}}^{11}, \ddot{\mathfrak{w}}^{11}, \ddot{\mathfrak{z}}^{11}) : \Theta_{\mathcal{N}}{}^{1}], \\ [(\ddot{\mathfrak{p}}^{12}, \ddot{\mathfrak{q}}^{12}, \ddot{\mathfrak{r}}^{12}, \ddot{\mathfrak{s}}^{1}, \ddot{\mathfrak{t}}^{12}, \ddot{\mathfrak{u}}^{12}, \ddot{\mathfrak{v}}^{12}, \ddot{\mathfrak{w}}^{12}, \ddot{\mathfrak{z}}^{12}) : \xi_{\mathcal{N}}{}^{1}], \\ [(\ddot{\mathfrak{p}}^{13}, \ddot{\mathfrak{q}}^{13}, \ddot{\mathfrak{r}}^{13}, \ddot{\mathfrak{s}}^{13}, \ddot{\mathfrak{t}}^{13}, \ddot{\mathfrak{u}}^{13}, \ddot{\mathfrak{v}}^{13}, \ddot{\mathfrak{w}}^{13}, \ddot{\mathfrak{z}}^{13}) : \varpi_{\mathcal{N}}{}^{1}] \end{pmatrix}$$

$$\overline{\aleph\aleph\aleph}_2 = \begin{pmatrix} [(\ddot{\mathfrak{p}}^{21}, \ddot{\mathfrak{q}}^{21}, \ddot{\mathfrak{r}}^{21}, \ddot{\mathfrak{s}}^{21}, \ddot{\mathfrak{t}}^{21}, \ddot{\mathfrak{u}}^{21}, \ddot{\mathfrak{v}}^{21}, \ddot{\mathfrak{w}}^{21}, \ddot{\mathfrak{z}}^{21}) : \Theta_{\mathcal{N}}{}^{2}], \\ [(\ddot{\mathfrak{p}}^{22}, \ddot{\mathfrak{q}}^{22}, \ddot{\mathfrak{r}}^{22}, \ddot{\mathfrak{s}}^{22}, \ddot{\mathfrak{t}}^{22}, \ddot{\mathfrak{u}}^{22}, \ddot{\mathfrak{v}}^{22}, \ddot{\mathfrak{w}}^{22}, \ddot{\mathfrak{z}}^{22}) : \xi_{\mathcal{N}}{}^{2}], \\ [(\ddot{\mathfrak{p}}^{23}, \ddot{\mathfrak{q}}^{23}, \ddot{\mathfrak{r}}^{23}, \ddot{\mathfrak{s}}^{23}, \ddot{\mathfrak{t}}^{23}, \ddot{\mathfrak{u}}^{23}, \ddot{\mathfrak{v}}^{23}, \ddot{\mathfrak{w}}^{23}, \ddot{\mathfrak{z}}^{23}) : \varpi_{\mathcal{N}}{}^{2}] \end{pmatrix}$$

- Addition

$$\overline{\aleph\aleph\aleph}_1 \oplus \overline{\aleph\aleph\aleph}_2 = (\{[\ddot{\mathfrak{p}}^{11} + \ddot{\mathfrak{p}}^{21} - \ddot{\mathfrak{p}}^{11}\ddot{\mathfrak{p}}^{21}, \ \ddot{\mathfrak{q}}^{11} + \ddot{\mathfrak{q}}^{21} - \ddot{\mathfrak{q}}^{11}\ddot{\mathfrak{q}}^{21}, \ddot{\mathfrak{r}}^{11} + \ddot{\mathfrak{r}}^{21} - \ddot{\mathfrak{r}}^{11}\ddot{\mathfrak{r}}^{21}, \ddot{\mathfrak{s}}^{11} + \ddot{\mathfrak{s}}^{21} - \ddot{\mathfrak{s}}^{11}\ddot{\mathfrak{s}}^{21}, \ddot{\mathfrak{t}}^{11} + \ddot{\mathfrak{t}}^{21} - \ddot{\mathfrak{t}}^{11}\ddot{\mathfrak{t}}^{21}, \ddot{\mathfrak{u}}^{11} + \mathfrak{u}^{21} - \ddot{\mathfrak{u}}^{11}\ddot{\mathfrak{u}}^{21}, \ddot{\mathfrak{v}}^{11} + \ddot{\mathfrak{v}}^{21} - \ddot{\mathfrak{v}}^{11}\ddot{\mathfrak{v}}^{21}, \ddot{\mathfrak{w}}^{11} + \mathfrak{w}^{21} - \ddot{\mathfrak{w}}^{11}\mathfrak{w}^{21}, \ \ddot{\mathfrak{z}}^{11} + \ddot{\mathfrak{z}}^{21} - \ddot{\mathfrak{z}}^{11}\ddot{\mathfrak{z}}^{21}],) < \{[\ddot{\mathfrak{p}}^{11} + \ddot{\mathfrak{p}}^{21} - \ddot{\mathfrak{p}}^{11}\ddot{\mathfrak{p}}^{21}, \ \ddot{\mathfrak{q}}^{11} + \ddot{\mathfrak{q}}^{21} - \ddot{\mathfrak{q}}^{11}\ddot{\mathfrak{q}}^{21}, \ddot{\mathfrak{r}}^{11} + \ddot{\mathfrak{r}}^{21} - \ddot{\mathfrak{r}}^{11}\ddot{\mathfrak{r}}^{21}, \ddot{\mathfrak{s}}^{11} + \ddot{\mathfrak{s}}^{21} - \ddot{\mathfrak{s}}^{11}\ddot{\mathfrak{s}}^{21}, \ddot{\mathfrak{t}}^{11} + \ddot{\mathfrak{t}}^{21} - \ddot{\mathfrak{t}}^{11}\ddot{\mathfrak{t}}^{21}, \ddot{\mathfrak{u}}^{11} + \mathfrak{u}^{21} - \ddot{\mathfrak{u}}^{11}\ddot{\mathfrak{u}}^{21}, \ddot{\mathfrak{v}}^{11} + \ddot{\mathfrak{v}}^{21} - \ddot{\mathfrak{v}}^{11}\ddot{\mathfrak{v}}^{21}, \ddot{\mathfrak{w}}^{11} + \mathfrak{w}^{21} - \ddot{\mathfrak{w}}^{11}\mathfrak{w}^{21}, \ \ddot{\mathfrak{z}}^{11} + \ddot{\mathfrak{z}}^{21} - \ddot{\mathfrak{z}}^{11}\ddot{\mathfrak{z}}^{21}], \ [\ddot{\mathfrak{p}}^{12}\ddot{\mathfrak{p}}^{22}, \ddot{\mathfrak{q}}^{12}\ddot{\mathfrak{q}}^{22}, \ddot{\mathfrak{r}}^{12}\ddot{\mathfrak{r}}^{22}, \ddot{\mathfrak{s}}^{11}\ddot{\mathfrak{s}}^{22}, \ddot{\mathfrak{t}}^{12}\ddot{\mathfrak{t}}^{22}, \ddot{\mathfrak{u}}^{12}\ddot{\mathfrak{u}}^{22}, \ddot{\mathfrak{v}}^{12}\ddot{\mathfrak{v}}^{22}, \ddot{\mathfrak{w}}^{12}\ddot{\mathfrak{w}}^{22}, \ddot{\mathfrak{z}}^{12}\ddot{\mathfrak{z}}^{22}], [\ddot{\mathfrak{p}}^{13}\ddot{\mathfrak{p}}^{23}, \ddot{\mathfrak{q}}^{13}\ddot{\mathfrak{q}}^{23}, \ddot{\mathfrak{r}}^{13}\ddot{\mathfrak{r}}^{23}, \ddot{\mathfrak{s}}^{13}\ddot{\mathfrak{s}}^{23}, \ddot{\mathfrak{t}}^{13}\ddot{\mathfrak{t}}^{23}, \ddot{\mathfrak{u}}^{13}\ddot{\mathfrak{u}}^{23}, \ddot{\mathfrak{v}}^{13}\ddot{\mathfrak{v}}^{23}, \ddot{\mathfrak{w}}^{13}\ddot{\mathfrak{w}}^{23}, \ddot{\mathfrak{z}}^{13}\ddot{\mathfrak{z}}^{23}]\} >$$

