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Performance Analysis of Neutrosophic Multi-Server Queuing-Inventory System under Catastrophic Conditions

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Abstract

In this study, we introduce a Neutrosophic-based framework for analyzing a finite capacity multi-server queuing-inventory system (MQIS) that integrates attraction-retention mechanisms for impatient customers and addresses catastrophic inventory disruptions. Recognizing the limitations of purely deterministic and probabilistic models in uncertain environments, we adopt Single-Valued Trapezoidal Neutrosophic Numbers (SVTNNs) to represent key system parameters such as arrival rates, service rates, attraction intensities, retention probabilities, and catastrophe occurrences. The system operates under Markovian assumptions, with arrivals modeled by a Poisson process, service and vacation times following exponential distributions, and inventory replenished via an (s, S) policy. To facilitate steady-state analysis, the Neutrosophic model is transformed into equivalent deterministic models through the application of Zadeh's extension principle combined with the (α, β, γ)-cut method. Furthermore, a cost optimization problem is formulated to identify optimal service configurations, solved using a genetic algorithm. A numerical illustration is provided to demonstrate the model's ability to capture uncertainty and improve decision-making in complex service environments.

Keywords: Neutrosophic Logic; Multi-Server Queuing-Inventory System; Customer Attraction and Retention; Impatient Customers; Inventory Optimization; Asynchronous Server Vacations.

1 Introduction

Queuing-inventory systems (QIS) are fundamental in modeling service operations where the dynamics of customer arrivals and inventory depletion interact. In high-demand sectors such as telecommunications, logistics, and healthcare, managing both customer flow and stock availability under operational constraints is critical for maintaining service quality and cost efficiency. Classical queuing-inventory models, such as those developed by Buzacott and Shanthikumar [1], typically assume exponential interarrival and service times and deterministic system parameters. While mathematically tractable, these assumptions often fail to capture the complex uncertainties observed in real-world systems, such as fluctuating arrival rates, unpredictable service times, and variable customer behavior.

The inherent imprecision, ambiguity, and inconsistency in service environments have motivated researchers to seek more flexible modeling approaches. Fuzzy set theory, introduced by Zadeh [2], offers one such framework by permitting degrees of membership between 0 and 1, allowing systems to model vagueness in customer arrivals, service rates, and inventory levels. Applications of fuzzy logic to queueing inventory systems, such as those of Buckley [3] and Kao and Liu [4], have enhanced the robustness of system analysis under uncertain conditions. However, fuzzy models are inherently limited in their ability to accommodate conflicting or indeterminate information, as they treat uncertainty primarily as gradual vagueness rather than acknowledging the coexistence of truth, indeterminacy, and falsity.

In response to these limitations, intuitionistic fuzzy sets were introduced by Atanassov [5], extending fuzzy sets to explicitly model both membership and nonmembership degrees. However, even intuitionistic models are restricted when dealing with complex, inconsistent, and incomplete information environments. To overcome these challenges, Smarandache [6] proposed the Neutrosophic set theory, where each element is characterized by independent degrees of truth, indeterminacy, and falsity. This tripartite structure offers a significantly richer framework for capturing the multifaceted uncertainty inherent in service systems.

In recent years, neutrosophic approaches have been applied more and more to queueing models. Researchers such as Maji and Roy [7] demonstrated that neutrosophic models could more accurately represent arrival rates and service rates when subject to indeterminate and inconsistent information. Compared to other neutrosophic representations, single-valued trapezoidal neutrosophic numbers (SVTNN) have proven particularly useful due to their computational simplicity and intuitive geometric interpretation, as evidenced by recent studies (Dayana et al. [11]).

However, most existing neutrosophic queueing models focus on single-server systems without fully integrating inventory dynamics, asynchronous server operations, or catastrophic disruptions. Real-world service systems, especially in large-scale telecommunications and healthcare applications, often involve multiple parallel servers, inventory constraints, customer impatience, active attraction-retention efforts, and occasional large-scale system shocks (e.g., technical failures, supply chain disruptions).

This research addresses these gaps by proposing Neutrosophic-based framework for analyzing a finite-capacity, multi-server queueing-inventory system with asynchronous server vacations, attraction-retention mechanisms, and catastrophic inventory events. In the model:

- Customers arrive according to a Poisson process and are served by one of C identical servers;

- Each service consumes one unit of inventory, with replenishment governed by an (s, S) policy;
- Servers independently take asynchronous vacations when idle, reflecting realistic operational policies;
- Customer attraction and retention strategies influence arrival and abandonment decisions under uncertainty;
- Catastrophic events are incorporated as neutrosophic random shocks impacting both customers and inventory levels.

System parameters such as arrival rates, service rates, attraction-retention probabilities, and catastrophe rates are modeled using SVTNNs, allowing the model to capture the full spectrum of uncertainty, indeterminacy, and inconsistency in operational environments. The steady-state analysis is conducted by transforming the neutrosophic model into a family of crisp Markovian models through the application of Zadeh's extension principle and the (α, β, γ)-cut method.

Moreover, a cost optimization problem is formulated, incorporating service costs, holding costs, shortage costs, and abandonment penalties, and is solved using a genetic algorithm to determine optimal service rates and vacation policies.

