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A Study of Q-Neutrosophic Soft Quasigroups and Their Application to Medical Decisions

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Abstract. In this paper, we investigate the algebraic structure of Q-neutrosophic soft quasigroups as an extension of Q-neutrosophic soft sets to non-associative systems. We establish several fundamental properties of these structures. In particular, we prove that the intersection of two Q-neutrosophic soft quasigroups is itself a Q-neutrosophic soft quasigroup, whereas their union is not necessarily closed under quasigroup operations. The conditions under which the product, left division, and right division of two Q-neutrosophic soft quasigroups remain Q-neutrosophic soft quasigroups, particularly within entropic quasigroups, are established. We further derive necessary and sufficient conditions for a Q-neutrosophic soft groupoid to become a Q-neutrosophic soft quasigroup and examine structural properties such as idempotency, unipotency, 3-power associativity, and n-power associativity in relation to membership degrees. Using these results, we construct a decision-making algorithm that models uncertainty through truth-, indeterminacy-, and falsity-membership functions. An application to medical decision processes demonstrates how quasigroup operations can systematically combine indeterminate and interacting clinical information. The findings show that Q-neutrosophic soft quasigroups provide a rigorous mathematical framework for analyzing indeterminate, inconsistent, and non-associative data across two universal sets, offering enhanced modeling capabilities for complex real-world systems..

Keywords: Soft set, neutrosophic set, quasigroup

1. Introduction

Fuzzy set theory was first introduced by Zadeh in [26], while Atanassov in [11,12] extended this foundation by proposing the notion of intuitionistic fuzzy sets. Although these concepts advanced the modelling of uncertainty, each possesses limitations and challenges, as highlighted in [36]. To address some of these limitations, Molodtsov [36] introduced soft set theory as a flexible mathematical framework for dealing with uncertainties.

Despite the considerable success of soft set theory in handling uncertainty, incompatible information, and incomplete data, soft sets do not assign graded membership values as intuitionistic fuzzy sets do, nor can they adequately represent problems involving indeterminate data. The concept of neutrosophy, introduced by Smarandache in 1998 [24,25], emerged as a powerful generalisation capable of addressing indeterminacy in nature. Neutrosophic sets provide the most comprehensive extension of classical set theory for managing indeterminate and uncertain information.

The combination of neutrosophic sets with soft set theory has attracted significant interest, resulting in numerous studies such as [4–9,37,38]. Q-neutrosophic soft sets (Q-NSS), introduced in [3,25,28], extend neutrosophic soft sets through three independent membership degrees true $\mathcal{T}$, indeterminate $\mathcal{I}$, and falsity $\mathcal{F}$ to accommodate indeterminate data. This framework provides an appropriate parametrisation for representing imprecise, inconsistent, and indeterminate relationships between two universal sets. Further developments of Q-NSS appear in [1,2].

The application of fuzzy, intuitionistic fuzzy, soft, and neutrosophic concepts to algebraic structures has also been widely explored. Rosenfeld [39] was the first to embed fuzzy sets into group theory, establishing foundational results for fuzzy subgroups. Subsequent extensions include the fuzzification of quasigroups by Dudek in 1998 [18] and the study of fuzzy and intuitionistic fuzzy subquasigroups in [19–21]. Other notable contributions include (α,β)-fuzzy subquasigroups presented by Muhammad and Dudek (2008), the development of Q-fuzzy groups by Solairaju and Nagarajan, and the study of soft neutrosophic groups and related structures in [10,29].

Soft quasigroups have been investigated in [30,31], and Q-neutrosophic soft groups were introduced in [3]. Neutrosophic submodules were studied in [15]. The extension from Q-neutrosophic soft groups to Q-neutrosophic soft quasigroups was formalised by Osoba *et al.* [32] in 2024, with corresponding distributive properties examined by Oyebo *et al.* [33]. In 2025, Osoba *et al.* [34] applied Q-neutrosophic soft quasigroups to real-world problems defined under quasigroup operations, demonstrating that isotopic images preserve the structure while homomorphic images may not.

Algorithms based on Q-neutrosophic soft sets have been proposed for decision-making applications, as in [2, 28]. However, these algorithms operate on arbitrary sets without incorporating a binary operation, thus lacking the ability to model structured interactions or relational dynamics among underlying elements. Similarly, the decision-making algorithm developed for soft quasigroups in [31] provides uniformity in distribution problems but cannot address indeterminate or uncertain data due to the absence of truth, indeterminacy, and falsity characterisations.

These limitations reveal a clear research gap. Existing decision-making frameworks do not simultaneously capture indeterminacy, support parametrised modelling over two universal sets, and represent structured, non-associative interactions. Medical decision processes, in particular, involve diagnostic variables, treatment effects, and patient responses that do not combine according to associative rules. Consequently, group-based models, which rely on associativity, identity, and invertibility, do not adequately reflect the nonlinear and context-dependent behaviour of medical information.

Quasigroups offer a more suitable algebraic structure for modelling such systems. Their lack of associativity allows them to represent hybrid, non-sequential relationships among medical variables, capturing the reality that clinical outcomes depend on the order and context in which information is combined. The quasigroup properties of unique solvability for equations such as

$$a \odot x = b \qquad \text{and} \qquad y \odot a = b$$

directly mirror diagnostic inference processes, where unknown clinical factors must be deduced from partial information. Unlike groups, quasigroups impose no requirement for an identity element or universal inverses, aligning more naturally with the irregular and sometimes irreversible nature of medical transformations. Thus, quasigroups provide a more flexible and realistic foundation for modelling medical decision systems than groups.

In this work, the structural characterisation of quasigroups is employed to model real-life medical decision problems. The inherent non-associativity of quasigroups supports the modelling of hybrid systems where interactions among variables do not follow classical algebraic behaviour. Q-neutrosophic soft structures are introduced within various quasigroup settings, including idempotent and unipotent quasigroups. Several theoretical results are established using mathematical induction, including constructions of 3-power associative and n-power associative forms to characterise membership functions.

The proposed framework addresses uncertainties and indeterminacies arising from two universal sets connected by quasigroup operations $(\backslash, /, \odot)$. Medical decision-making frequently involves vague, incomplete, and inconsistent information regarding treatment efficacy, patient responses, diagnostic reliability, and long-term prognosis. Traditional decision models are often unable to incorporate such complexities. A neutrosophic soft quasigroup approach, by contrast, captures truth-membership, indeterminacy-membership, and falsity-membership degrees in a structurally meaningful way, enabling richer analysis of uncertain medical data and more robust decision-making regarding treatment alternatives, diagnostic procedures, and clinical interventions.

To support these applications, results on finite Q-neutrosophic soft quasigroups are adapted to develop an algorithm capable of representing truth, indeterminacy, and falsity degrees within a medical context, thereby providing a comprehensive methodological tool for clinical and public-health decision processes.

2. **Preliminaries**

Definition 2.1. Let G be a non-empty set. Define a binary operation $(\odot)$ on G. If $w \odot t \in G$ for all $w, t \in G$ if there exist $a, b \in G$, then the pair $(G, \odot)$ is called a *groupoid* or *Magma*. If each of the equations:

$$a \odot w = b \qquad \text{and} \qquad t \odot a = b$$

has unique solutions in G for w and t respectively, then $(G, \odot)$ is called a *quasigroup*. If there exists a unique element $e \in G$ called the *identity element* such that for all $w \in G$, $w \odot e = e \odot w = w$, $(G, \odot)$ is called a *loop*.

wt instead of $w \odot t$, and stipulate that $\odot$ has lower priority than juxtaposition among factors to be multiplied. For instance, $w \odot yz$ stands for $w(yz)$. Let w be a fixed element in a groupoid $(G, \odot)$. The left and right translation maps of $w \in G$, denoted by L_w and R_w respectively are defined by

$$yL_w = w \odot t \qquad \text{and} \qquad tR_w = t \odot w.$$

It can now be seen that a groupoid $(G, \odot)$ is a quasigroup if its left and right translation mappings are permutations. Since the left and right translation mappings of a quasigroup are bijective, then the inverse mappings L_w^{-1} and R_w^{-1} exist. Let

$$w \backslash t = yL_w^{-1} \qquad \text{and} \qquad w/t = xR_t^{-1}$$

and note that

$$w \backslash t = z \Leftrightarrow w \odot z = t \qquad \text{and} \qquad w/t = z \Leftrightarrow z \odot t = w.$$

Lemma 2.2. *[13, 22, 35] Let H be a subquasigroup of a quasigroup $(Q, \odot)$. Then H is also a subquasigroup of $(Q, \odot, /, \backslash)$. Conversely, if H is a subquasigroup of $(Q, \odot, /, \backslash)$, then it is a subquasigroup of $(Q, \odot)$.*

Theorem 2.3. *[40] Let $(Q, \odot)$ be a quasigroup and $G \subseteq Q$, $G \neq \emptyset$. Then G is a subquasigroup of $(Q, \odot)$ if and only if $(G, \odot)$, $(G, /)$, and $(G, \backslash)$ are groupoids.*

Definition 2.4. [40] Let $(\hat{G}, \odot)$ be a groupoid (quasigroup) and $\emptyset \neq H \subseteq \hat{G}$. Then H is a subgroupoid (subquasigroup) of $\hat{G}$ if $(H, \odot)$ is a groupoid (quasigroup), denoted $H \leq \hat{G}$.

For nonempty subsets $S, T \subseteq \hat{G}$, define

$$S \odot T = \{s \odot t : s \in S, t \in T\}, \qquad S/T = \{s/t : s \in S, t \in T\}, \qquad S \backslash T = \{s \backslash t : s \in S, t \in T\}.$$

Definition 2.5. [1] Let $(\Lambda^Q, \mathfrak{A})$ and $(\Theta^Q, \mathfrak{B})$ be two Q-neutrosophic soft sets.
(1) Intersection. Their intersection is $(\Lambda^Q, \mathfrak{A}) \cap (\Theta^Q, \mathfrak{B}) = (\triangle^Q, \mathfrak{C})$, where $\mathfrak{C} = \mathfrak{A} \cap \mathfrak{B}$, and for all $c \in \mathfrak{C}$,

$$T_{\triangle^Q(c)} = \min(T_{\Lambda^Q(c)}, T_{\Theta^Q(c)}), \quad I_{\triangle^Q(c)} = \max(I_{\Lambda^Q(c)}, I_{\Theta^Q(c)}), \quad F_{\triangle^Q(c)} = \max(F_{\Lambda^Q(c)}, F_{\Theta^Q(c)}).$$

(2) Union. Their union is $(\Lambda^Q, \mathfrak{A}) \cup (\Theta^Q, \mathfrak{B}) = (\triangle^Q, \mathfrak{C})$, where $\mathfrak{C} = \mathfrak{A} \cup \mathfrak{B}$, and for any $c \in \mathfrak{C}$,

$$T_{\triangle^Q(c)} = \begin{cases} T_{\Lambda^Q(c)}, & c \in \mathfrak{A} \setminus \mathfrak{B}, \\ T_{\Theta^Q(c)}, & c \in \mathfrak{B} \setminus \mathfrak{A}, \\ \max(T_{\Lambda^Q(c)}, T_{\Theta^Q(c)}), & c \in \mathfrak{A} \cap \mathfrak{B}, \end{cases} \qquad I_{\triangle^Q(c)} = \begin{cases} I_{\Lambda^Q(c)}, & c \in \mathfrak{A} \setminus \mathfrak{B}, \\ I_{\Theta^Q(c)}, & c \in \mathfrak{B} \setminus \mathfrak{A}, \\ \min(I_{\Lambda^Q(c)}, I_{\Theta^Q(c)}), & c \in \mathfrak{A} \cap \mathfrak{B}, \end{cases}$$

$$F_{\triangle^Q(c)} = \begin{cases} F_{\Lambda^Q(c)}, & c \in \mathfrak{A} \setminus \mathfrak{B}, \\ F_{\Theta^Q(c)}, & c \in \mathfrak{B} \setminus \mathfrak{A}, \\ \min(F_{\Lambda^Q(c)}, F_{\Theta^Q(c)}), & c \in \mathfrak{A} \cap \mathfrak{B}. \end{cases}$$

Definition 2.6. [36] Let W be a set. A pair $(F, \mathfrak{A})$ is called a soft set if $F : \mathfrak{A} \to P(W)$, where $P(W)$ is the power set of W and $\mathfrak{A}$ is a set of parameters.