- Multiplication

$$\overline{\aleph\aleph\aleph}_1 \otimes \overline{\aleph\aleph\aleph}_2] = \{[\ddot{\mathfrak{p}}^{11}\ddot{\mathfrak{p}}^{21}, \ddot{\mathfrak{q}}^{11}\ddot{\mathfrak{q}}^{21}, \ddot{\mathfrak{r}}^{11}\ddot{\mathfrak{r}}^{21}, \ddot{\mathfrak{s}}^{11}\ddot{\mathfrak{s}}^{21}, \ddot{\mathfrak{t}}^{11}\ddot{\mathfrak{t}}^{21}, \ddot{\mathfrak{u}}^{11}\mathfrak{u}^{21}, \ddot{\mathfrak{v}}^{11}\ddot{\mathfrak{v}}^{21}, \ddot{\mathfrak{w}}^{11}\mathfrak{w}^{21}, \ddot{\mathfrak{z}}^{11}\ddot{\mathfrak{z}}^{21}], [(\ddot{\mathfrak{p}}^{12} + \ddot{\mathfrak{p}}^{22}, \ddot{\mathfrak{q}}^{12} + \ddot{\mathfrak{q}}^{22}, \ddot{\mathfrak{r}}^{12} + \ddot{\mathfrak{r}}^{22}, \ddot{\mathfrak{s}}^{11} +, \ddot{\mathfrak{s}}^{22}, \ddot{\mathfrak{t}}^{12} + \ddot{\mathfrak{t}}^{22}, \ddot{\mathfrak{u}}^{12} + \ddot{\mathfrak{u}}^{22}, \ddot{\mathfrak{v}}^{12} + \ddot{\mathfrak{v}}^{22}, \ddot{\mathfrak{w}}^{12} + \ddot{\mathfrak{w}}^{22}, \ddot{\mathfrak{z}}^{12} + \ddot{\mathfrak{z}}^{22}], [\ddot{\mathfrak{p}}^{13} + \ddot{\mathfrak{p}}^{23}, \ddot{\mathfrak{q}}^{13} + \ddot{\mathfrak{q}}^{23}, \ddot{\mathfrak{r}}^{13} + \ddot{\mathfrak{r}}^{23}, \ddot{\mathfrak{s}}^{13} + \ddot{\mathfrak{s}}^{23}, \ddot{\mathfrak{t}}^{13} + \ddot{\mathfrak{t}}^{23}, \ddot{\mathfrak{u}}^{13} + \ddot{\mathfrak{u}}^{23}, \ddot{\mathfrak{v}}^{13} + \ddot{\mathfrak{v}}^{23}, \ddot{\mathfrak{w}}^{13} + \ddot{\mathfrak{w}}^{23}, \ddot{\mathfrak{z}}^{13} + \ddot{\mathfrak{z}}^{23}]\}$$

5. Accuracy Function

The defuzzification process converts nonagonal neutrosophic numbers (NNNs) into crisp values using the accuracy function. The procedure is outlined as follows:

- Defuzzification of the trueness component:

The defuzzified value for the truth membership of an NNN is given by:

$$\mathfrak{D}\mathfrak{N}(\Theta_{\mathcal{N}})^{T}_{nn} = \frac{\ddot{\mathfrak{p}}^{11} + \ddot{\mathfrak{q}}^{11} + \ddot{\mathfrak{r}}^{11} + \ddot{\mathfrak{s}}^{11} + \ddot{\mathfrak{t}}^{11} + \ddot{\mathfrak{u}}^{11} + \ddot{\mathfrak{v}}^{11} + \ddot{\mathfrak{w}}^{11} + \ddot{\mathfrak{z}}^{11}}{9}$$

- Defuzzification of the indeterminacy component:

The defuzzified value for the indeterminacy membership of an NNN is computed as:

$$\mathfrak{DN}(\xi_{\mathcal{N}})^{I}_{nn} = \frac{\ddot{p}^{12} + \ddot{q}^{12} + \ddot{r}^{12} + \ddot{s}^{12} + \ddot{t}^{12} + \ddot{u}^{12} + \ddot{v}^{12} + \ddot{w}^{12} + \ddot{z}^{12}}{9}$$

- Defuzzification of falsity component

The defuzzified value for the falsity membership is given by:

$$\mathfrak{DN}(\varpi_{\mathcal{N}})^{F}_{nn} = \frac{\ddot{p}^{13} + \ddot{q}^{13} + \ddot{r}^{13} + \ddot{s}^{13} + \ddot{t}^{13} + \ddot{u}^{13} + \ddot{v}^{13} + \ddot{w}^{13} + \ddot{z}^{13}}{9}$$

Combined Representation of the Defuzzified Neutrosophic Number:

