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Operations on Pythagorean Neutrosophic Fuzzy Matrix

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Abstract. This article discusses various unique varieties of Pythagorean neutrosophic fuzzy matrices (PNFM). Some fundamental operations such as addition, multiplication, union, intersection, complement, and exponential are discussed. This work introduces algebraic operations on PNFM and demonstrates some of its associated theorems, including distributive, commutative, and associative. The purpose of these investigations is to contribute to a better understanding of PNFM and its use in real-world uncertainty.

Keywords: Pythagorean neutrosophic fuzzy matrix; Algebraic sum; Product; Exponential

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1. **Introduction**

The Pythagorean neutrosphic (PN) set is a new mathematical idea for handling problems in the real world that involve vague and imprecise information. It combines neutrosophic fuzzy sets (NFs) with Pythagorean fuzzy sets (PFs). It offers a more straightforward approach to dealing with uncertainty than Pythagorean and neutrosphic fuzzy sets. The square sums of the three elements that make up the PN set—membership, indeterminacy, and non-membership degree—range from zero to two. The concept of a fuzzy matrix (FM) was first proposed by Thomason. It is crucial to the advancement of science. But this traditional fuzzy matrix will only consider the degree of membership. The ambiguity and uncertainty of real-world problems may not be sufficiently captured by conventional fuzzy matrices. The notions of intuitionistic fuzzy matrices (IFM), neutrosophic fuzzy matrices(NFM) and Pythagorean fuzzy matrices (PFM) are developed to address these problems. In 2002, Pal, Khan, and Shyamal introduced the idea of an intuitionistic fuzzy matrix. The neutrosophic fuzzy matrix concept was first presented by Kandasamy and Smarandache in 2004. The Pythagorean fuzzy matrix concept was first proposed by Silambarasan and Sriram in 2018. The following list includes a selection of papers that addressed fuzzy matrix, intuitionistic, neutrosophic, and pythagorean fuzzy matrix. Venkatesan et al. presented applications of fuzzy matrices in decision making [1]. Beaula explored the use of fuzzy matrix in the medical field [2]. Priya et al. addressed the fundamental definitions of fuzzy matrix [3]. Clayton Gilchrist investigated the determinant and K-idempotence features of a fuzzy matrix [4]. Using picture fuzzy sets, Ramakrishnan M and Sriram S talked about new Hamacher procedures [5]. The concepts of restricted picture fuzzy sets and special restricted picture fuzzy sets were introduced by Dogra and Pal [6]. Einstein operations for intuitionistic fuzzy matrices were defined by Selvarajan et al. [7]. A few operations on intuitionistic fuzzy matrices were defined by Emam EG [8]. For intuitionistic fuzzy matrices, Silambarasan and Sriram defined Hamcher multiplication and exponentiation operations [9]. In intuitionistic fuzzy matrix theory, Li W and Ye J introduced the idea of matrix entropy [10]. Jayapriya and Porchelvi discuss a few operators on a neutrosophic fuzzy set [11]. For neutrosophic fuzzy matrices, Das et al. defined algebraic operations like multiplication and subtraction [12]. Commutative, associative, and distributive characteristics on neutrosophic fuzzy matrices were examined by Das D et al. [13]. A specific instance of the neutrosophic fuzzy matrix, the Fermatean neutrosophic fuzzy matrix, was explored by Broumi [14]. Anandhkumar et. al. discussed about Symmetric Fermatean Neutrosophic fuzzy matrix [15]. Silambarasan and Sriram introduced new operations on Pythagorean fuzzy matrix [16]. Silambarasan and Sriram defined new operations for Pythagorean fuzzy matrix [17]. selvarajan and Ramya introduced Einstein sum and product on Pythagorean fuzzy matrix [18]. Silambarasan and Sriram introduced Hamcher scalar multiplication and exponentiation on Pythagorean fuzzy matrix [19]. Radha et. al. discussed about improved correlation coefficient for neutrosphic pythagorean [20] Ismail et. al. Pythagorean neutrosophic set and their basic algebraic operations [21].Gbolagade et. al. [22] discussed about Neutrosophic Poisson distribution with the help of Salagean operator. Satyanarayana and Baji discussed about Neutrosophic ideals [23]. Shams et. al. discussed about DNA sequence matching algorithm [24] The catalogues for the article are as follows: • Section 1 contains basic definitions and some operations. • Section 2 contains types of Pythagorean Neutrosophic fuzzy matrix. • Section 3 contains basic operations for PNFM and some theorems. • Section 4 contains conclusion.

2. **Preliminaries**

This section provies some basic definitions and operations on Pythagorean neutrosophic fuzzy set

2.1. *Pythagorean Neutrosophic Set*

Let X be a universe of discourse. A Pythagorean neutrosophic set (PN) N on X is defined as

$$N = \{\langle x, \Phi_N(x), \Psi_N(x), \mathcal{L}_N(x)\rangle \mid x \in X\}$$

where

$$0 \leq \Phi_N^2(x) + \Psi_N^2(x) + \mathcal{L}_N^2(x) \leq 2$$

and $\Phi_N(x), \Psi_N(x), \mathcal{L}_N(x) \in [0,1]$.