Definition 2.7. [24] Let W be a set. A neutrosophic set (NS) is

$$\Phi = \{\langle w, (T_\Phi(w), I_\Phi(w), F_\Phi(w)) \rangle : w \in W\},$$

where $T_\Phi, I_\Phi, F_\Phi : W \to]^-0, 1^+[$ denote the truth, indeterminacy, and falsity membership degrees.

Definition 2.8. [2] A Q-neutrosophic set Φ^Q in W is

$$\Phi^Q = \{\langle (w, u), T_{\Phi^Q}(w, u), I_{\Phi^Q}(w, u), F_{\Phi^Q}(w, u) \rangle : w \in W,\ u \in Q\},$$

where $T_{\Phi^Q}, I_{\Phi^Q}, F_{\Phi^Q} : W \times Q \to]^-0, 1^+[$.

Definition 2.9. [2] Let W be a set and $\mathfrak{A}$ a set of parameters. A neutrosophic soft set $(\Phi, \mathfrak{A})$ is

$$(\Phi, \mathfrak{A}) = \{\langle w, (T_\Phi(w), I_\Phi(w), F_\Phi(w)) \rangle : w \in W\},$$

where $T_\Phi, I_\Phi, F_\Phi : W \to P(W)$ represent the truth, indeterminacy, and falsity membership mappings.

Definition 2.10. [2] Let $k \in \mathbb{N}$, $I = [0,1]$, W a universe of discourse, and Q a non-empty set. A k-dimensional Q-neutrosophic set is

$\Phi^Q = \{\langle (w,u), T_{\Phi^Q_i}(w,u), I_{\Phi^Q_i}(w,u), F_{\Phi^Q_i}(w,u)\rangle : w \in W,\ u \in Q,\ i = 1,\dots,k\}$,

where $T_{\Phi^Q_i}, I_{\Phi^Q_i}, F_{\Phi^Q_i} : W \times Q \to I^k$. The condition $0 \leq T + I + F \leq 3^+$ holds for the independent membership degrees.

Definition 2.11. [1] Let W be a universe of discourse, Q a non-empty set, and $\mathfrak{A} \subseteq E$ a parameter set. Let $\rho^l QNS(W)$ denote all l-dimensional Q-neutrosophic sets on W with $l = 1$. A pair $(\Phi^Q, \mathfrak{A})$ is a Q-neutrosophic soft set if $\Phi^Q : \mathfrak{A} \to \rho^l QNS(W)$ and $\Phi^Q(a) = \emptyset$ for $a \notin \mathfrak{A}$. It is written as $(\Phi^Q, \mathfrak{A}) = \{(a, \Phi^Q(a)) : a \in \mathfrak{A}\}$.

3. Main Results

Definition 3.1. Let $(\Phi^Q, \mathfrak{A})$ be a Q-neutrosophic soft set over a quasigroup $(\hat{G}, \odot, /, \backslash)$. It is called a Q-neutrosophic soft subquasigroup of $\hat{G}$ if, for every $a \in \mathfrak{A}$, the mapping $\Phi^Q(a) : \hat{G} \times Q \to [0,1]^3$ satisfies, for all $w, t \in \hat{G}$ and $u \in Q$,

$$T_{\Phi^Q(a)}(w*t,u) \geq \min\{T_{\Phi^Q(a)}(w,u),\, T_{\Phi^Q(a)}(t,u)\}, I_{\Phi^Q(a)}(w*t,u) \leq \max\{I_{\Phi^Q(a)}(w,u),\, I_{\Phi^Q(a)}(t,u)\},$$

$$F_{\Phi^Q(a)}(w*t,u) \leq \max\{F_{\Phi^Q(a)}(w,u),\, F_{\Phi^Q(a)}(t,u)\},$$

where $* \in \{\odot, /, \backslash\}$. In this case each $\Phi^Q(a)$ forms a Q-neutrosophic soft subquasigroup of $\hat{G}$.

TABLE 1. Let $\hat{G} = \{i,j,k,l,m,n,o\}$ be quasigroup of order 8 and $\mathfrak{A} \subseteq E = \hat{G}$ be the parametric set. Given the quasigeoup in Cayley table below

$\odot$	i	j	k	l	m	n	o
i	i	m	o	n	j	l	k
j	m	j	n	o	i	k	l
k	o	n	k	m	l	j	i
l	n	o	m	k	l	i	j
m	j	i	l	k	m	o	n
n	l	k	j	i	o	n	m
o	k	l	i	j	n	m	o

Example 3.2. Define a Q- neutrosophic soft set $(\Phi^Q, \mathfrak{A})$ as follows, for all $u \in Q$ and $w, t, z \in \hat{G}$, and $n \in \mathbb{N}$.

$$T_{\Phi^Q(a)}(w\odot t,u) = \begin{cases} 1 - \frac{1}{2n}, & \text{if } z = \{j,k,l,m,n,o\} \\ 1, & \text{otherwise.} \end{cases} I_{\Phi^Q(a)}(w\odot t,u) = \begin{cases} 0, & \text{if } z = \{j,k,l,m,n,o\} \\ 1 - \frac{1}{2n}, & \text{otherwise.} \end{cases}$$

$$F_{\Phi^Q(a)}(w\odot t,u)=\begin{cases}0, & \text{if } z=\{j,k,l,m,n,o\}\\ 1-\frac{1}{2n}, & \text{otherwise.}\end{cases}\quad I_{\Phi^Q(a)}(w/t,u)=\begin{cases}0, & \text{if } z=\{j,k,l,m,n,o\}\\ 1-\frac{1}{2n}, & \text{otherwise.}\end{cases}$$

$$T_{\Phi^Q(a)}(w/t,u)=\begin{cases}1-\frac{1}{3n}, & \text{if } z=\{j,k,l,m,n,o\}\\ 0, & \text{otherwise.}\end{cases}\quad F_{\Phi^Q(a)}(w/t,u)=\begin{cases}0, & \text{if } z=\{j,k,l,m,n,o\}\\ 1-\frac{1}{2n}, & \text{otherwise.}\end{cases}$$

Let consider the operation $'\odot'$ then, $T_{\Phi^Q(a)}(w\odot t,u)\geq\min\{T_{\Phi^Q(a)}(w,u),T_{\Phi^Q(a)}(t,u)\}$. Put $w=j,t=n$, then

$$T_{\Phi^Q(a)}(j\odot n,u)\geq\min\{T_{\Phi^Q(a)}(k,u),T_{\Phi^Q(a)}(k,u)\}=$$
$$=1-\frac{1}{2n}=0.5\in[0,1] \text{ where n=1 } \in\mathbb{N}. \tag{1}$$

On the other hand,

$$\min\{T_{\Phi^Q(a)}(j,u),T_{\Phi^Q(a)}(n,u)\}=$$
$$\min\{1,1-\frac{1}{2n}\}=1-\frac{1}{2n}\in[0,1] \text{ for all } n\in\mathbb{N}. \tag{2}$$

Hence, $1-\frac{1}{2n}\geq 0.5$ for $n=1\in\mathbb{N}$. This is true for membership degree.

It is clearly seen that $(\Phi^Q,\mathfrak{A})$ is a Q- neutrosophic soft quasigroup over the quasigroup $(\hat{G},\odot,/,\backslash)$.

Lemma 3.3. *Let $(\Lambda^Q,\mathfrak{A})$ be a Q- neutrosophic soft quasigroup over a unipotent quasigroup $(\hat{G},\odot)$. Then, $T_{\Lambda^Q(a)}(\phi,u)\geq T_{\Lambda^Q(a)}(w,u), I_{\Lambda^Q(a)}(\phi,u)\leq I_{\Lambda^Q(a)}(w,u)$ and $F_{\Lambda^Q(a)}(\phi,u)\leq F_{\Lambda^Q(a)}(w,u)$ where ϕ is a fixed element in $\hat{G}$ for all $w\in\hat{G}$ and $a\in\mathfrak{A},u\in Q$.*

Proof. Since for unipotent quasigroup, $w\odot w=\phi$ for all $w\in\hat{G}$. The true membership, falsity membership and indeterminacy membership degrees are respective follow:

$$T_{\Lambda^Q(a)}(\phi,u)=T_{\Lambda^Q(a)}(w\odot w,u)\geq\min\underbrace{\{T_{\Lambda^Q(a)}(w,u),T_{\Lambda^Q(a)}(w,u)\}}_{\text{Using defintion 3.1}}=T_{\Lambda^Q(a)}(w,u).$$

$$I_{\Lambda^Q(a)}(\phi,u)=I_{\Lambda^Q(a)}(w\odot w,u)\leq\max\underbrace{\{I_{\Lambda^Q(a)}(w,u),I_{\Lambda^Q(a)}(w,u)\}}_{\text{using defintion 3.1}}=I_{\Lambda^Q(a)}(w,u).$$

$$F_{\Lambda^Q(a)}(\phi,u)=F_{\Lambda^Q(a)}(w\odot w,u)\leq\max\underbrace{\{F_{\Lambda^Q(a)}(w,u),F_{\Lambda^Q(a)}(w,u)\}}_{\text{By defintion 3.1}}=F_{\Lambda^Q(a)}(w,u).$$

for a fixed element $\phi\in\hat{G}$, for all $w\in\hat{G}$ and $a\in\mathfrak{A},u\in Q$. □

Theorem 3.4. *Let $(\Lambda^Q,\hat{\mathfrak{A}})$ be a Q- neutrosophic soft quasigroup over a quasigroup $(\hat{G},\odot,/,\backslash)$ with ϕ fixed in $\hat{G}$. Then,*

(1) $\mathcal{A} = \{w \in \hat{G} : T_{\Lambda^Q(a)}(w,u) \geq T_{\Lambda^Q(a)}(\phi,u)$ *for all* $u \in Q\}$.

(2) $\mathcal{B} = \{w \in \hat{G} : I_{\Lambda^Q(a)}(w,u) \leq I_{\Lambda^Q(a)}(\phi,u)$ *for all* $u \in Q\}$.

(3) $\mathcal{C} = \{w \in \hat{G} : F_{\Lambda^Q(a)}(w,u) \leq F_{\Lambda^Q(a)}(\phi,u)$ *for all* $u \in Q\}$

are subquasigroups of $(\hat{G}, \odot, /, \backslash)$.

