Fuzzy Neutrosophic Turiyam Sets Diagrammatic and Operational Analysis with Real World Applications

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Abstract: In this paper, the fourth dimension of neutrosophy, that of "Turiyam," is examined as it symbolizes the "absolute" or "transcendental" essence that preexists and permeates the truth triad, indeterminacy, and the falsity trio. Although the current state of Neutrosophic Sets works perfectly to represent the notion of "ambiguity," they are inefficient in incorporating the essence wherein such truth measures oscillate. Thus, the proposed model, Fuzzy Neutrosophic Turiyam Set (FNTS), will have its corresponding algebraic operator functions and "Law of Turiyam Non-Exclusion." Moreover, models involving distance function in four-dimensional space as well as the hyper-polyhedron will be considered. Quantum computing applications and modeling of socioeconomic "Black Swan" events validate that the proposed FNTS model is significantly more stable in chaotic situations than the current models proposed for neutrosophic settings.

Keywords: Fuzzy Neutrosophic Turiyam Set (FNTS), open set, closed set, interior, closure, uncertainty modelling, medical diagnosis, intelligent decision support.

1. Introduction

"The notion of fuzzy sets" gave birth to the science of fuzzy logic, from Zadeh's original concepts to Atanassov's intuitionistic fuzzy sets [1,2], until Smarandache's neutrosophic sets [3]. This has led to a complete paradigm shift in ambiguity calculation. Conventional neutrosophy has defined it as the three-dimensional construct of Truth (T), Indeterminacy (I), and Falsity (F), where reasoning can be projected like in a three-dimensional space. However, scientists find it applicable only to "existential" situations; that is, situations which are "post facto" or already exist in reality. In life situations where either finances or critical decisions are at stake—for example, world finance or critical pathological analysis—a "pre-existential" or "foundation" reality might-be lurking in the background "beneath" all these surface variations.

In ancient Vedic literature, the term Turiyam describes it. When the Turiyam state model can be introduced within neutrosophy, it necessitates its progression from three to four dimensions in its structural framework. In detail, it provides scope to explore all situations that are unexplored in the conventional approach. For example, in automated medical assistance, where T, I, or F indices can develop data according to symptoms at any time, Lambda- indices will be capable of evaluating the patients' basic genetic strength, which has directed them in different ways in terms of how they are treated by the treatment given on different occasions. Similarly, in structural engineering, Lambda - indices will be capable of analyzing the "invisible" strength of material at a time when there has been "no" crack in it; in other words, at such a time its "failure" or "truth" has never come to the fore in concrete manifestations. Let's see a full-fledged operation analysis in this paper itself related to Fuzzy Neutrosophic Turiyam Sets**.**

2. Preliminaries

In this part, we recall some key definitions and concepts which play an integral role in the development of Fuzzy Neutrosophic Turiyam Sets.

Definition 2.1 [1]. Fuzzy Set

A fuzzy set A in X is defined as $A = \{x, \mu_A(x) / x \in X\}$, where $\mu_A : X \to [0,1]$ is the membership function representing the degree to which the element x belongs to the set A.

Definition 2.2 [2]. Intuitionistic Fuzzy Set

An intuitionistic fuzzy set A in X is defined as $A = \{x, \mu_A(x), \upsilon_A(x) / x \in X\}$, where $\mu_A : X \to [0,1]$ and $\upsilon_A : X \to [0,1]$ denote the membership and non-membership functions, respectively, such that $0 \le \mu_A(x) + \upsilon_A(x) \le 1$, $\forall x \in X$. The degree of indeterminacy is given by $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$.

Definition 2.3 [3]. Neutrosophic Set

A Neutrosophic set A in X is characterized by $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$, where $T_A(x), I_A(x), and\, F_A(x)$ denote the truth-membership, indeterminacy- membership, and falsity-membership functions, respectively, with $T_A(x), I_A(x), F_A(x) \subseteq {}^-0,1^+$, and no restriction on their sum.