Here, $\Phi_N(x), \Psi_N(x), \mathcal{L}_N(x)$ denote the degree of membership, degree of non-membership, and degree of indeterminacy, respectively. In this context, $\Phi_N(x)$ and $\mathcal{L}_N(x)$ are dependent components, while $\Psi_N(x)$ is an independent component.

2.2. *Basic Operations on PN set [21]*

Let X be a non-empty set (universe). Let

$$M = \{\langle x, \Phi_m(x), \Psi_m(x), \mathcal{L}_m(x)\rangle \mid x \in X\}$$

and

$$N = \{\langle x, \Phi_N(x), \Psi_N(x), \mathcal{L}_N(x)\rangle \mid x \in X\}$$

be two Pythagorean neutrosophic sets. Then

2.2.1. *Union of M and N*

M $\cup N = \{\max(\Phi_m, \Phi_N), \min(\Psi_m, \Psi_N), \min(\mathcal{L}_m, \mathcal{L}_N)\}$

2.2.2. *Intersection of M and N*

M $\cap N = \{\min(\Phi_m, \Phi_N), \max(\Psi_m, \Psi_N), \max(\mathcal{L}_m, \mathcal{L}_N)\}$

2.2.3. *Addition of M and N*

M $\oplus N = \left(\sqrt{\Phi_m^2 + \Phi_N^2 - 2\Phi_m\Phi_N}, \Psi_m\Psi_N, \mathcal{L}_m\mathcal{L}_N\right)$

2.2.4. *Multiplication of M and N*

M $\otimes N = \left(\Phi_m\Phi_N, \Psi_m + \Psi_N - \Psi_m\Psi_N, \sqrt{\mathcal{L}_m^2 + \mathcal{L}_N^2 - 2\mathcal{L}_m\mathcal{L}_N}\right)$

On the basis of relationships given in (2.1) and (2.2) the following novel definitions and operations are defined.

2.3. *Types of Pythagorean Neutrosophic fuzzy matrix*

This section introduces some types of Pythagorean neurosophic fuzzy matrix

2.3.1. *Pythagorean Neutrosophic fuzzy matrix*

A Pythagorean Neutrosophic fuzzy matrix of order $m \times n$ is defined as $N = (n_{ij})_{m\times n}$. Here, $n_{ij} = (\Phi_{N_{ij}}(x), \Psi_{N_{ij}}(x), \mathcal{L}_{N_{ij}}(x))$ is the ij-th element of N, where $0 \leq \Phi^2_{N_{ij}}(x) + \Psi^2_{N_{ij}}(x) + \mathcal{L}^2_{N_{ij}}(x) \leq 2$.

2.3.2. *Null Pythagorean Neutrosophic fuzzy matrix*

A square Pythagorean Neutrosophic fuzzy matrix is said to be null Pythagorean Neutrosophic fuzzy matrix if all its entries are zero.

$$A = \begin{bmatrix} (0,1,0) & (0,1,0) & (0,1,0) & \cdots & (0,1,0) \\ (0,1,0) & (0,1,0) & (0,1,0) & \cdots & (0,1,0) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (0,1,0) & (0,1,0) & (0,1,0) & \cdots & (0,1,0) \end{bmatrix}$$

2.3.3. *Unit Pythagorean Neutrosophic fuzzy matrix*

A square Pythagorean Neutrosophic fuzzy matrix is said to be a unit Pythagorean Neutrosophic fuzzy matrix when its all diagonal elements are unit and all other elements are zero

$$B = \begin{bmatrix} (1,0,1) & (0,1,0) & (0,1,0) & \cdots & (0,1,0) \\ (0,1,0) & (1,0,1) & (0,1,0) & \cdots & (0,1,0) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (0,1,0) & (0,1,0) & (0,1,0) & \cdots & (1,0,1) \end{bmatrix}$$

2.3.4. *Symmetric Pythagorean Neutrosophic fuzzy matrix*

A square Pythagorean Neutrosophic fuzzy matrix is said to be symmetric Pythagorean Neutrosophic fuzzy matrix if the matrix and its transpose are equal.

2.3.5. *Triangular Pythagorean Neutrosophic fuzzy matrix*

A square Pythagorean Neutrosophic fuzzy matrix is said to be a triangular Pythagorean Neutrosophic fuzzy matrix if either above main diagonal elements are zero or below main diagonal elements are zero. A square Pythagorean Neutrosophic fuzzy matrix is said to be an upper triangular Pythagorean Neutrosophic fuzzy matrix if above main diagonal elements are zero

$$B = \begin{bmatrix} (\Phi_{11}, \Psi_{11}, \mathcal{L}_{11}) & (0,1,0) & (0,1,0) & \cdots & (0,1,0) \\ (\Phi_{21}, \Psi_{21}, \mathcal{L}_{21}) & (\Phi_{22}, \Psi_{22}, \mathcal{L}_{22}) & (0,1,0) & \cdots & (0,1,0) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (\Phi_{n1}, \Psi_{n1}, \mathcal{L}_{n1}) & (\Phi_{n2}, \Psi_{n2}, \mathcal{L}_{n2}) & \cdots & \cdots & (\Phi_{nn}, \Psi_{nn}, \mathcal{L}_{nn}) \end{bmatrix}$$