To validate the proposed framework, a numerical case study based on operational data from Ethio Telecom is presented. The results demonstrate that the neutrosophic model provides more resilient and realistic performance estimates compared to traditional approaches, offering valuable insights for service managers operating under high uncertainty.

The remainder of this paper is organized as follows: Section 2 introduces the preliminaries and fundamental definitions of neutrosophic sets. Section 3 describes the system assumptions and detailed model structure of the neutrosophic multi-server queueing-inventory system. Section 4 presents the neutrosophic NM/NM/C/K queueing-inventory model. Section 5 provides numerical results and discussion. Finally, Section 6 concludes the study and outlines directions for future research.

2 Preliminaries

Definition 2.1 (Neutrosophic Set [12]) *A Neutrosophic set N defined over a universe τ assigns to each element $\tau \in \tau$ three membership functions: truth-membership $T_A(\tau)$, indeterminacy-membership $I_A(\tau)$, and falsity-membership $F_A(\tau)$, where*

$$N = \{(\tau, (T_A(\tau), I_A(\tau), F_A(\tau))) : \tau \in \tau\}.$$

Each membership function maps to the interval $[0, 1]$, satisfying the constraint:

$$0 \leq \sup T_A(\tau) + \sup I_A(\tau) + \sup F_A(\tau) \leq 3.$$

Definition 2.2 (Single-Valued Neutrosophic Set (SVNS) [12]) *A Single-Valued Neutrosophic Set (SVNS) is a specific case of a Neutrosophic Set where each of the membership functions $T_A(\tau)$, $I_A(\tau)$, and $F_A(\tau)$ take crisp values in $[0, 1]$ only. An SVNS is given by:*

$$N = \{(\tau, (T_A(\tau), I_A(\tau), F_A(\tau))) : \tau \in \tau\},$$

under the condition that:

$$0 \leq \sup T_A(\tau) + \sup I_A(\tau) + \sup F_A(\tau) \leq 3.$$

Definition 2.3 (Single-Valued Trapezoidal Neutrosophic Number (SVTNN) [12]) *A Single-Valued Trapezoidal Neutrosophic Number (SVTNN) represents the truth, indeterminacy, and falsity membership functions using trapezoidal forms. For A, the truth-membership function $T_A(\tau)$ is expressed as:*

$$T_A(\tau) = \begin{cases} \frac{\tau_T - r_T^1}{r_T^2 - r_T^1}, & r_T^1 \leq \tau_T \leq r_T^2, \\ 1, & r_T^2 \leq \tau_T \leq r_T^3, \\ \frac{r_T^4 - \tau_T}{r_T^4 - r_T^3}, & r_T^3 \leq \tau_T \leq r_T^4, \\ 0, & \textit{otherwise}, \end{cases}$$

where $r_T^1 \leq r_T^2 \leq r_T^3 \leq r_T^4$. Similar forms define $I_A(\tau)$ and $F_A(\tau)$ for the indeterminacy and falsity memberships, respectively.

Definition 2.4 ((α, β, γ)-Cut for SVTNN [13]) *The (α, β, γ)-cut of a trapezoidal single-valued neutrosophic number partitions it into crisp intervals:*

$$D_{\alpha,\beta,\gamma} = \left([D_1(\alpha), D_2(\alpha)], [D_1^*(\beta), D_2^*(\beta)], [D_1^{**}(\gamma), D_2^{**}(\gamma)]\right),$$

where

$$\begin{aligned} [D_1(\alpha), D_2(\alpha)] &= [r_T^1 + \alpha(r_T^2 - r_T^1), r_T^4 - \alpha(r_T^4 - r_T^3)], \\ [D_1^*(\beta), D_2^*(\beta)] &= [r_I^2 - \beta(r_I^2 - r_I^1), r_I^3 + \beta(r_I^4 - r_I^3)], \\ [D_1^{**}(\gamma), D_2^{**}(\gamma)] &= [r_F^2 - \gamma(r_F^2 - r_F^1), r_F^3 + \gamma(r_F^4 - r_F^3)], \end{aligned}$$

with $0 \leq \alpha + \beta + \gamma \leq 3$.

Definition 2.5 (Arithmetic Operations on Intervals [14]) *Given two real intervals $[p_1, p_2]$ and $[p_3, p_4]$, their basic operations are defined as:*

- *Addition:* $[p_1, p_2] + [p_3, p_4] = [p_1 + p_3, p_2 + p_4]$,
- *Subtraction:* $[p_1, p_2] - [p_3, p_4] = [p_1 - p_4, p_2 - p_3]$,
- *Multiplication:* $[p_1, p_2] \times [p_3, p_4] = [\min\{p_1p_3, p_1p_4, p_2p_3, p_2p_4\}, \max\{p_1p_3, p_1p_4, p_2p_3, p_2p_4\}]$,
- *Division (assuming $0 \notin [p_3, p_4]$):*

$$[p_1, p_2] \div [p_3, p_4] = \left[\min\left\{\frac{p_1}{p_3}, \frac{p_1}{p_4}, \frac{p_2}{p_3}, \frac{p_2}{p_4}\right\}, \max\left\{\frac{p_1}{p_3}, \frac{p_1}{p_4}, \frac{p_2}{p_3}, \frac{p_2}{p_4}\right\}\right].$$

