P-Spaces, weak P-Spaces and almost P-Spaces on Fuzzy Neutrosophic Topological Spaces.

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Abstract: In this paper we introduce the concepts of fuzzy neutrosophic P-space, weak fuzzy neutrosophic P-space and Fuzzy neutrosophic almost P-space. Also we discuss several characterizations of fuzzy neutrosophic P-space, weak Fuzzy neutrosophic P-space and Fuzzy neutrosophic almost P-space. Several examples are given to illustrate the concepts introduced in this paper.
Keywords: Fuzzy neutrosophic regular-open(closed) sets; Fuzzy neutrosophic P-space; weak fuzzy neutrosophic P-space; fuzzy neutrosophic almost P-space and fuzzy neutrosophic Baire space.

1. Introduction

The fuzzy idea was invaded all branches of science as far back as the presentation of fuzzy sets by L. A. Zadeh [21]. The important concept of fuzzy topological space was offered by C.L. Chang [3]. The idea of fuzzy σ −- Baire Spaces was introduced by G. Thangaraj and E. Poongothai [13]. The concept of neutrosophic sets was defined with membership, non-membership and indeterminacy degrees. In 2017, Veereswari [20] introduced fuzzy neutrosophic topological spaces. The idea of fuzzy neutrosophic Baire spaces was introduced by E. Poongothai and E. Padmavathi [10].

In this paper we introduce the concepts of fuzzy neutrosophic P-space, weak fuzzy neutrosophic P-space and Fuzzy neutrosophic almost P-space. Also we discuss several characterizations of fuzzy neutrosophic P-space, weak Fuzzy neutrosophic P-space and Fuzzy neutrosophic almost P-space. Several examples are given to illustrate the concepts introduced in this paper.

2. Preliminaries

Definition 2.1. [2] A fuzzy neutrosophic set A on the universe of discourse X is defined as $\boldsymbol{A} = \langle \boldsymbol{x}, \boldsymbol{T_A}(\boldsymbol{x}), \boldsymbol{I_A}(\boldsymbol{x}), \boldsymbol{F_A}(\boldsymbol{x})\rangle, \boldsymbol{x} \in \boldsymbol{X}$ where $\boldsymbol{T}, \boldsymbol{I}, \boldsymbol{F}: \boldsymbol{X} \to [\boldsymbol{0}, \boldsymbol{1}]$ and $\boldsymbol{0} \leq \boldsymbol{T_A}(\boldsymbol{x}) + \boldsymbol{I_A}(\boldsymbol{x}) + \boldsymbol{F_A}(\boldsymbol{x}) \leq \boldsymbol{3}$.

Definition 2.2. [2] A fuzzy neutrosophic set A isa subset of a fuzzy neutrosophic set B (i.e.,) $\boldsymbol{A} \subseteq \boldsymbol{B}$ for all x if $\boldsymbol{T_A}(\boldsymbol{x}) \leq \boldsymbol{T_B}(\boldsymbol{x}), \boldsymbol{I_A}(\boldsymbol{x}) \leq \boldsymbol{I_B}(\boldsymbol{x}), \boldsymbol{F_A}(\boldsymbol{x}) \geq \boldsymbol{F_B}(\boldsymbol{x})$.

Definition 2.3. [2] Let X be a non-empty set, and $\boldsymbol{A} = \langle \boldsymbol{x}, \boldsymbol{T_A}(\boldsymbol{x}), \boldsymbol{I_A}(\boldsymbol{x}), \boldsymbol{F_A}(\boldsymbol{x})\rangle$, $\boldsymbol{B} = \langle \boldsymbol{x}, \boldsymbol{T_B}(\boldsymbol{x}), \boldsymbol{I_B}(\boldsymbol{x}), \boldsymbol{F_B}(\boldsymbol{x})\rangle$ be two fuzzy neutrosophic sets. Then

$\boldsymbol{A} \cup \boldsymbol{B}$=$\langle \boldsymbol{x}, \boldsymbol{max}(\boldsymbol{T_A}(\boldsymbol{x}), \boldsymbol{T_B}(\boldsymbol{x})), \boldsymbol{max}(\boldsymbol{I_A}(\boldsymbol{x}), \boldsymbol{I_B}(\boldsymbol{x})), \boldsymbol{min}(\boldsymbol{F_A}(\boldsymbol{x}), \boldsymbol{F_B}(\boldsymbol{x}))\rangle$

$\boldsymbol{A \cap B}$=$\langle \boldsymbol{x}, \boldsymbol{min}(\boldsymbol{T_A}(\boldsymbol{x}), \boldsymbol{T_B}(\boldsymbol{x})), \boldsymbol{min}(\boldsymbol{I_A}(\boldsymbol{x}), \boldsymbol{I_B}(\boldsymbol{x})), \boldsymbol{max}(\boldsymbol{F_A}(\boldsymbol{x}), \boldsymbol{F_B}(\boldsymbol{x}))\rangle$

Definition 2.4. [2] The difference between two fuzzy neutrosophic sets A and B is defined as $A \backslash B(x)$=$\langle x, min(T_A(x), F_B(x)), min(I_A(x), 1 - I_B(x)), max(F_A(x), T_B(x))\rangle$.

Definition 2.5. [2] A fuzzy neutrosophic set A over the universe X is said to be null or empty fuzzy neutrosophic set if $T_A(x) = 0, I_A(x) = 0, F_A(x) = 1$ for all $x \in X$. It is denoted by $0_{N.}$

Definition 2.6. [2] A fuzzy neutrosophic set A over the universe X is said to be absolute (universe) fuzzy neutrosophic set if $T_A(x) = 1, I_A(x) = 1, F_A(x) = 0$ for all $x \in X$. It is denoted by $1_{N.}$

Definition 2.7. [2] The complement of a fuzzy neutrosophic set A is denoted by A^c and is defined as $A^c = \langle x, T_{A^c}(x), I_{A^c}(x), F_{A^c}(x)\rangle$ where $T_{A^c}(x) = F_A(x), I_{A^c}(x) = 1 - I_A(x), F_{A^c}(x) = T_A(x)$. The complement of fuzzy neutrosophic set A can also be defined as $A^c = 1_N - A$.

Definition 2.8. [1] A fuzzy neutrosophic topology on a non-empty set X is a τ of fuzzy neutrosophic sets in X saisfying the following axioms.
(i) $0_N, 1_N \in \tau$
(ii) $A_1 \cap A_2 \in \tau$ *for any* $A_1, A_2 \in \tau$
(iii) $\cup A_i \in \tau$ *for any arbitrary family* $\{A_i : i \in J\} \in \tau$
In this case the pair (X, τ) is called fuzzy neutrosophic topological space and any fuzzy neutrosophic set in τ is known as fuzzy neutrosophic open set in X.