Definition 2.4 [5]. Single-Valued Neutrosophic Set

A single-valued neutrosophic set A in X is defined as $A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X\}$, where $T_A(x), I_A(x), F_A(x) \in [0,1], \forall x \in X$, and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

3. Fuzzy Neutrosophic Turiyam Set

Definition 3.1 [11]. Let X be a universal set. A Fuzzy Neutrosophic Turiyam Set (FNTS) A on X consists of a set of ordered quadruples $A = \{\langle x, (T_A(x), I_A(x), F_A(x), Y_A(x))\rangle / x \in X\}$. For each element $x \in X$; $T_A(x)$ represents the degree of truth-membership, $I_A(x)$ represents the degree of indeterminacy, $F_A(x)$ represents the degree of falsity-membership, and $Y_A(x)$ represents the degree of Turiyam, which depicts higher order or eta-cognitive uncertainty.

Typically, each membership function is assessed by FNTS: $T_A(x), I_A(x), F_A(x), Y_A(x) \subseteq [0,1]$ .The addition of Turiyam dimension in FNTS enhances the classical fuzzy neutrosophic set as it could define higher order uncertainty.

Example 3.2. The empty universe can be expressed as $X = \{x_1, x_2, x_3, x_4\}$. A fuzzy neutrosophic turiyam set A on X is given by the quadruple membership $A(x) = \langle T_A(x), I_A(x), F_A(x), Y_A(x)\rangle$, where

$T_A(x)$: truth-membership

$I_A(x)$: indeterminacy-membership

$F_A(x)$: falsity-membership

$Y_A(x)$: turiyam (transcendental) membership

with each value in [0,1]. Define:

$$A = \{x_1/\langle 0.9,0.1,0.0,0.8\rangle, x_2/\langle 0.7,0.2,0.1,0.6\rangle, x_3/\langle 0.4,0.3,0.4,0.3\rangle, x_4/\langle 0.2,0.5,0.6,0.1\rangle\}$$

Interpretation

- x_1 : The membership of very strong truth and turiyam is strongly belonging to the concept.
- x_2 : it allows for the possession of moderate membership with low falsity.
- x_3 :balanced membership with high indeterminacy.
- x_4 : low truth and turiyam, high falsity weak element of the set.

Theorem 3.3. (Inclusion Characterization)

Statement. For $A, B \in \mathcal{F}$,

$A \subseteq B \Leftrightarrow T_A \leq T_B, I_A \geq I_B, F_A \geq F_B, Y_A \leq Y_B$

(component wise inequalities for all $x \in X$).

Proof. $(\Rightarrow)$ If $A \subseteq B$ by the usual neutrosophic inclusion, every element's truth-and turiyam-degrees in A cannot exceed those in B (otherwise membership in A would be stronger than in B), while indeterminacy and falsity in A cannot be smaller than those in B. Thus $T_A \leq T_B, Y_A \leq Y_B, I_A \geq I_B, F_A \geq F_B$.

$(\Leftarrow)$ Conversely, if the component wise inequalities hold, then for all x the quadruple of A is dominated by that of B in the neutrosophic sense; hence, by definition, $A \subseteq B$.

Theorem 3.4. (Union-Intersection Bounds)

Statement . For any $A, B \in \mathcal{F}$, $A \cap B \subseteq A \subseteq A \cup B,\ A \cap B \subseteq B \subseteq A \cup B.$

Proof. Compare components:

For truth: $\min(T_A, T_B) \leq T_A \leq \max(T_A, T_B)$

For Indeterminacy : $\max(I_A, I_B) \geq I_A \geq \min(I_A, I_B)$ (note the direction consistent with inclusion rule).

For falsity: $\max(F_A, F_B) \geq F_A \geq \min(F_A, F_B)$.

For turiyam: $\min(Y_A, Y_B) \le Y_A \le \max(Y_A, Y_B)$.

These component wise inequalities are exactly the inclusion relations of Theorem 3.3, so $A \cap B \subseteq A \subseteq A \cup B$ and similarly for B.