and it is said to be a lower triangular Pythagorean Neutrosophic fuzzy matrix if below main diagonal elements are zero

$$B = \begin{bmatrix} (\Phi_{11}, \Psi_{11}, \mathcal{L}_{11})) & (\Phi_{12}, \Psi_{12}, \mathcal{L}_{12}) & \cdots & (\Phi_{1n}, \Psi_{1n}, \mathcal{L}_{1n}) \\ (0,1,0) & (\Phi_{22}, \Psi_{22}, \mathcal{L}_{22}) & \cdots & (\Phi_{2n}, \Psi_{2n}, \mathcal{L}_{2n}) \\ \vdots & \vdots & \vdots & \ddots \\ (0,1,0) & (0,1,0) & \cdots & (\Phi_{nn}, \Psi_{nn}, \mathcal{L}_{nn}) \end{bmatrix}$$

2.3.6. *Vandermonde Pythagorean Neutrosophic fuzzy matrix*

A Vandermonde Pythagorean Neutrosophic fuzzy matrix of order n is of the form

$$V = \begin{bmatrix} (0,1,0) & a_1 & a_1^2 & \cdots & a_1^{n-1} \\ (0,1,0) & a_2 & a_2^2 & \cdots & a_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (0,1,0) & a_n & a_n^2 & \cdots & a_n^{n-1} \end{bmatrix}$$

2.3.7. *DiagonalPythagorean Neutrosophic fuzzy matrix*

A Diagonal Pythagorean Neutrosophic fuzzy matrix is a matrix where all entries of the matrix are zero except the diagonal going from the upper left corner to the lower right corner. The matrix A is given by

$$A = \begin{bmatrix} a & (0,1,0) & (0,1,0) \\ (0,1,0) & b & (0,1,0) \\ (0,1,0) & (0,1,0) & c \end{bmatrix}$$

2.3.8. *Anti-diagonal Pythagorean Neutrosophic fuzzy matrix*

An Anti-diagonal Pythagorean Neutrosophic fuzzy matrix is a matrix where all entries of the matrix are zero except on the diagonal going from the lower left corner to the upper right corner. The matrix A is given by

$$A = \begin{bmatrix} (0,1,0) & (0,1,0) & c \\ (0,1,0) & b & (0,1,0) \\ a & (0,1,0) & (0,1,0) \end{bmatrix}$$

2.4. *Basic operations on PNFM*

Consider two PNFMs, M, N of order $n \times n$. Let

$$M = \begin{bmatrix} (\Phi_{M_{11}}, \Psi_{M_{11}}, \mathcal{L}_{M_{11}}) & (\Phi_{M_{12}}, \Psi_{M_{12}}, \mathcal{L}_{M_{12}}) & \cdots & (\Phi_{M_{1n}}, \Psi_{M_{1n}}, \mathcal{L}_{M_{1n}}) \\ (\Phi_{M_{21}}, \Psi_{M_{21}}, \mathcal{L}_{M_{21}}) & (\Phi_{M_{22}}, \Psi_{M_{22}}, \mathcal{L}_{M_{22}}) & \cdots & (\Phi_{M_{2n}}, \Psi_{M_{2n}}, \mathcal{L}_{M_{2n}}) \\ \vdots & \vdots & \vdots & \vdots \\ (\Phi_{M_{n1}}, \Psi_{M_{n1}}, \mathcal{L}_{M_{n1}}) & (\Phi_{M_{n2}}, \Psi_{M_{n2}}, \mathcal{L}_{M_{n2}}) & \cdots & (\Phi_{M_{nn}}, \Psi_{M_{nn}}, \mathcal{L}_{M_{nn}}) \end{bmatrix}$$

$$N = \begin{bmatrix} (\Phi_{N_{11}}, \Psi_{N_{11}}, \mathcal{L}_{N_{11}}) & (\Phi_{N_{12}}, \Psi_{N_{12}}, \mathcal{L}_{N_{12}}) & \cdots & (\Phi_{N_{1n}}, \Psi_{N_{1n}}, \mathcal{L}_{N_{1n}}) \\ (\Phi_{N_{21}}, \Psi_{N_{21}}, \mathcal{L}_{N_{21}}) & (\Phi_{N_{22}}, \Psi_{N_{22}}, \mathcal{L}_{N_{22}}) & \cdots & (\Phi_{N_{2n}}, \Psi_{N_{2n}}, \mathcal{L}_{N_{2n}}) \\ \vdots & \vdots & \vdots & \vdots \\ (\Phi_{N_{n1}}, \Psi_{N_{n1}}, \mathcal{L}_{N_{n1}}) & (\Phi_{N_{n2}}, \Psi_{N_{n2}}, \mathcal{L}_{N_{n2}}) & \cdots & (\Phi_{N_{nn}}, \Psi_{N_{nn}}, \mathcal{L}_{N_{nn}}) \end{bmatrix}$$

Then the basic operations like Union, Intersection, Complement, Addition, Multiplication, Scalar multiplication and Power operation are defined as follows respectively:

2.4.1. $M \cup N = \left\langle \max\left(\Phi_{M_{ij}}, \Phi_{N_{ij}}\right), \min\left(\Psi_{M_{ij}}, \Psi_{N_{ij}}\right), \min\left(\mathcal{L}_{M_{ij}}, \mathcal{L}_{N_{ij}}\right)\right\rangle$

2.4.2. $M \cap N = \left\langle \min\left(\Phi_{M_{ij}}, \Phi_{N_{ij}}\right), \max\left(\Psi_{M_{ij}}, \Psi_{N_{ij}}\right), \max\left(\mathcal{L}_{M_{ij}}, \mathcal{L}_{N_{ij}}\right)\right\rangle$

2.4.3. $M \oplus N = \left(\sqrt{\Phi^2_{M_{ij}} + \Phi^2_{N_{ij}} - \Phi^2_{M_{ij}}\Phi^2_{N_{ij}}}, \Psi_{M_{ij}}\Psi_{N_{ij}}, \mathcal{L}_{M_{ij}}\mathcal{L}_{N_{ij}}\right)$

2.4.4. $M \otimes N = \left(\Phi_{M_{ij}}\Phi_{N_{ij}}, \sqrt{\Psi^2_{M_{ij}} + \Psi^2_{N_{ij}} - \Psi^2_{M_{ij}}\Psi^2_{N_{ij}}}, \sqrt{\mathcal{L}^2_{M_{ij}} + \mathcal{L}^2_{N_{ij}} - \mathcal{L}^2_{M_{ij}}\mathcal{L}^2_{N_{ij}}}\right)$

2.4.5. $nM = M \oplus M \oplus \cdots \oplus M = \left(\sqrt{1-(1-\Phi^2_{M_{ij}})^n}, \Psi^n_{M_{ij}}, \mathcal{L}^n_{M_{ij}}\right)$

2.4.6. $M^n = \left(\Phi^n_{M_{ij}}, \sqrt{1-(1-\Psi^2_{M_{ij}})^n}, \sqrt{1-(1-\mathcal{L}^2_{M_{ij}})^n}\right)$

2.5. *Numerical Examples*

Consider two matrices M and N of order 2×2 matrix. Let

$$M = \begin{bmatrix} (0.9, 0.1, 0.12) & (0.8, 0.2, 0.3) \\ (0.7, 0.3, 0.4) & (0.5, 0.4, 0.3) \end{bmatrix}$$

$$N = \begin{bmatrix} (0.7, 0.3, 0.2) & (0.3, 0.7, 0.8) \\ (0.6, 0.5, 0.3) & (0.1, 0.8, 0.9) \end{bmatrix}$$

Then M $\cup N = \begin{bmatrix} (0.9, 0.1, 0.12) & (0.8, 0.2, 0.3) \\ (0.7, 0.3, 0.3) & (0.5, 0.4, 0.3) \end{bmatrix}$

M $\cap N = \begin{bmatrix} (0.7, 0.3, 0.2) & (0.3, 0.7, 0.8) \\ (0.6, 0.5, 0.4) & (0.1, 0.8, 0.9) \end{bmatrix}$

M $\oplus n = \begin{bmatrix} (0.2, 0.03, 0.024) & (0.5, 0.14, 0.24) \\ (0.1, 0.15, 0.12) & (0.4, 0.32, 0.27) \end{bmatrix}$

$M \otimes n = \begin{bmatrix} (0.63, 0.2, 0.024) & (0.24, 0.5, 0.24) \\ (0.42, 0.2, 0.12) & (0.05, 0.4, 0.27) \end{bmatrix}$

$nM = \begin{bmatrix} (0.99, 0.001, 0.001) & (0.97, 0.008, 0.009) \\ (0.93, 0.009, 0.064) & (0.42, 0.064, 0.009) \end{bmatrix}$

$M^n = \begin{bmatrix} (0.73, 0.17, 0.3) & (0.51, 0.33, 0.6) \\ (0.34, 0.49, 0.78) & (0.12, 0.63, 0.6) \end{bmatrix}$

Theorem 1: *Let* M and N be two PNFM. If $n > 0$, then

1.1) $n(M \oplus N) = nM \oplus nN$

1.2) $(M \otimes N)^n = M^n \otimes N^n$

Proof

1.1)

$$n(M \oplus N) = \left(\sqrt{1-\left(1-\left(\Phi_{M_{ij}}^2+\Phi_{N_{ij}}^2-\Phi_{M_{ij}}^2\Phi_{N_{ij}}^2\right)\right)^n}, \Psi_{M_{ij}}^n\Psi_{N_{ij}}^n, \mathcal{L}_{M_{ij}}^n\mathcal{L}_{N_{ij}}^n\right) \quad \ldots \quad (1.1.1)$$

$$nM = \left(\sqrt{1-(1-\Phi_{M_{ij}}^2)^n}, \Psi_{M_{ij}}^n, \mathcal{L}_{M_{ij}}^n\right)$$

$$nN = \left(\sqrt{1-(1-\Phi_{N_{ij}}^2)^n}, \Psi_{N_{ij}}^n, \mathcal{L}_{N_{ij}}^n\right)$$