3 Description of the Model

We consider a finite-capacity neutrosophic multi-server queuing-inventory system (NMQIS) with attraction-retention mechanisms for impatient customers, where C removable servers operate under an asynchronous vacation policy with random lead times. The system is modeled using neutrosophic logic to represent the degrees of truth, indeterminacy, and falsity associated with system components and behaviors under uncertainty. The assumptions and neutrosophic characterizations are as follows:

1. **System Configuration:** The system consists of C removable servers, a waiting room with capacity N, and a maximum inventory level S. Each service consumes one inventory item. Let (T_1, I_1, F_1) denote the neutrosophic representation of the belief in accurate inventory deduction after each service, where T_1 is the degree of truth, I_1 the degree of indeterminacy, and F_1 the degree of falsity.

2. **Customer Arrival Process:** Customers arrive according to a neutrosophic Poisson process with arrival rate $\lambda_N = (\lambda_T, \lambda_I, \lambda_F)$. Upon arrival, a customer either joins or balks with neutrosophic probability $b_n^N = (b_n^T, b_n^I, b_n^F)$ given by:

$$b_n^T = \begin{cases} 1, & 0 \leq n \leq C - D - 1 \\ \frac{N-n}{N}, & C - D \leq n \leq N \end{cases}, \quad b_n^I = f_1(n), \quad b_n^F = 1 - b_n^T - b_n^I$$

   where $f_1(n)$ represents the uncertainty in the decision-making process of arriving customers due to behavioral or informational ambiguity.

3. **Behavioral Control Mechanisms:** Behavioral control mechanisms are applied based on system congestion:

   - When $n \leq \frac{3N}{4}$, an attraction policy is used, increasing the arrival rate to $\lambda_N^{\text{attr}} = (\lambda(1+\beta), \beta_I, \beta_F)$.
   - When $n > \frac{3N}{4}$, a retention policy is used to reduce abandonment, represented by a retention rate $r_N = (r_T, r_I, r_F)$.

4. **Service Process:** Customers are served in a single queue using the FCFS discipline. Service times are i.i.d. and follow a neutrosophic exponential distribution with rate $\mu_N = (\mu_T, \mu_I, \mu_F)$, such that $s_N(t) = (\mu_T e^{-\mu_T t}, \mu_I(t), \mu_F(t))$, where $\mu_I(t)$ and $\mu_F(t)$ reflect ambiguity or system failures.

5. **Customer Patience:** Customers possess a neutrosophic patience time distributed exponentially with reversion rate $\alpha_N = (\alpha_T, \alpha_I, \alpha_F)$. The neutrosophic relapse rate $R_N(n)$ depends on the length of the queue and is defined as:

$$R_N(n) = \begin{cases} ((n - C + D)(1 - r_T)\alpha_T, R_I(n), R_F(n)), & C - D < n < N \\ (0, 0, 0), & 0 \leq n \leq C - D \end{cases}$$

   where $R_I(n)$ and $R_F(n)$ account for uncertainty and contradiction in abandonment behavior.

6. **Server Vacations:** When the system is empty, D servers go on asynchronous neutrosophic vacation with exponential duration $v_N(t) = (\xi_T e^{-\xi_T t}, \xi_I(t), \xi_F(t))$, where $\xi_N = (\xi_T, \xi_I, \xi_F)$ is the vacation rate under neutrosophic parameters.

7. **Inventory Management:** The inventory is managed through a neutrosophic (s, S) policy:

   - When the inventory level drops to s, a replenishment is triggered to restore inventory to S.
   - The replenishment lead time follows a neutrosophic exponential distribution with rate $\eta_N = (\eta_T, \eta_I, \eta_F)$.

- The condition $D < s$ holds with a neutrosophic degree (T_7, I_7, F_7) ensuring operational integrity.

8. **Inventory Depletion:** If the inventory is depleted:

- The service rate becomes neutrosophically zero: $(0, \mu_I^{\text{stock}}, \mu_F^{\text{stock}})$.
- Customers wait for restocking or leave due to indefinite wait times, governed by $R_N^{\text{stock}}(n)$.
- The system remains idle in a neutrosophic inertial state until replenishment is completed.

9. **Catastrophic Events:**

- Catastrophic events, such as system failures, natural disasters, or other disruptions, occur according to a neutrosophic Poisson process with rate $\gamma = (\gamma_T, \gamma_I, \gamma_F)$. This rate reflects the occurrence of these events with truth degree γ_T, indeterminacy degree γ_I, and falsity degree γ_F, capturing the uncertainty and unpredictability of these events.
- Upon the occurrence of a catastrophe, the inventory is immediately emptied, and the on-hand inventory is reset to zero. This is modeled by a neutrosophic state:
$$\text{Inventory}_{\text{catastrophe}} = (1, I_{\text{cat}}, F_{\text{cat}}),$$
where I_{cat} and F_{cat} represent the uncertainty and contradiction regarding the full restoration of inventory.
- Customers whose service is interrupted by a catastrophic event may either **rejoin the queue** or **leave the system**. The probability of a customer's decision is represented by a neutrosophic probability:
$$P_{\text{rejoin}} = (P_T, P_I, P_F),$$
where P_T is the truth degree (probability of rejoining the queue), P_I is the indeterminacy (uncertainty), and P_F is the falsity (probability of leaving the system).
- During the catastrophic event, all **servers become inoperative**, and the system enters a state of inactivity. This is modeled by the neutrosophic state of the server's operational status:
$$\text{Server Status}_{\text{catastrophe}} = (S_T, S_I, S_F) = (0, S_{\text{cat}}, 1),$$
where S_T is the truth degree (full inactivity), S_I represents uncertainty in the downtime, and S_F is the falsity degree (failure of servers to function).
- The system undergoes a **restoration process**, where the service is gradually resumed. The restoration time follows a neutrosophic exponential distribution with rate $\kappa = (\kappa_T, \kappa_I, \kappa_F)$. The restoration process is given by:
$$R_{\text{restore}}(t) = (\kappa_T e^{-\kappa_T t}, \kappa_I(t), \kappa_F(t)),$$
where $R_{\text{restore}}(t)$ represents the neutrosophic restoration time.