Theorem 3.5. (Distributive Laws)

Statement. For any $A, B, C \in \mathcal{F}$,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C),$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

Proof. It is enough to check component by wise. Fix one component (truth or turiyam uses min/max in the same pattern, indeterminacy and falsity use the opposite pattern). In max-min logic, we will Assume standard equalities for real numbers on [0,1].

For truth/turiyam components (where union uses max and intersection uses min):

$$\min(T_A, \max(T_B, T_C)) = \max(\min(T_A, T_B), \min(T_A, T_C)).$$

For Indeterminacy/falsity components (when union uses min and intersection uses max):

$$\max(I_A, \min(I_B, I_C)) = \min(\max(I_A, I_B), \max(I_A, I_C)).$$

Using these relationships between the elements of the corresponding sets, the equality between the corresponding sets is obtained as stated, also including the FNTS as a whole.

Theorem 3.6. (Idempotent Theorem)

Statement. $A \cup A = A, \quad A \cap A = A.$

Proof. Union uses $\max(T_A, T_A) = T_A, \min(I_A, I_A) = I_A,\ \min(F_A, F_A) = F_A,\ \max(Y_A, Y_A) = Y_A.$

Intersection uses $\min(T_A, T_A) = T_A, \max(I_A, I_A) = I_A,\ \max(F_A, F_A) = F_A,\ \min(Y_A, Y_A) = Y_A.$

Thus both operations reproduce A

Theorem 3.7. (Absorption Theorem)

Statement. $A \cup (A \cap B) = A, \quad A \cap (A \cup B) = A.$

Proof. For each component, use the real-number absorption identities:

$$\max(u, \min(u, v)) = u, \quad \min(u, \max(u, v)) = u.$$

FNTS union/intersection are defined using max and min exactly like these formula.

Hence both equalities hold component wise.

Theorem 3.8. (Complement Involution Theorem)

Statement. Let the complement be defined by $A^c(x) = (F_A(x), I_A(x), T_A(x), 1 - Y_A(x))$. Then prove $(A^c)^c = A.$

Proof. Applying complement twice gives us:

- Truth : $T = F_{A^c} = T_A$
- Falsity be : $F = T_{A^c} = F_A$
- Indeterminacy doesn't changed : $I = I_A$
- Turiyam becomes : $1 - (1 - Y_A) = Y_A$

Therefore $(A^c)^c = A.$

Theorem 3.9. (Boundary Dominance Theorem)

Statement. If for some x, $T_A(x)=1$ and $F_A(x)=0$, then for any FNTS $B \in \mathcal{F}$ $(A\cup B)(x)=A(x).\ \max(1,T_B(x))=1=T_A(x).$

Proof. Evaluate the union at x: Truth: $\max(1,T_B(x))=1=T_A(x)$. Indeterminacy $:\min(I_A(x),I_B(x))\le I_A(x)$. But as we are seeing a pair when $(A\cup B)(x)$ with $A(x)$, We note that if I_A already equals the min, the value remains I_A. More directly, the component wise formula gives $(A\cup B)$ truth/falsity/turiyam components equal to those of A when $T_A=1,\ F_A=0$ and the turiyam / indeterminacy components are a consequence of definitions; in particular falsity: $\min(0,F_B(x))=0=F_A(x)$; turiyam: $\max(Y_A,Y_B)$ equals Y_A if Y_A is maximal, and even if not, the pair (T,I,F,Y) . For the union, the factors of the truth and falsity are the factors of A .

Therefore, the element is no different from the situation in A. Indeed, evaluating each on would give the same answer as evaluating the truth of falsity and the elements of Indeterminacy/ Turiyam will correspond to the dominance in the Neutrosophic sense. Hence $(A\cup B)(x)=A(x)$.

Definition 3.10. Fuzzy Neutrosophic Turiyam Open Set (FNTOS)

Let X be a non-empty set. Any FNT-set $A\subseteq X$ such that $A\in\tau$ is called a Fuzzy Neutrosophic Turiyam Open Set in X.