The defuzzified neutrosophic number in terms of trueness, indeterminacy, and falsity components is represented as:

$$\mathfrak{DN}\left(\Theta_{\mathcal{N}}, \xi_{\mathcal{N}}, \varpi_{\mathcal{N}}\right)^{(T,I,F)}_{nn} = \left(\begin{array}{c} \dfrac{\ddot{p}^{11} + \ddot{q}^{11} + \ddot{r}^{11} + \ddot{s}^{11} + \ddot{t}^{11} + \ddot{u}^{11} + \ddot{v}^{11} + \ddot{w}^{11} + \ddot{z}^{11}}{9}, \\ \dfrac{\ddot{p}^{12} + \ddot{q}^{12} + \ddot{r}^{12} + \ddot{s}^{12} + \ddot{t}^{12} + \ddot{u}^{12} + \ddot{v}^{12} + \ddot{w}^{12} + \ddot{z}^{12}}{9}, \\ \dfrac{\ddot{p}^{13} + \ddot{q}^{13} + \ddot{r}^{13} + \ddot{s}^{13} + \ddot{t}^{13} + \ddot{u}^{13} + \ddot{v}^{13} + \ddot{w}^{13} + \ddot{z}^{13}}{9} \end{array} \right)$$

Graded Mean Defuzzification to a Crisp Number:

Finally, the graded mean formula is used to convert the defuzzified neutrosophic values into a single crisp value:

$$(\mathfrak{GN})^{\aleph}_{nn} = \frac{\mathfrak{DN}(\Theta_{\mathcal{N}})^{T}_{nn} + 4\mathfrak{DN}(\xi_{\mathcal{N}})^{I}_{nn} + \mathfrak{DN}(\varpi_{\mathcal{N}})^{F}_{nn}}{4}$$

6. Nonagonal Neutrosophic Transportation Problem (NNTP)

In many practical scenarios, transportation-related data such as costs, supply, and demand are often uncertain, imprecise, and influenced by indeterminate factors. Classical models fail to address this level of complexity effectively. To overcome these limitations, the present work employs NNNs, which extend the neutrosophic set theory by incorporating nine parameters to better characterize uncertainty through truth, indeterminacy, and falsity components. By embedding NNNs into the classical transportation model, we develop a more flexible and expressive approach.

Here we consider two types in which the decision-maker is uncertain about the exact values of key parameters—specifically, the transportation cost from the i[th] source to the j[th] destination, as well as the certainty or uncertainty associated with the supply and demand of goods. To handle such indeterminacy and vagueness, we introduce a new class of transportation problem, referred to as the NNTP, where the cost, supply, and demand parameters are all represented using NNNs.

This paper focuses on solving an NNTP characterized by m supply centers and n demand centers. Let $c_{\tilde{ij}} = c_{\tilde{ij}}^{\tilde{N}\tilde{N}\tilde{N}}$ denote the nonagonal neutosophic number corresponding to the transportation cost of sending one unit of goods from i[th] source to j[th] destination. Let $a_i = a_i^{\tilde{N}\tilde{N}\tilde{N}}$ and $b_j = b_j^{\tilde{N}\tilde{N}\tilde{N}}$ represent supplies and demands, respectively. The mathematical formulation of the NNTP is then presented as follows.

Minimize $Z^{**} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{\tilde{ij}}^{\tilde{N}\tilde{N}\tilde{N}} \, x_{ij}^{**}$

Subject to

$$\sum_{i=1}^{m} x_{ij}^{**} = a_i^{\tilde{N}\tilde{N}\tilde{N}}, i = 1,2,\cdots,m$$

$$\sum_{j=1}^{n} x_{ij}^{**} = b_j^{\tilde{N}\tilde{N}\tilde{N}}, j = 1,2,\cdots,n$$

$$x_{ij}^{**} \geq 0$$

x_{ij}^{**} denotes the number of units transported from the source i to the destination j.

Table 1: Uncertain transportation table

	D_1	**D_2**	**D_3**	**Supply**
S_1	$c_{\tilde{11}}^{\tilde{N}\tilde{N}\tilde{N}}$	$c_{\tilde{12}}^{\tilde{N}\tilde{N}\tilde{N}}$	$c_{\tilde{13}}^{\tilde{N}\tilde{N}\tilde{N}}$	$a_1^{\tilde{N}\tilde{N}\tilde{N}}$
S_2	$c_{\tilde{21}}^{\tilde{N}\tilde{N}\tilde{N}}$	$c_{\tilde{22}}^{\tilde{N}\tilde{N}\tilde{N}}$	$c_{\tilde{23}}^{\tilde{N}\tilde{N}\tilde{N}}$	$a_2^{\tilde{N}\tilde{N}\tilde{N}}$
S_3	$c_{\tilde{31}}^{\tilde{N}\tilde{N}\tilde{N}}$	$c_{\tilde{32}}^{\tilde{N}\tilde{N}\tilde{N}}$	$c_{\tilde{33}}^{\tilde{N}\tilde{N}\tilde{N}}$	$a_3^{\tilde{N}\tilde{N}\tilde{N}}$
Demand	$b_1^{\tilde{N}\tilde{N}\tilde{N}}$	$b_2^{\tilde{N}\tilde{N}\tilde{N}}$	$b_3^{\tilde{N}\tilde{N}\tilde{N}}$	

6.1 Nonagonal Neutrosophic Transportation Problem of type 1

A transportation problem where costs between the origins and destinations are represented using NNNs, while the supply at each source and demand at each destination are considered as deterministic values is called NNTP of type 1. This model helps capture uncertainty in transportation expenses due to fluctuating market conditions, fuel prices, or other unpredictable factors. Mathematically, an NNTP of Type 1 is formulated as follows:

Minimize $Z^{**} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{\widetilde{ij}}^{\,\widetilde{\aleph\aleph\aleph}}\, x_{ij}^{**}$

Subject to

$$\sum_{i=1}^{m} x_{ij}^{**} = a_i, i = 1,2,\cdots,m$$

$$\sum_{j=1}^{n} x_{ij}^{**} = b_j, j = 1,2,\cdots,n$$

$$x_{ij}^{**} \geq 0$$

6.2 Nonagonal Neutrosophic Transportation Problem of type 2

A transportation problem is that in which the supply at origins and demand at destinations are expressed as NNNs, while the transportation costs remain crisp. This model is suitable for scenarios where the quantity of goods available or required is uncertain due to variability in production, consumption, or forecasting errors.

