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Automobile Evaluation Using an Extended PROMETHEE Method under Neutrosophic Supra Topological Framework

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Abstract. Neutrosophic sets are widely used due to their ability to handle uncertainties and incomplete information. Recently, researchers have integrated neutrosophic sets with supra topological spaces, leading to the development of a new theory known as neutrosophic supra topological space. The PROMETHEE method is a well-established approach for solving multi-criteria decision-making problems. Selecting the best car under uncertainty and multiple criteria is a significant challenge for a buyer. This decision-making problem motivated us to develop a novel PROMETHEE method with neutrosophic supra topological space to select the optimal car. This novel decision-making method incorporated the PROMETHEE method with neutrosophic supra topological space. Using the proposed method, a numerical example is provided to determine the car that outperforms other models in terms of fuel efficiency, emission standards, safety, overall cost, and future resale value. We also explore the sensitivity of the results to different parameter values within the model. Finally, we compare the proposed method with the existing methods.

Keywords: Car selection, PROMETHEE method, neutrosophic set, neutrosophic supra topological space,Decision making methods.

1. **Introduction**

The rapid urbanization and population growth in modern cities have led to a significant increase in the use of private vehicles. This surge in personal transportation contributes heavily to urban congestion and environmental degradation due to its dependence on fossil fuels, resulting in elevated carbon dioxide (CO_2) emissions in city centers. Conventional fossil-fuel-powered vehicles still dominate urban transportation, posing serious threats to sustainable development goals, particularly those aimed at mitigating climate change and reducing greenhouse gas (GHG) emissions. In response, electric vehicles (EVs) have emerged as a cleaner alternative, playing a vital role in the transition towards low-carbon and sustainable urban transportation systems. Despite their promise, EVs present notable limitations such as limited driving range, high initial cost, insufficient charging infrastructure, and longer charging times. As consumers become more environmentally conscious, selecting the optimal vehicle, be it electric, hybrid, or conventional, becomes more important. It requires the consideration of multiple conflicting factors, including price, performance, safety, energy efficiency, infrastructure availability, and environmental impact. Given the complexity and ambiguity involved in such decision scenarios, Multi-Criteria Decision-Making (MCDM) methods are indispensable tools for systematically evaluating alternatives based on diverse criteria. Traditional decision-making approaches often struggle with uncertainty and vagueness inherent in real-world problems. Fuzzy set theory (FS) [31], while helpful, can still fall short in capturing complex scenarios with incomplete information. To resolve this, the concept of intuitionistic fuzzy set (IFS) [5] is introduced by Atanassov. Neutrosophic sets (NS) [7], introduced by Florentin Smarandache, extend the capabilities of fuzzy sets by incorporating a third component, "indeterminacy", alongside truth and falsity values. This additional dimension allows for a more subtle representation of human judgment and subjective evaluations. The PROMETHEE (Preference Ranking Organization of Methods for Enrichment Evaluations) method, developed by Brans [3, 6], is a widely used MCDM technique. It focuses on pairwise comparisons between alternatives based on predefined preference functions, making it flexible and adaptable to various decision problems. PROMETHEE methods have been applied in numerous contexts. Nassereddine et al. [11] integrated a new PROMETHEE preference function and synergy criteria to evaluate emergency response systems, addressing the critical need for inter-agency collaboration in disaster management. Qi et al. [21] introduced a dynamic weighting approach based on preference expectations and ordered weighted averaging to accommodate interdependencies and prioritizations among criteria. Zhao et al. [20] introduced extended PROMETHEE methods utilizing 2-dimension linguistic term sets (2DLEs) to solve Multi-Attribute Decision-Making (MADM) problems, enhancing preference functions with a possibility degree for 2DLEs, effectively handling both comparable and incomparable evaluations. Yu et al. [19] introduced an enhanced failure mode and effects analysis model for submarine pipeline risk analysis, improving interval-valued intuitionistic fuzzy rough number theory for expert opinion collection and combining Exponential TODIM (an acronym in Portuguese for interactive and multicriteria decision-making) with PROMETHEE-II and analytical hierarchical process (AHP) for robust failure mode ranking and risk value calculation. Zhao et al. [16] presented a modified PROMETHEE II method that simplifies computation by integrating multiple steps, thereby reducing complexity and database interactions.

Recently, the integration of extended fuzzy set theory with topological concepts has emerged as a promising approach for addressing complex problems characterized by uncertainty and indeterminacy. Garg et al. [28] introduced the TOPSIS (Technique of Order Preference by Similarity to an Ideal Solution) method under a spherical fuzzy soft environment for solving decision-making problems [28]. While classical topology is a powerful tool, often it struggles in modeling situations involving vague, imprecise, or contradictory information. To address these limitations, the concept of neutrosophic topological space [27] was introduced, extending traditional topological notions to accommodate the inherent ambiguity present in real-world scenarios.

Building on this foundation, neutrosophic supra topological spaces (NSTPS) [22] have been developed and solved a decision-making problem on medical diagnosis. Also, neutrosophic support soft topological space is introduced, and a decision-making problem via neutrosophic support soft topological space [29] is solved. Supra topological spaces [26], by relaxing the constraints of traditional topologies, offer increased flexibility in modeling diverse structures. Some weak and strong forms of sets are introduced, and their properties in neutrosophic supra topological spaces [30] are studied.

This generalization is particularly valuable in applications where the data exhibits a high degree of uncertainty, such as in decision-making, pattern recognition, and information fusion. By leveraging the expressive power of neutrosophic sets within a supra topological framework, we can construct more robust and adaptable models that better reflect the complexities of real-world phenomena. This work explores multi-criteria decision-making models by incorporating the PROMETHEE method and neutrosophic supra topological spaces. This integrated approach emerges as a powerful tool for decision-making under uncertainty, aiming to demonstrate its potential in the context of automobile evaluation.

1.1. *Motivations*

The following points outline the motivations for proposing this novel model to solve multi-criteria decision-making problem:

- Neutrosophic sets allow for more effective modeling of imprecise, incomplete, and inconsistent data. Supra-topological structures provide a generalized flexible framework for comparing preferences more effectively.
- While PROMETHEE is widely used in practice, its theoretical foundation is limited when handling highly uncertain or indeterminate data. Neutrosophic supra topological spaces, however, offer a theoretical framework for managing such data. By combining the two, this approach bridges the gap between theoretical advancements and practical applications, resulting in a more comprehensive and adaptable decision-making tool.