Definition 2.9. [1] The complement A^c of a fuzzy neutrosophic set A in a fuzzy neutrosophic topological space (X, τ) is called fuzzy neutrosophic closed set in X.

Definition 2.10. [1] Let (X, τ) be a fuzzy neutrosophic topological space and $A = \langle x, T_A(x), I_A(x), F_A(x)\rangle$ be a fuzzy neutrosophic set in X. Then the closure and interior of A are defined by

$$int\,(A) = \cup \{G: G\ is\ a\ fuzzy\ neutrosophic\ open\ set\ in\ X\ and\ G \subseteq A\}$$
$$cl(A) = \cap \{G: G\ is\ a\ fuzzy\ neutrosophic\ closed\ set\ in\ X\ and\ A \subseteq G\}$$

Definition 2.11. [1] Let (X, τ) be a fuzzy neutrosophic topological space over X. Then the following properties hold. $(i)\ cl(A^c) = (int\ A)^c$, $(ii)\ int\ (A)^c = (cl\ A)^c$.

Definition 2.12. [10] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (P, τ_N) is called a fuzzy neutrosophic F_σ −set if A_N =$\vee_{i=1}^{\infty} A_{N_i}$, where $\overline{A_{N_i}} \in \tau_N$ for $i \in I$.

Definition 2.13. [10] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (P, τ_N) is called a fuzzy neutrosophic G_δ −set if A_N =$\wedge_{i=1}^{\infty} A_{N_i}$, where $A_{N_i} \in \tau_N$ for $i \in I$.

Definition 2.14. [10] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (P, τ_N) is called a fuzzy neutrosophic dense if there exist no fnCS B_N in (P, τ_N) s.t $A_N \subset B_N \subset 1_X$. That is, $fn(A_N)^- = 1_N$.

Definition 2.15. [10] A fuzzy neutrosophic set A_N in a fuzzy neutrosophic topological space (P, τ_N) is called a fuzzy neutrosophic nowhere dense set if there exist no non zero fnOS B_N in (P, τ_N) s.t $B_N \subset fn(A_N)^-$. That is, $fn(((A_N)^-)^+) = 0_N$.

Definition 2.16. [10] Let (P, τ_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set A_N in (P, τ_N) is called fuzzy neutrosophic one category set if A_N =$\vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere dense sets in (P, τ_N). Any other fuzzy neutrosophic set in (P, τ_N) is said to be of fuzzy neutrosophic two category.

Definition 2.17. [10] A fuzzy neutrosophic topological space (P, τ_N) is called fuzzy neutrosophic one category space if the fuzzy neutrosophic set 1_x is a fuzzy neutrosophic one category set in (P, τ_N). That is, $1_x = \vee_{i=1}^{\infty} A_{N_i}$, where A_{N_i}'s are fuzzy neutrosophic nowhere dense sets in (P, τ_N). Otherwise (P, τ_N) will be called a fuzzy neutrosophic two category space.

Definition 2.18. [10] Let A_N be a fuzzy neutrosophic one category set in (P, τ_N). Then $\overline{A_N}$ is called fuzzy neutrosophic residual set in (P, τ_N).

Definition 2.19. [10] A fuzzy neutrosophic topological space (P, τ_N) is called fuzzy neutrosophic Baire space if $\mathrm{fn}\left(\vee_{i=1}^{\infty} (A_{N_i})\right)^+ = 0_N$,where A_{N_i}'s are fuzzy neutrosophic nowhere dense sets in (P, τ_N).

Definition 2.20. [10] Let (P, τ_N) be a fuzzy neutrosophic topological space. Then the following are equivalent.
(1) (P, τ_N) is a fuzzy neutrosophic Baire Space.
(2) $fn(A_N)^+ = 0_N$, for every fuzzy neutrosophic one category set A_N in (P, τ_N).
(3) $fn(B_N)^+ = 1_N$, for every fuzzy neutrosophic residual set B_N in (P, τ_N).

Definition 2.21.[5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set λ_N in (X_N, T_N) is called a fuzzy neutrosophic σ −nowhere dense set if λ_N is a fuzzy neutrosophic F_σ −set in (X_N, T_N) such that $int(\lambda_N) = 0_N$.

Definition 2.22. [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. A fuzzy neutrosophic set λ_N in (X_N, T_N) is called a fuzzy neutrosophic σ −first category set if $\lambda_N = \vee_{i=1}^{\infty} (\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic σ −nowhere dense sets in (X_N, T_N). Any other fuzzy neutrosophic set in (X_N, T_N) is said to be fuzzy neutrosophic σ −second category sets in (X_N, T_N).

Definition 2.23. [5] Let λ_N be a fuzzy neutrosophic σ −first category set in (X_N, T_N). Then $1_N - \lambda_N$ is called a fuzzy neutrosophic σ −residual set in (X_N, T_N).

Definition 2.24. [5] A fuzzy neutrosophic topological space (X_N, T_N) is called fuzzy neutrosophic σ −first category space if the fuzzy neutrosophic set 1_{X_N} isa fuzzy neutrosophic σ −first category set in (X_N, T_N). That is $1_{X_N} = \vee_{i=1}^{\infty} (\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic σ −nowhere dense sets in (X_N, T_N). Otherwise (X_N, T_N) will be called a fuzzy neutrosophic σ −second category space.

Definition 2.25. [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then (X_N, T_N) is called a fuzzy neutrosophic σ −Baire Space if $int(\vee_{i=1}^{\infty} (\lambda_{N_i})) = 0_N$, where (λ_{N_i})'s are fuzzy neutrosophic σ −nowhere dense sets in (X_N, T_N).

Theorem 2.26. [5] Let (X_N, T_N) be a fuzzy neutrosophic topological space. Then the following are equivalent.

(1) (X_N, T_N) is a fuzzy neutrosophic σ −Baire Space.
(2) $int(\lambda_N) = 0_N$, for every fuzzy neutrosophic σ −first category set λ_N in (X_N, T_N).
(3) $cl(\mu_N) = 1_N$, for every fuzzy neutrosophic σ −residual set μ_N in (X_N, T_N).

Definition 2.27. [4] FNS λ_N in FNTS (X,τ) is called Fuzzy Neutrosophic Regular-Open set (Briefly, FNR-open) if $\lambda_N = FNInt(FNcl(\lambda_N))$.

Definition 2.28. [4] FNS λ_N in FNTS (X,τ) is called Fuzzy Neutrosophic Regular-Closed set (Briefly, FNR-closed) if $\lambda_N = FNInt(FNint(\lambda_N))$.