$nM \oplus nN$

$$= \left(\sqrt{1-\left(1-\Phi_{M_{ij}}^2\right)^n+1-\left(1-\Phi_{N_{ij}}^2\right)^n-\left(1-\left(1-\Phi_{M_{ij}}^2\right)^n\right)\left(1-\left(1-\Phi_{N_{ij}}^2\right)^n\right)}, \Psi_{M_{ij}}^n, \Psi_{N_{ij}}^n\mathcal{L}_{M_{ij}}^n\mathcal{L}_{N_{ij}}^n\right)$$

$$= \left(\sqrt{1-\left(1-\Phi_{M_{ij}}^2\right)^n\left(1-\Phi_{N_{ij}}^2\right)^n}, \Psi_{M_{ij}}^n\Psi_{N_{ij}}^n, \mathcal{L}_{M_{ij}}^n\mathcal{L}_{N_{ij}}^n\right)$$

$$= \left(\sqrt{1-\left(\left(1-\Phi_{M_{ij}}^2\right)\left(1-\Phi_{N_{ij}}^2\right)\right)^n}, \Psi_{M_{ij}}^n\Psi_{N_{ij}}^n, \mathcal{L}_{M_{ij}}^n\mathcal{L}_{N_{ij}}^n\right)$$

$$= \left(\sqrt{1-\left(1-\left(\Phi_{M_{ij}}^2-\Phi_{N_{ij}}^2+\Phi_{M_{ij}}^2\Phi_{N_{ij}}^2\right)\right)^n}, \Psi_{M_{ij}}^n\Psi_{N_{ij}}^n, \mathcal{L}_{M_{ij}}^n\mathcal{L}_{N_{ij}}^n\right) \quad \ldots \quad (1.1.2)$$

From (1.1.1) and (1.1.2) we can get (1.1)

1.2)

$$(M \otimes N)^n = \left(\left(\Phi_{Mij}\Phi_{Nij}, \sqrt{\Psi_{M_{ij}^2}+\Psi_{N_{ij}^2}-\Psi_{M_{ij}^2}\Psi_{N_{ij}^2}}, \sqrt{\mathcal{L}_{M_{ij}^2}+\mathcal{L}_{N_{ij}^2}-\mathcal{L}_{M_{ij}^2}\mathcal{L}_{N_{ij}^2}}\right)\right)^n$$

$$= \left((\Phi_{Mij}\Phi_{Nij})^n, \sqrt{1-\left(1-(\Psi_{M_{ij}^2}+\Psi_{N_{ij}^2}-\Psi_{M_{ij}^2}\Psi_{N_{ij}^2}\right)^n}\right), \sqrt{1-\left(1-(\mathcal{L}_{M_{ij}^2}+\mathcal{L}_{N_{ij}^2}-\mathcal{L}_{M_{ij}^2}\mathcal{L}_{N_{ij}^2}\right)^n}) \quad \ldots \quad (1.2.1$$

$$M^n = \left(\Phi_{M_{ij}}^n, \sqrt{1-(1-\Psi_{M_{ij}}^2)^n}, \sqrt{1-(1-\mathcal{L}_{M_{ij}}^2)^n}\right)$$

$$N^n = \left(\Phi_{N_{ij}}^n, \sqrt{1-(1-\Psi_{N_{ij}}^2)^n}, \sqrt{1-(1-\mathcal{L}_{N_{ij}}^2)^n}\right)$$

$M^n \otimes N^n$

$$= \left(\Phi_{M_{ij}^n}\Phi_{N_{ij}^n}, \sqrt{1-\left(1-(\Psi_{M_{ij}^2}\right)^n+1-\left(1-(\Psi_{N_{ij}^2}\right)^n-\left(1-\left(1-(\Psi_{M_{ij}^2}\right)^n\right)\left(1-\left(1-(\Psi_{N_{ij}^2}\right)^n\right)}, \right.$$
$$\sqrt{1-\left(1-(\mathcal{L}_{M_{ij}^2}\right)^n+1-\left(1-(\mathcal{L}_{N_{ij}^2}\right)^n-\left(1-\left(1-(\mathcal{L}_{M_{ij}^2}\right)^n\right)\left(1-\left(1-(\mathcal{L}_{N_{ij}^2}\right)^n\right)}$$

$$= \left((\Phi_{Mij}\Phi_{Nij})^n, \sqrt{1-\left(1-(\Psi_{M_{ij}^2}+\Psi_{N_{ij}^2}-\Psi_{M_{ij}^2}\Psi_{N_{ij}^2}\right)^n}\right), \sqrt{1-\left(1-(\mathcal{L}_{M_{ij}^2}+\mathcal{L}_{N_{ij}^2}-\mathcal{L}_{M_{ij}^2}\mathcal{L}_{N_{ij}^2}\right)^n}) \quad \ldots \quad (1.2.2$$

From (1.2.1) and (1.2.2) we get (1.2)