- Despite the advancements in multi-criteria decision-making methodologies, the integration of neutrosophic supra topological spaces with the PROMETHEE method remains unexplored. This research addresses the gap by proposing a novel framework that leverages the strengths of both approaches to solve complex decision problems.

The novelty of the proposed work is in extending the traditional PROMETHEE method by incorporating NSTPS, which is an innovative approach to modeling uncertainty and vagueness in decision-making. This is the first attempt to integrate neutrosophic supra topology into the PROMETHEE method. The study is employed to select the best car, emphasizing GHG emissions, fuel efficiency, safety, cost, and resale value. It aligns with the EU Transport White Paper goals of reducing GHG emissions by 60% by 2050, making it highly relevant for policymakers and consumers. The work provides a structured way to help car buyers prioritize sustainability and economic feasibility in their decision-making process.

2. **Related Studies**

Multi-criteria decision-making often involves complex situations where choices must be made based on conflicting or incommensurable criteria. Traditional methods struggle to handle the inherent vagueness and uncertainty of such situations. Mahmood et al. [15] investigated the applicability of the bipolar complex fuzzy rough set in cyber security. Fuzzy and intuitionistic fuzzy approaches offer elegant solutions to address these challenges by incorporating the subjective preferences and hesitation of decision-makers [12, 13, 17]. Hamurcu and Eren [14] proposed a hybrid multicriteria decision-making approach, combining AHP, TOPSIS, and goal programming to evaluate conflicting factors and determine the optimal electric vehicle choice. Ali et al. [22] proposed a technique that combines full Consistency for weight calculation and fuzzy TOPSIS for ranking, demonstrating enhanced accuracy and consistency compared to traditional methods. It offers a versatile tool for diverse alternative selection scenarios. Chand et al. [23] employed a Fuzzy AHP approach to rank sedan cars based on criteria such as performance, economy, and comfort, aiming to simplify the selection process by providing consumers with a clear understanding of their preferences.

Originally developed by Brans et al. [6] PROMETHEE ranks alternatives based on pairwise comparisons using preference functions and indifference thresholds. Fuzzy logic is integrated into PROMETHEE by representing preferences and thresholds as fuzzy sets, allowing for more flexible and nuanced decision-making. The advantages of Fuzzy PROMETHEE include ease of use, the ability to handle imprecise information, and consideration of both positive and negative outranking flows.

An extension of Fuzzy PROMETHEE utilizes IFS. IFS captures both the degree of membership and non-membership in a set, effectively representing hesitation or uncertainty in subjective judgments. The additional degree of non-membership (hesitation) in IFS provides richer information compared to Fuzzy PROMETHEE, leading to more comprehensive and accurate decision-making. Advantages include the ability to deal with ambiguity and conflicting information, visualize positive and negative outranking flows separately, and incorporate the importance of criteria using IFS weights. Applications encompass sustainable building material selection ([10]). Xu et al. [1] proposed an integrated method combining PROMETHEE and TODIM in a neutrosophic environment. This method introduces a new formula for ranking alternatives, highlighting the potential of combining decision-making methods under the neutrosophic framework. Xu et al. [2] introduce the concept of probabilistic simplified neutrosophic sets (PSNS) and develop a PROMETHEE-based decision-making approach for group decision problems. PSNS incorporates probabilistic elements into neutrosophic evaluations, further enhancing the method's ability to handle uncertainty. Xu et al. [4] propose an improved PROMETHEE method using multi-valued neutrosophic sets, expanding upon the traditional single-valued approach. This allows for richer information representation and potentially more accurate decision-making in complex scenarios.

2.1. *Research Questions*

In light of the challenges posed by uncertainty and indeterminacy in MCDM, this study addresses the following research questions:

- How can the PROMETHEE method be effectively integrated with neutrosophic supra-topological spaces to handle uncertainty and imprecision in decision-making?
- Can expert opinions or decision matrices be compared using existing models?
- What are the consequences of applying aggregation operators to neutrosophic sets that include null or absolute neutrosophic values?
- Does any existing research apply the PROMETHEE method within the framework of supra topological spaces?

Existing studies using the PROMETHEE method and other decision-making techniques lack a mechanism for comparing expert opinions or decision matrices directly. Furthermore, when aggregation operators are applied to neutrosophic sets containing null or absolute values, they often produce trivial results. Additionally, no prior research has integrated the PROMETHEE method with neutrosophic supra topological spaces for enhanced decision-making.

To address this research gap and respond to the questions outlined above, we propose a novel decision-making framework that integrates the PROMETHEE method with neutrosophic supra topological structures. This approach allows for a robust comparison of expert opinions and effectively manages both null and absolute neutrosophic values, thereby improving the reliability and depth of the decision-making process.

2.2. *Objectives and Contributions of the Study*

The objective and major contributions of this article are presented below:

- The primary objective of this study is to integrate the PROMETHEE method with NSTPS to address MCDM problems characterized by uncertainty, indeterminacy, and imprecise information.
- Additionally, we seek to demonstrate the practical applicability of the proposed model through a numerical example, testing its performance across various parameter values to ensure reliability and consistency.
- We proposed a hybrid decision-making framework that combines the PROMETHEE method with NSTPSs, enhancing its ability to manage uncertainty and vagueness in decision criteria.
- The viability and applicability of the proposed approach are demonstrated through a numerical application in the context of automobile evaluation, showcasing its effectiveness in real-world scenarios.
- A detailed sensitivity analysis is conducted to assess the impact of varying parameters on the results, ensuring the consistency and robustness of the proposed model.

By bridging the gap between theoretical advancements and practical applications, this study makes a significant contribution to the field of MCDM, offering a powerful tool for decision-making in uncertain and complex environments. The key contributions in this article include a new decision-making model for finding the best car based on multiple criteria. The main difference between the proposed model and other existing PROMETHEE methods under fuzzy and its extension sets is that the proposed model compares the expert's opinion or decision matrix and provides non-trivial results if the null and absolute neutrosophic values are present in the expert's opinion or decision matrix. The subsequent sections of this paper are organized as follows:

- The "Basic definitions" section provides a comprehensive review of fundamental concepts, including the definitions of fuzzy sets, intuitionistic fuzzy sets, and neutrosophic sets, along with their respective operations. Additionally, it introduces the concept of neutrosophic supra-topological spaces and discusses score and accuracy functions, which are essential for understanding the proposed framework.
- The "Decision-Making: PROMETHEE Method" section presents a novel hybrid decision-making model that integrates the PROMETHEE method with neutrosophic supra-topological spaces. The proposed framework is designed to address multi-criteria decision-making problems under conditions of uncertainty and indeterminacy. A detailed flowchart is presented to illustrate the step-by-step implementation of the model, ensuring clarity and ease of application for practitioners.