Minimize $Z^{**} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{\widetilde{ij}}\; x_{ij}^{**}$

Subject to

$$\sum_{i=1}^{m} x_{ij}^{**} = a_i^{\widetilde{\aleph\aleph\aleph}}, i = 1,2,\cdots,m$$

$$\sum_{j=1}^{n} x_{ij}^{**} = b_j^{\widetilde{\aleph\aleph\aleph}}, j = 1,2,\cdots,n$$

$$x_{ij}^{**} \geq 0$$

7. Computational method

7.1 The steps of the proposed algorithms are given below:

7.1.1 Defuzzify each nonagonal neutrosophic cost $c_{\widetilde{ij}}^{\,\widetilde{\aleph\aleph\aleph}}$, nonagonal neutrosophic supply $a_i^{\tilde{\aleph}\tilde{\aleph}\tilde{\aleph}}$,, and nonagonal neutrosophic demand $b_j^{\tilde{\aleph}\tilde{\aleph}\tilde{\aleph}}$ of NTP in cost matrix to their corresponding crisp data using the accuracy functions to enable further computational analysis.

7.1.2 Verify the given NNT table is balanced or not.

$$\text{If } \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \text{ , then TP is balanced}$$

$$\text{If } \sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j \text{ , then TP is unbalanced}$$

7.1.3 If the given TP is balanced then go to step 7.1.4, Otherwise, it should be balanced by adding a dummy row or a dummy column with zero entries in cost matrix as required.

7.1.4 In this step, we search the initial basic feasible solutions of the crisp transportation problem by using by VAM method ensuring efficient cost approximation in the starting solution.

7.1.5 Locate all unallocated (idle) cells in the transportation table whose transportation costs are less than the highest cost among the currently allocated cells

7.1.6 For each identified idle cells, construct a closed loop that begins and ends at the same cell, following only horizontal and vertical movements with all corners are passing through the allocated cells.

7.1.7 Assign alternating '+' and '-' signs to the allocated values at each corner cell of the loop, starting with a '+' sign at the idle cell being evaluated.

7.1.8 Calculate the trade-off cost for each loop by summing the allocated values of its corner cells taking into account their respective signs.

7.1.9 Identify the loop associated with the maximum trade-off cost and perform reallocation as follows:

a) Within this loop, identify the adjacent cell to the idle cell that has the highest transportation cost.

b) Adjust the allocation in the loop by alternately adding and subtracting the allocated value of this highest- cost cell along the loop, starting with the idle cell

7.1.10 Continue the process iteratively until no further improvements can be made, and the transportation cost is minimized. The resulting allocation constitutes the optimal solution to the transportation problem under the nonagonal neutrosophic framework.

7.1.11 The optimal solution of the objective function is calculated by

$$Z_{\overline{(\aleph\aleph\aleph)}} = c_{\check{i}\check{j}} \times x_{ij}^{**}.$$

7.2 Nonagonal Neutrosophic Vogel's approximation method

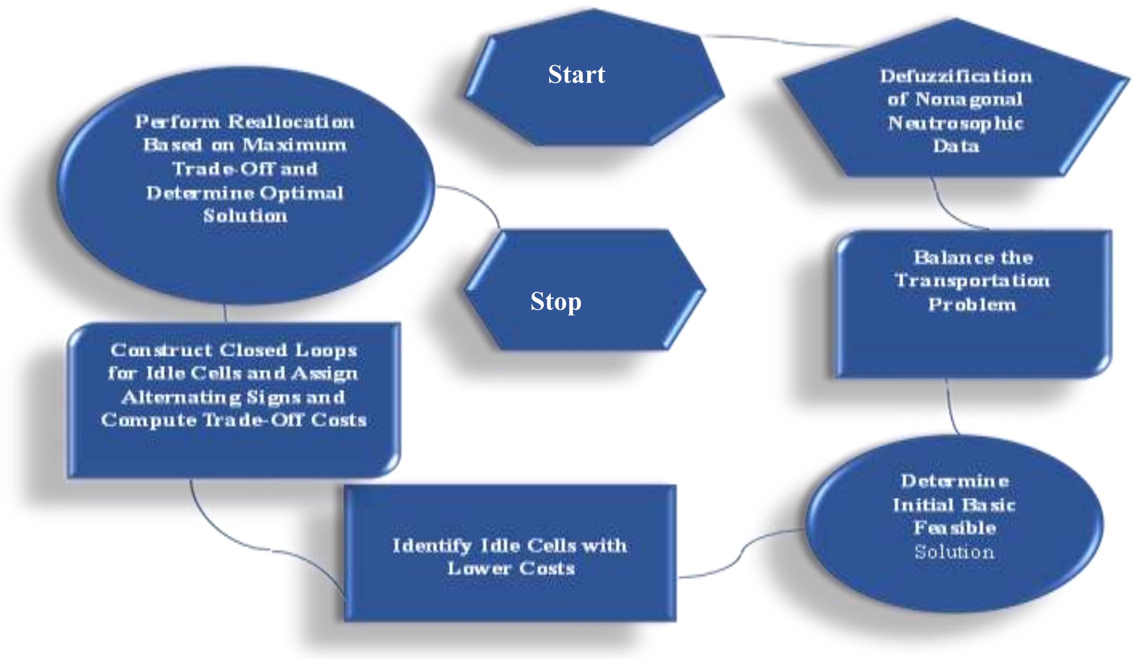
Among the various methods available, Vogel's Approximation Method (VAM) is widely used to determine the initial basic feasible solution of a transportation problem. The computational steps of VAM are as follows:

7.2.1 For each row and column in the NNTP, identify the smallest and the next smallest cost values. Then, compute the penalty p_i and p_j for each row, $i = 1,2,\cdots,m$ and each column $j = 1,2,\cdots,n$ defined as the difference between these two values.

7.2.2 Identify the largest penalty among all rows and columns. If a tie occurs, select any one arbitrarily. Suppose the greatest penalty corresponding to the mth row and $c_{\widetilde{mj}}$ be the smallest cost in that row. Then, allocate $x_{mj}{}^{**} = \min(a_i, b_j)$ in the (m, j)th cell.

7.2.3 Update the corresponding supply and demand, adjust the transportation table accordingly, and recalculate the row and column penalties. Repeat this process iteratively (i.e., return to Step 7.2.1) until all supply and demand constraints are fully satisfied.