3. Fuzzy Neutrosophic P-Spaces

Definition 3.1. [12] A fuzzy neutrosophic topological space $(\boldsymbol{X_N}, \boldsymbol{T_N})$ is called a fuzzy neutrosophic P-Space if countable intersection of fuzzy neutrosophic open sets in $(\boldsymbol{X_N}, \boldsymbol{T_N})$ is fuzzy neutrosophic open. That is, every non-zero fuzzy neutrosophic $\boldsymbol{G_\delta}$ −set in $(\boldsymbol{X_N}, \boldsymbol{T_N})$ is fuzzy neutrosophic open in $(\boldsymbol{X_N}, \boldsymbol{T_N})$.

Example 3.2. Let $\boldsymbol{X_N} = \{\boldsymbol{x_1}, \boldsymbol{x_2}\}$. The fuzzy neutrosophic sets $\boldsymbol{\lambda_N}$ and $\boldsymbol{\mu_N}$ are defined on $\boldsymbol{X_N}$ as follows:

$\boldsymbol{\lambda_N}: \boldsymbol{X_N} \to [\boldsymbol{0_N}, \boldsymbol{1_N}]$ is defined as $\boldsymbol{\lambda_N(x_1) = (1, 0.2, 0.1)}, \boldsymbol{\lambda_N(x_2) = (0.6, 0.3, 0.1)}$. $\boldsymbol{\mu_N}: \boldsymbol{X_N} \to [\boldsymbol{0_N}, \boldsymbol{1_N}]$ is defined as $\boldsymbol{\mu_N(x_1) = (0.8, 0.1, 0.1)}, \boldsymbol{\mu_N(x_2) = (1, 0, 0)}$. Then, $\boldsymbol{T_N = \{0, \lambda_N, \mu_N, X_N\}}$ is a fuzzy neutrosophic topology on $\boldsymbol{X_N}$.

Now, the intersection of fuzzy neutrosophic sets are $\boldsymbol{\lambda_N \wedge \mu_N \in T_N}$, then every fuzzy neutrosophic $\boldsymbol{G_\delta}$ −set is fuzzy neutrosophic open. Hence, $(\boldsymbol{X_N}, \boldsymbol{T_N})$ is a fuzzy neutrosophic P-Space.

Example 3.3. Let $\boldsymbol{X_N} = \{\boldsymbol{x_1}, \boldsymbol{x_2}\}$. The fuzzy neutrosophic sets $\boldsymbol{\alpha_N}$ and $\boldsymbol{\beta_N}$ are defined on $\boldsymbol{X_N}$ as follows:

$\boldsymbol{\alpha_N}: \boldsymbol{X_N} \to [\boldsymbol{0_N}, \boldsymbol{1_N}]$ is defined as $\boldsymbol{\alpha_N(x_1) = (1, 0.2, 0.1)}, \boldsymbol{\alpha_N(x_2) = (0.5, 0.3, 0.2)}$.

$\boldsymbol{\beta_N}: \boldsymbol{X_N} \to [\boldsymbol{0_N}, \boldsymbol{1_N}]$ is defined as $\boldsymbol{\beta_N(x_1) = (0.6, 0.1, 0.3)}, \boldsymbol{\beta_N(x_2) = (0.7, 0.2, 0.2)}$.

Then, $\boldsymbol{T_N = \{0, \lambda_N, \mu_N, X_N\}}$ is a fuzzy neutrosophic topology on $\boldsymbol{X_N}$. Now, the fuzzy neutrosophic sets are $\boldsymbol{\alpha_N}, \boldsymbol{\beta_N}$ is $\boldsymbol{\eta_N = \alpha_N \wedge \beta_N}$ is a fuzzy neutrosophic $\boldsymbol{G_\delta}$ −set. But $\boldsymbol{\eta_N \notin T_N}$ in $(\boldsymbol{X_N}, \boldsymbol{T_N})$. Hence the fuzzy neutrosophic topological space $(\boldsymbol{X_N}, \boldsymbol{T_N})$ is not a fuzzy neutrosophic P-Space.

Proposition 3.4. If $\boldsymbol{\lambda_N}$ is a non-zero fuzzy neutrosophic $\boldsymbol{F_\sigma - set}$ in a fuzzy neutrosophic P-Space $(\boldsymbol{X_N}, \boldsymbol{T_N})$, then $\boldsymbol{\lambda_N}$ is a fuzzy neutrosophic closed set in $(\boldsymbol{X_N}, \boldsymbol{T_N})$.

Proof. Given that $\boldsymbol{\lambda_N}$ is a non-zero fuzzy neutrosophic $\boldsymbol{F_\sigma - set}$ in $(\boldsymbol{X_N}, \boldsymbol{T_N})$, $\boldsymbol{\lambda_N} = \vee_{i=1}^{\infty} (\boldsymbol{\lambda_{N_i}})$, where the fuzzy neutrosophic sets $(\boldsymbol{\lambda_{N_i}})$'s are fuzzy neutrosophic closed in $(\boldsymbol{X_N}, \boldsymbol{T_N})$. Then $\boldsymbol{1_N - \lambda_N = 1_N -} (\vee_{i=1}^{\infty} (\boldsymbol{\lambda_{N_i}})) = \wedge_{i=1}^{\infty} (\boldsymbol{1_N - \lambda_{N_i}})$. Now, $(\boldsymbol{\lambda_{N_i}})$'s are fuzzy neutrosophic closed in $(\boldsymbol{X_N}, \boldsymbol{T_N})$, implies that $(\boldsymbol{1_N - \lambda_{N_i}})$'s are fuzzy neutrosophic open in $(\boldsymbol{X_N}, \boldsymbol{T_N})$. Hence we have $\boldsymbol{1_N - \lambda_N} = \wedge_{i=1}^{\infty} (\boldsymbol{1_N - \lambda_{N_i}})$, where $\boldsymbol{1_N - \lambda_{N_i} \in T_N}$. Then $\boldsymbol{1_N - \lambda_N}$ is a fuzzy neutrosophic $\boldsymbol{G_\delta}$ −set in $(\boldsymbol{X_N}, \boldsymbol{T_N})$. Since $(\boldsymbol{X_N}, \boldsymbol{T_N})$ is a fuzzy neutrosophic P-Space, $\boldsymbol{1_N - \lambda_N}$ is a fuzzy neutrosophic open in $(\boldsymbol{X_N}, \boldsymbol{T_N})$. Therefore $\boldsymbol{\lambda_N}$ is a fuzzy neutrosophic closed set in $(\boldsymbol{X_N}, \boldsymbol{T_N})$.

Proposition 3.5. If the fuzzy neutrosophic topological space $(\boldsymbol{X_N}, \boldsymbol{T_N})$ is a fuzzy neutrosophic P-Space, then $\boldsymbol{cl}(\vee_{i=1}^{\infty} (\boldsymbol{\lambda_{N_i}})) = \vee_{i=1}^{\infty} \boldsymbol{cl}(\boldsymbol{\lambda_{N_i}})$, where $(\boldsymbol{\lambda_{N_i}})$'s are non-zero fuzzy neutrosophic closed sets in $(\boldsymbol{X_N}, \boldsymbol{T_N})$.