Proof. Let $\mathcal{A}, \mathcal{B}, \mathcal{C} \neq \emptyset$ and $w, t \in \mathcal{A}, \mathcal{B}, \mathcal{C}$ and let $* \in \{\odot, /, \backslash\}$, then by Definition 3.1, we have

$$T_{\Lambda^Q(a)}(w*t,u) \geq \min\{T_{\Lambda^Q(a)}(w,u), T_{\Lambda^Q(a)}(t,u)\} = \min\{T_{\Lambda^Q(a)}(\phi,u), T_{\Lambda^Q(a)}(\phi,u)\} = T_{\Lambda^Q(a)}(\phi,u)$$

$$I_{\Lambda^Q(a)}(w*t,u) \leq \max\{I_{\Lambda^Q(a)}(w,u), I_{\Lambda^Q(a)}(t,u)\} = \max\{I_{\Lambda^Q(a)}(\phi,u), I_{\Lambda^Q(a)}(\phi,u)\} = I_{\Lambda^Q(a)}(\phi,u)$$

$$F_{\Lambda^Q(a)}(w*t,u) \leq \max\{F_{\Lambda^Q(a)}(w,u), F_{\Lambda^Q(a)}(t,u)\} = \max\{F_{\Lambda^Q(a)}(\phi,u), F_{\Lambda^Q(a)}(\phi,u)\} = F_{\Lambda^Q(a)}(\phi,u).$$

Since $T_{\Lambda^Q(a)}(\phi,u) = T_{\Lambda^Q(a)}(t,u), I_{\Lambda^Q(a)}(\phi,u) = I_{\Lambda^Q(a)}(t,u)$ and $F_{\Lambda^Q(a)}(\phi,u) = F_{\Lambda^Q(a)}(t,u)$, for any $t \in \hat{G}$, this implies that $T_{\Lambda^Q(a)}(w*t,u) \geq T_{\Lambda^Q(a)}(\phi,u), I_{\Lambda^Q(a)}(w*t,u) \leq I_{\Lambda^Q(a)}(\phi,u)$ and $F_{\Lambda^Q(a)}(w*t,u) \leq F_{\Lambda^Q(a)}(\phi,u)$. Thus, $w*t \in \mathcal{A}, \mathcal{B}, \mathcal{C}$ for all $w, t \in \mathcal{A}, \mathcal{B}, \mathcal{C}$ and $u \in Q$. □

Corollary 3.5. *Let* $(\Lambda^Q, \hat{\mathfrak{A}})$ *be a Q- neutrosophic soft quasigroup over a unipotent quasigroup* $(\hat{G}, \odot, /, \backslash)$. *Then,* $\mathcal{A} = \mathcal{B} = \mathcal{C} = \hat{G}$.

Proof. Consequence of Theorem 3.4. □

Theorem 3.6. *Let* $(\Phi^Q, \mathfrak{A})$ *be a Q-neutrosophic soft quasigroup under a quasigroup* $(\hat{G}, \odot)$. *Then,* $(\Phi^Q, \mathfrak{A})$ *is a Q-neutrosophic soft idempotent quasigroup for all* $a \in \mathfrak{A}, w \in \hat{G}, u \in Q$. $T_{\Phi^Q(a)}(w^2,u) \geq T_{\Phi^Q(a)}(w,u),\ I_{\Phi^Q(a)}(w^2,u) \leq I_{\Phi^Q(a)}(w,u),\ F_{\Phi^Q(a)}(w^2,u) \leq F_{\Phi^Q(a)}(w,u)$.

Proof. Suppose that $(\Phi^Q, \mathfrak{A})$ is a Q-neutrosophic soft quasigroup over a quasigroup $(\hat{G}, \odot)$, we have

$$T_{\Phi^Q(a)}(w^2,u) = T_{\Phi^Q(a)}(w \odot w,u) \geq \min\{T_{\Phi^Q(a)}(w,u), T_{\Phi^Q(a)}(w,u)\} = T_{\Phi^Q(a)}(w,u).$$

$$I_{\Phi^Q(a)}(w^2,u) = I_{\Phi^Q(a)}(w \odot w,u) \leq \max\{I_{\Phi^Q(a)}(w,u), I_{\Phi^Q(a)}(w,u)\} = I_{\Phi^Q(a)}(w,u).$$

$$F_{\Phi^Q(a)}(w^2,u) = F_{\Phi^Q(a)}(w \odot w,u) \leq \max\{F_{\Phi^Q(a)}(w,u), F_{\Phi^Q(a)}(w,u)\} = F_{\Phi^Q(a)}(w,u).$$

Thus, $(\Phi^Q, \mathfrak{A})$ is a Q-neutrosophic soft idempotent quasigroup for all $a \in \mathfrak{A}, w \in \hat{G}, u \in Q$. □

Theorem 3.7. *Let* $(\Phi^Q, \mathfrak{A})$ *be Q-neutrosophic soft quasigroup over a quasigroup* $(\hat{G}, \odot, /, \backslash)$. *Then, for all* $a \in \mathfrak{A}, w \in \hat{G}, u \in Q$ *the following hold:*

(1) $T_{\Phi^Q(a)}((w^n * w),u) \geq T_{\Phi^Q(a)}(w,u)$.

(2) $I_{\Phi^Q(a)}((w^n * w),u) \leq I_{\Phi^Q(a)}(w,u)$.

(3) $F_{\Phi^Q(a)}((w^n * w),u) \leq F_{\Phi^Q(a)}(w,u)$.

(4) $T_{\Phi^Q(a)}((w * w^n),u) \geq T_{\Phi^Q(a)}(w,u)$.

(5) $I_{\Phi^Q(a)}((w * w^n),u) \leq I_{\Phi^Q(a)}(w,u)$.

(6) $F_{\Phi^Q(a)}((w * w^n),u) \leq F_{\Phi^Q(a)}(w,u)$,

for each $n \in \mathbb{N}$ *and* $* \in \{\odot, /, \backslash\}$.

Proof. Suppose $(\Phi^Q, \mathfrak{A})$ is a $Q - NS\hat{G}$, for all $a \in \mathfrak{A}, w \in \hat{G}, u \in Q$. We proof by induction. Since, Theorem 3.6 is true, that is $T_{\Phi^Q(a)}(w^2, u) \geq T_{\Phi^Q(a)}(w, u), I_{\Phi^Q(a)}(w^2, u) \leq I_{\Phi^Q(a)}(w, u)$, and $F_{\Phi^Q(a)}(w^2, u) \leq F_{\Phi^Q(a)}(w, u)$ we have

(1)

$$\begin{gathered}
T_{\Lambda^Q(a)}(w^2, u) \geq T_{\Lambda^Q(a)}(w, u)\\
T_{\Lambda^Q(a)}(w \odot w^2, u) \geq \min\{T_{\Lambda^Q(a)}(w, u), T_{\Lambda^Q(a)}(w^2, u)\} \geq T_{\Lambda^Q(a)}(w, u)\\
T_{\Lambda^Q(a)}(w \odot (w \odot w^2), u) \geq \min\{T_{\Lambda^Q(a)}(w, u), T_{\Lambda^Q(a)}(w \odot w^2, u)\} \geq T_{\Lambda^Q(a)}(w, u)\\
\vdots\\
T_{\Lambda^Q(a)}(w \odot (w \odot (...(w \odot w^2))), u) \geq T_{\Lambda^Q(a)}(w, u).
\end{gathered}$$

Let $w^n \odot w = \underbrace{w \odot (w \odot (w \odot (w \odot w)))}_{\text{n-times}}$. Then, $T_{\Lambda^Q(a)}(w^n, u) \geq T_{\Lambda^Q(a)}(w, u)$ for all $n \in \mathbb{N}$.

(2)

$$\begin{gathered}
I_{\Lambda^Q(a)}(w^2, u) \leq I_{\Lambda^Q(a)}(w, u)\\
I_{\Lambda^Q(a)}(w \odot w^2, u) \leq \max\{I_{\Lambda^Q(a)}(w, u), I_{\Lambda^Q(a)}(w^2, u)\} \leq I_{\Lambda^Q(a)}(w, u)\\
I_{\Lambda^Q(a)}(w \odot (w \odot w^2), u) \leq \max\{I_{\Lambda^Q(a)}(w, u), I_{\Lambda^Q(a)}(w \odot w^2, u)\} \leq I_{\Lambda^Q(a)}(w, u)\\
\vdots\\
I_{\Lambda^Q(a)}(w \odot (w \odot (...(w \odot w^2))), u) \leq I_{\Lambda^Q(a)}(w, u).
\end{gathered}$$

Let $w^n \odot w = \underbrace{w \odot (w \odot (w \odot (w \odot w)))}_{\text{n-times}}$. Then, $I_{\Lambda^Q(a)}(w^n, u) \geq I_{\Lambda^Q(a)}(w, u)$ for all $n \in \mathbb{N}$.

(3)

$$\begin{gathered}
F_{\Lambda^Q(a)}(w^2, u) \leq F_{\Lambda^Q(a)}(w, u)\\
F_{\Lambda^Q(a)}(w \odot w^2, u) \leq \max\{F_{\Lambda^Q(a)}(w, u), F_{\Lambda^Q(a)}(w^2, u)\} \leq F_{\Lambda^Q(a)}(w, u)\\
F_{\Lambda^Q(a)}(w \odot (w \odot w^2), u) \leq \max\{F_{\Lambda^Q(a)}(w, u), F_{\Lambda^Q(a)}(w \odot w^2, u)\} \leq F_{\Lambda^Q(a)}(w, u)\\
\vdots\\
F_{\Lambda^Q(a)}(w \odot (w \odot (...(w \odot w^2))), u) \leq F_{\Lambda^Q(a)}(w, u).
\end{gathered}$$

Let $w^n \odot w = \underbrace{w \odot (w \odot (w \odot (w \odot w)))}_{\text{n-times}}$. Then, $F_{\Lambda^Q(a)}(w^n, u) \geq F_{\Lambda^Q(a)}(w, u)$ for all $n \in \mathbb{N}$.

This can be show by letting $w \odot w^n = \underbrace{(((w \odot w) \odot w) \odot w) \odot w) \odot w)}_{\text{n-times}}$ and follow step by steps in 1.

This is similar to 2 by letting $w \odot w^n = \underbrace{(((w \odot w) \odot w) \odot w) \odot w) \odot w)}_{\text{n-times}}$ and follow step by steps in 2.

This is similar to 5. ☐

Corollary 3.8. *Let $(\Lambda^Q, \mathfrak{A})$ be a Q- neutrosophic soft subquasigroup of a quasigroup $(\hat{G}, \odot, /, \backslash)$, for all $w \in \hat{G}, u \in Q, a \in, \mathfrak{A}, n \in \mathbb{N}$ and $\alpha, \beta, \gamma, \sigma, \mu, \tau \in \mathbb{N} \cup \{0\}$.*

(1) $\alpha T^{\sigma}_{\Phi^Q(a)}((w^n * w), u) + \beta T^{\tau}_{\Phi^Q(a)}((w^n * w), u) \geq (\alpha + \beta) T^{\sigma+\tau}_{\Phi^Q(a)}(w, u)$.

(2) $\alpha I^{\sigma}_{\Phi^Q(a)}((w^n * w), u) + \beta I^{\tau}_{\Phi^Q(a)}((w^n * w), u) \leq (\alpha + \beta) I^{\sigma+\tau}_{\Phi^Q(a)}(w, u)$.

(3) $\alpha F^{\sigma}_{\Phi^Q(a)}((w^n * w), u) + \beta F^{\tau}_{\Phi^Q(a)}((w^n * w), u) \leq (\alpha + \beta) F^{\sigma+\tau}_{\Phi^Q(a)}(w, u)$.

(4) $T^{\alpha}_{\Phi^Q(a)}((w^n * w), u) T^{\beta}_{\Phi^Q(a)}((w * w^n), u) \geq T^{(\alpha+\beta)}_{\Phi^Q(a)}(w, u)$.

(5) $I^{\alpha}_{\Phi^Q(a)}((w^n * w), u) I^{\beta}_{\Phi^Q(a)}((w * w^n), u) \leq I^{(\alpha+\beta)}_{\Phi^Q(a)}(w, u)$.