- The ”Numerical Example” section demonstrates the practical applicability of the proposed model; this section provides a detailed solution to an automobile evaluation problem. The example highlights the effectiveness of the hybrid approach in handling real-world decision-making scenarios. Furthermore, a sensitivity analysis is conducted to evaluate the impact of varying parameters on the results, ensuring the robustness and consistency of the proposed model.
- The ”Conclusions” section summarizes the key contributions of the study, outlines the potential directions for future research, and discusses the limitations of the proposed model.

3. Basic Definitions

This section presents essential definitions and operations related to our study.

Definition 3.1. [31] A fuzzy set $\mathbb{M}$ in a universe of discourse X is defined as a set of ordered pairs:

$$\mathbb{M} = \{(x, \mathfrak{m}_{\mathbb{M}}(x))|x \in X\}$$

where the membership function $\mathfrak{m}_{\mathbb{M}} : X \to [0, 1]$ for each element x in X.

Definition 3.2. [5] An intuitionistic fuzzy set $\mathbb{M}$ in X is defined as

$$\mathbb{M} = \{\langle x, \mathfrak{m}_{\mathbb{M}}(x), \mathfrak{n}_{\mathbb{M}}(x)\rangle|x \in X\},$$

where the degree of membership and non-membership function respectively denoted as

$$\mathfrak{m}_{\mathbb{M}} : X \to [0, 1]$$

and

$$\mathfrak{n}_{\mathbb{M}} : X \to [0, 1]$$

for each x in X, and $0 \leq \mathfrak{m}_{\mathbb{M}}(x) + \mathfrak{n}_{\mathbb{M}}(x) \leq 1$.

Definition 3.3. [7] A neutrosophic set $\mathbb{M}$ in the universe of discourse X is of the form

$$\mathbb{M} = \{\langle \alpha, \mathfrak{m}_{\mathbb{M}}(\alpha), \mathfrak{e}_{\mathbb{M}}(\alpha), \mathfrak{n}_{\mathbb{M}}(\alpha)\rangle : \alpha \in X\}$$

and $\mathfrak{m}_{\mathbb{M}}(\alpha), \mathfrak{e}_{\mathbb{M}}(\alpha), \mathfrak{n}_{\mathbb{M}}(\alpha)$ are standard or non-standard subsets of $]0, 1[$
where $\mathfrak{m}_{\mathbb{M}}(\alpha), \mathfrak{e}_{\mathbb{M}}(\alpha), \mathfrak{n}_{\mathbb{M}}(\alpha)$ represents the degree of favorable, degree of indeterminacy and the degree of non-favorable function provided there is no restriction in the addition of $\mathfrak{m}_{\mathbb{M}}(\alpha), \mathfrak{e}_{\mathbb{M}}(\alpha)$ *and* $\mathfrak{n}_{\mathbb{M}}(\alpha)$, So $^{-}0 \leq sup\mathfrak{m}_{\mathbb{M}}(\alpha) + sup\mathfrak{e}_{\mathbb{M}}(\alpha) + sup\mathfrak{n}_{\mathbb{M}}(\alpha) \leq 3^{+}$

Definition 3.4. [7] Let $\mathbb{L}$ and $\mathbb{M}$ be two neutrosophic sets of the form
$\mathbb{L} = \{\langle \alpha, \mathfrak{m}_{\mathbb{L}}(\alpha), \mathfrak{e}_{\mathbb{L}}(\alpha), \mathfrak{n}_{\mathbb{L}}(\alpha)\rangle : \alpha \in X\}$, $\mathbb{M} = \{\langle \alpha, \mathfrak{m}_{\mathbb{M}}(\alpha), \mathfrak{e}_{\mathbb{M}}(\alpha), \mathfrak{n}_{\mathbb{M}}(\alpha)\rangle : \alpha \in X\}$. Then

(a) A neutrosophic set $\mathbb{L}$ is said to be a subset of another neutrosophic set $\mathbb{M}$, denoted by $\mathbb{L} \subseteq \mathbb{M}$ if for all $\alpha in X$: $\mathfrak{m}_{\mathbb{L}}(\alpha) \leq \mathfrak{m}_{\mathbb{M}}(\alpha)$, $\mathfrak{e}_{\mathbb{L}}(\alpha) \geq \mathfrak{e}_{\mathbb{M}}(\alpha)$ and $\mathfrak{n}_{\mathbb{L}}(\alpha) \geq \mathfrak{n}_{\mathbb{M}}(\alpha)$ for all $\alpha \in X$.

(b) A neutrosophic set $\mathbb{L}$ is said to be equal to another neutrosophic set $\mathbb{M}$, denoted by $\mathbb{L} = \mathbb{M}$ if for all $\alpha in X$: $\mathfrak{m}_{\mathbb{L}}(\alpha) = \mathfrak{m}_{\mathbb{M}}(\alpha)$, $\mathfrak{e}_{\mathbb{L}}(\alpha) = \mathfrak{e}_{\mathbb{M}}(\alpha)$ and $\mathfrak{n}_{\mathbb{L}}(\alpha) = \mathfrak{n}_{\mathbb{M}}(\alpha)$.