Proof. Let $(\boldsymbol{\lambda_{N_i}})$'s be a non-zero fuzzy neutrosophic closed sets in a fuzzy neutrosophic P-space $(\boldsymbol{X_N}, \boldsymbol{T_N})$. Then $\boldsymbol{\lambda_N} = \vee_{i=1}^{\infty} (\boldsymbol{\lambda_{N_i}})$, is a non-zero fuzzy neutrosophic $\boldsymbol{F_\sigma - set}$ in $(\boldsymbol{X_N}, \boldsymbol{T_N})$. By proposition 3.1., $\boldsymbol{\lambda_N}$ is a fuzzy neutrosophic closed set in $(\boldsymbol{X_N}, \boldsymbol{T_N})$. Hence $\boldsymbol{cl(\lambda_N) = \lambda_N}$, which implies that $\boldsymbol{cl}(\vee_{i=1}^{\infty} (\boldsymbol{\lambda_{N_i}})) = \vee_{i=1}^{\infty} (\boldsymbol{\lambda_{N_i}}) = \vee_{i=1}^{\infty} \boldsymbol{cl}(\boldsymbol{\lambda_{N_i}})$. [Since $(\boldsymbol{\lambda_{N_i}})$'s are $(\boldsymbol{\lambda_{N_i}})$'s are fuzzy neutrosophic closed,

$cl(\lambda_{N_i}) = \lambda_{N_i}$]. Therefore $cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty} cl(\lambda_{N_i})$, where (λ_{N_i}) 's are non-zero fuzzy neutrosophic closed sets in (X_N, T_N).

Proposition 3.6. If (λ_{N_i})'s are fuzzy neutrosophic regular-closed sets in a fuzzy neutrosophic P-space (X_N, T_N), then $cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty}(\lambda_{N_i})$.

Proof. Let (λ_{N_i})'s are fuzzy neutrosophic regular-closed sets in a fuzzy neutrosophic P-space (X_N, T_N). Then (λ_{N_i})'s are fuzzy neutrosophic closed sets in (X_N, T_N), which implies that $(1_N - \lambda_{N_i})$'s are fuzzy neutrosophic open sets in (X_N, T_N). Let $\mu_N = \wedge_{i=1}^{\infty}[1_N - \lambda_{N_i}]$. Then μ_N is a non-zero fuzzy neutrosophic G_δ −set in (X_N, T_N). Since the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic P-space, $int(\mu_N) = \mu_N$, which implies that $int(\wedge_{i=1}^{\infty}[1_N - \lambda_{N_i}]) = \wedge_{i=1}^{\infty}[1_N - \lambda_{N_i}]$. Then $1_N - cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = 1_N - \vee_{i=1}^{\infty}(\lambda_{N_i})$. Hence we have $cl(\vee_{i=1}^{\infty}(\lambda_{N_i})) = \vee_{i=1}^{\infty}(\lambda_{N_i})$.

Proposition 3.7. If the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic P-space and if λ_N is a fuzzy neutrosophic first category set in (X_N, T_N), then λ_N is a fuzzy neutrosophic dense set in (X_N, T_N).

Proof. Assume that the contrary. Suppose that λ_N is a fuzzy neutrosophic first category set in (X_N, T_N) such that $cl(\lambda_N) = 1_N$. Then $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where the fuzzy neutrosophic sets (λ_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X_N, T_N). Now, $1_N - cl(\lambda_{N_i})$ is a fuzzy neutrosophic open set in (X_N, T_N). Let $\mu_N = \wedge_{i=1}^{\infty}[1_N - cl(\lambda_{N_i})]$. Then μ_N is a non-zero fuzzy neutrosophic G_δ −set in (X_N, T_N). Now, we have $(\wedge_{i=1}^{\infty}[1_N - cl(\lambda_{N_i})]) = 1_N - \vee_{i=1}^{\infty}(cl(\lambda_{N_i})) \le 1_N - \vee_{i=1}^{\infty}(\lambda_{N_i}) = 1_N - \lambda_N$. Hence $\mu_N \le 1_N - \lambda_N$ .Then $int(\mu_N) \le int(1_N - \lambda_N) = 1_N - cl(\lambda_N) = 1_N - 1_N = 0_N$. That is, $int(\mu_N) = 0_N$. Since (X_N, T_N) is a fuzzy neutrosophic P-space, $\mu_N = int(\mu_N)$ which implies that $\mu_N = 0_N$, a contradiction to μ_N being a non-zero fuzzy neutrosophic G_δ −set in (X_N, T_N). Hence $cl(\lambda_N) \ne 1_N$.

Proposition 3.8. If the fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic P-space and if λ_N is a fuzzy neutrosophic first category set in (X_N, T_N), then λ_N is not a fuzzy neutrosophic nowhere dense set in (X_N, T_N).

Proof. Let λ_N be a fuzzy neutrosophic first category set in a fuzzy neutrosophic P-space (X_N, T_N). Then, we have $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$, where (λ_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X_N, T_N). Now, intcl(λ_N) = intcl($\vee_{i=1}^{\infty}(\lambda_{N_i})$) ≥ int[$\vee_{i=1}^{\infty}$cl($\lambda_{N_i}$)] and [$\vee_{i=1}^{\infty}$cl($\lambda_{N_i}$)] is a fuzzy neutrosophic F_σ−set in (X_N, T_N). Since $(X_N, T_N)$is a fuzzy neutrosophic P-space, by proposition 3.1, [$\vee_{i=1}^{\infty}$cl(λ_{N_i})] is a non-zero fuzzy neutrosophic closed set in (X_N, T_N). Also interior of a fuzzy neutrosophic closed is a fuzzy neutrosophic regular-open set, int[$\vee_{i=1}^{\infty}$cl(λ_{N_i})] is a non-zero fuzzy neutrosophic regular-open set in (X_N, T_N). Hence we have$0_N \ne$ int[$\vee_{i=1}^{\infty}$cl(λ_{N_i})] ≤ intcl(λ_N), implies that intcl(λ_N) $\ne 0_N$. Therefore λ_N is not a fuzzy neutrosophic nowhere dense set in (X_N, T_N).

Proposition 3.9. If λ_N is a fuzzy neutrosophic first category set in a fuzzy neutrosophic P-space (X_N, T_N) such that $\mu_N \le 1_N - \lambda_N$, where μ_N is a non-zero fuzzy neutrosophic dense G_δ −set in (X_N, T_N), then λ_N is a fuzzy neutrosophic nowhere dense set in (X_N, T_N).