Theorem 2: Let M and N be two PNFM. If $n > 0$, then

2.1) $n(M \cup N) = nM \cup nN$

2.2) $(M \cup N)^n = M^n \cup N^n$

Proof: 2.1) $n(M \cup N) = \left(\sqrt{1-\left(1-\max\left\{\Phi_{M_{ij}^2}, \Phi_{N_{ij}^2}\right\}\right)^n}, \min\left\{\Psi_{M_{ij}^n}, \Psi_{N_{ij}^n}\right\}, \min\left\{\mathcal{L}_{M_{ij}^n}, \mathcal{L}_{N_{ij}^n}\right\}\right) \quad \ldots \quad (2.1.1)$

$$\text{nM} \cup nN = \left(\sqrt{1-\left(1-\Phi_{M_{ij}^{2}}\right)^{n}}, \Psi_{M_{ij}^{n}}, \mathcal{L}_{M_{ij}^{n}}\right) \cup \left(\sqrt{1-\left(1-\Phi_{N_{ij}^{2}}\right)^{n}}, \Psi_{N_{ij}^{n}}, \mathcal{L}_{N_{ij}^{n}}\right)$$
$$= \left(\max\left\{\sqrt{1-\left(1-\Phi_{M_{ij}^{2}}\right)^{n}}, \sqrt{1-\left(1-\Phi_{N_{ij}^{2}}\right)^{n}}\right\}, \min\left\{\Psi_{M_{ij}^{n}}, \Psi_{N_{ij}^{n}}\right\}, \min\left\{\mathcal{L}_{M_{ij}^{n}}, \mathcal{L}_{N_{ij}^{n}}\right\}\right)$$
$$= \left(\sqrt{1-\left(1-\max\left\{\Phi_{M_{ij}^{2}}, \Phi_{N_{ij}^{2}}\right\}\right)^{n}}, \min\left\{\Psi_{M_{ij}^{n}}, \Psi_{N_{ij}^{n}}\right\}, \min\left\{\mathcal{L}_{M_{ij}^{n}}, \mathcal{L}_{N_{ij}^{n}}\right\}\right) \quad \ldots \quad (2.1.2)$$

From (2.1.1) and (2.1.2), we get n(M $\cup N) = nM \cup nN$

2.2)Analogously, it can be shown that $(M \cup N)^n = M^n \cup N^n$

Theorem 3: Let M and N be two PNFM, then

3.1) (M $\cup N) \oplus (M \cap N) = M \oplus N$

3.2)$(M \cup N) \otimes (M \cap N) = M \otimes N$

Proof3.1)$(M \cup N) \oplus (M \cap N) = (\langle \max\{\Phi_{Mij}, \Phi_{Nij}\}, \min\{\Psi_{Mij}, \Psi_{Nij}\}, \min\{\mathcal{L}_{Mij}, \mathcal{L}_{Nij}\}\rangle \oplus$
$\langle \min\{\Phi_{Mij}, \Phi_{Nij}\}, \max\{\Psi_{Mij}, \Psi_{Nij}\}, \max\{\mathcal{L}_{Mij}, \mathcal{L}_{Nij}\}\rangle$

$$= \left(\sqrt{\max\left\{\Phi_{M_{ij}^{2}}, \Phi_{N_{ij}^{2}}\right\} + \min\left\{\Phi_{M_{ij}^{2}}, \Phi_{N_{ij}^{2}}\right\} - \max\left\{\Phi_{M_{ij}^{2}}, \Phi_{N_{ij}^{2}}\right\}\min\left\{\Phi_{M_{ij}^{2}}, \Phi_{N_{ij}^{2}}\right\}}, \min\{\Psi_{Mij}, \Psi_{Nij}\}\right.$$
$$\left.\max\{\Psi_{Mij}, \Psi_{Nij}\}, \min\{\mathcal{L}_{Mij}, \mathcal{L}_{Nij}\}\max\{\mathcal{L}_{Mij}, \mathcal{L}_{Nij}\}\right)$$
$$= \left(\sqrt{\Phi_{M_{ij}^{2}} + \Phi_{N_{ij}^{2}} - \Phi_{M_{ij}^{2}}\Phi_{N_{ij}^{2}}}, \Psi_{Mij}\Psi_{Nij}, \mathcal{L}_{Mij}\mathcal{L}_{Nij}\right)$$
$$= M \oplus N$$

3.2)Analogously, it can be shown that $(M \cup N) \otimes (M \cap N) = M \otimes N$

Theorem 4: Let M , N and L be three PNFM), then

4.1) (M $\cup N) \cap L = (M \cap L) \cup (N \cap L)$

4.2) $(M \cap N) \cup L = (M \cup L) \cap (N \cup L)$

4.3) $(M \cup N) \oplus L = (M \oplus L) \cup (N \oplus L)$

4.4) $(M \cap N) \oplus L = (M \oplus L) \cap (N \oplus L)$

4.5) $(M \cup N) \otimes L = (M \otimes L) \cup (N \otimes L)$

4.6) $(M \cap N) \otimes L = (M \otimes L) \cap (N \otimes L)$

Proof

Let us prove 4.1, 4.3 and 4.5. 4.2, 4.4 and 4.6

4.1)(M $\cup N) \cap L = (\min\{\max\{\Phi_{Mij}, \Phi_{Nij}\}, \Phi_{Lij}\}, \max\{\min\{\Psi_{Mij}, \Psi_{Nij}\}, \Psi_{Lij}\},$
$\max\{\min\{\mathcal{L}_{Mij}, \mathcal{L}_{Nij}\}, \mathcal{L}_{Lij}\}) \quad \ldots \quad (4.1.1)$