(c) Complement of neutrosophic set $\mathbb{L}$, denoted and defined as $\mathbb{L}^C = \{\langle \alpha, \mathfrak{n}_{\mathbb{L}}(\alpha), 1 - \mathfrak{e}_{\mathbb{L}}(\alpha), \mathfrak{m}_{\mathbb{L}}(\alpha)\rangle : \alpha \in X\}$;

(d) The intersection of two neutrosophic sets denoted and defined as $\mathbb{M} \cap \mathbb{N} = \{\langle \alpha, \mathfrak{m}_{\mathbb{L}}(\alpha) \wedge \mathfrak{m}_{\mathbb{M}}(\alpha), \mathfrak{e}_{\mathbb{L}}(\alpha) \vee \mathfrak{e}_{\mathbb{M}}(\alpha), \mathfrak{n}_{\mathbb{L}}(\alpha) \vee \mathfrak{n}_{\mathbb{M}}(\alpha)\rangle : \alpha \in X\}$;

(e) The union of two neutrosophic sets denoted and defined as $\mathbb{M} \cup \mathbb{N} = \{\langle \alpha, \mathfrak{m}_{\mathbb{L}}(\alpha) \vee \mathfrak{m}_{\mathbb{M}}(\alpha), \mathfrak{e}_{\mathbb{L}}(\alpha) \wedge \mathfrak{e}_{\mathbb{M}}(\alpha), \mathfrak{n}_{\mathbb{L}}(\alpha) \wedge \mathfrak{n}_{\mathbb{M}}(\alpha)\rangle : \alpha \in X\}$;

(f) For a scalar $\omega \in [0, 1]$, the scalar multiplication of a neutrosophic set A, denoted and defined as: $\omega\mathbb{L} = \{\langle \alpha, \omega\mathfrak{m}_{\mathbb{L}}(\alpha), \omega\mathfrak{e}_{\mathbb{L}}(\alpha), \omega\mathfrak{n}_{\mathbb{L}}(\alpha)\rangle : \alpha \in X\}$

Definition 3.5. [7] Let X be a universal set.

(i). A NS is called an absolute NS over X and it is denoted by 1^X, if $\forall a \in X, \mathfrak{m}_{1^{\mathbb{X}}}(a) = 1, \mathfrak{e}_{1^{\mathbb{X}}}(a) = 0, \mathfrak{n}_{1^{\mathbb{X}}}(a) = 0$.

(ii). A NS is called an null NS over X and it is denoted by $1^{\emptyset}$, if $\forall a \in X, \mathfrak{m}_{1^{\emptyset}}(a) = 0, \mathfrak{e}_{1^{\emptyset}}(a) = 1, \mathfrak{n}_{1^{\emptyset}}(a) = 1$.

Definition 3.6. [26] Let X be a non-empty universal set and τ be a collection of subsets of X. The pair (X, τ) is called a supra topological space, if

(i). Empty set and the entire set is in τ.

(ii). The union of any collection of supra open set is also a supra open set. i.e.,$\{U_i | i \in I\} \subseteq \tau$, then $\bigcup_{i\in I} U_i \in \tau$.

Each element in the collection τ is called an open set. Here I is the index set.

Definition 3.7. [22] Let X be a non-empty universal set and τ be a collection of neutrosophic sets of X. The pair (X, τ) is called a neutrosophic supra topological space, if it satisfies the following axioms:

(i). The absolute and null neutrosophic set belong to τ.

(ii). The union of any collection of neutrosophic sets in τ is also in τ. i.e., $\{U_i | i \in I\} \subseteq \tau$, then $\bigcup_{i\in I} U_i \in \tau$.

Each neutrosophic set in τ is called a neutrosophic open set and I is the index set.

Definition 3.8. [23] Let $\mathbb{L}$ be a single-valued NS and the score function of $\mathbb{L}$ is denoted and defined by $S_{\mathbb{L}} = \frac{2+\mathfrak{m}_{\mathbb{L}}-\mathfrak{e}_{\mathbb{L}}-\mathfrak{n}_{\mathbb{L}}}{3}$.

Definition 3.9. [23] Let $\mathbb{L}$ be a single-valued NS and the accuracy function of $\mathbb{L}$ is denoted and defined by $E_{\mathbb{L}} = \mathfrak{m}_{\mathbb{L}} - \mathfrak{n}_{\mathbb{L}}$.

Definition 3.10. [23] Let $\mathbb{L}$ and $\mathbb{M}$ be single-valued NSs and the score function of $\mathbb{L}$ is less than the score function of $\mathbb{M}$ if $\mathbb{L}$ is less than $\mathbb{M}$. If the score function of both single-valued NSs is equal then we consider the following constraints :

i. if $E_{\mathbb{L}} < E_{\mathbb{M}}$ then $\mathbb{L}$ is less than $\mathbb{M}$.
ii. if $E_{\mathbb{L}} = E_{\mathbb{M}}$ then $\mathbb{L}$ is equal to $\mathbb{M}$.

4. **Decision Making: Extended PROMETHEE Method**

The procedure for the extended PROMETHEE method and flowchart (refer to figure 1) are given below:

Let the set of alternatives be $\mathcal{R} = \{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_3 ....\mathcal{R}_n\}$ and the set of criteria be $\mathbb{A} = \{\mathbb{A}_1, \mathbb{A}_2, ...\mathbb{A}_m\}$. Let $\mathcal{G} = \{\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3 ....\mathcal{G}_k\}$ denote the set of decision makers.

Step 1: Create the weight parametric matrix.

Decision makers created the weighted parametric matrix $\mathbb{G}_w$, whose entries are the values of each criterion assigned by each decision maker considering linguistic terms as shown in table 1.

Linguistic Terms	Weights
Extremely important	0.9
Very strongly important	0.8
very important	0.6
important	0.4
slightly important	0.2

Table 1. Linguistic terms to determine the alternatives

$$G_w = \begin{bmatrix} \rho_{11} & \rho_{12} & .... & \rho_{1n} \\ \rho_{21} & \rho_{22} & .... & \rho_{2n} \\ .. & .. & .. & .. \\ .. & .. & .. & .. \\ \rho_{m1} & \rho_{m2} & .... & \rho_{mn} \end{bmatrix}$$

Step 2: Normalize the weighted parametric matrix.

Since each attribute does not necessarily have the same weight, we need to normalize the weight of each criterion.

$$N = \begin{bmatrix} \mathfrak{a}_{11} & \mathfrak{a}_{12} & .... & \mathfrak{a}_{1n} \\ \mathfrak{a}_{21} & \mathfrak{a}_{22} & .... & \mathfrak{a}_{2n} \\ .. & .. & .. & .. \\ .. & .. & .. & .. \\ \mathfrak{a}_{m1} & \mathfrak{a}_{m2} & .... & \mathfrak{a}_{mn} \end{bmatrix}$$

where

$$\mathfrak{a}_{ij} = \frac{\rho_{ij}}{\sqrt{\sum_{i=1}^{m} \rho_{ij}^2}} \tag{1}$$

Step 3: Calculate the weight vector.