Proof. Let λ_N be a fuzzy neutrosophc first category set in (X_N, T_N). Then, $\lambda_N = \vee_{i=1}^{\infty}(\lambda_{N_i})$,where (λ_{N_i})'s are fuzzy neutrosophic nowhere dense sets in (X_N, T_N). Now, $1_N - cl(\lambda_{N_i})$ is a fuzzy neutrosophic open set in (X_N, T_N). Let $\mu_N = \wedge_{i=1}^{\infty}[1_N - cl(\lambda_{N_i})]$. Then μ_N is a non-zero fuzzy neutrosophic dense G_δ−set in (X_N, T_N). Now, we have $\wedge_{i=1}^{\infty}[1_N - cl(\lambda_{N_i})] = 1_N - \vee_{i=1}^{\infty} cl(\lambda_{N_i}) \le 1_N - \vee_{i=1}^{\infty}(\lambda_{N_i}) = 1_N - \lambda_N$. Hence $\mu_N \le (1_N - \lambda_N)$. Then, we have $\lambda_N \le (1_N - \mu_N)$. Now, intcl($\lambda_N$) ≤

intcl($\mathbf{1}_N - \boldsymbol{\mu}_N$), which implies that intcl($\boldsymbol{\lambda}_N$) ≤ $\mathbf{1}_N$ − clint($\boldsymbol{\mu}_N$). Since $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a fuzzy neutrosophic P-space, the fuzzy neutrosophic $\boldsymbol{G}_\delta$−set $\boldsymbol{\mu}_N$ is fuzzy neutrosophic open in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ and int($\boldsymbol{\mu}_N$) = $\boldsymbol{\mu}_N$. Therefore intcl($\boldsymbol{\lambda}_N$) ≤ $\mathbf{1}_N$ − cl($\boldsymbol{\mu}_N$) =$\mathbf{1}_N - \mathbf{1}_N = \mathbf{0}_N$. (Since $\boldsymbol{\mu}_N$ is a fuzzy neutrosophic dense). Then intcl($\boldsymbol{\lambda}_N$) = $\mathbf{0}_N$ and hence $\boldsymbol{\lambda}_N$ is a fuzzy neutrosophic nowhere dense set in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$.

Proposition 3.10. If $\boldsymbol{\lambda}_N$ is a fuzzy neutrosophic first category set in a fuzzy neutrosophic P-space $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ such that $\boldsymbol{\mu}_N \le \mathbf{1}_N - \boldsymbol{\lambda}_N$, where $\boldsymbol{\mu}_N$ is a non-zero fuzzy neutrosophic dense $\boldsymbol{G}_\delta$-set in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$, then $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a fuzzy neutrosophic Baire Space.

Proof. Let $\boldsymbol{\lambda}_N$ be a fuzzy neutrosophic first category set in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$. As in the proof of the proposition 3.6., we have intcl($\boldsymbol{\lambda}_N$) = $\mathbf{0}_N$. Then int($\boldsymbol{\lambda}_N$) ≤ intcl($\boldsymbol{\lambda}_N$), implies that int($\boldsymbol{\lambda}_N$) =$\mathbf{0}_N$ and hence by Theorem 2.1., $(\boldsymbol{X}_N, \boldsymbol{T}_N)$is a fuzzy neutrosophic Baire space.

Proposition 3.11. If the fuzzy neutrosophic topological space$(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a fuzzy neutrosophic P-space and if $\boldsymbol{\lambda}_N$ is a fuzzy neutrosophic dense and fuzzy neutrosophic first category set in$(\boldsymbol{X}_N, \boldsymbol{T}_N)$, then there is no non-zero fuzzy neutrosophic $\boldsymbol{G}_\delta$−set $\boldsymbol{\mu}_N$ in$(\boldsymbol{X}_N, \boldsymbol{T}_N)$ such that $\boldsymbol{\mu}_N \le \mathbf{1}_N - \boldsymbol{\lambda}_N$.

Proof. Let $\boldsymbol{\lambda}_N$ be a fuzzy neutrosophic first category set in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$. As in the proof of the proposition 3.6., we have a fuzzy neutrosophic $\boldsymbol{G}_\delta$−set $\boldsymbol{\mu}_N$ in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ such that $\boldsymbol{\mu}_N \le \mathbf{1}_N - \boldsymbol{\lambda}_N$. Then int($\boldsymbol{\mu}_N$) ≤ int($\mathbf{1}_N - \boldsymbol{\lambda}_N$), implies that int($\boldsymbol{\mu}_N$) ≤ $\mathbf{1}_N$ − cl($\boldsymbol{\lambda}_N$) = $\mathbf{1}_N - \mathbf{1}_N = \mathbf{0}_N$. [Since $\boldsymbol{\lambda}_N$ is a fuzzy neutrosophic dense, cl($\boldsymbol{\lambda}_N$) = $\mathbf{1}_N$]. That is, int($\boldsymbol{\mu}_N$) = $\mathbf{0}_N$. Since $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a fuzzy neutrosophic P-space, int($\boldsymbol{\mu}_N$) = $\boldsymbol{\mu}_N$ and hence we have $\boldsymbol{\mu}_N = \mathbf{0}_N$. Hence, if $\boldsymbol{\lambda}_N$ is a fuzzy neutrosophic dense and fuzzy neutrosophic first category set in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$, then there is no non-zero fuzzy neutrosophic $\boldsymbol{G}_\delta$−set $\boldsymbol{\mu}_N$ in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ such that $\boldsymbol{\mu}_N \le \mathbf{1}_N - \boldsymbol{\lambda}_N$.

4. Weak Fuzzy Neutrosophic P-Spaces

Definition 4.1. A fuzzy neutrosophic topological space $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is called a weak fuzzy neutrosophic P-space if the countable intersection of fuzzy neutrosophic regular-open sets in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a fuzzy neutrosophic regular-open set in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$. That is, $\wedge_{i=1}^{\infty}(\boldsymbol{\lambda}_{N_i})$ is fuzzy neutrosophic regular-open in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$, where $(\boldsymbol{\lambda}_{N_i})$'s are fuzzy neutrosophic regular-open sets in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$.

Example 4.2. Let $\boldsymbol{X}_N = \{\boldsymbol{x}_1, \boldsymbol{x}_2\}$. The fuzzy neutrosophic sets, $\boldsymbol{\lambda}_N$, $\boldsymbol{\mu}_N$ are defined on $\boldsymbol{X}_N$ as follows:

$\boldsymbol{\lambda}_N : \boldsymbol{X}_N \to [\mathbf{0}_N, \mathbf{1}_N]$ is defined as $\boldsymbol{\lambda}_N(\boldsymbol{x}_1)$ = (0.6, 0.2, 0.2), $\boldsymbol{\lambda}_N(\boldsymbol{x}_2)$ = (0.3, 0.5, 0.2).