- During the restoration period, **new customers continue to arrive**, with the arrival rate $\lambda_{\text{restore}} = (\lambda_T, \lambda_I, \lambda_F)$, adjusted according to the uncertainty and restoration progress.
- Once the restoration process is complete, the system transitions back to normal operation, with **servers becoming operative** and inventory levels being replenished. The system recovery is modeled by the neutrosophic state:

$$\text{System Recovery} = (T_{\text{recovery}}, I_{\text{recovery}}, F_{\text{recovery}}),$$

where T_{recovery} indicates the truth degree of recovery, I_{recovery} reflects the indeterminacy, and F_{recovery} represents the falsity degree in the recovery process.

4 Neutrosophic NM/NM/C/K QIS with Attraction-Retention and Catastrophes

This section develops a comprehensive neutrosophic queuing-inventory model denoted by NM/NM/C/K, tailored for real-world service systems where both queuing dynamics and inventory levels are subject to uncertainty. In addition to traditional customer arrival and service processes, the model incorporates (i) inventory management through a neutrosophic (s, S) policy, (ii) behavioral mechanisms for customer attraction and retention, and (iii) the occurrence of catastrophic events that may suddenly deplete inventory or disrupt service.

In multi-server service environments such as telecommunications centers, retail warehouses, and healthcare systems, customer impatience, stockouts, and operational disruptions are common and interconnected phenomena. Classical queueing-inventory models often fail to simultaneously capture these dynamics, especially under uncertain, inconsistent, or incomplete information. To bridge this gap, we formulate a neutrosophic NM/NM/C/K model using Single-Valued Trapezoidal Neutrosophic Numbers (SVTNNs), enabling the joint modeling of arrival rates, service times, inventory levels, and behavioral probabilities with associated degrees of truth, indeterminacy, and falsity.

This model provides a flexible structure for evaluating system performance and risk, including metrics such as expected queue length, inventory shortfall probabilities, and abandonment rates, all under varying levels of uncertainty. The integration of customer attraction-retention strategies and catastrophic events further enhances its applicability to modern service systems facing unpredictable and high-stakes operating conditions.

4.1 Classical M/M/C/K Queueing Model Overview

The classical M/M/C/K model involves a queuing system with:

- Customers arriving according to a Poisson process with rate λ,
- Exponentially distributed service times with rate μ,
- C parallel servers,
- Finite system capacity K, including both servers and queue space,
- A first-come, first-served (FCFS) service discipline.

Performance Measures

The key performance metrics of the classical model include:

Probability of Zero Customers in the System:

$$p_0 = \left[\sum_{n=0}^{C-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^C}{C!} \cdot \frac{1 - \left(\frac{\lambda}{C\mu}\right)^{K-C+1}}{1 - \left(\frac{\lambda}{C\mu}\right)} \right]^{-1}$$

Probability of n Customers in the System:

$$p_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} p_0, & 0 \leq n < C \\ \frac{(\lambda/\mu)^n}{C^{n-C} C!} p_0, & C \leq n \leq K \end{cases}$$

Expected Queue Length:

$$L_q = \sum_{n=C}^{K} (n - C) p_n$$

Expected Number of Customers in the System:

$$L_s = \sum_{n=0}^{K} n p_n$$

Average Waiting Time in Queue:

$$W_q = \frac{L_q}{\lambda(1 - p_K)}$$

Average Waiting Time in the System:

$$W_s = W_q + \frac{1}{\mu}$$

4.2 Neutrosophic Arrival and Service Times

In the neutrosophic setting, let the arrival and service times be modeled as follows:

$$A = \{(b, T_A(b), I_A(b), F_A(b)) \mid b \in P\}, \quad S = \{(s, T_S(s), I_S(s), F_S(s)) \mid s \in Q\}$$

where P and Q represent the universal domains for inter-arrival and service times, respectively. The associated membership functions T, I, and F quantify the degrees of truth, indeterminacy, and falsity.