The weight vector is calculated from step 2 by the following equation

$$\mathbb{W}_i = \frac{\sum_{i=1}^{m} \mathfrak{a}_{ij}}{m} \text{ and } V_{w_i} = \frac{\mathbb{W}_i}{\sum_{i=1}^{n} \mathbb{W}_i}$$

provided the sum of weight V_{w_i} equal to unity.

Step 4: Construct neutrosophic supra topology.

Each decision maker's report is based on the criteria for each alternative. Such reports are given in matrix form, where entries are the neutrosophic values. Let $D_1, D_2 ...., D_k$ denote the decision matrix. Construct the neutrosophic supra topology by combining NSs, given each decision-maker.

Step 5: Aggregation of decision matrix.

The decision matrix is aggregated by taking the average for each alternative for each criterion.

$$D_{agg} = \frac{D_1 + D_2 + ... + D_k}{k}$$

Step 6: Normalize the decision matrix.

Convert the aggregated decision matrix into a normalized decision matrix by taking the complement of the cost factor and keeping the remaining unchanged.

Step 7: Construct the preference function.

Construct the preference function $P_j(B_i, B_r)$ of scheme B_i relative to B_r under the criteria G_j by the following formula:

$$P_j(B_i, B_r) = \begin{cases} 0 & d \leq p \\ \frac{d-p}{q-p} & p \leq d \leq q \\ 1 & d \geq q \end{cases} \tag{2}$$

The range of the preference function is from 0 to 1. If $P_j(B_i, B_r) = 0$, then there is no difference between B_i and B_r. If $P_j(B_i, B_r)$ is nearly zero, then the difference between B_i and B_r is relatively small. Suppose $P_j(B_i, B_r)$ is nearly 1; then B_i is possibly better than B_r. If $P_j(B_i, B_r) = 1$, then B_i is strongly better than B_r. d is the priority function parameter and the difference between the criterion value of B_i and B_r.

Step 8: The priority index of the scheme B_i relative to B_r is denoted and defined by

$$\pi(B_i, B_r) = \sum_{j=1}^{n} V_{w_j} P_j(B_i, B_r) \tag{3}$$

Step 9: Compute the inflow, outflow, and net flow.

Within the context of scheme evaluation, the concept of "flow" is used to measure how a particular scheme compares to others. Inflow, denoted by ϕ_i^+, represents the degree to which scheme B_i surpasses other schemes in terms of performance or other relevant criteria. Conversely, outflow, denoted by ϕ_i^-, reflects the extent to which other schemes outperform scheme B_i. The net flow, calculated as the difference between inflow and outflow, provides an in-depth look at scheme B_i's relative position compared to its peers. It is denoted by ϕ_i.

$$\phi_i^+ = \frac{1}{n-1} \sum_{r=1}^{n} \pi(B_i, B_r) \tag{4}$$

$$\phi_i^- = \frac{1}{n-1} \sum_{r=1}^{n} \pi(B_r, B_i) \tag{5}$$

Step 10: Rank the alternatives.

Arrange the net flow values in ascending order. The greatest ϕ_i is the best alternative.

The following numerical example demonstrates how the extended PROMETHEE method with a neutrosophic supra topological environment can be employed in a real-world decision-making scenario. The step-by-step computational process, including the construction of decision matrices, calculation of criteria weights, and final ranking of alternatives, is presented to illustrate the practical applicability and robustness of the proposed model.

4.1. *Numerical Example*

Transport greenhouse gas (GHG) emissions increased by 26% from 1990 to 2016, despite improvements in vehicle efficiency. Emissions continue to rise due to economic growth and increased transportation usage. Road transport accounts for 72% of total transport GHG emissions. Car ownership rates have grown significantly, which has led to larger car fleets and higher emissions. Many countries aim to reduce transport GHG emissions by 60% by 2050 compared to 1990 levels. Its primary focus is on increasing transport system efficiency, promoting low-emission alternative energy, and transitioning to zero-emission vehicles. To support and reduce GHG emissions, car buyers have to consider this criterion as a primary one. Imagine a customer facing a crucial decision in selecting a car that runs on petrol, diesel, electricity, or CNG (Compressed Natural Gas). Their primary focus lies in five key factors that will shape their choice: fuel efficiency, emissions standards, safety, overall cost, and future resale value.