8. Diagram Representation of NNTP



9. Case Study

9.1 Type 1

A food bank distributes surplus food from various donation centers to community kitchens, where the donation centers serve as the sources and the local shelters act as the destinations. In this scenario, the transportation cost associated with each cell is represented using nonagonal neutrosophic numbers, capturing the inherent uncertainty and indeterminacy in real-world logistics. Notably, the transportation cost obtained through the proposed approach is found to be lower than that achieved by existing methods.The supply values are 15,25 and 30.The demand values are 24, 19 and 27. The input values for the NNTP are organized in Table 2 below:

Table 2 Nonagonal Neutrosophic Transportation Table

	D_1	D_2	D_3
S_1	((1,6,7,2,3,5,2.5,6,8.3), (1.2,2.4,9,7,4.4,6,7.8,8,9) (3,5,8,1,5,3.6,5,3,2))	((1,4,7,1,3,5,3.5,6,7.5) (0.5,2.5,4.5,1,2,3, 1.5,3.5,5.5), (3,5,8,1,5,3.6,5,3,5.7))	((1,3,5,0.5,1.5,3.5, 2,4,6), (1,2,3, 0.5,1.5,2.5, 1.5,2.5,3.5) (1,1.5,4, 0.5,1,2.5,1.25,3,4.25))
S_2	((1.5,2.5,3.5,1,1.5,3,2,3,4), (2,4,6,1.5,2.5,4.5,3,5,7) (1,5,8,1.5,4.5,7.5,4,6.5,9))	((1,6.5,7.2,2,4,5,3.5,6,9.3), (2.2,2.4,8,6,4.4,9,7.8,8,7), (6,5,9,3,5,3.6,5,8,9))	((1,6,7,2,3,5,2.5,6,8.3) (1,1.5,4, 0.5,1,2.5, 1.25,3,4.25) (1.5,2.5,3.5, 1,1.5,3, 2,3,4))
S_3	((2,4,6, 1.5,2.5,4.5, 3,5,3), (1.2,2.4,9,7,4.4,6,7.8,8,9), (1,5,8, 1.5,4.5,7.5, 4,6.5,9))	((1.1,5,8, 1.5,3,6.5, 4,7,9), (1.5,2,7,7,4.4,9,6.8,8,9), (4,6,9,2,5,4.6,5,6.3,5.7))	((1.6,9,8,2.6,3,5,5.5,6,8.7) (5.2,2.7,9,7,8.4,6,7.6,8,9) (7,5.5,8.7,1.7,5,3.6,5,3,5.7))

9.1.1 Conversion of NNNs to crisp values using score function

Step 7.1.1 The score function can be employed to convert the nonagonal neutrosophic (NN) cost values into crisp values. The resulting crisp cost values corresponding to Table 2 are presented as follows:

$$\mathfrak{DN}\left(\Theta_{\mathcal{N}},\xi_{\mathcal{N}},\varpi_{\mathcal{N}}\right)_{nn}^{(T,I,F)}$$

$$= \left(\begin{array}{c} \dfrac{\ddot{p}^{11}+\ddot{q}^{11}+\ddot{r}^{11}+\ddot{s}^{11}+\ddot{t}^{11}+\ddot{u}^{11}+\ddot{v}^{11}+\ddot{w}^{11}+\ddot{z}^{11}}{9}, \\ \dfrac{\ddot{p}^{12}+\ddot{q}^{12}+\ddot{r}^{12}+\ddot{s}^{12}+\ddot{t}^{12}+\ddot{u}^{12}+\ddot{v}^{12}+\ddot{w}^{12}+\ddot{z}^{12}}{9}, \\ \dfrac{\ddot{p}^{13}+\ddot{q}^{13}+\ddot{r}^{13}+\ddot{s}^{13}+\ddot{t}^{13}+\ddot{u}^{13}+\ddot{v}^{13}+\ddot{w}^{13}+\ddot{z}^{13}}{9} \end{array} \right)$$

$$(\mathfrak{G}\mathfrak{R})^{\aleph}_{nn} = \frac{\mathfrak{D}\mathfrak{R}(\Theta_{\mathcal{N}})^{T}_{nn} + 4\mathfrak{D}\mathfrak{R}(\xi_{\mathcal{N}})^{I}_{nn} + \mathfrak{D}\mathfrak{R}(\varpi_{\mathcal{N}})^{F}_{nn}}{4}$$

Here,

$$\mathfrak{D}\mathfrak{R}\left(c_{\widetilde{11}}{}^{\widetilde{\aleph\aleph\aleph}}\right)^{(T,I,F)}_{nn} = \left\langle\begin{pmatrix}(1,6,7,2,3,5,2.5,6,8.3),\\(1.2,2.4,9,7,4.4,6,7.8,8,9)\\(3,5,8,1,5,3.6,5,3,2)\end{pmatrix}\right\rangle = \begin{pmatrix}\frac{(1+6+7+2+3+5+2.5+6+8.3)}{9},\\\frac{(1.2+2.4+9+7+4.4+6+7.8+8+9)}{9}\\\frac{(3+5+8+1+5+3.6+5+3+2)}{9}\end{pmatrix} = \begin{pmatrix}4.53\\6.09\\3.96\end{pmatrix}$$

$$\mathfrak{G}\mathfrak{R}\left(\begin{pmatrix}4.53\\6.09\\3.96\end{pmatrix}\right)^{\aleph}_{nn} = \frac{4.53 + 4\times 6.09 + 3.96}{4} = 8.21$$

Following the conversion of NNTP cost values into crisp form using the score function, the corresponding cost matrix is shown in Table 3.

Table 3 Crisp transportation table

	D_1	D_2	D_3	SUPPLY
S1	8.21	5.09	3.27	15
S2	5.86	8.81	3.78	25
S_3	8.26	8.64	9.61	30
DEMAND	24	19	27	

Step 7.1.2 - 7.1.3 The given transportation problem is balanced since the total supply equals the total demand, i.e., $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 70$

Step 7.1.4: Applying the VAM Method to find the initial basic feasible solution. To begin, identify the smallest and the next smallest elements in each row and each column of Table 3.

For rows:

$$c_{13} < c_{12} < c_{11} \Rightarrow least\ and\ second\ least\ cost\ are:\ c_{13},\ c_{12}$$

$$c_{23} < c_{21} < c_{22} \Rightarrow least\ and\ second\ least\ cost\ are:\ c_{23}, c_{21}$$

$$c_{31} < c_{32} < c_{33} \Rightarrow least\ and\ second\ least\ cost\ are:\ c_{31}, c_{32}$$

For columns:

$$c_{21} < c_{11} < c_{31} \Rightarrow least\ and\ second\ least\ cost\ are:\ c_{21}, c_{11}$$

$$c_{12} < c_{32} < c_{22} \Rightarrow least\ and\ second\ least\ cost\ are:\ c_{12}, c_{32}$$

$$c_{13} < c_{23} < c_{33} \Rightarrow least\ and\ second\ least\ cost\ are:\ c_{13}, c_{23}$$

Next, select the row or column with the highest penalty. Since the second column has the highest penalty, it is chosen for allocation. Proceeding in the same manner, the initial basic feasible solution is obtained, as shown in Table 4.