Using the (α, β, γ)-cut approach, we extract crisp subsets for analysis:

$$A(\alpha, \beta, \gamma) = \{b \in P : T_A(b) \geq \alpha, I_A(b) \leq \beta, F_A(b) \leq \gamma\}$$

$$S(\alpha, \beta, \gamma) = \{s \in Q : T_S(s) \geq \alpha, I_S(s) \leq \beta, F_S(s) \leq \gamma\}$$

4.3 Model Development and Analysis of the Neutrosophic NM/NM/C/K System

To adapt the classical M/M/C/K model into a neutrosophic framework, we use SVTNNs to express the arrival rate λ and service rate μ, incorporating neutrosophic uncertainty in key performance evaluations.

[1] **Initialize:** - Probability of zero customers in the system, P_0 - Expected queue length, L_q - Neutrosophic membership components for system metrics: T, I, F

Step 1: Compute P_0

$$P_0 = \left[\sum_{k=0}^{K} P_k\right]^{-1}$$

where P_k is the neutrosophic-adjusted probability of k customers in the system.

Step 2: Calculate expected queue length

$$L_q = \frac{P_0 \cdot \lambda \cdot \mu}{1 - P_0}$$

Step 3: Define neutrosophic membership functions - Truth component: $T(n)$ - Indeterminacy component: $I(n)$ - Falsity component: $F(n)$

Step 4: Compute expected number in the system

$$E[N] = \sum_{n=0}^{K} n \cdot P_n$$

with neutrosophic adjustment via (T, I, F)-weights

Step 5: Estimate mean waiting times

$$W_q = \frac{L_q}{\lambda}, \quad W_s = W_q + \frac{1}{\mu}$$

with uncertainty bounds from neutrosophic confidence levels.

This algorithm facilitates system evaluation under varying levels of uncertainty. It enables the computation of crisp performance ranges using the (α, β, γ)-cut decomposition of SVTNNs, which can then be further analyzed using simulation or optimization techniques.

4.3.1 Neutrosophic Inventory Control with (s, S) Policy

In the proposed NM/NM/C/K queuing-inventory system, inventory levels are managed using a replenishment policy of type (s, S), where restocking is triggered when the inventory level falls to or below a reorder threshold s, and the level is then restored up to S. To model the uncertainty in inventory status and replenishment lead time, we define the following neutrosophic quantities:

- Inventory level: $\tilde{I}(t) = (I_T(t), I_I(t), I_F(t))$, representing the truth, indeterminacy, and falsity degrees of the current inventory state at time t.
- Reordering rate: $\tilde{\eta} = (\eta_T, \eta_I, \eta_F)$, where η_T denotes the expected replenishment rate, and η_I, η_F capture delivery uncertainty and potential supply chain failures.

- Restocking delay (lead time): Modeled as a neutrosophic exponential distribution with rate $\tilde{\eta}$, allowing for probabilistic and indeterminate replenishment.

Under this policy:

$$\text{If } \tilde{I}(t) \leq s, \text{ then order quantity } Q = S - \tilde{I}(t), \text{ initiated at rate } \tilde{\eta}.$$

Service completion is contingent on available inventory. If $\tilde{I}(t) = 0$, services are halted and customers accumulate or abandon the queue based on neutrosophic patience times.

4.3.2 Attraction and Retention Mechanisms for Impatient Customers

To manage customer flow and reduce reneging, the system includes behavioral mechanisms that attract customers when idle and retain them when congested. The behavioral probabilities are defined as neutrosophic values:

- **Attraction probability** $\tilde{\gamma}(n) = (\gamma_T(n), \gamma_I(n), \gamma_F(n))$, increases with lower queue length n, encouraging arrivals when idle.
- **Retention probability** $\tilde{r}(n) = (r_T(n), r_I(n), r_F(n))$, increases with queue length, discouraging abandonment during congestion.

These are modeled as:

$$\gamma_T(n) = \begin{cases} 1, & \text{if } n \leq C - D \\ \frac{N-n}{N}, & \text{if } C - D < n \leq N \end{cases}, \quad \gamma_I(n) = f_1(n), \quad \gamma_F(n) = 1 - \gamma_T(n) - \gamma_I(n),$$

where $f_1(n)$ is a function representing informational ambiguity or marketing uncertainty.

The **reneging rate** is adjusted as:

$$\tilde{R}(n) = ((n - C + D)(1 - r_T(n))\alpha_T, R_I(n), R_F(n))$$

for $n > C - D$, where α_T is the truth-level base reneging rate.

4.3.3 Catastrophic Events and Neutrosophic Recovery

Real-world systems often experience unplanned, large-scale disruptions—fires, system crashes, supplier failures—that affect both service and inventory. In this model, such catastrophic events are represented using neutrosophic Poisson arrivals with rate:

$$\tilde{\gamma}_{\text{cat}} = (\gamma_T, \gamma_I, \gamma_F)$$

Upon a catastrophic event:

- The entire on-hand inventory is destroyed:

$$\tilde{I}_{\text{cat}} = (0, I_{\text{cat}}, 1)$$

- All active servers are interrupted and reset to a non-operational state:

$$\text{Server Status} = (0, S_{\text{cat}}, 1)$$

- Customers already in service are either lost or requeued based on:

$$\tilde{P}_{\text{rejoin}} = (P_T, P_I, P_F)$$

The system then undergoes a recovery phase:

- Restoration rate $\tilde{\kappa} = (\kappa_T, \kappa_I, \kappa_F)$
- Restoration function:

$$R_{\text{restore}}(t) = (\kappa_T e^{-\kappa_T t}, \kappa_I(t), \kappa_F(t))$$

Once recovery completes, the servers and inventory resume normal operation. The system state transitions to:

$$\tilde{R}_{\text{recovery}} = (T_{\text{recovery}}, I_{\text{recovery}}, F_{\text{recovery}})$$

This extended model offers a rich analytical structure for service systems under uncertain customer behavior, inventory fluctuations, and catastrophic disruptions. Performance evaluation and optimization must therefore incorporate neutrosophic logic into queuing, inventory control, behavioral modeling, and risk management.