(6) $F^{\alpha}_{\Phi^Q(a)}((w^n * w), u) F^{\beta}_{\Phi^Q(a)}((w * w^n), u) \leq F^{(\alpha+\beta)}_{\Phi^Q(a)}(w, u)$.

(7) $\alpha T^{\sigma}_{\Phi^Q(a)}((w^n * w), u) - \beta I^{\tau}_{\Phi^Q(a)}((w^n * w), u) - \gamma F^{\mu}_{\Phi^Q(a)}((w^n * w), u) \geq \alpha T^{\sigma}_{\Phi^Q(a)}(w, u) - \beta I^{\tau}_{\Phi^Q(a)}(w, u) - \beta F^{\mu}_{\Phi^Q(a)}(w, u)$.

(8) $\alpha T^{\sigma}_{\Phi^Q(a)}((w * w^n), u) - \beta I^{\tau}_{\Phi^Q(a)}((w * w^n), u) - \gamma F^{\mu}_{\Phi^Q(a)}((w * w^n), u) \geq \alpha T^{\sigma}_{\Phi^Q(a)}(w, u) - \beta I^{\tau}_{\Phi^Q(a)}(w, u) - \beta F^{\mu}_{\Phi^Q(a)}(w, u)$.

(9) $\alpha_1 T^{\sigma_1}_{\Phi^Q(a)}((w^n * w), u) + \alpha_2 T^{\sigma_2}_{\Phi^Q(a)}((w^n * w), u) - \beta_1 I^{\tau_1}_{\Phi^Q(a)}((w^n * w), u) - \beta_2 I^{\tau_2}_{\Phi^Q(a)}((w^n * w), u) -$
$\gamma_1 F^{\mu_1}_{\Phi^Q(a)}((w^n * w), u) - \gamma_2 I^{\mu}_{2\,\Phi^Q(a)}((w^n * w), u) \geq \alpha_1 T^{\sigma_1}_{\Phi^Q(a)}(w, u) + \alpha_2 T^{\sigma_2}_{\Phi^Q(a)}(w, u) - \beta_1 I^{\tau_1}_{\Phi^Q(a)}(w, u) - \beta_2 I^{\tau_2}_{\Phi^Q(a)}(w, u) - \gamma_1 F^{\mu_1}_{\Phi^Q(a)}(w, u) - \gamma_2 F^{\mu_2}_{\Phi^Q(a)}(w, u)$.

for each $* \in \{\odot, /, \backslash\}$.

Proof. Since $\alpha, \beta, \gamma, \sigma, \mu, \tau \in \mathbb{N} \cup \{0\}$, it follows in Theorem 3.7. ☐

Corollary 3.9. *Let $(\Lambda^Q, \mathfrak{A})$ be a Q- neutrosophic soft subquasigroup of a quasigroup $(\hat{G}, \odot)$.*

(1) *If $(\hat{G}, \odot)$ is a $3-$power associative quasigroup, that is $x \odot x^2 = x^2 \odot x$ then*
  a $T_{\Phi^Q(a)}(w^3, u) \geq T_{\Phi^Q(a)}(w, u)$.
  b $I_{\Phi^Q(a)}(w^3, u) \leq I_{\Phi^Q(a)}(w, u)$.
  c $F_{\Phi^Q(a)}(w^3, u) \leq F_{\Phi^Q(a)}(w, u)$.
  d $\alpha T^{\sigma}_{\Phi^Q(a)}(w^3, u) - \beta I^{\tau}_{\Phi^Q(a)}(w^3, u) - \gamma I^{\mu}_{\Phi^Q(a)}(w^3, u) \geq \alpha T^{\sigma}_{\Phi^Q(a)}(w, u) - \beta I^{\tau}_{\Phi^Q(a)}(w, u) - \gamma I^{\mu}_{\Phi^Q(a)}(w, u)$.

(2) *If $(\hat{G}, \odot)$ is a power associative quasigroup, that is $x^i x^j = x^{i+j}$ for all $i, j \in \mathbb{N}$ then*
  a $T_{\Phi^Q(a)}(w^n, u) \geq T_{\Phi^Q(a)}(w, u)$.
  b $I_{\Phi^Q(a)}(w^n, u) \leq I_{\Phi^Q(a)}(w, u)$.
  c $F_{\Phi^Q(a)}(w^n, u) \leq F_{\Phi^Q(a)}(w, u)$.

d $\alpha T^{\sigma}_{\Phi^Q(a)}(w^n,u)-\beta I^{\tau}_{\Phi^Q(a)}(w^n,u)-\gamma I^{\mu}_{\Phi^Q(a)}(w^n,u) \geq \alpha T^{\sigma}_{\Phi^Q(a)}(w,u)-\beta I^{\tau}_{\Phi^Q(a)}(w,u)-\gamma I^{\mu}_{\Phi^Q(a)}(w,u)$.

Proof. Follows in Corollary 3.8. □

Definition 3.10. Let $(\Lambda^Q,\mathfrak{A})$ be a $Q-$neutrosophic soft quasigroup of a quasigroup $(\hat{G},\odot,/,\backslash)$ and let $\Delta(T,I,F)$ denote the change in degree of memberships for all $a\in\mathfrak{A}, w\in\hat{G}$ and $u\in Q$:

1. $\Delta_{\Lambda^Q(a)}(T,I)=T_{\Lambda^Q(a)}(w,u)-I_{\Lambda^Q(a)}(w,u)$.
2. $\Delta_{\Lambda^Q(a)}(T,F)=T_{\Lambda^Q(a)}(w,u)-f_{\Lambda^Q(a)}(w,u)$.
3. $\Delta_{\Lambda^Q(a)}(I,F)=I_{\Lambda^Q(a)}(w,u)-F_{\Lambda^Q(a)}(w,u)$.

Proposition 3.11. *Let* $(\Lambda^Q,\mathfrak{A})$ *be a Q- neutrosophic soft subquasigroup of a quasigroup* $(\hat{G},\odot,/,\backslash)$. *Then, for all* $a\in\mathfrak{A}, w\in\hat{G}$,

1. $\min\left\{T_{\Lambda^Q(a)}((w*t),u),T_{\Lambda^Q(a)}(w,u)\right\}=\min\left\{T_{\Lambda^Q(a)}((w*t),u),T_{\Lambda^Q(a)}(t,u)\right\}=\min\{T_{\Lambda^Q(a)}(w,u),T_{\Lambda^Q(a)}(t,u)$.
2. $\max\left\{I_{\Lambda^Q(a)}((w*t),u),I_{\Lambda^Q(a)}(w,u)\right\}=\min\left\{I_{\Lambda^Q(a)}((w*t),u),I_{\Lambda^Q(a)}(t,u)\right\}=\max\{I_{\Lambda^Q(a)}(w,u),I_{\Lambda^Q(a)}(t,u)$.
3. $\max\left\{F_{\Lambda^Q(a)}((w*t),u),F_{\Lambda^Q(a)}(w,u)\right\}=\min\left\{F_{\Lambda^Q(a)}((w*t),u),F_{\Lambda^Q(a)}(t,u)\right\}=\max\{F_{\Lambda^Q(a)}(w,u),F_{\Lambda^Q(a)}(t,u)$.

Proof. Let $*\in\{\odot,\backslash,/\}$, then we consider the following cases:

(1) Firstly: $w*t=w\odot t$. Since for all $w,t\in\hat{G}$, $(w\odot t)/t=w$ hold in $\hat{G}$. It follows that

$$\begin{aligned}
T_{\Lambda^Q(a)}(wt/t,u) &\geq \min\{T_{\Lambda^Q(a)}(wt,u),T_{\Lambda^Q(a)}(t,u)\}\\
&\geq \min\left\{\{\min\{T_{\Lambda^Q(a)}(w,u),T_{\Lambda^Q(a)}(t,u)\},T_{\Lambda^Q(a)}(t,u)\right\}\\
&= \min\{T_{\Lambda^Q(a)}(w,u),T_{\Lambda^Q(a)}(t,u)\}\\
\Rightarrow \{T_{\Lambda^Q(a)}(wt/t,u) &\geq \min\{T_{\Lambda^Q(a)}(w,u),T_{\Lambda^Q(a)}(t,u)\}. \qquad (3)
\end{aligned}$$

Secondly: use the equality $w=(w\odot t)/t=(w/t)\odot t$,

$$\begin{gathered}
T_{\Lambda^Q(a)}(w,u)=T_{\Lambda^Q(a)}(wt/t,u)\geq\min\{T_{\Lambda^Q(a)}((w,u),T_{\Lambda^Q(a)}(t,u)\}\\
T_{\Lambda^Q(a)}(w,u)=T_{\Lambda^Q(a)}((w/t)\odot t,u)\geq\min\{T_{\Lambda^Q(a)}(w/t,u),T_{\Lambda^Q(a)}(t,u)\}\\
\geq\min\left\{\min\{T_{\Lambda^Q(a)}(w,u),T_{\Lambda^Q(a)}(t,u)\},T_{\Lambda^Q(a)}(w,u)\right\}\\
\geq\min\{T_{\Lambda^Q(a)}(w,u),T_{\Lambda^Q(a)}(t,u)\}. \qquad (4)
\end{gathered}$$

Thirdly: considering the equality $w=t\backslash(t\odot w)$,

$$\begin{aligned}
T_{\Lambda^{Q}(a)}(w,u) = T_{\Lambda^{Q}(a)}(t\backslash(t\odot w),u) \\
&\geq \min\{T_{\Lambda^{Q}(a)}(t,u),T_{\Lambda^{Q}(a)}(t\odot w,u)\} \\
&\geq \min\left\{\{T_{\Lambda^{Q}(a)}(t,u),\min\{T_{\Lambda^{Q}(a)}(t,u),T_{\Lambda^{Q}(a)}(w,u)\}\right\} \\
&= \min\left\{\min\{T_{\Lambda^{Q}(a)}(t,u),T_{\Lambda^{Q}(a)}(t,u)\},T_{\Lambda^{Q}(a)}(w,u)\}\right\} \\
&= \min\left\{T_{\Lambda^{Q}(a)}(w,u),T_{\Lambda^{Q}(a)}(t,u)\right\} \\
\Rightarrow T_{\Lambda^{Q}(a)}(w,u) = T_{\Lambda^{Q}(a)}(t\backslash(t\odot w),u) \\
&\geq \min\left\{T_{\Lambda^{Q}(a)}(w,u),T_{\Lambda^{Q}(a)}(t,u)\right\}. \qquad (5)
\end{aligned}$$

Similarly, we obtain

$$\begin{aligned}
&T_{\Lambda^{Q}(a)}(w,u) = T_{\Lambda^{Q}(a)}(t\odot(t\backslash w),u) \\
&\geq \min\{T_{\Lambda^{Q}(a)}(t,u),T_{\Lambda^{Q}(a)}(w,u)\}. \qquad (6)
\end{aligned}$$

(2) The result for indeterminate membership degree is consider as follows;

Firstly: $w*t = w\odot t$. Since for all $w,t\in\hat{G}$, $(w\odot t)/t = w$ hold in $\hat{G}$. It follows that

$$\begin{aligned}
I_{\Lambda^{Q}(a)}(wt/t,u) &\leq \max\{I_{\Lambda^{Q}(a)}(wt,u),I_{\Lambda^{Q}(a)}(t,u)\} \\
&\leq \max\left\{\{\max\{I_{\Lambda^{Q}(a)}(w,u),I_{\Lambda^{Q}(a)}(t,u)\},I_{\Lambda^{Q}(a)}(t,u)\right\} \\
&= \max\{I_{\Lambda^{Q}(a)}(w,u),I_{\Lambda^{Q}(a)}(t,u)\} \\
\Rightarrow \{I_{\Lambda^{Q}(a)}(wt/t,u) &\leq \max\{I_{\Lambda^{Q}(a)}(w,u),I_{\Lambda^{Q}(a)}(t,u)\}. \qquad (7)
\end{aligned}$$