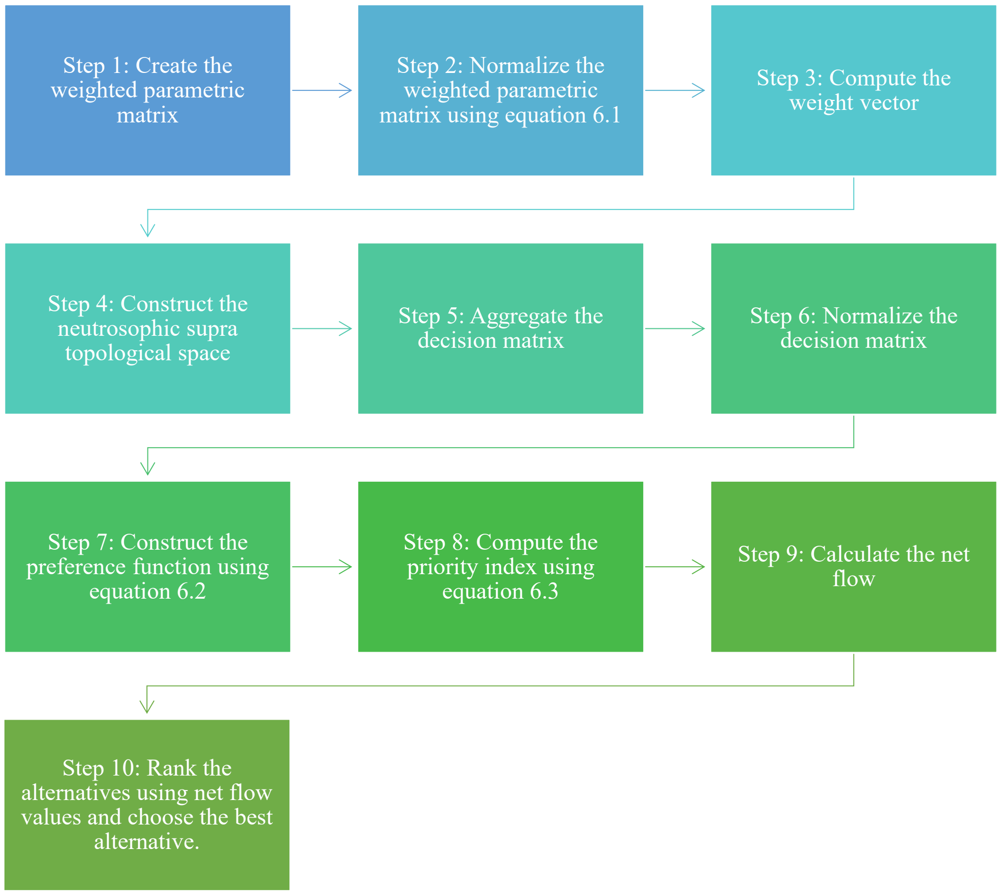

Figure 1. Flowchart of PROMETHEE method

1. Fuel efficiency: Cars with higher fuel efficiency contribute less to GHG emissions.
2. Emissions Standards: Compliance with EU CO2 emission standards ensures lower environmental impact.
3. Safety: Speaking about car safety, it includes airbags, anti-lock braking systems, traction control, and electronic stability control.
4. Overall cost: The cost of a car includes purchase cost, maintenance, and insurance.
5. Future resale value: Resale value is important in the future, if we sell the car.

Therefore, the customer investigated the various automobile engineers and got their opinion. These engineers are the decision-makers for our problem.

Let $\mathcal{R} = \{\mathcal{R}_1 = Petrol\ car, \mathcal{R}_2 = diesel\ car, \mathcal{R}_3 = electric\ car, \mathcal{R}_4 = CNG\ car\}$ be the alternatives, $\mathbb{A} = \{\mathbb{A}_1 = Fuel\ efficiency, \mathbb{A}_2 = Emissions\ standards, \mathbb{A}_3 = safety, \mathbb{A}_4 = Over\ all\ cost, \mathbb{A}_5 = future\ resale\ value\}$ be the criteria. Let $\mathcal{G} = \{\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3, \mathcal{G}_4, \mathcal{G}_5\}$ be the set of decision makers.

Step 1: Decision makers assign weights to each criterion, and the weighted parametric matrix G_w is given below to determine the weight of the criteria.

$$G_w = \begin{array}{c} \\ \mathcal{G}_1 \\ \mathcal{G}_2 \\ \mathcal{G}_3 \\ \mathcal{G}_4 \\ \mathcal{G}_5 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbb{A}_1 & \mathbb{A}_2 & \mathbb{A}_3 & \mathbb{A}_4 & \mathbb{A}_5 \end{array} \\ \begin{pmatrix} 0.9 & 0.6 & 0.9 & 0.8 & 0.4 \\ 0.8 & 0.6 & 0.8 & 0.9 & 0.6 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.6 \\ 0.9 & 0.6 & 0.9 & 0.9 & 0.8 \\ 0.8 & 0.6 & 0.8 & 0.9 & 0.6 \end{pmatrix} \end{array}$$

Step 2: Normalized weighted parametric matrix N is calculated by the equation (6.1).

$$N = \begin{bmatrix} 0.51 & 0.55 & 0.48 & 0.52 & 0.46 \\ 0.46 & 0.37 & 0.43 & 0.39 & 0.23 \\ 0.46 & 0.37 & 0.43 & 0.52 & 0.69 \\ 0.34 & 0.55 & 0.48 & 0.39 & 0.23 \\ 0.46 & 0.37 & 0.43 & 0.39 & 0.46 \end{bmatrix}$$

Step 3: The weight vector V_w is $V_w = \{0.203, 0.201, 0.204, 0.203, 0.189\}$

Step 4: NSTS is constructed by arranging the decision maker's reports. Let $D_1, D_2....D_5$ denote the decision matrix of the decision-maker's report.

$$D_1 = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbb{A}_1 & \mathbb{A}_2 & \mathbb{A}_3 & \mathbb{A}_4 & \mathbb{A}_5 \end{array} \\ \begin{pmatrix} (0.6, 0.4, 0.2) & (0.6, 0.3, 0.4) & (0.7, 0.2, 0.3) & (0.8, 0.2, 0.2) & (0.7, 0.2, 0.3) \\ (0.7, 0.2, 0.3) & (0.5, 0.4, 0.5) & (0.6, 0.3, 0.4) & (0.7, 0.2, 0.3) & (0.6, 0.3, 0.4) \\ (0.9, 0.1, 0.1) & (0.8, 0.2, 0.2) & (0.9, 0.1, 0.1) & (0.5, 0.4, 0.5) & (0.8, 0.3, 0.2) \\ (0.7, 0.2, 0.3) & (0.7, 0.2, 0.3) & (0.6, 0.3, 0.4) & (0.6, 0.3, 0.4) & (0.6, 0.3, 0.4) \end{pmatrix} \end{array}$$

$$D_2 = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbb{A}_1 & \mathbb{A}_2 & \mathbb{A}_3 & \mathbb{A}_4 & \mathbb{A}_5 \end{array} \\ \begin{pmatrix} (0.6,0.3,0.2) & (0.7,0.2,0.3) & (0.8,0.2,0.2) & (0.7,0.2,0.3) & (0.7,0.3,0.3) \\ (0.7,0.2,0.3) & (0.6,0.3,0.4) & (0.7,0.2,0.3) & (0.6,0.4,0.4) & (0.6,0.4,0.4) \\ (0.9,0.2,0.1) & (0.9,0.2,0.1) & (0.9,0.1,0.1) & (0.5,0.3,0.5) & (0.8,0.3,0.2) \\ (0.8,0.2,0.2) & (0.8,0.3,0.2) & (0.6,0.4,0.4) & (0.6,0.4,0.4) & (0.6,0.3,0.4) \end{pmatrix} \end{array}$$

$$D_3 = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbb{A}_1 & \mathbb{A}_2 & \mathbb{A}_3 & \mathbb{A}_4 & \mathbb{A}_5 \end{array} \\ \begin{pmatrix} (0.6,0.3,0.2) & (0.7,0.2,0.3) & (0.8,0.2,0.2) & (0.8,0.2,0.2) & (0.7,0.2,0.3) \\ (0.7,0.2,0.2) & (0.6,0.3,0.4) & (0.7,0.2,0.3) & (0.7,0.2,0.3) & (0.6,0.3,0.4) \\ (0.9,0.1,0.1) & (0.9,0.2,0.1) & (0.9,0.1,0.1) & (0.5,0.3,0.5) & (0.8,0.3,0.2) \\ (0.8,0.2,0.2) & (0.8,0.2,0.2) & (0.6,0.3,0.4) & (0.6,0.3,0.4) & (0.6,0.3,0.4) \end{pmatrix} \end{array}$$

$$D_4 = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbb{A}_1 & \mathbb{A}_2 & \mathbb{A}_3 & \mathbb{A}_4 & \mathbb{A}_5 \end{array} \\ \begin{pmatrix} (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) \\ (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) \\ (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) \\ (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) & (0.0,1.0,1.0) \end{pmatrix} \end{array}$$

$$D_5 = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbb{A}_1 & \mathbb{A}_2 & \mathbb{A}_3 & \mathbb{A}_4 & \mathbb{A}_5 \end{array} \\ \begin{pmatrix} (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) \\ (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) \\ (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) \\ (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) & (1.0,0.0,0.0) \end{pmatrix} \end{array}$$

The collection of decision matrix $\{D_1, D_2, D_3, D_4, D_5\}$ generates the NSTS.