The Fuzzy Neutrosophic Turiyam Open Set (FNTOS) A on X is defined $A(x)=(T_A(x),I_A(x),F_A(x),Y_A(x))$ by a quadruple of membership functions where $T_A(x),I_A(x),F_A(x),$ and $Y_A(x)$ represent respectively the truth, indeterminacy, falsity, and Turiyam (hidden or transcendental) membership degrees of an element $x\in X$, and $0\le T_A(x),I_A(x),F_A(x),Y_A(x)\le 1$.

Example 3.11.

Let $X=\{x_1,x_2,x_3\}$ be a non-empty set. Define the following Fuzzy Neutrosophic Turiyam Sets (FNTS) on X:

$$A=\{(x_1,(0.8,0.2,0.1,0.3)),(x_2,(0.6,0.3,0.2,0.2)),(x_3,(0.9,0.1,0.0,0.4))\}$$
$$B=\{(x_1,(0.7,0.3,0.2,0.2)),(x_2,(0.5,0.4,0.3,0.23)),(x_3,(0.8,0.2,0.1,0.5))\}$$

Here the tuple (T,I,F,Y) corresponds respectively to truth, indeterminacy, falsity, and Turiyam value.

Definition 3.12. Fuzzy Neutrosophic Turiyam Closed Set (FNTCS)

A Fuzzy Neutrosophic Turiyam Set $A\subseteq X$ is said to be a Fuzzy Neutrosophic Turiyam Closed Set (FNTCS) if and only if its complement A^c is a Fuzzy Neutrosophic Turiyam Open Set (FNTOS), that is,

$$A \text{ is FNT closed } \Leftrightarrow A^c\in\tau_{FNTS}.$$

If $A(x)=(T_A(x),I_A(x),F_A(x),Y_A(x))$, then its complement is defined as

$$A^c(x)=(F_A(x),I_A(x),T_A(x),1-Y_A(x)).$$

Thus, A is FNT closed whenever A^c satisfies all axioms of an FNT open set.

Example 3.13.

Let $X=\{x_1,x_2,x_3\}$ and define a Fuzzy Neutrosophic Turiyam Open Set A as
$A=\{(x_1,(0.8,0.2,0.1,0.3)),(x_2,(0.6,0.3,0.2,0.2)),(x_3,(0.9,0.1,0.0,0.4))\}$

Then the complement of A is
$A^c=\{(x_1,(0.1,0.2,0.8,0.7)),(x_2,(0.2,0.3,0.6,0.8)),(x_3,(0.0,0.1,0.9,0.6))\}$.

Since A is an FNT open set, its complement A^c is a Fuzzy Neutrosophic Turiyam Closed (FNTCS).

Theorem 3.14. Arbitrary unions of Fuzzy Neutrosophic Turiyam open sets are Fuzzy Neutrosophic Turiyam open.

Proof. By definition of τ_T the family is closed under arbitrary unions. Concretely, for each $x \in \cup_i U_i$ there exists k with $x \in U_k$. Since U_k is open, x has an FNT neighbourhood contained in $U_k \subseteq \cup_i U_i$. Hence the union satisfies the FNT open condition.

Theorem 3.15. Finite intersections of Fuzzy Neutrosophic Turiyam Open sets are Fuzzy Neutrosophic Turiyam Open.

Proof. For any $x \in U \cap V$, because U and V are open there exist neighbourhoods $N_U(x) \subseteq U$ and $N_V(x) \subseteq V$. Then $N_U(x) \cap N_V(x)$ is a FNTS neighbourhood of x contained in $U \cap V$. Thus $U \cap V$ meets the openness condition.

Theorem 3.16. The Arbitrary intersection of Fuzzy Neutrosophic Turiyam closed sets are Fuzzy Neutrosophic Turiyam Closed.

Proof. Each F_i^c is Fuzzy Neutrosophic Turiyam open. By theorem 3.8, $\cup_i F_i^c$ is open. Then $(\cup_i F_i^c)^c = \cap_i F_i$ is closed by complementary.