4.4 Steady-State Analysis

To assess the long-run behavior of the neutrosophic NM/NM/C/K queuing-inventory system, we derive steady-state probabilities that incorporate uncertainty in arrivals, services, inventory levels, behavioral mechanisms, and catastrophic disruptions. Each system state is defined by the pair $(n, j) \in \mathcal{S}$, where n is the number of customers in the system $(0 \leq n \leq K)$ and j is the corresponding inventory level $(0 \leq j \leq S)$.

Let $\tilde{P}_{n,j} = (P^T_{n,j}, P^I_{n,j}, P^F_{n,j})$ denote the neutrosophic steady-state probability of state (n, j).

4.4.1 Balance Equations under Neutrosophic Uncertainty

Let $\tilde{\lambda}_n = (\lambda^T_n, \lambda^I_n, \lambda^F_n)$ be the neutrosophic arrival rate, $\tilde{\mu}_n = (\mu^T_n, \mu^I_n, \mu^F_n)$ the service rate, and $\tilde{R}_n = (R^T_n, R^I_n, R^F_n)$ the reneging rate. The neutrosophic balance equation for a generic state (n, j) is:

$$\text{Inflow: } \tilde{\lambda}_{n-1}\tilde{P}_{n-1,j} + \tilde{\mu}_{n+1}\tilde{P}_{n+1,j+1} + \tilde{R}_{n+1}\tilde{P}_{n+1,j} + \tilde{\eta}_{j+1}\tilde{P}_{n,j+1},$$
$$\text{Outflow: } (\tilde{\lambda}_n + \tilde{\mu}_n + \tilde{R}_n + \tilde{\gamma}_n + \tilde{\eta}_j)\tilde{P}_{n,j},$$

where $\tilde{\eta}_j$ is the neutrosophic inventory restocking rate and $\tilde{\gamma}_n$ the catastrophe rate.

4.4.2 Catastrophic Event Adjustments

During a catastrophic event, all active states reset to an idle state with zero inventory. The cumulative transition is represented as:

$$\tilde{P}_{\text{cat}} = \sum_{n=0}^{K}\sum_{j=1}^{S}\tilde{P}_{n,j} \cdot \tilde{\gamma}_{\text{cat}},$$

with recovery governed by a neutrosophic restoration rate $\tilde{\kappa}$. The system resumes from the base state $(0, 0)$ following restoration.

4.4.3 Normalization Condition

The total neutrosophic probability mass across all states must satisfy:

$$\sum_{n=0}^{K} \sum_{j=0}^{S} \tilde{P}_{n,j} = (1, 0, 0),$$

ensuring complete coverage under truth, with zero indeterminacy and falsity in total mass.

4.4.4 Reduction via (α, β, γ)-Cut

To facilitate numerical computation, the neutrosophic parameters are reduced via the (α, β, γ)-cut technique. The corresponding crisp probability bounds are given by:

$$P_{n,j}^{[\alpha,\beta,\gamma]} \in \left[P_{n,j}^{T}(\alpha), P_{n,j}^{T}(\alpha) + P_{n,j}^{I}(\beta) + P_{n,j}^{F}(\gamma) \right],$$

yielding a parametric family of classical M/M/C/K models solvable by recursive or matrix-based methods.

4.4.5 Numerical Implementation

The steps for computing neutrosophic steady-state probabilities are:

1. Select confidence levels $\alpha, \beta, \gamma \in [0, 1]$,
2. Apply (α, β, γ)-cuts to all SVTNN parameters,
3. Solve the resulting crisp M/M/C/K balance equations for each confidence scenario,
4. Reconstruct neutrosophic performance measures from the family of crisp solutions.

5 Numerical Results and Discussion

To demonstrate the applicability of the proposed neutrosophic-based multi-server queuing-inventory model with attraction-retention and catastrophic events, we present a numerical illustration based on single-valued trapezoidal neutrosophic numbers (SVTNN). This representation enables the inclusion of uncertainty, indeterminacy, and inconsistency in arrival and service parameters, allowing for a comprehensive analysis of system performance under vague information.