$\boldsymbol{\mu}_N : \boldsymbol{X}_N \to [\mathbf{0}_N, \mathbf{1}_N]$ is defined as $\boldsymbol{\mu}_N(\boldsymbol{x}_1)$ = (0.5, 0.3, 0.2), $\boldsymbol{\mu}_N(\boldsymbol{x}_2)$ = (0.2, 0.2, 0.6).

Then, $\boldsymbol{T}_N = \{\mathbf{0}_N, \boldsymbol{\lambda}_N, \boldsymbol{\mu}_N, \boldsymbol{\lambda}_N \vee \boldsymbol{\mu}_N, \boldsymbol{\lambda}_N \wedge \boldsymbol{\mu}_N, \mathbf{1}_N\}$ is a fuzzy neutrosophic topology on $\boldsymbol{X}_N$.

Now, $\boldsymbol{\lambda}_N$, $\boldsymbol{\mu}_N$, $\boldsymbol{\lambda}_N \vee \boldsymbol{\mu}_N$ and $\boldsymbol{\lambda}_N \wedge \boldsymbol{\mu}_N$ are fuzzy neutrosophic regular-open sets in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$

and $(\boldsymbol{\lambda}_N) \wedge (\boldsymbol{\mu}_N) \wedge (\boldsymbol{\lambda}_N \vee \boldsymbol{\mu}_N) \wedge (\boldsymbol{\lambda}_N \wedge \boldsymbol{\mu}_N) = \boldsymbol{\lambda}_N \wedge \boldsymbol{\mu}_N$ is a fuzzy neutrosophic regular-open set in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$. Hence $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a weak fuzzy neutrosophic P-space.

Proposition 4.3. A fuzzy neutrosophic topological space $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a weak fuzzy neutrosophic P-space if and only if $\vee_{i=1}^{\infty}(\boldsymbol{\mu}_{N_i})$, where $(\boldsymbol{\mu}_{N_i})$'s are fuzzy neutrosophic regular-closed sets in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is fuzzy neutrosophic regular-closed in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$.

Proof. Let $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ be a weak fuzzy neutrosophic P-space. Then intcl($\wedge_{i=1}^{\infty}(\boldsymbol{\lambda}_{N_i})$) =$\wedge_{i=1}^{\infty}(\boldsymbol{\lambda}_{N_i})$, where $(\boldsymbol{\lambda}_{N_i})$'s are fuzzy neutrosophic regular-open sets in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$. Now, $\mathbf{1}_N - [\boldsymbol{intcl}(\wedge_{i=1}^{\infty}(\boldsymbol{\lambda}_{N_i}))]$, implies that clint[($\vee_{i=1}^{\infty}$ $(\mathbf{1}_N - \boldsymbol{\lambda}_{N_i})$)] = $\vee_{i=1}^{\infty}$ $(\mathbf{1}_N - \boldsymbol{\lambda}_{N_i})$.Let $\boldsymbol{\mu}_{N_i}$ = $(\mathbf{1}_N - \boldsymbol{\lambda}_{N_i})$. Since $\boldsymbol{\lambda}_{N_i}$ is a fuzzy neutrosophic regular-open set in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$), $\boldsymbol{\mu}_{N_i}$ is a fuzzy neutrosophic regular-closed set in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$. Then we have clint($\vee_{i=1}^{\infty}$ $(\boldsymbol{\mu}_{N_i})$) =$\vee_{i=1}^{\infty}$ $(\boldsymbol{\mu}_{N_i})$. Hence $\vee_{i=1}^{\infty}$ $(\boldsymbol{\mu}_{N_i})$ is a fuzzy neutrosophic regular-closed in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$. Conversely,suppose that clint($\vee_{i=1}^{\infty}$ $(\boldsymbol{\mu}_{N_i})$) = $\vee_{i=1}^{\infty}$ $(\boldsymbol{\mu}_{N_i})$, where ($\boldsymbol{\mu}_{N_i}$)'s are fuzzy neutrosophic regularclosed sets in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$. Then $\mathbf{1}_N$ − clint($\vee_{i=1}^{\infty}$ $(\boldsymbol{\mu}_{N_i})$) = $\mathbf{1}_N$ −$\vee_{i=1}^{\infty}$ $(\boldsymbol{\mu}_{N_i})$, which implies that intcl($\wedge_{i=1}^{\infty}(\mathbf{1}_N - \boldsymbol{\mu}_{N_i})$) = $\wedge_{i=1}^{\infty}(\mathbf{1}_N - \boldsymbol{\mu}_{N_i})$, where $(\mathbf{1}_N - \boldsymbol{\mu}_{N_i})$'s are fuzzy neutrosophic regularopen sets in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$. Therefore $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a weak fuzzy neutrosophic P-space.

Proposition 4.4. If a fuzzy neutrosophic topological space $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a fuzzy neutrosophic P-space, then $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a weak fuzzy neutrosophic P-space.

Proof. Let $(\boldsymbol{\lambda}_{N_i})$'s be fuzzy neutrosophic regular-closed sets in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$. Since $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a fuzzy neutrosophic P-space by propositionosition 3.3.,we have cl($\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$) = $\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$ .Now, clint($\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$) ≤ cl($\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$) = $\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$. That is clint($\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$)≤$\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$.....(1).

Since ($\boldsymbol{\lambda}_{N_i}$)'s are fuzzy neutrosophic regular-closed sets in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$, clint($\boldsymbol{\lambda}_{N_i}$) = $\boldsymbol{\lambda}_{N_i}$. Then$\vee_{i=1}^{\infty}$ $\boldsymbol{clint}(\boldsymbol{\lambda}_{N_i})$= $\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$, which implies that $\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$ ≤ clint($\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$).....(2).

From (1) and (2), we have, clint($\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$) = $\vee_{i=1}^{\infty}$ $(\boldsymbol{\lambda}_{N_i})$. Hence by proposition 4.1., $(\boldsymbol{X}_N, \boldsymbol{T}_N)$is a weak fuzzy neutrosophic P-space.

A weak fuzzy neutrosophic P-space need not be a fuzzy neutrosophic P-space. For consider the following example.