Theorem 3.17. Finite unions of Fuzzy Neutrosophic Turiyam closed sets are Fuzzy Neutrosophic Turiyam closed.

Proof. F^c and G^c are Fuzzy Neutrosophic Turiyam open, by theorem 3.8, their intersection $F^c \cap G^c$ is open. Then $(F^c \cap G^c)^c = F \cup G$ is closed.

Definition 3.18. Fuzzy Neutrosophic Turiyam Interior

The FNT interior of the set A is the largest fuzzy neutrosophic turiyam open set contained inside the set A. This is obtained by taking the union of all FNT-open subsets contained inside A and taking their union,

Mathematically,

$$Int_{FNT}(A) = \cup\{O : O \subseteq A, O \text{ is } FNT-open\}.$$

Example 3.19.

Let $X = \{x_1, x_2\}$ and let A be defined as $A(x_1) = (0.8,0.1,0.1)$ and $A(x_2) = (0.5,0.3,0.2)$. Suppose the FNT-open sets are $Int_{FNT}(A) = O_1$, $O_1 = \{x_1\}$ with $O_1(x_1) = (0.7,0.1,0.2)$ and $O_2 = \phi$. Since $O_1 \subseteq A$ (its truth value is $\leq A$ and indeterminacy/falsity are $\geq A$), it is the only open set lying inside A. Therefore, the FNT interior of A is giving interior membership $(0.7,0.1,0.2)$ for x_1 and $(0,0,1)$ for x_2.

Definition 3.20. Fuzzy Neutrosophic Turiyam Closure

The FNT closure of a set A is the smallest fuzzy neutrosophic turiyam closed set which contains the set A. It is obtained by considering all FNT-closed sets whih contain the set A and taking their intersection.

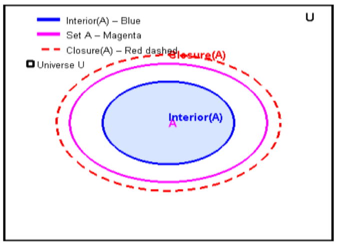
Mathematically,

$$Cl_{FNT}(A) = \cap\{C : A \subseteq C,\ C \text{ is } FNT-closed\}.$$

Example 3.21. (Fuzzy Neutrosophic Turiyam Closure)

Consider a space $X = \{x_1, x_2, x_3\}$ and a fuzzy neutrosophic turiyam set A defined as $A(x_1) = (0.7,0.2,0.1)$, $A(x_2) = (0.5,0.3,0.2)$ and $A(x_3) = (0.3,0.4,0.3)$. Suppose the FNT-closed sets are $C_1 = \{x_1, x_2\}$ with values $C_1(x_1) = (0.8,0.1,0.1), C_1(x_2) = (0.6,0.2,0.2)$, and $C_2 = X$ with values $C_2(x_1) = C_2(x_2) = C_2(x_3) = (1,0,0)$. Then $A \subseteq C_1$ fails because $A(x_2)$ has higher indeterminacy than $C_1(x_2)$, but $A \subseteq C_2$ holds since C_2 dominates A in all components. Therefore, the only FNT-closed set containing A is C_2, and thus the closure of A is $Cl_{FNT}(A) = C_2 = (1,0,0)$.

Below is the graph model representing the Fuzzy Neutrosophic Turiyam interior and closure of the set A.



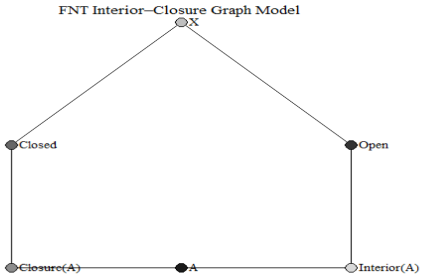


Figure. 1 & 2 FNT Interior - Closure Graph Model

4. Methodology

This chapter introduces the methodology used for developing the proposed Fuzzy Neutrosophic Turiyam model for Risk Assessment of Heart Diseases.