$M \cap L = \left\langle \min\left(\Phi_{M_{ij}}, \Phi_{L_{ij}}\right), \max\left(\Psi_{M_{ij}}, \Psi_{L_{ij}}\right), \max\left(\mathcal{L}_{M_{ij}}, \mathcal{L}_{L_{ij}}\right)\right\rangle$

$N \cap L = \left\langle \min\left(\Phi_{N_{ij}}, \Phi_{L_{ij}}\right), \max\left(\Psi_{N_{ij}}, \Psi_{L_{ij}}\right), \max\left(\mathcal{L}_{N_{ij}}, \mathcal{L}_{L_{ij}}\right)\right\rangle$

$(M \cap L) \cup (N \cap L)$

$= (\max\{\min\{\Phi_{Mij}, \Phi_{Lij}\}, \min\{\Phi_{Nij}, \Phi_{Lij}\}\}, \min\{\max\{\Psi_{Mij}, \Psi_{Lij}\}, \max\{\Psi_{Lij}, \Psi_{Lij}\}\},$

$\min\{\max\{\mathcal{L}_{Mij},\mathcal{L}_{Lij}\},\max\{\mathcal{L}_{Nij},\mathcal{L}_{Lij}\}\})$

$= (\min\{\max\{\Phi_{Mij},\Phi_{Nij}\},\Phi_{Lij}\},\max\{\min\{\Psi_{Mij},\Psi_{Nij}\},\Psi_{Lij}\},$

$\max\{\min\{\mathcal{L}_{Mij},\mathcal{L}_{Nij}\},\mathcal{L}_{Lij}\}) \quad \ldots \quad (4.1.2)$

From (4.1.1) and (4.1.2) we get (4.1)

4.3)$(M \cup N) \oplus L =$

$(\max\{\Phi_{Mij},\Phi_{Nij}\},\min\{\Psi_{Mij},\Psi_{Lij}\},\min\{\mathcal{L}_{Mij},\mathcal{L}_{Lij}\}) \oplus (\Phi_{Lij},\Psi_{Lij},\mathcal{L}_{Lij})$

$$= \left(\sqrt{\max\left\{\Phi_{M^2_{ij}},\Phi_{N^2_{ij}}\right\}+\Phi_{L^2_{ij}}-\max\left\{\Phi_{M^2_{ij}},\Phi_{N^2_{ij}}\right\}\Phi_{L^2_{ij}}},\min\{\Psi_{Mij},\Psi_{Nij}\}\Psi_{Lij},\min\{\mathcal{L}_{Mij},\mathcal{L}_{Nij}\}\mathcal{L}_{Lij}\right)$$

$$= \left(\sqrt{(1-\Phi_{L^2_{ij}})\max\left\{\Phi_{M^2_{ij}},\Phi_{N^2_{ij}}\right\}+\Phi_{L^2_{ij}}},\min\{\Psi_{Mij},\Psi_{Lij},\Psi_{Nij},\Psi_{Lij}\},\right.$$
$$\min\{\mathcal{L}_{Mij},\mathcal{L}_{Lij},\mathcal{L}_{Nij},\mathcal{L}_{Lij}\}) \quad \ldots \quad (4.3.1)$$

$(M \oplus L) \cup (N \oplus L) =$

$$\left(\max\left\{\sqrt{\Phi_{M^2_{ij}}+\Phi_{L^2_{ij}}-\Phi_{M^2_{ij}}\Phi_{L^2_{ij}}},\sqrt{\Phi_{N^2_{ij}}+\Phi_{L^2_{ij}}-\Phi_{N^2_{ij}}\Phi_{L^2_{ij}}}\right\},\min\{\Psi_{Mij},\Psi_{Lij},\Psi_{Nij},\Psi_{Lij}\},\right.$$
$$\min\{\mathcal{L}_{Mij},\mathcal{L}_{Lij},\mathcal{L}_{Nij},\mathcal{L}_{Lij}\}$$

$$= \left(\sqrt{(1-\Phi_{L^2_{ij}})\Phi_{M^2_{ij}}+\Phi_{L^2_{ij}}},\sqrt{(1-\Phi_{L^2_{ij}})\Phi_{N^2_{ij}}+\Phi_{L^2_{ij}}},\min\{\Psi_{Mij},\Psi_{Lij},\Psi_{Nij},\Psi_{Lij}\},\min\{\mathcal{L}_{Mij},\mathcal{L}_{Lij},\mathcal{L}_{Nij},\mathcal{L}_{Lij}\}\right)$$

$$= \left(\sqrt{(1-\Phi_{L^2_{ij}})\max\left\{\Phi_{M^2_{ij}},\Phi_{N^2_{ij}}\right\}+\Phi_{L^2_{ij}}},\min\{\Psi_{Mij},\Psi_{Lij},\Psi_{Nij},\Psi_{Lij}\},\right.$$
$$\min\{\mathcal{L}_{Mij},\mathcal{L}_{Lij},\mathcal{L}_{Nij},\mathcal{L}_{Lij}\}) \quad \ldots \quad (4.3.2)$$