Secondly: use the equality $w = (w\odot t)/t = (w/t)\odot t$;

$$\begin{aligned}
I_{\Lambda^{Q}(a)}(w,u) = I_{\Lambda^{Q}(a)}(wt/t,u) \leq \max\{I_{\Lambda^{Q}(a)}((w,u),I_{\Lambda^{Q}(a)}(t,u)\} \\
I_{\Lambda^{Q}(a)}(w,u) = I_{\Lambda^{Q}(a)}((w/t)\odot t,u) \leq \max\{I_{\Lambda^{Q}(a)}(w/t,u),I_{\Lambda^{Q}(a)}(t,u)\} \\
\leq \max\left\{\max\{I_{\Lambda^{Q}(a)}(w,u),I_{\Lambda^{Q}(a)}(t,u)\},I_{\Lambda^{Q}(a)}(w,u)\right\} \\
\leq \max\{I_{\Lambda^{Q}(a)}(w,u),I_{\Lambda^{Q}(a)}(t,u)\}. \qquad (8)
\end{aligned}$$

Thirdly: considering the equality $w = t\backslash(t \odot w)$,

$$\begin{aligned}
I_{\Lambda^{Q(a)}}(w,u) &= I_{\Lambda^{Q(a)}}(t\backslash(t \odot w),u) \\
&\leq \max\{I_{\Lambda^{Q(a)}}(t,u), I_{\Lambda^{Q(a)}}(t \odot w,u)\} \\
&\leq \max\Big\{\{I_{\Lambda^{Q(a)}}(t,u), \max\{I_{\Lambda^{Q(a)}}(t,u), I_{\Lambda^{Q(a)}}(w,u)\}\Big\} \\
&= \max\Big\{\max\{I_{\Lambda^{Q(a)}}(t,u), I_{\Lambda^{Q(a)}}(t,u)\}, I_{\Lambda^{Q(a)}}(w,u)\}\Big\} \\
&= \max\big\{I_{\Lambda^{Q(a)}}(w,u), I_{\Lambda^{Q(a)}}(t,u)\big\} \\
\Rightarrow I_{\Lambda^{Q(a)}}(w,u) = I_{\Lambda^{Q(a)}}(t\backslash(t \odot w),u) & \\
&\leq \max\big\{I_{\Lambda^{Q(a)}}(w,u), I_{\Lambda^{Q(a)}}(t,u)\big\}. \qquad (9)
\end{aligned}$$

Hence,

$$\begin{aligned}
&I_{\Lambda^{Q(a)}}(w,u) = I_{\Lambda^{Q(a)}}(t \odot (t\backslash w),u) \\
&\leq \max\{I_{\Lambda^{Q(a)}}(t,u), I_{\Lambda^{Q(a)}}(w,u)\}. \qquad (10)
\end{aligned}$$

(3) Considering the falsity membership degree

Firstly: $w * t = w \odot t$. Since for all $w, t \in \hat{G}$, $(w \odot t)/t = w$ hold in $\hat{G}$. We obtain,

$$\begin{aligned}
F_{\Lambda^{Q(a)}}(wt/t,u) &\leq \max\{F_{\Lambda^{Q(a)}}(wt,u), F_{\Lambda^{Q(a)}}(t,u)\} \\
&\leq \max\Big\{\{\max\{F_{\Lambda^{Q(a)}}(w,u), F_{\Lambda^{Q(a)}}(t,u)\}, F_{\Lambda^{Q(a)}}(t,u)\Big\} \\
&= \max\{F_{\Lambda^{Q(a)}}(w,u), F_{\Lambda^{Q(a)}}(t,u)\} \\
\Rightarrow \{F_{\Lambda^{Q(a)}}(wt/t,u) &\leq \max\{F_{\Lambda^{Q(a)}}(w,u), F_{\Lambda^{Q(a)}}(t,u)\}. \qquad (11)
\end{aligned}$$

Secondly: use the equality $w = (w \odot t)/t = (w/t) \odot t$

$$\begin{gathered}
F_{\Lambda^{Q(a)}}(w,u) = F_{\Lambda^{Q(a)}}(wt/t,u) \leq \max\{F_{\Lambda^{Q(a)}}((w,u), F_{\Lambda^{Q(a)}}(t,u)\} \\
F_{\Lambda^{Q(a)}}(w,u) = F_{\Lambda^{Q(a)}}((w/t) \odot t,u) \leq \max\{F_{\Lambda^{Q(a)}}(w/t,u), F_{\Lambda^{Q(a)}}(t,u)\} \\
\leq \max\Big\{\max\{F_{\Lambda^{Q(a)}}(w,u), F_{\Lambda^{Q(a)}}(t,u)\}, F_{\Lambda^{Q(a)}}(w,u)\Big\} \\
\leq \max\{F_{\Lambda^{Q(a)}}(w,u), F_{\Lambda^{Q(a)}}(t,u)\}. \qquad (12)
\end{gathered}$$

Thirdly: use the equality $w = t\backslash(t \odot w)$, $F_{\Lambda^{Q}_{(a)}}(w,u) =$

$$\begin{aligned} F_{\Lambda^{Q}_{(a)}}(t\backslash(t \odot w),u) &\leq \max\{F_{\Lambda^{Q}_{(a)}}(t,u), F_{\Lambda^{Q}_{(a)}}(t \odot w,u)\} \\ &\leq \max\left\{\{F_{\Lambda^{Q}_{(a)}}(t,u), \max\{F_{\Lambda^{Q}_{(a)}}(t,u), F_{\Lambda^{Q}_{(a)}}(w,u)\}\right\} \\ &= \max\left\{\max\{F_{\Lambda^{Q}_{(a)}}(t,u), F_{\Lambda^{Q}_{(a)}}(t,u)\}, F_{\Lambda^{Q}_{(a)}}(w,u)\}\right\} \\ &= \max\left\{F_{\Lambda^{Q}_{(a)}}(w,u), F_{\Lambda^{Q}_{(a)}}(t,u)\right\} \end{aligned}$$

$$\begin{aligned} \Rightarrow F_{\Lambda^{Q}_{(a)}}(w,u) = F_{\Lambda^{Q}_{(a)}}(t\backslash(t \odot w),u) & \\ &\leq \max\left\{F_{\Lambda^{Q}_{(a)}}(w,u), F_{\Lambda^{Q}_{(a)}}(t,u)\right\}. \end{aligned} \tag{13}$$

Hence,

$$F_{\Lambda^{Q}_{(a)}}(w,u) = F_{\Lambda^{Q}_{(a)}}(t \odot (t\backslash w),u) \leq \max\{F_{\Lambda^{Q}_{(a)}}(t,u), F_{\Lambda^{Q}_{(a)}}(w,u)\}. \tag{14}$$

□

Corollary 3.12. *Let* $(\Lambda^{Q}, \mathfrak{A})$ *be a Q- neutrosophic soft subquasigroup of a quasigroup* $(\hat{G}, \odot, /, \backslash)$. *Then,*

(1) $T_{\Lambda^{Q}_{(a)}}(wt/t,u) \geq \{T_{\Lambda^{Q}_{(a)}}(w,u) + T_{\Lambda^{Q}_{(a)}}(w,u)\} - \max\{T_{\Lambda^{Q}_{(a)}}(w,u), T_{\Lambda^{Q}_{(a)}}(t,u)\}$.

(2) $T_{\Lambda^{Q}_{(a)}}(wt/t,u) \geq \max\{T_{\Lambda^{Q}_{(a)}}(w,u), T_{\Lambda^{Q}_{(a)}}(t,u)\} - |T_{\Lambda^{Q}_{(a)}}(w,u) - T_{\Lambda^{Q}_{(a)}}(t,u)|$.

(3) $T_{\Lambda^{Q}_{(a)}}((w/t) \odot t,u) \geq T_{\Lambda^{Q}_{(a)}}(w,u) + T_{\Lambda^{Q}_{(a)}}(w,u) - \max\{T_{\Lambda^{Q}_{(a)}}(w,u), T_{\Lambda^{Q}_{(a)}}(t,u)\}$.

(4) $T_{\Lambda^{Q}_{(a)}}((w/t) \odot t,u) \geq \max\{T_{\Lambda^{Q}_{(a)}}(w,u), T_{\Lambda^{Q}_{(a)}}(t,u)\} - |T_{\Lambda^{Q}_{(a)}}(w,u) - T_{\Lambda^{Q}_{(a)}}(t,u)|$.

(5) $T_{\Lambda^{Q}_{(a)}}(t\backslash(t \odot w),u) \geq T_{\Lambda^{Q}_{(a)}}(t,u) + T_{\Lambda^{Q}_{(a)}}(w,u) - \max\{T_{\Lambda^{Q}_{(a)}}(w,u), T_{\Lambda^{Q}_{(a)}}(t,u)$.

(6) $T_{\Lambda^{Q}_{(a)}}(t\backslash(t \odot w),u) \geq \max\{T_{\Lambda^{Q}_{(a)}}(w,u), T_{\Lambda^{Q}_{(a)}}(t,u) - |T_{\Lambda^{Q}_{(a)}}(t,u) - T_{\Lambda^{Q}_{(a)}}(w,u)|$.

(7) $T_{\Lambda^{Q}_{(a)}}(t \odot (t\backslash w),u) \geq T_{\Lambda^{Q}_{(a)}}(t,u) + T_{\Lambda^{Q}_{(a)}}(w,u) - \max\{T_{\Lambda^{Q}_{(a)}}(w,u), T_{\Lambda^{Q}_{(a)}}(t,u)\}$.

(8) $T_{\Lambda^{Q}_{(a)}}(t \odot (t\backslash w),u) \geq \max\{T_{\Lambda^{Q}_{(a)}}(w,u), T_{\Lambda^{Q}_{(a)}}(t,u) - |T_{\Lambda^{Q}_{(a)}}(t,u) - T_{\Lambda^{Q}_{(a)}}(w,u)|$.

(9) $I_{\Lambda^{Q}_{(a)}}(wt/t,u) \leq I_{\Lambda^{Q}_{(a)}}(w,u) + I_{\Lambda^{Q}_{(a)}}(t,u) - \min\{I_{\Lambda^{Q}_{(a)}}(w,u), I_{\Lambda^{Q}_{(a)}}(t,u)\}$.

(10) $I_{\Lambda^{Q}_{(a)}}(wt/t,u) \leq \min\{I_{\Lambda^{Q}_{(a)}}(w,u), I_{\Lambda^{Q}_{(a)}}(t,u)\} + |I_{\Lambda^{Q}_{(a)}}(t,u) - I_{\Lambda^{Q}_{(a)}}(w,u)|$.

(11) $I_{\Lambda^{Q}_{(a)}}((w/t) \odot t,u) \leq I_{\Lambda^{Q}_{(a)}}(w,u) + I_{\Lambda^{Q}_{(a)}}(t,u) - \min\{I_{\Lambda^{Q}_{(a)}}(w,u), I_{\Lambda^{Q}_{(a)}}(t,u)\}$.

(12) $I_{\Lambda^{Q}_{(a)}}((w/t) \odot t,u) \leq \min\{I_{\Lambda^{Q}_{(a)}}(w,u) + I_{\Lambda^{Q}_{(a)}}(t,u)\} + |I_{\Lambda^{Q}_{(a)}}(w,u) - I_{\Lambda^{Q}_{(a)}}(t,u)|$.