Example 4.5. Let $\boldsymbol{X}_N$ = {a, b}. The fuzzy neutrosophic sets $\boldsymbol{\lambda}_N$ and $\boldsymbol{\mu}_N$ are defined on $\boldsymbol{X}_N$ as follows:

$\boldsymbol{\lambda}_N$: $\boldsymbol{X}_N$ → $[\mathbf{0}_N, \mathbf{1}_N$] is defined as $\boldsymbol{\lambda}_N$ = {⟨a,(0.5, 0.3, 0.4)⟩,⟨b,(0.4, 0.6, 0.4)⟩}

$\boldsymbol{\mu}_N$: $\boldsymbol{X}_N$ → $[\mathbf{0}_N, \mathbf{1}_N$] is defined as $\boldsymbol{\mu}_N$ = {⟨a,(0.5, 0.3, 0.3)⟩,⟨b,(0.4, 0.5, 0.4)⟩}

Then, $\boldsymbol{T}_N$ = {$\mathbf{0}_N$, $\boldsymbol{\lambda}_N$, $\boldsymbol{\mu}_N$,$\boldsymbol{\lambda}_N \vee \boldsymbol{\mu}_N$, $\boldsymbol{\lambda}_N \wedge \boldsymbol{\mu}_N$,$\mathbf{1}_N$ } is a fuzzy neutrosophic topology on $\boldsymbol{X}_N$ .Now, the fuzzy neutrosophic sets $\boldsymbol{\mu}_N$ and $\boldsymbol{\lambda}_N \vee \boldsymbol{\mu}_N$ are fuzzy neutrosophic regular-open sets in$(\boldsymbol{X}_N, \boldsymbol{T}_N)$ and $(\boldsymbol{\mu}_N$) ∧ $(\boldsymbol{\lambda}_N \vee \boldsymbol{\mu}_N$) = $\boldsymbol{\mu}_N$ is a fuzzy neutrosophic regular-open set in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$and hence $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is a weak fuzzy neutrosophic P-space. But $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is not a fuzzyneutrosophic P-space, since the fuzzy neutrosophic $\boldsymbol{G}_\delta$−set $(\boldsymbol{\lambda}_N \wedge \boldsymbol{\mu}_N$)∧$(\boldsymbol{\lambda}_N \vee \boldsymbol{\mu}_N$) is not fuzzy neutrosophic open in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$.

5. Fuzzy Neutrosophic Almost P-Spaces

Definition 5.1. A fuzzy neutrosophic topological space $(\boldsymbol{X}_N, \boldsymbol{T}_N)$ is called a fuzzy neutrosophic almost P-space if for every non-zero fuzzy $\boldsymbol{G}_\delta$−set $\boldsymbol{\lambda}_N$ in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$, int$(\boldsymbol{\lambda}_N) \neq \mathbf{0}_N$ in $(\boldsymbol{X}_N, \boldsymbol{T}_N)$.

Example 5.2. Let $\boldsymbol{X}_N$ = {a, b}. The fuzzy neutrosophic sets $\boldsymbol{\alpha}_N$, $\boldsymbol{\beta}_N$ and $\boldsymbol{\gamma}_N$ are defined on $\boldsymbol{X}_N$ as follows:

$\boldsymbol{\alpha}_N$: $\boldsymbol{X}_N$ → $[\mathbf{0}_N, \mathbf{1}_N$] is defined as $\boldsymbol{\alpha}_N$ = {⟨a,(0.5, 0.6, 0.4)⟩,⟨b,(0.6, 0.4, 0.5)⟩}

$\boldsymbol{\beta}_N$: $\boldsymbol{X}_N$ → $[\mathbf{0}_N, \mathbf{1}_N$] is defined as $\boldsymbol{\beta}_N$ = {⟨a,(0.4, 0.7, 0.5)⟩,⟨b,(0.5, 0.6, 0.5)⟩}

$\boldsymbol{\gamma}_N$: $\boldsymbol{X}_N$ → $[\mathbf{0}_N, \mathbf{1}_N$] is defined as $\boldsymbol{\gamma}_N$ = {⟨a,(0.5, 0.8, 0.6)⟩,⟨b,(0.8, 0.7, 0.5)⟩}

Then, T_N = {0_N , α_N , β_N ,γ_N , $\alpha_N \vee \beta_N$, $\beta_N \vee \gamma_N$, $\alpha_N \vee \gamma_N$, $\alpha_N \wedge \beta_N$, $\beta_N \wedge \gamma_N$, $\alpha_N \wedge \gamma_N$, 1_N } is a fuzzy neutrosophic topology on X_N . Now, for the fuzzy neutrosophic $G_\delta-$sets ($\alpha_N \wedge \beta_N$),($\beta_N \wedge \gamma_N$) and ($\alpha_N \wedge \gamma_N$) in (X_N, T_N), int[($\alpha_N \wedge \beta_N$) ∧ ($\beta_N \wedge \gamma_N$) ∧ ($\alpha_N \wedge \gamma_N$)] = ($\beta_N \wedge \gamma_N$) $\neq 0_N$ .Hence (X_N, T_N) is a fuzzy neutrosophic almost P-space.

(1) Clearly every fuzzy neutrosophic P-space is a fuzzy neutrosophic almost fuzzy neutrosophic P-space, since for every non-zero fuzzy neutrosophic $G_\delta-$set δ_N in (X_N, T_N), we have int(δ_N) = $\delta_N \neq 0_N$. But the converse need not be true. For in example 4.2., for every nonzero fuzzy neutrosophic $G_\delta-$set δ_N in (X_N, T_N), we have int(δ_N)$\neq 0_N$ in (X_N, T_N). Hence (X_N, T_N) is a fuzzy neutrosophic almost P-space, but ((X_N, T_N) is not a fuzzy neutrosophic P-space, since the fuzzy neutrosophic $G_\delta-$set {[$\lambda_N \vee \mu_N$]∧[$\lambda_N \vee \gamma_N$]∧[$\mu_N \vee \gamma_N$]} is not fuzzy neutrosophic open in (X_N, T_N).

(2) A fuzzy neutrosophic almost P-space need not be a weak fuzzy neutrosophic P-space. For, consider the following example.

Example 5.3. Let X_N = {a, b}. The fuzzy neutrosophic sets A_N , B_N and C_N are defined on X_N as follows:

$A_N : X_N \to [0_N, 1_N]$ is defined as A_N = {⟨a,(0.8, 0.6, 0.7)⟩,⟨b,(0.7, 0.6, 0.6)⟩}

$B_N : X_N \to [0_N, 1_N]$ is defined as B_N = {⟨a,(0.9, 0.8, 0.6)⟩,⟨b,(0.6, 0.5, 0.6)⟩}

$C_N : X_N \to [0_N, 1_N]$ is defined as C_N = {⟨a,(0.7, 0.5, 0.9)⟩,⟨b,(0.9, 0.6, 0.8)⟩}

Then, T_N = {0_N, A_N , B_N , C_N , $A_N \vee B_N$, $B_N \vee C_N$, $C_N \vee A_N$, $A_N \wedge B_N$, $B_N \wedge C_N$, $A_N \wedge C_N$, $A_N \vee B_N \vee C_N$, 1_N } is a fuzzy neutrosophic topology on X_N . In every non zero fuzzy neutrosophic G_δ-sets ($B_N \vee C_N$) ∧ ($B_N \wedge C_N$), we have int[($B_N \vee C_N$) ∧ ($B_N \wedge C_N$)] $\neq 0_N$ in (X_N, T_N). Hence (X_N, T_N) is a fuzzy neutrosophic almost P-space, but (X_N, T_N) is not a weak fuzzyneutrosophic P-space.