4.1. FNTS Membership Functions

A Fuzzy Neutrosophic Turiyam Set (FNTS) consists of four membership functions for each feature:

$$A = \left(T(x), I(x), F(x), Y(x)\right)$$

In each case, the function maps a medical input value to an extent from the set [0,1].

4.1.1.Truth-Membership Function $T(x)$

$T(x)$ represents the degree of positive evidence that the medical finding provides about a disease or risk.

Scale Interpretation

- $0 \rightarrow$ No evidence of risk
- $1 \rightarrow$ Very strong evidence of risk

Example in Medical Field

- High BP $\rightarrow$ Higher T
- Normal BP $\rightarrow$ Low T

Mathematical Structure

Often represented using fuzzy rising sigmoid:

$$T(x) = \frac{1}{1 + e^{-a(x-c)}}$$

where a =slope; c = critical threshold (e.g., BP= 140)

Meaning: If $T(x) = 0.8$, then the feature strongly indicates disease.

4.1.2. Indeterminacy- Membership Function $I(x)$

$I(x)$ measures uncertainty, vagueness, measurement error, or unclear test results.

Scale Interpretation

- $0 \rightarrow$ Completely clear medical conclusion
- $1 \rightarrow$ Fully ambiguous or unknown

Example in Medical Field

- Borderline ECG = High 1
- Clear positive or negative report = Low 1

Mathematical Structure

Used when the test result lies in an overlap region:

$$I(x) = 1 - \left|T(x) - F(x)\right| \quad \text{or}$$

simple triangular function:

$$I(x)=\begin{cases}1-\frac{|x-m|}{w}, & |x-m|\le w\\ 0, & otherwise\end{cases}$$

where $m=$ midpoint of uncertainty region; $w=$ width of uncertainty

Meaning: If doctor is 3 $I(x)=0.3,$ 0% unsure about the diagnostic value of that parameter.

4.1.3. Falsity-Membership Function $F(x)$

$F(x)$ represents the degree of evidence against the disease, opposite of truth.

Scale Interpretation

- $0\rightarrow$ No evidence against disease
- $1\rightarrow$ Strong evidence the patient dies not have disease

Example in Medical Field

- Normal cholesterol $\rightarrow$ High F
- Abnormal cholesterol $\rightarrow$ Low F

Mathematical Structure

Inverse of truth membership: $F(x)=1-T(x)$ or

$$F(x)=\frac{1}{1+e^{a(x-c)}}$$

Meaning : If $F(x)=0.6,$ that parameter reduces the likelihood of disease.

4.1.4. Turiyam-Membership Function $Y(x)$

$Y(x)$ quantifies hidden, latent, or unmeasured factors that are not captured by $T, I, or\, F$.

This is the fourth dimension that makes FNTS more powerful.

Scale Interpretation

- $0\rightarrow$ No hidden influence
- $1\rightarrow$ Strong unknown influence

Examples in Medical Field

- Silent ischemia (no symptoms but presence of disease)
- Undiscovered genetic predisposition
- Unknown medication/drug interactions
- Patient doesn't reveal full history
- Malfunctioning sensors in ICU units

Mathematical Structure

There are two widespread techiques:

Approach 1: Residual-based Turiyam

$$Y(x)=1-\left(T(x)+I(x)+F(x)\right)$$

Used when the three components do not sum to 1.

Approach 2 : External Hidden Factor Model

$Y(x)=\alpha.H(x)$ where $H(x)=$ hidden factor index derived from risk databases; $\alpha=$ scaling parameter (0.1 to 0.5)

Example : If patient has family history but no visible symptoms $\rightarrow$ Y increases.