Table 4: Initial basic feasible solution by VAM method

	D1	**D2**	**D3**	**SUPPLY**
S1	8.21	5.09 **15**	3.27	15
S2	5.86	8.81	3.78 **25**	25
S3	8.26 **24**	8.64 **4**	9.61 **2**	30
DEMAND	24	19	27	

The selected entries are represented by bold

$x_{12}^{**} = 15,\ x_{23}^{**} = 25,\ x_{31}^{**} = 24,\ x_{32}^{**} = 4,\ x_{33}^{**} = 2$

Step 7.1.5 -7.1.6 Identify all unallocated (idle) cells in the transportation table whose transportation costs are lower than the highest cost among the currently allocated cells. In this case, cells S_1D_1, S_1D_3, $S_2D_1$and S_2D_2 have transportation costs lower than that of the allocated cell S_3D_3. For each of the identified idle cells, construct a closed loop starting and ending at the same cell. The loop must consist of horizontal and vertical movements only, and every corner of the loop must pass through an allocated cell. The corresponding loops and allocations are illustrated in Table 5 and Table 6.

Table 5 Loop Allocation in the first stage

	D_1	D_2	D_3	SUPPLY
S1	8.21	5.09 **15**	3.27	15
S2	5.86	8.81	3.78 **25**	25
S_3	8.26 **24**	8.64 **4**	9.61 **2**	30
DEMAND	24	19	27	

Table 6 Second stage loop allocation

	D_1	D_2	D_3	SUPPLY
S1	8.21	5.09 **15**	3.27	15
S2	5.86	8.81	3.78 **25**	25
S_3	8.26 **24**	8.64 **4**	9.61 **2**	30
DEMAND	24	19	27	

Step 7.1.7 -7.1.8 Assign alternating '+' and '–' signs to the allocated cells at each corner of the constructed loop, beginning with a '+' at the idle cell under evaluation. This sign pattern must alternate as you traverse the loop. Next, compute the trade-off cost for each loop by summing the transportation costs of the allocated corner cells, applying the assigned signs appropriately.

The calculated trade-off costs for the identified idle cells are as follows:

Trade-off cost for S_1D_1 = -24+4-15 = -35

Trade-off cost for S_1D_3 = -13

Trade-off cost for S_2D_1 = -47

Trade-off cost for S_2D_2 = -27

Step 7.1.9 Identify the loop corresponding to the maximum trade-off cost and proceed with the reallocation as follows:

a) Within this loop, determine the allocated cell adjacent to the idle cell that has the highest transportation cost.

b) Adjust the allocations along the loop by alternately adding and subtracting the allocated value of this highest-cost cell, starting from the idle cell.

In this case, the cell S_1D_3 has the maximum trade-off cost. To initiate optimal reallocation, identify the adjacent allocated cell within the loop that has the highest transportation cost, which also corresponds to the cell S_1D_3 (idle cell). The loop is then updated by adding and subtracting the allocated value (= 2) alternately at the corner cells of the loop (Table 7).

Table 7 Loop allocation

	D1	D2	D3	SUPPLY
S1	8.21	5.09 **(15-2)**	~~3.27~~ **(2)**	15
S2	5.86	8.81	3.78 **25**	25
S3	8.26 **24**	8.64 (4+**2)**	~~9.61 **(2-2)**~~	30
DEMAND	24	19	27	

Step 7.1.10 In the current iteration, the idle cell with the highest trade-off cost is S_1D_1. Among its adjacent allocated cells, S_3D_1 possesses the highest cost value. However, upon attempting reallocation using this cell as part of the closed loop, the resulting adjustment yields a negative allocation, which is not permissible. As a result, no further feasible improvements can be made through reallocation. This indicates that the solution has reached its optimal state, and the iteration process can be concluded. The optimal crisp solution of TP is shown in table 8 as follows:

Table 8 Optimal transportation table

	D1	D2	D3	SUPPLY
S1	8.21	5.09 **13**	3.27 **2**	15
S2	5.86	8.81	3.78 **25**	25
S3	8.26 **24**	8.64 **6**	9.61	30
DEMAND	24	19	27	

The corresponding optimal solution of the Nonagonal Neutrosophic Transportation Problem, along with the allocation of NNNs, is presented in Table 9.

Table 9 Optimal solution of the Nonagonal Neutrosophic Transportation Problem

	D_1	D_2	D_3
S_1		$\begin{pmatrix}(1,4,7,1,3,5,3.5,6,7.5)\\(0.5,2.5,4.5,1,2,3,1.5,3.5,5.5),\\(3,5,8,1,5,3.6,5,3,5.7)\end{pmatrix}$ **13**	$\begin{pmatrix}(1,3,5,0.5,1.5,3.5,2,4,6),\\(1,2,3,0.5,1.5,2.5,1.5,2.5,3.5)\\(1,1.5,4,0.5,1,2.5,1.25,3,4.25)\end{pmatrix}$ **2**
S_2			$\begin{pmatrix}(1,6,7,2,3,5,2.5,6,8.3)\\(1,1.5,4,0.5,1,2.5,1.25,3,4.25)\\(1.5,2.5,3.5,1,1.5,3,2,3,4)\end{pmatrix}$ **25**
S_3	$\begin{pmatrix}(2,4,6,1.5,2.5,4.5,3,5,3),\\(1.2,2.4,9,7,4.4,6,7.8,8,9),\\(1,5,8,1.5,4.5,7.5,4,6.5,9)\end{pmatrix}$ **24**	$\begin{pmatrix}(1.1,5,8,1.5,3,6.5,4,7,9),\\(1.5,2,7,7,4.4,9,6.8,8,9),\\(4,6,9,2,5,4.6,5,6.3,5.7)\end{pmatrix}$ **6**	