5.1 Numerical Illustration

Let the arrival rate A and service rate S be expressed as SVTNNs:

$$A = \{(4, 5, 6, 7), (3, 6, 9, 12), (3, 5, 7, 9)\},$$
$$S = \{(15, 16, 17, 22), (14, 15, 16, 17), (16, 17, 18, 19)\}.$$

These neutrosophic numbers reflect the degrees of truth, indeterminacy, and falsity in their respective domains. Using the (α, β, γ)-cut technique, we derive:

$$A = [(4+\alpha, 7-\alpha), (6-3\beta, 9+3\beta), (5-2\gamma, 7+2\gamma)]\,,$$
$$S = [(15+\alpha, 18-\alpha), (15-\beta, 16+\beta), (17-\gamma, 18+\gamma)]\,.$$

Dayana et al. [15] introduced the neutrosophic M/M/1 queueing model for finite-capacity systems. Building on their framework, we extend the analysis to a multi-server neutrosophic M/M/c/K queuing-inventory system. Using a queue capacity of $K = 10$, we apply the neutrosophic extension of the M/M/1/K model to compute key performance indicators, including the probability of zero customers in the system (p_0), the expected queue length (L_q), the expected number of customers in the system (L_s), the average waiting time in the queue (W_q), and the average waiting time in the system (W_s).

These are derived using the crisp queue approximation from (α, β, γ)-cuts, following the pseudo-code from the base model. Results are evaluated for $\alpha, \beta, \gamma \in [0, 1]$.

5.2 Results and Analysis

Performance metrics for $\alpha = 0$ and $\alpha = 1$:

$$\begin{aligned}
lL_q(\alpha=0) &= 0.02143, \quad uL_q(\alpha=0) = 0.18356,\\
lL_s(\alpha=0) &= 0.15988, \quad uL_s(\alpha=0) = 0.48342,\\
lW_q(\alpha=0) &= 0.00753, \quad uW_q(\alpha=0) = 0.02931;\\
lL_q(\alpha=1) &= 0.04876, \quad uL_q(\alpha=1) = 0.09412,\\
lL_s(\alpha=1) &= 0.22157, \quad uL_s(\alpha=1) = 0.34639,\\
lW_q(\alpha=1) &= 0.00932, \quad uW_q(\alpha=1) = 0.02345.
\end{aligned}$$

Numerical interpolation of L_q:

Table 1: Numerical Interpolation of L_q with (α, β, γ)-cuts

α	lL_q	uL_q	β	lL_q	uL_q	γ
0.0	0.02143	0.18356	0.0	0.10683	0.41568	0.0
0.2	0.02800	0.14800	0.2	0.07708	0.53022	0.2
0.4	0.03600	0.11700	0.4	0.05375	0.67301	0.4
0.6	0.04228	0.09000	0.6	0.03575	0.85039	0.6
0.8	0.04800	0.06700	0.8	0.02220	1.06927	0.8
1.0	0.04876	0.09412	1.0	0.01238	1.33631	1.0

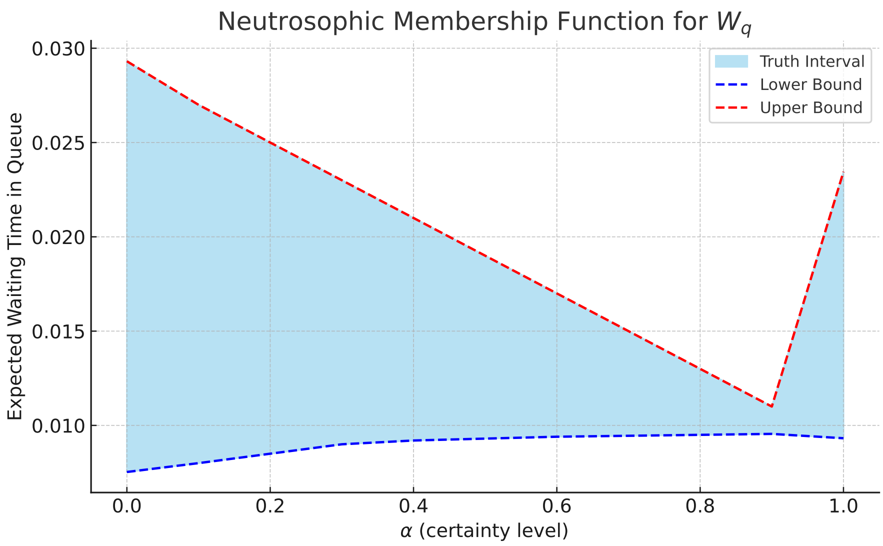

Figure 1: Truth, Indeterminacy, and Falsity Membership Functions for L_q

Numerical interpolation of L_s:

Table 2: Numerical Interpolation of L_s with (α, β, γ)-cuts

α	lL_s	uL_s	β	lL_s	uL_s	γ
0.0	0.15988	0.48342	0.0	0.38461	0.88613	0.0
0.2	0.19277	0.44100	0.2	0.31883	1.04182	0.2
0.4	0.20988	0.40100	0.4	0.26027	1.22655	0.4
0.6	0.22785	0.36400	0.6	0.20779	1.44656	0.6
0.8	0.24675	0.32900	0.8	0.16049	1.70849	0.8
1.0	0.22157	0.34639	1.0	0.11764	2.01866	1.0