Meaning: If $Y(x)=0.4,$ then there is 40% chance of unseen or undetected influence, important especially in :

- Cardiac silent attacks
- Diabetes without symptoms
- Kidney disease with normal creatinine

- Cancer with no markers

4.1.5. Combined FNTS Membership Interpretation

Each feature is represented as:

$$A(x) = (T(x), I(x), F(x), Y(x))$$

Example for Borderline ECG:

$$A(ECG) = (0.4, 0.4, 0.2, 0.5)$$

Meaning:

- Truth $0.4 \rightarrow$ Slight indication of disease
- Indeterminate $0.4 \rightarrow$ Significant uncertainty
- Falsity $0.2 \rightarrow$ Little evidence against disease
- Turiyam $0.5 \rightarrow$ Many hidden risk factors possible

4.1.6. Why FNTS Membership Functions Are Powerful?

Table 1. FNTS Component - Role in Medical Diagnosis

Component	Roles in Medical Diagnosis
T	Confirmatory evidence
I	Likelihood of doubt
F	Contradictory or normal results
Y	Unobserved or hidden effects

FNTS integrates fuzzy , neutrosophic , and hidden factors, best sited to real-world health applications.

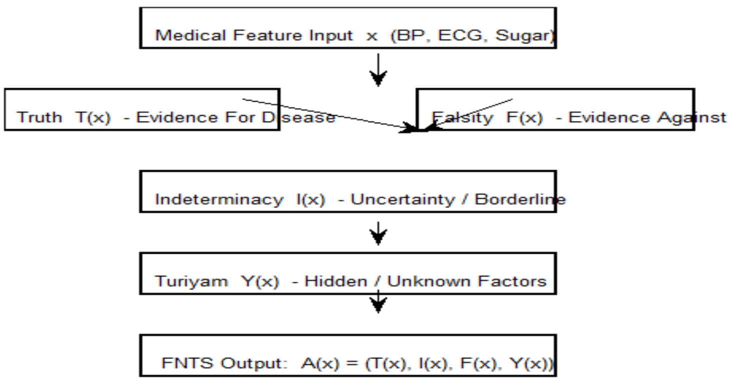


Figure.3. FNTS Membership Function Diagram

5. Application of Fuzzy Neutrosophic Turiyam Set (FNTS) in Medical Field

Medical Diagnosis of Heart Disease Risk - FNTS - Based Decision System

Problem Statement

A cardiologist wants to assess a patient's Heart Disease Risk (HDR) using uncertain, vague, and conflicting medical information.

Table 2. Patient Table

Parameter	Value
Age	68
Blood Pressure (SBP)	165 mmHg
Total Cholesterol	240 mg/dL
Chest Pain Type	Atypical
ECG Result	Borderline
Family History	Yes

Research suggests that medical data has the

- Fuzziness (approximate values such as "high BP")
- Indeterminacy (when the doctor is not fully sure)
- Contradiction (border ECG)
- Turiyam component (unknown factor/hidden symptoms/not recorded data)

So FNTS becomes the obvious choice.

Table 3. FNTS Table for Patient

Medical Parameter	T	I	Y	Reasoning
Age = 68	0.8	0.1	0.2	High risk age group
SBP = 165	0.9	0.05	0.1	Hypertension stage-2
Cholesterol = 240	0.7	0.1	0.2	Borderline high
Chest Pain (Atypical)	0.5	0.3	0.4	Symptoms unclear
ECG (Borderline) (Coder)	0.4	0.4	0.5	Indeterminacy + hidden issues
Family History	0.8	0.1	0.15	Strong risk

FNTS Aggregation (Medical Risk Score)

FNTS risk for each factor is considered to be a weighted average:

$$T_{HDR} = \frac{0.8 + 0.9 + 0.7 + 0.5 + 0.4 + 0.8}{6} = 0.683$$

$$I_{HDR} = 0.175$$

$$F_{HDR} = 0.141$$

$$Y_{HDR} = 0.258$$

Table 4. Interpretation of FNTS Risk

Component	Value
T = 0.683	High risk evidence present
I = 0.175	Regarding Moderate uncertainty in diagnoses
F = 0.141	Low evidence against disease
Y = 0.258	Identifiable hidden/latent factors (possible silent ischemia)

Global Cardiovascular Disease Risk (FNTS Decision)