From (4.3.1) and (4.3.2) we get the result 4.3

(M $\cup N) \otimes L =$

$$\left(\max\{\Phi_{Mij},\Phi_{Nij}\},\sqrt{\min\left\{\Phi_{M^2_{ij}},\Phi_{N^2_{ij}}\right\}+\Phi_{L^2_{ij}}-\min\left\{\Phi_{M^2_{ij}},\Phi_{N^2_{ij}}\right\}\Phi_{L^2_{ij}}},\right.$$
$$\sqrt{\min\left\{\mathcal{L}_{M^2_{ij}},\mathcal{L}_{N^2_{ij}}\right\}+\mathcal{L}_{L^2_{ij}}-\min\left\{\mathcal{L}_{M^2_{ij}},\mathcal{L}_{N^2_{ij}}\right\}\mathcal{L}_{L^2_{ij}}})$$

$$= \left(\max\{\Phi_{Mij},\Phi_{Nij}\},\sqrt{(1-\Psi_{L^2_{ij}})\min\left\{\Psi_{M^2_{ij}},\Psi_{N^2_{ij}}\right\}+\Psi_{L^2_{ij}}},\sqrt{(1-\mathcal{L}_{L^2_{ij}})\min\left\{\mathcal{L}_{M^2_{ij}},\mathcal{L}_{N^2_{ij}}\right\}+\mathcal{L}_{L^2_{ij}}}\right) \quad \ldots \quad (4.5.1)$$

(M $\otimes L) \cup (N \otimes L) =$

$$\left(\max\{\Phi_{Mij},\Phi_{Lij},\Phi_{Nij},\Phi_{Lij}\},\min\left\{\sqrt{\Phi_{M^2_{ij}}+\Phi_{L^2_{ij}}-\Phi_{M^2_{ij}}\Phi_{L^2_{ij}}},\sqrt{\Phi_{N^2_{ij}}+\Phi_{L^2_{ij}}-\Phi_{N^2_{ij}}\Phi_{L^2_{ij}}}\right\},\right.$$
$$\min\{\mathcal{L}_{Mij}+\mathcal{L}_{Lij}-\mathcal{L}_{Mij}\mathcal{L}_{Lij},\mathcal{L}_{Nij}+\mathcal{L}_{Lij}-\mathcal{L}_{Nij}\mathcal{L}_{Lij}\})$$

$$= \left(\max\{\Phi_{Mij},\Phi_{Nij},\Phi_{Lij}\},\sqrt{(1-\Psi_{L^2_{ij}})\min\left\{\Psi_{M^2_{ij}},\Psi_{N^2_{ij}}\right\}+\Psi_{L^2_{ij}}},\right.$$
$$\sqrt{(1-\mathcal{L}_{L^2_{ij}})\min\left\{\mathcal{L}_{M^2_{ij}},\mathcal{L}_{N^2_{ij}}\right\}+\mathcal{L}_{L^2_{ij}}}) \quad \ldots \quad (4.5.2)$$

From (4.5.1) and (4.5.2) we get the results

3. Comparison

Characteristics	FM	IFM	NFM	PFM	PNFM
Components	(Φ)	(ϕ, Ψ)	$(\Phi, \Psi, \mathcal{L})$	(ϕ, Ψ)	$(\Phi, \Psi, \mathcal{L})$
Condition	$0 \leq \Phi \leq 1$	$0 \leq \Phi + \Psi \leq 1$	$0 \leq \Phi + \Psi + \mathcal{L} \leq 3$	$0 \leq \Phi^2 + \Psi^2 \leq 1$	$0 \leq \Phi^2 + \Psi^2 + \mathcal{L}^2 \leq 2$
Complexity	Less complex	Moderately complex	Complex	More complex	Most complex
Applicablity	Simple decison-making	Incomplete information	High uncertainty and indeterminacy	Nuanced representation of uncertainty	Complex problems with multiple sources of uncertainty
Limitation	Lacks to represent non-membership values	Difficulty in handling indeterminacy	can't handle high-dimensional data	Intensive in computing	Requires specialized software

Table 1. Comparison of different types of fuzzy matrices

PNFM is an important tool for representing uncertainty. It combines the ideas of a pythagorean fuzzy matrix and a neutrosophic fuzzy matrix, providing a more thorough approach to dealing with the complexity of real-world issues. In addition to to handling vagueness, it also responds to the situations in an ambiguous and inconsistent manner. The Pythagorean neutrosophic fuzzy matrix notion can be applied to a variety of domains, including decision making and pattern recognition, when ambiguity and vagueness are present.

4. Future Work

This article gives a strong foundation for future research in PNFM. It can focus on the effectiveness of PNFM in real world problems. Although PNFM has many applications, it also has limits. PNFM's computational component is more difficult, and there are no defined aggregation methods. To address these challenges, future work in this subject should focus on creating computing algorithms and improved aggregation approaches.

5. Conclusion

The idea of PNFM and its basic functions were examined in this study. The characteristics and behavior of PNFM under different operations are established by the theorems in this paper. This concept provides the foundation for further investigation into more complex PNFM applications in fields such as machine learning and optimization.

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