Optimal Solution ,

$$Z_{\overline{(\aleph\aleph\aleph)}} =$$

$$\begin{pmatrix}(1,4,7,1,3,5,3.5,6,7.5)\\(0.5,2.5,4.5,1,2,3,1.5,3.5,5.5),\\(3,5,8,1,5,3.6,5,3,5.7)\end{pmatrix}\times 13+\begin{pmatrix}(1,3,5,0.5,1.5,3.5,2,4,6),\\(1,2,3,0.5,1.5,2.5,1.5,2.5,3.5)\\(1,1.5,4,0.5,1,2.5,1.25,3,4.25)\end{pmatrix}\times 2$$

$$+\begin{pmatrix}(1,6,7,2,3,5,2.5,6,8.3)\\(1,1.5,4,0.5,1,2.5,1.25,3,4.25)\\(1.5,2.5,3.5,1,1.5,3,2,3,4)\end{pmatrix}\times 25$$

$$+\begin{pmatrix}(2,4,6,1.5,2.5,4.5,3,5,3),\\(1.2,2.4,9,7,4.4,6,7.8,8,9),\\(1,5,8,1.5,4.5,7.5,4,6.5,9)\end{pmatrix}\times 24+\begin{pmatrix}(1.1,5,8,1.5,3,6.5,4,7,9),\\(1.5,2,7,7,4.4,9,6.8,8,9),\\(4,6,9,2,5,4.6,5,6.3,5.7)\end{pmatrix}\times 6$$

$$= (\mathbf{5.09\times 13}) + (\mathbf{3.27\times 2}) + (\mathbf{3.78\times 25}) + (\mathbf{8.26\times 24}) + (\mathbf{8.64\times 6})$$

$$= \mathbf{417.29}$$

9.2 Type 2

In this case, the supply and demand values for each cell are represented using Nonagonal Neutrosophic Numbers to effectively model uncertainty. The corresponding input values for the NNTP are:

$\mathcal{S}_1^{\aleph} =$
$(10,15,20,24,16,22,12,15,19), (20,25,30,24,26,32,22,25,29), (15,20,25,19,21,27,17,20,24)$

$\mathcal{S}_2^{\aleph} =$
$(13,18,23,17,19,25,15,18,22), (15,20,25,19,21,27,17,20,24), (17,22,27,21,23,29,19,22,26)$

$\mathcal{S}_3^{\aleph} =$
$(10,15,20,14,16,22,12,15,19)), (17,22,27,21,23,29,19,22,26), (15,20,25,19,21,27,17,20,24)$

$\mathcal{D}_1^{\aleph} =$
$(10,15,20,14,16,22,12,15,19)), (20,25,30,24,26,32,22,25,29), (15,20,25,19,21,27,17,20,24)$

$\mathcal{D}_2^{\aleph} =$
$(15,25,30,34,26,22,32,15,29), (10,15,20,14,36,22,12,35,29), (25,10,15,29,11,27,27,20,34)$

$\mathcal{D}_3^{\aleph} =$
$(20,15,30,24,26,22,22,15,29), (30,25,20,14,16,22,32,25,39), (25,20,25,39,21,27,27,20,24)$

After applying the score function to convert the nonagonal neutrosophic supply and demand values into crisp numbers, the corresponding cost matrix is displayed in Table 10.

Table 10 Crisp transportation table of type 2 problem

	D₁	**D₂**	**D₃**	**SUPPLY**
S1	13	22	34	35.36
S2	20	19	23	31.33
S₃	24	24	12	32.09
DEMAND	35.09	33.27	36.75	

The initial basic feasible solution for the Type-2 NNTP, obtained using the Vogel's Approximation Method, is presented in Table 11.

Table 11 IBFS by VAM

	D₁	D₂	D₃	SUPPLY
S1	13 **35.09**	22 **0.27**	34	35.36
S2	20	19 **26.67**	23 **4.66**	31.33
S₃	24	24	12 **32.09**	32.09
S4	0	0 **6.33**	0	6.33
DEMAND	35.09	33.27	36.75	

The Optimal allocation for the Type-2 Crisp NNTP is shown in Table 12.

Table 12 Optimal solution of the Type-2 Crisp NNTP

	D_1	D_2	D_3	SUPPLY
S1	13 **35.09**	22 **0.27**	34	35.36
S2	20	19 **31.33**	23	31.33
S_3	24	24	12 **32.09**	32.09
S4	0	0 **1.67**	0 **4.66**	6.33
DEMAND	35.09	33.27	36.75	

The Optimal Solution is given by

$$\boldsymbol{Z}_{\overline{(\aleph\aleph\aleph)}} = (13 \times 35.09) + (22 \times 0.27) + (19 \times 31.33) + (12 \times 32.09) + (0 \times 1.67) + (0 \times 4.66) = \mathbf{1442.46}$$

10. Comparative Study

Real-life applications of Nonagonal Neutrosophic Numbers in transportation problems have been addressed using various existing methods. To assess the effectiveness of the proposed solution technique, a comparative study was conducted across the methods: the Neutrosophic North-West Corner Rule (NNWCR), Neutrosophic Vogel's Approximation Method (NVAM), Method discussed in [23] and the Proposed Method. This comparison is performed on two types of NNTPs. The evaluation is based on total transportation cost, where all neutrosophic cost values were first converted into crisp values using score functions, ensuring a consistent and fair comparison across all methods.

In order to solve these problems, we identified limitations in the existing approaches and introduced a simplified ranking technique for Nonagonal Neutrosophic Numbers. Our proposed method demonstrates improved performance by effectively converting neutrosophic transportation problems into their crisp equivalents and achieving lower total transportation costs.

All computations were carried out under the same input data and after defuzzification, allowing a uniform basis for comparison. Table 13 highlights the results of the comparative analysis, demonstrating the superior efficiency of the proposed method in terms of cost minimization.

Table 13 Comparison table

Type	**Nonagonal Neutrosophic North-West Corner NTP**		**Nonagonal Neutrosophic VAM**		**Method proposed by [23]**		**Proposed Method**	
Type 1	solutions	Minimum Cost	solutions	Minimum Cost	solutions	Minimum Cost	solutions	Minimum Cost
	$x_{11}^{**} = 15$ $x_{21}^{**} = 9$ $x_{22}^{**} = 16$ $x_{32}^{**} = 3$ $x_{33}^{**} = 27$	602.24	$x_{12}^{**} = 15$ $x_{23}^{**} = 25$ $x_{31}^{**} = 24$ $x_{32}^{**} = 4$ $x_{33}^{**} = 2$	422.87	$x_{12}^{**} = 15$ $x_{23}^{**} = 25$ $x_{31}^{**} = 24$ $x_{32}^{**} = 4$ $x_{33}^{**} = 2$	422.87	$x_{12}^{**} = 13$ $x_{13}^{**} = 2$ $x_{23}^{**} = 25$ $x_{31}^{**} = 24$ $x_{32}^{**} = 6$	417.29
Type 2	$x_{11}^{**} = 35.09$ $x_{12}^{**} = 0.27$ $x_{32}^{**} = 31.33$ $x_{32}^{**} = 1.67$ $x_{33}^{**} = 30.42$ $x_{43}^{**} = 6.33$	1462.5	$x_{11}^{**} = 35.09$ $x_{12}^{**} = 0.27$ $x_{22}^{**} = 26.67$ $x_{23}^{**} = 4.66$ $x_{33}^{**} = 32.09$ $x_{42}^{**} = 6.33$	1461.1	$x_{11}^{**} = 35.09$ $x_{12}^{**} = 0.27$ $x_{22}^{**} = 31.33$ $x_{32}^{**} = 1.67$ $x_{33}^{**} = 30.42$ $x_{43}^{**} = 6.33$	1462.5	$x_{11}^{**} = 35.09$ $x_{12}^{**} = 0.27$ $x_{22}^{**} = 31.33$ $x_{33}^{**} = 32.09$ $x_{42}^{**} = 1.67$ $x_{43}^{**} = 4.66$	1442.46