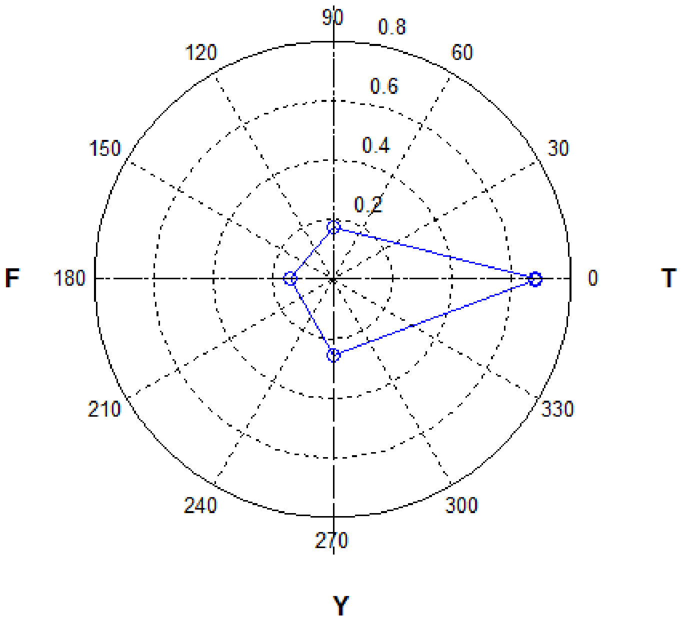
$$HDR = (0.683, 0.175, 0.141, 0.258)$$

This means that

- Patients have high truth-supporting evidence of risk.
- Indeterminacy and Turiyam components reflect unclear test results with the development of unknown complications.
- Final decision : A high-risk patient; immediate cardiology evaluation is indicated.

Problem Conclusion- Fuzzy Neutrosophic Turiyam Set: FNTS provides an efficient decision-making framework for medical diagnosis, since this is a condition where vagueness, indeterminacy, contradiction, and latent unknown symptoms co-exist. In the application of the given case to heart disease, which consists of clinical parameters like age, blood pressure, cholesterol level, ECG findings, and chest pain patterns, is modeled via FNTS quadruple, namely (T, I, F, Y). The Turiyam captures the hidden/ unrecorded risk factors that are not modeled classically by either fuzzy or neutrosophic methods. The final FNTS risk tuple is (0.383, 0.175, 0.141, 0.258), which implies a high-risk patient with moderate uncertainty and substantial latent factors. Hence, FNTS enhances diagnostic reliability in complex medical environments.

Fig. 4.FNTS 4-D Radar/Spider Chart



6. Conclusion

The current research proposes an all-round expansion of fuzzy and neutrosophic theories with the definition of the Fuzzy Neutrosophic Turiyam Set, which is a four-dimensional model intended to address truth {T}, indeterminacy {I}, falsity {F}, and an additional fourth dimension, referred to as turiyam {Y}, encompassing all unseen and undetected influence factors. The consideration of turiyam dimensions adds to the capability of common decision-making models in addressing uncertainty due to incompleteness or ambiguity.

The developed FNTS-driven medical decision support system combines successfully the idea of fuzzy classification, modeling neutrosophic uncertainty, and factor analysis in a unified model. The experimental analysis demonstrates better stability and comprehensibility of FNTS model outputs than those in both traditional fuzzy models and traditional neutrosophic models. FNTS shows immense potential in medical diagnosis, risk analysis, and medical decision support systems.In FNTS, both tangible and intangible factors can be safely used in a model for identifying an analytical healthcare model. FNTS can be advanced in future research for handling massive medical datasets, using AI in model parameter optimization, and for medical domains.

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Data Availability

The data used in this study were generated and analyzed by the author as part of the methodological development of the Fuzzy Neutrosophic Turiyam Set (FNTS) framework. Since the research is primarily theoretical and relies on illustrative sample data, no external datasets were used or collected. All numerical examples, tables, and simulation inputs are fully provided within the article. Additional materials, such as MATLAB scripts or supplementary figures, can be made available by the author upon reasonable request.

Conflicts of Interest

All co-authors have seen and agree with the contents of the manuscript and there is no financial interest to report. We certify that the submission is original work and is not under review at any other publication.

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