Step 5: The collection of the neutrosophic matrix is aggregated by taking the average of each alternative with respect to the criteria. The aggregated neutrosophic decision matrix D_{agg} is shown below:

$$D_{agg} = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbb{A}_1 & \mathbb{A}_2 & \mathbb{A}_3 & \mathbb{A}_4 & \mathbb{A}_5 \end{array} \\ \begin{pmatrix} (0.56,0.4,0.32) & (0.6,0.34,0.4) & (0.66,0.32,0.34) & (0.66,0.32,0.34,) & (0.62,0.34,0.38) \\ (0.62,0.34,0.34) & (0.54,0.4,0.46) & (0.6,0.34,0.4) & (0.6,0.36,0.4) & (0.56,0.4,0.44) \\ (0.74,0.28,0.26) & (0.72,0.32,0.28) & (0.74,0.26,0.26) & (0.5,0.4,0.5) & (0.68,0.38,0.32) \\ (0.66,0.32,0.34) & (0.66,0.34,0.34) & (0.56,0.4,0.44) & (0.56,0.4,0.44) & (0.56,0.38,0.44) \end{pmatrix} \end{array}$$

The aggregated decision matrix is further normalized depending on the cost and beneficial criteria.

Step 6: The normalized decision matrix is denoted by N_D. It is calculated by taking the complement of the cost criteria and keeping the beneficial criteria unchanged. The normalized neutrosophic decision matrix N_D is presented below:

$$N_D = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \mathbb{A}_1 & \mathbb{A}_2 & \mathbb{A}_3 & \mathbb{A}_4 & \mathbb{A}_5 \end{array} \\ \begin{pmatrix} (0.32, 0.6, 0.56) & (0.6, 0.34, 0.4) & (0.34, 0.68, 0.66) & (0.66, 0.32, 0.34,) & (0.62, 0.34, 0.38) \\ (0.34, 0.66, 0.62) & (0.54, 0.4, 0.46) & (0.4, 0.66, 0.6) & (0.6, 0.36, 0.4) & (0.56, 0.4, 0.44) \\ (0.26, 0.72, 0.74) & (0.72, 0.32, 0.28) & (0.26, 0.74, 0.74) & (0.5, 0.4, 0.5) & (0.68, 0.38, 0.32) \\ (0.34, 0.68, 0.66) & (0.66, 0.34, 0.34) & (0.44, 0.6, 0.56) & (0.56, 0.4, 0.44) & (0.56, 0.38, 0.44) \end{pmatrix} \end{array}$$

Step 7: Here we set p = 0 and q = 1. The preference functions P_1, P_2, P_3, P_4 and P_5 are calculated for the four alternatives from the normalized neutrosophic decision matrix using the equation (6.2) and the preference functions P_1, P_2, P_3, P_4, and P_5 are shown below:

$$P_1 = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{R}_1 & \mathcal{R}_2 & \mathcal{R}_3 & \mathcal{R}_4 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0.006772537 & 0 & 0 & 0 \\ 0.024381134 & 0.024381134 & 0 & 0.013545074 \\ 0.010836059 & 0.004063522 & 0 & 0 \end{pmatrix} \end{array}$$

$$P_2 = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{R}_1 & \mathcal{R}_2 & \mathcal{R}_3 & \mathcal{R}_4 \end{array} \\ \begin{pmatrix} 0 & 0.012038133 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.017388415 & 0.029426548 & 0 & 0.009362993 \\ 0.008025422 & 0.020063556 & 0 & 0 \end{pmatrix} \end{array}$$

$$P_3 = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{R}_1 & \mathcal{R}_2 & \mathcal{R}_3 & \mathcal{R}_4 \end{array} \\ \begin{pmatrix} 0 & 0.009539854 & 0 & 0.019079708 \\ 0 & 0 & 0 & 0.009539854 \\ 0.014991199 & 0.024531053 & 0 & 0.034070907 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$P_4 = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{R}_1 & \mathcal{R}_2 & \mathcal{R}_3 & \mathcal{R}_4 \end{array} \\ \begin{pmatrix} 0 & 0.010809583 & 0.027023956 & 0.018916769 \\ 0 & 0 & 0.016214374 & 0.008107187 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0.008107187 & 0 \end{pmatrix} \end{array}$$

$$P_5 = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{R}_1 & \mathcal{R}_2 & \mathcal{R}_3 & \mathcal{R}_4 \end{array} \\ \begin{pmatrix} 0 & 0 & 0.005042219 & 0 \\ 0.011344993 & 0 & 0.016387212 & 0.001260555 \\ 0 & 0 & 0 & 0 \\ 0.010084438 & 0 & 0.015126658 & 0 \end{pmatrix} \end{array}$$

Step 8: The priority index $\pi(B_i, B_r)$ is computed using the equation (6.3), and the result is shown below:

$$\pi(B_i, B_r) = \begin{array}{c} \\ \mathcal{R}_1 \\ \mathcal{R}_2 \\ \mathcal{R}_3 \\ \mathcal{R}_4 \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{R}_1 & \mathcal{R}_2 & \mathcal{R}_3 & \mathcal{R}_4 \end{array} \\ \begin{pmatrix} 0 & 0.03238757 & 0.032066176 & 0.037996477 \\ 0.01811753 & 0 & 0.032601586 & 0.018907596 \\ 0.056760747 & 0.078338734 & 0 & 0.056978973 \\ 0.02894592 & 0.024127078 & 0.023233844 & 0 \end{pmatrix} \end{array}$$

Step 9: Inflow, outflow, and net flow are determined using equations (6.4) and (6.5), and the values are displayed in table 2.