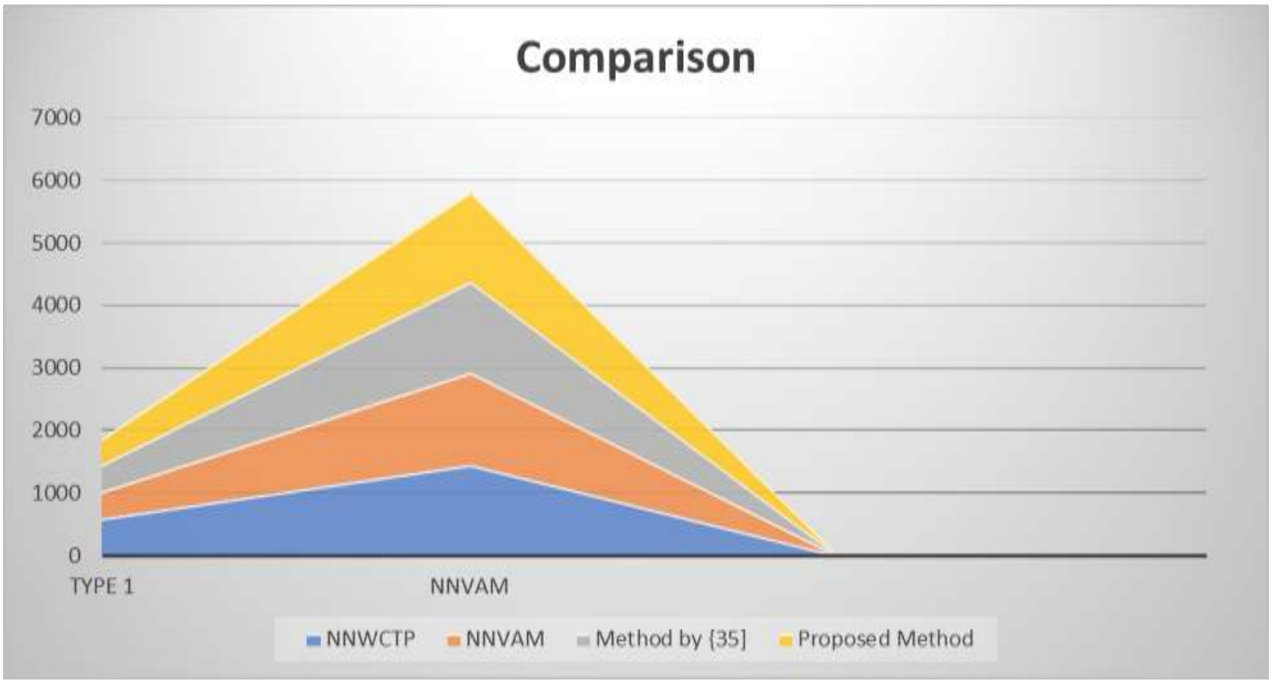


Figure 1: Comparison of results with proposed method and existing methods

This comparison clearly indicates that the Proposed Optimal Method is not only well-suited for handling complex neutrosophic data but also consistently delivers more cost-effective solutions. Therefore, it can be regarded as a superior alternative to the methods as North-West Corner Rule, Vogel's Approximation Method, and the approach discussed in [23] for solving transportation problems under uncertainty.

11. Results and Limitations

11.1 Results

The proposed method, developed for solving transportation problems under uncertainty using Nonagonal Neutrosophic Numbers, was tested across two distinct models and compared with established approaches including the North-West Corner Rule, Vogel's Approximation Method, and the method proposed by Pachamuthu and Rabinson (2021). All cost parameters represented in NNNs were converted to crisp values using the newly introduced score functions.

The results demonstrate that the proposed method consistently yields the lowest transportation cost across both types:

- Type 1: Achieved a minimum cost of 417.29, outperforming other methods (NWCR: 602.24, VAM: 422.87).
- Type 2: Achieved a minimum cost of 1442.46, lower than all compared methods (NWCR: 1462.5, VAM: 1461.1).

These results validate the efficiency of the proposed score functions and optimization framework in handling imprecise, indeterminate, and inconsistent data through the NNN structure.

11.1 Advantages and Limitations

No study is without constraints, and this research, despite its strengths, includes some limitations as well. The major advantages of the proposed method and their corresponding limitations are summarized in Table 14.

Table 14

Advantages	Limitations
Provides a more detailed representation of uncertainty using Nonagonal Neutrosophic Numbers.	Increased computational complexity due to the nine-parameter structure
Yields lower transportation cost compared to classical methods	Requires domain expertise to properly interpret and implement NNNs

12. Conclusion and Future Work

In recent times, the concept of neutrosophy has been effectively established as a robust approach for handling uncertainties and indeterminacies in real-life applications. This study investigates both balanced and unbalanced transportation problems using nonagonal neutrosophic numbers by introducing a novel method for determining the optimal solution. All relevant parameters in the problem are expressed in nonagonal neutrosophic terms, allowing for a more comprehensive and realistic representation of uncertainty. The proposed ranking function provides a more practical and structured approach to decision-making under uncertainty. The effectiveness of the method has been demonstrated through comparisons with existing approaches, showing improvements in flexibility and solution quality.

While the current research marks a significant advancement in transportation modeling under uncertainty using Nonagonal Neutrosophic Numbers, there are several promising directions for future exploration. One potential extension involves addressing time-minimizing or multi-objective transportation problems, where both cost and time parameters are considered simultaneously within the neutrosophic framework. Additionally, the methodology can be adapted for other domains such as supply chain optimization, disaster relief logistics, and energy distribution networks, where uncertainty plays a critical role. Lastly, the development of dedicated software tools for modeling and solving Nonagonal Neutrosophic Transportation Problems (NNTPs) would support broader adoption and practical implementation of the proposed approach.

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