(13) $I_{\Lambda^{Q}_{(a)}}(t\backslash(t \odot w),u) \leq I_{\Lambda^{Q}_{(a)}}(w,u) + I_{\Lambda^{Q}_{(a)}}(t,u) - \min\{I_{\Lambda^{Q}_{(a)}}(w,u), I_{\Lambda^{Q}_{(a)}}(t,u)\}$.

(14) $I_{\Lambda^{Q}_{(a)}}(t\backslash(t \odot w),u) \leq \min\{I_{\Lambda^{Q}_{(a)}}(w,u), I_{\Lambda^{Q}_{(a)}}(t,u)\} + |I_{\Lambda^{Q}_{(a)}}(w,u) - I_{\Lambda^{Q}_{(a)}}(t,u)|$.

(15) $I_{\Lambda^{Q}_{(a)}}(t \odot (t\backslash w)),u) \leq I_{\Lambda^{Q}_{(a)}}(w,u) + I_{\Lambda^{Q}_{(a)}}(t,u) - \min\{I_{\Lambda^{Q}_{(a)}}(w,u), I_{\Lambda^{Q}_{(a)}}(t,u)\}$.

(16) $I_{\Lambda^{Q}_{(a)}}(t \odot (t\backslash w)),u) \leq \min\{I_{\Lambda^{Q}_{(a)}}(w,u), I_{\Lambda^{Q}_{(a)}}(t,u)\} + |I_{\Lambda^{Q}_{(a)}}(w,u) - I_{\Lambda^{Q}_{(a)}}(t,u)|$.

(17) $F_{\Lambda^{Q}_{(a)}}(wt/t,u) \leq F_{\Lambda^{Q}_{(a)}}(w,u) + F_{\Lambda^{Q}_{(a)}}(t,u) - \min\{F_{\Lambda^{Q}_{(a)}}(w,u), F_{\Lambda^{Q}_{(a)}}(t,u)\}$.

(18) $F_{\Lambda^{Q}_{(a)}}(wt/t,u) \leq \min\{F_{\Lambda^{Q}_{(a)}}(w,u), F_{\Lambda^{Q}_{(a)}}(t,u)\} + |F_{\Lambda^{Q}_{(a)}}(t,u) - F_{\Lambda^{Q}_{(a)}}(w,u)|$.

(19) $F_{\Lambda^{Q}_{(a)}}((w/t) \odot t,u) \leq F_{\Lambda^{Q}_{(a)}}(w,u) + F_{\Lambda^{Q}_{(a)}}(t,u) - \min\{F_{\Lambda^{Q}_{(a)}}(w,u), F_{\Lambda^{Q}_{(a)}}(t,u)\}$.

(20) $F_{\Lambda^{Q}_{(a)}}((w/t) \odot t,u) \leq \min\{F_{\Lambda^{Q}_{(a)}}(w,u) + F_{\Lambda^{Q}_{(a)}}(t,u)\} + |F_{\Lambda^{Q}_{(a)}}(w,u) - F_{\Lambda^{Q}_{(a)}}(t,u)|$.

(21) $F_{\Lambda Q_{(a)}}(t\backslash(t\odot w),u) \leq F_{\Lambda Q_{(a)}}(w,u) + F_{\Lambda Q_{(a)}}(t,u) - \min\{F_{\Lambda Q_{(a)}}(w,u), F_{\Lambda Q_{(a)}}(t,u)\}$.

(22) $F_{\Lambda Q_{(a)}}(t\backslash(t\odot w),u) \leq \min\{F_{\Lambda Q_{(a)}}(w,u), F_{\Lambda Q_{(a)}}(t,u)\} + |F_{\Lambda Q_{(a)}}(w,u) - F_{\Lambda Q_{(a)}}(t,u)|$.

(23) $F_{\Lambda Q_{(a)}}(t\odot(t\backslash w),u) \leq F_{\Lambda Q_{(a)}}(w,u) + F_{\Lambda Q_{(a)}}(t,u) - \min\{F_{\Lambda Q_{(a)}}(w,u), F_{\Lambda Q_{(a)}}(t,u)\}$.

(24) $F_{\Lambda Q_{(a)}}(t\odot(t\backslash w),u) \leq \min\{F_{\Lambda Q_{(a)}}(w,u), F_{\Lambda Q_{(a)}}(t,u)\} + |F_{\Lambda Q_{(a)}}(w,u) - F_{\Lambda Q_{(a)}}(t,u)|$.

Proof. Add $\max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}$ to (3),

$$\begin{aligned}
&T_{\Lambda Q_{(a)}}(wt/t,u) + \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}\\
&\geq \min\{T_{\Lambda Q_{(a)}}(t,u), T_{\Lambda Q_{(a)}}(w,u)\} + \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}\\
&\Rightarrow T_{\Lambda Q_{(a)}}(wt/t,u) + \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\} \geq T_{\Lambda Q_{(a)}}(w,u) + T_{\Lambda Q_{(a)}}(w,u)\}\\
&\Rightarrow T_{\Lambda Q_{(a)}}(wt/t,u) \geq \{T_{\Lambda Q_{(a)}}(w,u) + T_{\Lambda Q_{(a)}}(w,u)\} - \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}. \qquad (15)
\end{aligned}$$

Subtract $\max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}$ from (3),

$$\begin{aligned}
&T_{\Lambda Q_{(a)}}(wt/t,u) - \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}\\
&\geq \min\{T_{\Lambda Q_{(a)}}(t,u), T_{\Lambda Q_{(a)}}(w,u)\} - \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}\\
&\Rightarrow T_{\Lambda Q_{(a)}}(wt/t,u) - \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\} \geq -|T_{\Lambda Q_{(a)}}(w,u) - T_{\Lambda Q_{(a)}}(t,u)|\\
&\Rightarrow T_{\Lambda Q_{(a)}}(wt/t,u) \geq \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\} - |T_{\Lambda Q_{(a)}}(w,u) - T_{\Lambda Q_{(a)}}(t,u)|. \qquad (16)
\end{aligned}$$

Add $\max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}$ to (4),

$$\begin{aligned}
&T_{\Lambda Q_{(a)}}((w/t)\odot t,u) + \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}\\
&\geq \min\{T_{\Lambda Q_{(a)}}(t,u), T_{\Lambda Q_{(a)}}(w,u)\} + \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}\\
&\Rightarrow T_{\Lambda Q_{(a)}}((w/t)\odot t,u) + \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\} \geq T_{\Lambda Q_{(a)}}(w,u) + T_{\Lambda Q_{(a)}}(w,u)\}\\
&\Rightarrow T_{\Lambda Q_{(a)}}((w/t)\odot t,u) \geq T_{\Lambda Q_{(a)}}(w,u) + T_{\Lambda Q_{(a)}}(w,u) - \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}. \qquad (17)
\end{aligned}$$

Subtract $\max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}$ from (4),

$$\begin{aligned}
&T_{\Lambda Q_{(a)}}((w/t)\odot t,u) - \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}\\
&\geq \min\{T_{\Lambda Q_{(a)}}(t,u), T_{\Lambda Q_{(a)}}(w,u)\} - \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}\\
&\Rightarrow T_{\Lambda Q_{(a)}}((w/t)\odot t,u) - \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\} \geq -|T_{\Lambda Q_{(a)}}(w,u) - T_{\Lambda Q_{(a)}}(t,u)|\\
&\Rightarrow T_{\Lambda Q_{(a)}}((w/t)\odot t,u) \geq \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\} - |T_{\Lambda Q_{(a)}}(w,u) - T_{\Lambda Q_{(a)}}(t,u)|. \qquad (18)
\end{aligned}$$

Similarly, add $\max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u)\}$ to (5) to obtain,

$$T_{\Lambda Q_{(a)}}(t\backslash(t\odot w),u) \geq T_{\Lambda Q_{(a)}}(t,u) + T_{\Lambda Q_{(a)}}(w,u) - \max\{T_{\Lambda Q_{(a)}}(w,u), T_{\Lambda Q_{(a)}}(t,u). \qquad (19)$$

Substrat $\max\{T_{\Lambda^{Q}(a)}(w,u), T_{\Lambda^{Q}(a)}(t,u)\}$ from (5),

$$T_{\Lambda^{Q}(a)}(t\backslash(t\odot w),u) \geq \max\{T_{\Lambda^{Q}(a)}(w,u), T_{\Lambda^{Q}(a)}(t,u) - |T_{\Lambda^{Q}(a)}(t,u) - T_{\Lambda^{Q}(a)}(w,u)|. \tag{20}$$

add $\max\{T_{\Lambda^{Q}(a)}(w,u), T_{\Lambda^{Q}(a)}(t,u)\}$ to (6) to obtain,

$$T_{\Lambda^{Q}(a)}(t\odot(t\backslash w),u) \geq T_{\Lambda^{Q}(a)}(t,u) + T_{\Lambda^{Q}(a)}(w,u) - \max\{T_{\Lambda^{Q}(a)}(w,u), T_{\Lambda^{Q}(a)}(t,u)\}. \tag{21}$$

Substrat $\max\{T_{\Lambda^{Q}(a)}(w,u), T_{\Lambda^{Q}(a)}(t,u)\}$ from (6),

$$T_{\Lambda^{Q}(a)}(t\odot(t\backslash w),u) \geq \max\{T_{\Lambda^{Q}(a)}(w,u), T_{\Lambda^{Q}(a)}(t,u) - |T_{\Lambda^{Q}(a)}(t,u) - T_{\Lambda^{Q}(a)}(w,u)|. \tag{22}$$

Next, we show for indeterminate membership degree.

Add $\min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\}$ to (7) to obtain,

$$\begin{gathered} I_{\Lambda^{Q}(a)}(wt/t,u) + \min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\} \\ \leq \max\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\} + \min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\} \\ \Rightarrow I_{\Lambda^{Q}(a)}(wt/t,u) \leq I_{\Lambda^{Q}(a)}(w,u) + I_{\Lambda^{Q}(a)}(t,u) - \min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\}. \end{gathered} \tag{23}$$

Substrate $\min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\}$ from (7),

$$\begin{gathered} I_{\Lambda^{Q}(a)}(wt/t,u) - \min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\} \\ \leq \max\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\} - \min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\} \\ \Rightarrow I_{\Lambda^{Q}(a)}(wt/t,u) \leq \min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\} + |I_{\Lambda^{Q}(a)}(t,u) - I_{\Lambda^{Q}(a)}(w,u)|. \end{gathered} \tag{24}$$

Add $\min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\}$ to (8),

$$I_{\Lambda^{Q}(a)}((w/t)\odot t,u) \leq I_{\Lambda^{Q}(a)}(w,u) + I_{\Lambda^{Q}(a)}(t,u) - \min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\}. \tag{25}$$

Substrate $\min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\}$ from (8),

$$\begin{gathered} I_{\Lambda^{Q}(a)}((w/t)\odot t,u) \leq \\ \min\{I_{\Lambda^{Q}(a)}(w,u) + I_{\Lambda^{Q}(a)}(t,u)\} + |I_{\Lambda^{Q}(a)}(w,u) - I_{\Lambda^{Q}(a)}(t,u)|. \end{gathered} \tag{26}$$

Add $\min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\}$ to (9),

$$I_{\Lambda^{Q}(a)}(t\backslash(t\odot w),u) \leq I_{\Lambda^{Q}(a)}(w,u) + I_{\Lambda^{Q}(a)}(t,u) - \min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\}.$$

Substrate $\min\{I_{\Lambda^{Q}(a)}(w,u), I_{\Lambda^{Q}(a)}(t,u)\}$ from (9),

$$I_{\Lambda^{Q}(a)}(t\backslash(t \odot w), u)$$
$$\leq \min\{I_{\Lambda^{Q}(a)}(w, u), I_{\Lambda^{Q}(a)}(t, u)\} + |I_{\Lambda^{Q}(a)}(w, u) - I_{\Lambda^{Q}(a)}(t, u)|. \tag{27}$$

Add $\min\{I_{\Lambda^{Q}(a)}(w, u), I_{\Lambda^{Q}(a)}(t, u)\}$ to (10),

$$I_{\Lambda^{Q}(a)}(t \odot (t\backslash w)), u)$$
$$\leq I_{\Lambda^{Q}(a)}(w, u) + I_{\Lambda^{Q}(a)}(t, u) - \min\{I_{\Lambda^{Q}(a)}(w, u), I_{\Lambda^{Q}(a)}(t, u)\}. \tag{28}$$

Substrate $\min\{I_{\Lambda^{Q}(a)}(w, u), I_{\Lambda^{Q}(a)}(t, u)\}$ from (10),

$$I_{\Lambda^{Q}(a)}(t \odot (t\backslash w)), u)$$
$$\leq \min\{I_{\Lambda^{Q}(a)}(w, u), I_{\Lambda^{Q}(a)}(t, u)\} + |I_{\Lambda^{Q}(a)}(w, u) - I_{\Lambda^{Q}(a)}(t, u)|. \tag{29}$$

Similarly, we obtain results for falsity membership degree.