Inflow(ϕ_i^+)	Outflow (ϕ_i^-)	Net flow (ϕ_i)
0.10245	0.10382	-0.0014
0.06963	0.13485	-0.0652
0.19208	0.0879	0.10418
0.07631	0.11388	-0.0376

Table 2. Inflow, outflow and net flow values

Step 10: From table 2, the net flow of alternative 3 is greater than alternatives 1, 2, and 4. This indicates that alternative 3 is the best one. That is, an electric car is better than other cars.

4.2. *Sensitivity Analysis*

The alternatives and their weights remain the same for convenience. The values of the parameters p and q are changed, and the result is displayed in table 3, and its corresponding chart is given in figure 2.

parameter p,q	Net flow(ϕ_i)	Ranking
p=0,q=0.75	$\phi_1 = -0.0018, \phi_2 = -0.087, \phi_3 = 0.1389, \phi_4 = -0.0501$	$\mathcal{R}_3 > \mathcal{R}_1 > \mathcal{R}_4 > \mathcal{R}_2$
p=0,q=1	$\phi_1 = -0.0014, \phi_2 = -0.0652, \phi_3 = 0.10418, \phi_4 = -0.0376$	$\mathcal{R}_3 > \mathcal{R}_1 > \mathcal{R}_4 > \mathcal{R}_2$

Table 3. The ranking of the alternatives under different parameters

From the table 3, Alternative 3 is the best choice. The sensitivity analysis indicated that the ranking of alternatives remained unchanged, even when preference threshold (p) and indifference threshold (q) values were changed significantly. This stability indicates that the relative strength of preferences between alternatives is consistent. This behavior implies that

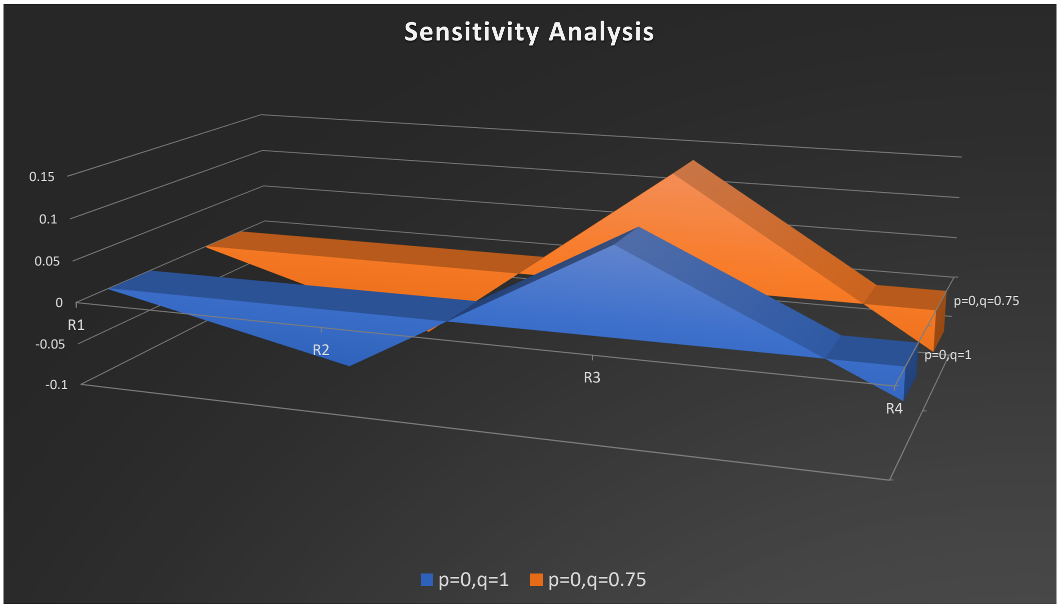

FIGURE 2. Net flow with different parameters

Methods	Rank
Single-valued neutrosophic weighted averaging operator [?]	$R_4 > R_2 > R_3 > R_1$
Single-valued neutrosophic weighted geometric operator [?]	$R_3 > R_1 > R_4 > R_2$
Extended PROMETHEE method	$R_1 > R_4 > R_2 > R_3$

TABLE 4. Comparison of the proposed method with the existing model

the model is robust. For decision-makers, this analysis provides confidence that the selected alternative will remain optimal under slightly different preference assumptions.

4.3. *Comparative Analysis*

Neutrosophic PROMETHEE method is compared with the single-valued neutrosophic weighted averaging operator and single-valued neutrosophic weighted geometric operator.

The results shown in table 4, the rankings from the single-valued neutrosophic weighted geometric operator, and the extended PROMETHEE method with neutrosophic supra topological structure are the same and match up well. This alignment reinforces the validity and stability of the proposed model. On the other hand, the ranking obtained through the single-valued neutrosophic weighted averaging operator shows slight variation. This variation is due to employing the single-valued neutrosophic aggregation operators in the aggregated decision matrix. If these operators were applied directly to the initial decision matrices, it would result in trivial outcomes.

5. Conclusion

While neutrosophic sets offer a way to handle uncertainties in decision-making, classical methods like PROMETHEE may not fully capture their nuances. This study proposes an extended PROMETHEE method within a neutrosophic supra topological space for MCDM. We explored the use of a linear preference function with parameters p and q set to 0 and 1, demonstrating that these values do not significantly impact the results. The advantage of the proposed method is that the proposed model is an easy-to-understand ranking of alternatives through preference flows. This method uses neutrosophic logic, which makes it easier to address the uncertainty and vagueness in expert opinions. While the traditional methods have trouble with this. The neutrosophic supra topological structure offers a systematic framework for clustering, comparing, and analyzing a variety of opinions. The use of neutrosophic supra topological space in MCDM enables layered aggregation of decision matrices, ensuring that varying perspectives contribute meaningfully to the final ranking.

A key limitation of the proposed approach is that the decision-maker's judgment significantly affects the outcome, which may introduce bias. Computational complexity will arise for large-scale problems with numerous alternatives and criteria. The future direction of this study is to develop a hybrid model that integrates the proposed neutrosophic decision-making framework with machine learning techniques to enhance the evaluation and selection of green vehicles. This integration aims to improve the model's ability to handle complex, large uncertain data and to support more accurate, data-driven decision-making in the context of sustainable automobile choices.

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