(3)A weak fuzzy neutrosophic P-space need not be a fuzzy neutrosophic almost P-space. For, consider the following example:

Example 5.4. Let X_N = {a, b}. The fuzzy neutrosophic sets λ_N , μ_N and γ_N are defined on X_N as follows :

$\lambda_N : X_N \to [0_N, 1_N]$ is defined as λ_N = {⟨a,(0, 0.4, 0.5)⟩,⟨b,(0.4, 0.5, 0.6)⟩}

$\mu_N : X_N \to [0_N, 1_N]$ is defined as μ_N = {⟨a,(0.6, 0, 0.4)⟩,⟨b,(0.4, 0.6, 0.6)⟩}

$\gamma_N : X_N \to [0_N, 1_N]$ is defined as γ_N = {⟨a,(0.5, 0.6, 0)⟩,⟨b,(0.6, 0, 0.5)⟩}

Then, T_N = {0_N , λ_N , μ_N ,γ_N , $\lambda_N \vee \mu_N$, $\mu_N \vee \gamma_N$, $\lambda_N \vee \gamma_N$, $\lambda_N \wedge \mu_N$, $\mu_N \wedge \gamma_N$, $\lambda_N \wedge \gamma_N$, $\lambda_N \vee \mu_N \vee \gamma_N$, $\lambda_N \wedge \mu_N \wedge \gamma_N$, 1_N } is a fuzzy neutrosophic topology on X_N . Now, ($\lambda_N \wedge \mu_N$),($\lambda_N \wedge \gamma_N$),($\lambda_N \wedge \mu_N \wedge \gamma_N$),($\lambda_N \vee \gamma_N$) are fuzzy neutrosophic regular-open sets in (X_N, T_N) and[($\lambda_N \wedge \mu_N$) ∧ ($\lambda_N \wedge \gamma_N$) ∧ ($\lambda_N \wedge \mu_N \wedge \gamma_N$) ∧ ($\lambda_N \vee \gamma_N$)] = ($\lambda_N \wedge \gamma_N$) is a fuzzy neutrosophicregular-open set in (X_N, T_N) and hence (X_N, T_N) is a weak fuzzy neutrosophic P-space. But(X_N, T_N) is not a fuzzy neutrosophic almost P-space, since for the non-zero fuzzy neutrosophic $G_\delta-$set {($\lambda_N \vee \mu_N$) ∧ ($\mu_N \vee \gamma_N$) ∧ ($\lambda_N \vee \gamma_N$) ∧ ($\lambda_N \wedge \mu_N$)}, we have int{[($\lambda_N \vee \mu_N$) ∧ ($\mu_N \vee \gamma_N$) ∧ ($\lambda_N \vee \gamma_N$) ∧ ($\lambda_N \wedge \mu_N$)]} = 0_N .

Proposition 5.5. If a fuzzy neutrosophic topological space (X_N, T_N) is a fuzzy neutrosophic almost P-space if and only if the only fuzzy neutrosophic F_σ−set λ_N such that cl(λ_N) = 1_N in(X_N, T_N) is 1_{X_N}.

Proof. Let $\lambda_N(\neq 1_N)$ be a fuzzy neutrosophic F_σ−set such that cl(λ_N) =1_N in (X_N, T_N).Then, $1_N - \lambda_N$ is a non-zero fuzzy neutrosophic G_δ−set in (X_N, T_N). Now, 1_N −cl(λ_N) = 0_N ,implies that int($1_N - \lambda_N$) =0_N , which is a contradiction to (X_N, T_N) being a fuzzy neutrosophic almost P-space in which int(δ_N) $\neq 0_N$ for any non-zero fuzzy neutrosophic G_δ−set δ_N in (X_N, T_N). Hence our assumption that cl(λ_N) = 1_N does not hold. Therefore there is no non-zero fuzzy neutrosophic F_σ−set (other than 1_{X_N}) in (X_N, T_N) such that cl(λ_N) = 1_N.

Conversely, let us assume that the only fuzzy neutrosophic F_σ−set λ_N in (X_N, T_N) such that cl(λ_N) = 1_N is 1_{X_N} . Let λ_N be a fuzzy neutrosophic G_δ−set in (X_N, T_N). Then λ_N =$\wedge_{i=1}^{\infty}$ (λ_{N_i}), where (λ_{N_i})'s are fuzzy neutrosophic open sets in (X_N, T_N). Now, $1_N - \lambda_N$ =$\vee_{i=1}^{\infty}$ $(1_N - \lambda_{N_i})$. Then $1_N - \lambda_N$ is a fuzzy neutrosophic F_σ −set in (X_N, T_N). By hypothesis, cl($1_N - \lambda_N$) $\neq 1_N$. Then, 1_N − int(λ_N)$\neq 1_N$, this implies that int(λ_N) $\neq 0_N$. Hence for the non-zero fuzzy neutrosophic G_δ−set λ_N in (X_N, T_N), we have int(λ_N) $\neq 0_N$ and therefore (X_N, T_N) is a fuzzy neutrosophic almost P-space.

If a fuzzy neutrosophic topological space (X_N, T_N) has non-zero fuzzy neutrosophic nowhere dense fuzzy neutrosophic G_δ−sets, then (X_N, T_N) is not a fuzzy neutrosophic almost P-space. For consider the following proposition.

Proposition 5.6. If λ_N is a non-zero fuzzy neutrosophic nowhere dense fuzzy neutrosophic G_δ−set in a fuzzy neutrosophic topological space (X_N, T_N), then (X_N, T_N) is not a fuzzy neutrosophic almost P-space.

Proof. Let λ_N be a non-zero fuzzy neutrosophic nowhere dense fuzzy neutrosophic G_δ−set λ_N in (X_N, T_N). Then int(λ_N) ≤ intcl(λ_N) and intcl(λ_N) = 0_N , implies that int(λ_N) =0_N .Hence for the non-zero fuzzy neutrosophic G_δ−set λ_N in (X_N, T_N), int(λ_N) = 0_N in (X_N, T_N).Therefore (X_N, T_N) is not a fuzzy neutrosophic almost P-space.

6. Applications

They can be applied in medical diagnosis to represent uncertain symptoms and test results, helping doctors make better decisions. In decision-making and management systems, fuzzy neutrosophic weak P-spaces help analyze situations where information may be incomplete or inconsistent. They are also useful in artificial intelligence and expert systems for representing knowledge and reasoning under uncertainty.

7. Conclusions

In conclusion Fuzzy neutrosophic P-space → Fuzzy neutrosophic weak P-space ⇆/ Fuzzy neutrosophic almost P-space. The systematic development and analysis of these fuzzy neutrosophic spaces enhance our ability to model and study complex systems under uncertainty, supporting advancements in applied Mathematics, artificial intelligence and decision support systems.

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