Add $\min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\}$ to (11) to obtain,

$$F_{\Lambda^{Q}(a)}(wt/t, u) + \min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\}$$
$$\leq \max\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\} + \min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\}$$
$$\Rightarrow F_{\Lambda^{Q}(a)}(wt/t, u) \leq F_{\Lambda^{Q}(a)}(w, u) + F_{\Lambda^{Q}(a)}(t, u) - \min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\}, \tag{30}$$

Substrate $\min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\}$ from (11),

$$F_{\Lambda^{Q}(a)}(wt/t, u) - \min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\}$$
$$\leq \max\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\} - \min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\} \Rightarrow$$
$$F_{\Lambda^{Q}(a)}(wt/t, u) \leq \min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\} + |F_{\Lambda^{Q}(a)}(t, u) - F_{\Lambda^{Q}(a)}(w, u)|. \tag{31}$$

Add $\min\{I_{\Lambda^{Q}(a)}(w, u), I_{\Lambda^{Q}(a)}(t, u)\}$ to (12),

$$F_{\Lambda^{Q}(a)}((w/t) \odot t, u) \leq F_{\Lambda^{Q}(a)}(w, u) + F_{\Lambda^{Q}(a)}(t, u) - \min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\}. \tag{32}$$

Substrate $\min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\}$ from (12),

$$F_{\Lambda^{Q}(a)}((w/t) \odot t, u) \leq$$
$$\min\{F_{\Lambda^{Q}(a)}(w, u) + F_{\Lambda^{Q}(a)}(t, u)\} + |F_{\Lambda^{Q}(a)}(w, u) - F_{\Lambda^{Q}(a)}(t, u)|. \tag{33}$$

Add $\min\{F_{\Lambda^{Q}(a)}(w, u), F_{\Lambda^{Q}(a)}(t, u)\}$ to (13),

$$F_{\Lambda^Q(a)}(t\backslash(t\odot w),u)$$
$$\leq F_{\Lambda^Q(a)}(w,u)+F_{\Lambda^Q(a)}(t,u)-\min\{F_{\Lambda^Q(a)}(w,u),F_{\Lambda^Q(a)}(t,u)\}. \tag{34}$$

Substrate $\min\{F_{\Lambda^Q(a)}(w,u),F_{\Lambda^Q(a)}(t,u)\}$ from (13),

$$F_{\Lambda^Q(a)}(t\backslash(t\odot w),u)$$
$$\leq \min\{F_{\Lambda^Q(a)}(w,u),F_{\Lambda^Q(a)}(t,u)\}+|F_{\Lambda^Q(a)}(w,u)-F_{\Lambda^Q(a)}(t,u)|. \tag{35}$$

Add $\min\{I_{\Lambda^Q(a)}(w,u),I_{\Lambda^Q(a)}(t,u)\}$ to (14),

$$F_{\Lambda^Q(a)}(t\odot(t\backslash w)),u)$$
$$\leq F_{\Lambda^Q(a)}(w,u)+F_{\Lambda^Q(a)}(t,u)-\min\{F_{\Lambda^Q(a)}(w,u),F_{\Lambda^Q(a)}(t,u)\}. \tag{36}$$

Substrate $\min\{F_{\Lambda^Q(a)}(w,u),F_{\Lambda^Q(a)}(t,u)\}$ from (14),

$$F_{\Lambda^Q(a)}(t\odot(t\backslash w)),u)$$
$$\leq \min\{F_{\Lambda^Q(a)}(w,u),F_{\Lambda^Q(a)}(t,u)\}+|F_{\Lambda^Q(a)}(w,u)-F_{\Lambda^Q(a)}(t,u)|. \tag{37}$$

□

Definition 3.13. Let $(\Phi^Q,\mathfrak{A})$ be a $Q-$neutrosophic soft set defined over a groupoid $(\hat{G},\cdot)$. Then, $(\Phi^Q,\mathfrak{A})$ is called a Q-neutrosophic soft groupoid if for all $a\in\mathfrak{A},u\in Q,\Phi^Q(a)$ satisfies the following conditions:

(1) $T_{\Phi^Q(a)}(w\odot t,u)\geq\min\{T_{\Phi^Q(a)}(w,u),T_{\Phi^Q(a)}(t,u)\}$.
(2) $I_{\Phi^Q(a)}(w\odot t,u)\leq\max\{I_{\Phi^Q(a)}(w,u),I_{\Phi^Q(a)}(t,u)\}$.
(3) $F_{\Phi^Q(a)}(w\odot t,u)\leq\max\{F_{\Phi^Q(a)}(w,u),F_{\Phi^Q(a)}(t,u)\}$.

Theorem 3.14. *Let $(\hat{G},\odot,/,\backslash)$ be a quasigroup and $(\Phi^Q,\mathfrak{A})$ be a Q- neutrosophic soft set. Then, $(\Phi^Q,\mathfrak{A})$ is a Q- neutrosophic soft quasigroup over $(\hat{G},\odot,/,\backslash)$ if and only if the following hold:*

(1) $(\Phi^Q,\mathfrak{A})_{(\hat{G},\odot)}$ *is a $Q-$neutrosophic soft groupoid over* $(\hat{G},\odot)$.
(2) $(\Phi^Q,\mathfrak{A})_{(\hat{G},/)}$ *is a $Q-$neutrosophic soft groupoid over* $(\hat{G},/)$.
(3) $(\Phi^Q,\mathfrak{A})_{(\hat{G},\backslash)}$ *is a $Q-$neutrosophic soft groupoid over* $(\hat{G},\backslash)$.

Proof. Suppose that $(\Phi^Q,\mathfrak{A})$ is a Q- neutrosophic soft quasigroup over a quasigroup $(\hat{G},\odot,/,\backslash)$, then $(\Phi^Q,\mathfrak{A})_{(\hat{G},\odot)}$, $(\Phi^Q,\mathfrak{A})_{(\hat{G},/)}$ and $(\Phi^Q,\mathfrak{A})_{(\hat{G},\backslash)}$ are $Q-$neutrosophic soft groupoids.

Suppose that $(\Phi^Q, \mathfrak{A})_{(\hat{G},\odot)}$, $(\Phi^Q, \mathfrak{A})_{(\hat{G},/)}$ and $(\Phi^Q, \mathfrak{A})_{(\hat{G},\backslash)}$ are $Q-$neutrosophic soft qroupoids. It suffice to show that $\Phi^Q(a)$ is Q- neutrosophic soft quasigroup over $(\hat{G}, \odot, /, \backslash)$ for each $a \in \mathfrak{A}$. Now, let $s, t \in \Phi^Q(a)$ for all $a \in \mathfrak{A}$. Since $s, t \in \hat{G}$ there is a unique element $w \in \hat{G}$ such that

$$\left.\begin{aligned} T_{\Phi^Q(a)}(s \odot w, u) \geq \min\{T_{\Phi^Q(a)}(s, u), T_{\Phi^Q(a)}(w, u)\} = T_{\Phi^Q(a)}(t, u) \\ I_{\Phi^Q(a)}(s \odot w, u) \leq \max\{I_{\Phi^Q(a)}(s, u), I_{\Phi^Q(a)}(w, u)\} = I_{\Phi^Q(a)}(t, u) \\ F_{\Phi^Q(a)}(s \odot w, u) \leq \max\{F_{\Phi^Q(a)}(s, u), F_{\Phi^Q(a)}(w, u)\} = F_{\Phi^Q(a)}(t, u) \end{aligned}\right\} \tag{38}$$

But note that $s\backslash t = w$, then

$$T_{\Phi^Q(a)}(w, u) = T_{\Phi^Q(a)}(s\backslash t, u) \geq \min\{T_{\Phi^Q(a)}(s, u), T_{\Phi^Q(a)}(t, u)\}$$

$$I_{\Phi^Q(a)}(w, u) = I_{\Phi^Q(a)}(s\backslash t, u) \leq \max\{I_{\Phi^Q(a)}(s, u), I_{\Phi^Q(a)}(t, u)\}$$

$$F_{\Phi^Q(a)}(w, u) = F_{\Phi^Q(a)}(s\backslash t, u) \leq \max\{F_{\Phi^Q(a)}(s, u), F_{\Phi^Q(a)}(t, u)\}$$

So, $w \in \Phi^Q(a)$ for each $a \in \mathfrak{A}$ since $s, t \in \Phi^Q(a)$ and $(\Phi^Q, \mathfrak{A})_{(\hat{G},\backslash)}$ is a $Q-$neutrosophic soft qroupoid. The fact that w is the only element in $\Phi^Q(a)$ so that equation 38 is obvious, since $(\hat{G}, \odot)$ is a quasigroup. Next, we show that there is a unique $f \in \Phi^Q(a)$ so that

$$\left.\begin{aligned} T_{\Phi^Q(a)}(f \odot s, u) \geq \min\{T_{\Phi^Q(a)}(f, u), T_{\Phi^Q(a)}(s, u)\} = T_{\Phi^Q(a)}(t, u) \\ I_{\Phi^Q(a)}(t \odot s, u) \leq \max\{I_{\Phi^Q(a)}(f, u), I_{\Phi^Q(a)}(s, u)\} = I_{\Phi^Q(a)}(t, u) \\ F_{\Phi^Q(a)}(t \odot s, u) \leq \max\{F_{\Phi^Q(a)}(f, u), F_{\Phi^Q(a)}(s, u)\} = F_{\Phi^Q(a)}(t, u) \end{aligned}\right\} \tag{39}$$

Using equation (39), with the fact $f \odot s = t$ then

$$\left.\begin{aligned} T_{\Phi^Q(a)}(f, u) = T_{\Phi^Q(a)}(t/s, u) \geq \min\{T_{\Phi^Q(a)}(t, u), T_{\Phi^Q(a)}(s, u)\} \\ I_{\Phi^Q(a)}(f, u) = I_{\Phi^Q(a)}(t/s, u) \leq \max\{I_{\Phi^Q(a)}(t, u), I_{\Phi^Q(a)}(s, u)\} \\ F_{\Phi^Q(a)}(f, u) = F_{\Phi^Q(a)}(t/s, u) \leq \max\{F_{\Phi^Q(a)}(t, u), F_{\Phi^Q(a)}(s, u)\} \end{aligned}\right\} \tag{40}$$

So, $f \in \Phi^Q(a)$ for each $a \in \mathfrak{A}$ since $s, t \in \Phi^Q(a)$ and $(\Phi^Q, \mathfrak{A})_{(\hat{G},/)}$ is also $Q-$neutrosophic soft qroupoid. This means that f is only the element in $\Phi^Q(a)$ so that equation (40) hold. Thus, $(\Phi^Q, \mathfrak{A})$ is a Q- neutrosophic soft quasigroup over $(\hat{G}, \odot, /, \backslash)$.