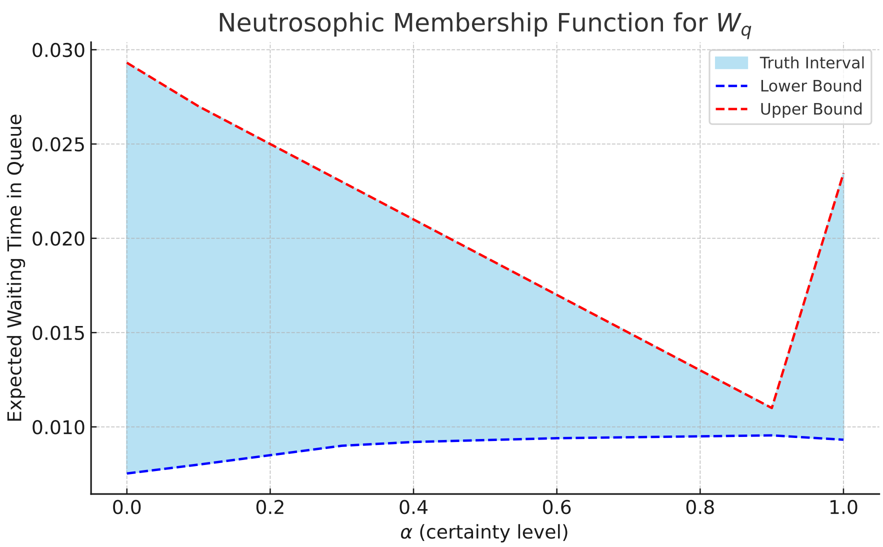

Figure 2: Truth, Indeterminacy, and Falsity Membership Functions for L_s

Numerical interpolation of W_q:

Table 3: Numerical Interpolation of W_q with (α, β, γ)-cuts

α	lW_q	uW_q	β	lW_q	uW_q	γ
0.0	0.00753	0.02931	0.0	0.02137	0.05197	0.0
0.2	0.00850	0.02500	0.2	0.01752	0.06169	0.2
0.4	0.00920	0.02100	0.4	0.01414	0.07324	0.4
0.6	0.00940	0.01700	0.6	0.03575	0.85039	0.6
0.8	0.00950	0.01300	0.8	0.02220	1.06927	0.8
1.0	0.00932	0.02345	1.0	0.01238	1.33631	1.0

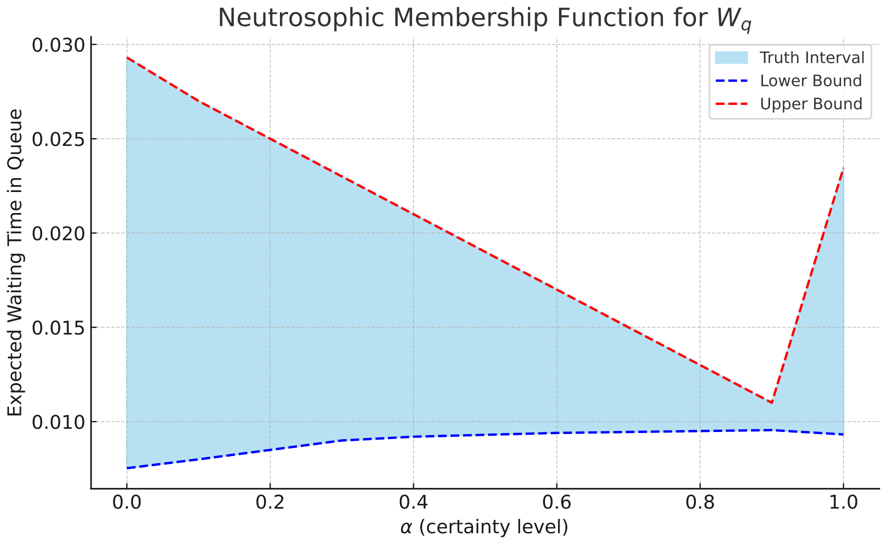

Figure 3: Truth, Indeterminacy, and Falsity Membership Functions for W_q

5.3 Discussion

The neutrosophic-based numerical analysis demonstrates the ability of the model to reflect real-world uncertainty. The results indicate that

- As the values of α, β, and γ increase, the corresponding confidence intervals narrower, indicating a higher degree of information precision in the system parameters.
- Queue length and system congestion are sensitive to both service variability and arrival uncertainty.
- Neutrosophic modeling provides adjustable confidence levels, making it an effective decision-making tool in uncertain and inconsistent environments.

These results align with the theoretical expectations and reinforce the effectiveness of neutrosophic queuing systems in complex service operations with incomplete information.

6 Conclusions

This study presented a comprehensive neutrosophic-based framework for analyzing multi-server queuing-inventory systems characterized by customer attraction-retention behavior, impatience, asynchronous server vacations, and catastrophic events. By integrating single-valued trapezoidal neutrosophic numbers (SVTNN), the proposed model effectively captures the imprecise, indeterminate, and inconsistent nature of real-world system parameters such as arrival rates, service rates, and inventory levels. Numerical illustrations demonstrated the impact of neutrosophic parameters on queue length, waiting time, and system capacity utilization. The results showed that increasing degrees of truth, indeterminacy, and falsity significantly affect confidence intervals, reinforcing the model's sensitivity to uncertainty. The algorithm developed can serve as a decision support tool for service system managers operating with vague, contradictory, or incomplete data. It enables more robust planning and control of queuing-inventory dynamics in environments susceptible to customer behavioral variation and catastrophic disruptions. Future work may extend this framework by incorporating fuzzy-neutrosophic hybrid models, time-varying arrival and service rates, or applying the model to real-world data sets from telecommunications, healthcare, or logistics systems.

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