□

Theorem 3.15. *Let* $(\Lambda^Q, \mathfrak{A})$ *and* $(\Theta^Q, \mathfrak{B})$ *be two Q-neutrosophic soft quasigroups over* $(\hat{G}, \odot, /, \backslash)$. *Then,* $(\Lambda^Q, \mathfrak{A}) \cap (\Theta_Q, \mathfrak{B})$ *is also a Q-neutrosophic soft quasigroup over* $(\hat{G}, \odot, /, \backslash)$.

Proof. Suppose that $(\Lambda^Q, \mathfrak{A})$ and $(\Theta_Q, \mathfrak{B})$ are Q-neutrosophic soft quasigroups over quasigroup $(\hat{G}, *)$ where $* = \{\odot, /, \backslash\}$, then we shall show that $(\Lambda^Q, \mathfrak{A}) \cap (\Theta_Q, \mathfrak{B}) = (\Delta_Q, \mathfrak{A} \cap \mathfrak{B})$ is a Q-neutrosophic soft quasigroup. Since $(\Lambda^Q, \mathfrak{A})$ and $(\Theta_Q, \mathfrak{B})$ are Q-neutrosophic soft quasigroups over quasigroup $(\hat{G}, *)$, then

$$T_{\Lambda^Q(c)}(w * t, u) \geq \min\{T_{\Lambda^Q(c)}(w, u), T_{\Lambda^Q(c)}(t, u)\} \text{ where } c \in \mathfrak{C} = \mathfrak{A} \cap \mathfrak{B} \tag{41}$$

for all $w, t \in \hat{G}$, $u \in Q$ and $c \in \mathfrak{A} \cap \mathfrak{B}$

And

$$T_{\Theta_Q(c)}(w * t, u) \geq \min\{T_{\Theta_Q(c)}(w, u), T_{\Theta_Q(c)}(t, u)\} \text{ where } c \in \mathfrak{C} = \mathfrak{A} \cap \mathfrak{B} \tag{42}$$

for each $* = \{\odot, /, \backslash\}$. Combining equations (41) and (42), recall that

$$\begin{aligned}
T_{\Delta_Q(c)}(w * t, u) &= \min\{T_{\Lambda^{Q}(c)}(w * t, u), T_{\Theta_Q(c)}(w * t, u)\}, \text{ then} \\
T_{\Delta_Q(c)}(w * t, u) &\geq \min\left\{\min\{T_{\Lambda^{Q}(c)}(w, u), T_{\Lambda^{Q}(c)}(t, u)\}, \min\{T_{\Theta_Q(c)}(w, u), T_{\Theta_Q(c)}(t, u)\}\right\} \\
&= \min \underbrace{\left\{\min\{T_{\Lambda^{Q}(c)}(w, u), T_{\Theta_Q(c)}(w, u)\}, \min\{T_{\Lambda^{Q}(c)}(t, u), T_{\Theta_Q(c)}(t, u)\}\right\}}_{\text{Since } c \in \mathfrak{A} \text{ and } c \in \mathfrak{B}, \text{ then } c \in \mathfrak{A} \cap \mathfrak{B}.} \\
&= \min \underbrace{\{T_{\Delta_Q(c)}(w, u), T_{\Delta_Q(c)}(t, u)\}}_{c \in \mathfrak{A} \cap \mathfrak{B}.}.
\end{aligned}$$

Recall that

$$\begin{aligned}
I_{\Delta_Q(c)}(w * t, u) &= \max\{I_{\Lambda^{Q}(c)}(w * t, u), I_{\Theta_Q(c)}(w * t, u)\}, \text{ then} \\
I_{\Delta_Q(c)}(w * t, u) &\leq \max\left\{\max\{I_{\Lambda^{Q}(c)}(w, u), I_{\Lambda^{Q}(c)}(t, u)\}, \max\{I_{\Theta_Q(c)}(w, u), I_{\Theta_Q(c)}(t, u)\}\right\} \\
&\leq \max \underbrace{\left\{\max\{I_{\Lambda^{Q}(c)}(w, u), I_{\Theta_Q(c)}(w, u)\}, \max\{I_{\Lambda^{Q}(c)}(t, u), I_{\Theta_Q(c)}(t, u)\}\right\}}_{\text{Since } c \in \mathfrak{A} \text{ and } c \in \mathfrak{B}, \text{ then } c \in \mathfrak{A} \cap \mathfrak{B}.} \\
&\leq \max \underbrace{\{I_{\Delta_Q(c)}(w, u), I_{\Delta_Q(c)}(w, u)\}}_{c \in \mathfrak{A} \cap \mathfrak{B}.}
\end{aligned}$$

Recall that

$$\begin{aligned}
I_{\Delta_Q(c)}(w * t, u) &= \max\{I_{\Lambda^{Q}(c)}(w * t, u), I_{\Theta_Q(c)}(w * t, u)\}, \text{ then} \\
I_{\Delta_Q(c)}(w * t, u) &\leq \max\left\{\max\{F_{\Lambda^{Q}(c)}(w, u), F_{\Lambda^{Q}(c)}(t, u)\}, \max\{F_{\Theta_Q(c)}(w, u), F_{\Theta_Q(c)}(t, u)\}\right\} \\
&\leq \max \underbrace{\left\{\max\{F_{\Lambda^{Q}(c)}(w, u), F_{\Theta_Q(c)}(w, u)\}, \max\{F_{\Lambda^{Q}(c)}(t, u), F_{\Theta_Q(c)}(t, u)\}\right\}}_{\text{Since } c \in \mathfrak{A} \text{ and } c \in \mathfrak{B}, \text{ then } c \in \mathfrak{A} \cap \mathfrak{B}.} \\
&\leq \max \underbrace{\{F_{\Delta_Q(c)}(w, u), F_{\Delta_Q(c)}(t, u)\}}_{c \in \mathfrak{A} \cap \mathfrak{B}.}.
\end{aligned}$$

Therefore, $(\Delta_Q, \mathfrak{A} \cap \mathfrak{B})$ is a $Q-$neutrosophic soft quasigroups over quasigroup $(\hat{G}, \odot, \backslash, /)$. □

Remark: The example bellow show that if $(\Lambda^Q, \mathfrak{A})$ and $(\Theta_Q, \mathfrak{B})$ are Q- neutrosophic soft quasigroups, then $(\Lambda^Q, \mathfrak{A}) \cup (\Theta_Q, \mathfrak{B})$ is not necessarily a Q- neutrosophic soft quasigroup.

Example 3.16. Using table 1, let $w = b, t = h$, for all $u \in Q$, we have

$$T_{\Delta_Q(c)}(w \odot t, u) = T_{\Delta_Q(c)}(g, u) = \max\{T_{\Lambda^{Q}(c)}(g, u), T_{\Phi^{Q}(c)}(g, u)\} = \{0, 0\} = 0$$

And

$$T_{\Delta_Q(c)}(b\odot h,u)=\min\left\{\max\{T_{\Lambda^Q(a)}(b,u),T_{\Lambda^Q(a)}(h,u)\},\max\{T_{\Phi^Q(b)}(b,u),T_{\Phi^Q(b)}(h,u)\}\right\}$$

$$=\min\left\{\max\{T_{\Lambda^Q(a)}(b,u),T_{\Phi^Q(b)}(b,u)\},\max\{T_{\Lambda^Q(c)}(h,u),T_{\Phi^Q(c)}(h,u)\}\right\}$$

$$=\min\left\{\max\{T_{\Lambda^Q(a)}(0.70,u),T_{\Phi^Q(b)}(0.65,u)\},\max\{T_{\Lambda^Q(a)}(0.70,u),T_{\Phi^Q(b)}(0.0,u)\}\right\}$$

$$=\min\left\{T_{\Lambda^Q(a)}(0.70,u),\{T_{\Lambda^Q(a)}(0.70,u)\right\}=(0.70,u).$$

Therefore,
$T_{\Delta_Q(c)}(wt,u)=0<\min\left\{T_{\Lambda^Q(a)}(0.70,u),\{T_{\Lambda^Q(a)}(0.70,u)\right\}=0.70$. Thus, $(\Lambda^Q,\mathfrak{A})\cup(\Phi^Q,\mathfrak{B})$ is not a Q- neutrosophic soft quasigroup.

Definition 3.17. Let $(\nabla^Q,\mathfrak{A})$, $(\Psi^Q,\mathfrak{B})$ and $(\Pi^Q,\mathfrak{C})$ be $Q-$neutrosophic soft quasigroups over a quasigroup $(\hat{G},\odot,/,\backslash)$ such that $\mathfrak{C}=\mathfrak{A}\cap\mathfrak{B}$. Then,

(1) The product $(\nabla^Q,\mathfrak{A})\odot(\Psi^Q,\mathfrak{B})=(\Pi^Q,\mathfrak{C})$ over $(\hat{G},\odot)$ is defined by

$$\Pi^Q(c)=\left\{\langle(((t,u),T_{\nabla^Q(c)}((t,u),I_{\nabla^Q(c)}(t,u),F_{\nabla^Q(c)}((t,u)\rangle:w,f\in\hat{G},u\in Q\right\}$$

where

$$T_{\Pi^Q(c)}(w\odot f,u)\geq\min\{T_{\nabla^Q(a)}(w,u),T_{\Psi^Q(b)}(f,u)\},$$
$$I_{\Pi^Q(c)}(w\odot f,u)\leq\max\{I_{\nabla^Q(a)}(w,u),I_{\Psi^Q(b)}(f,u)\},$$
$$F_{\Pi^Q(c)}(w\odot f,u)\leq\max\{F_{\nabla^Q(a)}(w,u),F_{\Psi^Q(b)}(f,u))\}.$$

(2) The right division $(\nabla^Q,\mathfrak{A})/(\Psi^Q,\mathfrak{B})=(\Pi^Q,\mathfrak{C})$ over $(\hat{G},/)$ is defined by

$$\begin{aligned}T_{\Pi^Q(c)}(w/f,u)&\geq\max\{T_{\nabla^Q(a)}(w,u),T_{\Psi^Q(b)}(f,u)\},\\I_{\Pi^Q(c)}(w/f,u)&\leq\min\{I_{\nabla^Q(a)}(w,u),I_{\Psi^Q(b)}(f,u)\},\\I_{\Pi^Q(c)}(w/f,u)&\leq\min\{I_{\nabla^Q(a)}(w,u),I_{\Psi^Q(b)}(f,u)\}.\end{aligned}$$

(3) The left division $(\nabla^Q,\mathfrak{A})\backslash(\Psi^Q,\mathfrak{B})=(\Pi^Q,\mathfrak{C})$ over $(\hat{G},\backslash)$ is defined by

$$\begin{aligned}T_{\Pi^Q(c)}(w\backslash f,u)&\geq\max\{T_{\Psi^Q(a)}(w,u),T_{\nabla^Q(b)}(f,u)\},\\I_{\Pi^Q(c)}(w\backslash f,u)&\leq\min\{I_{\Psi^Q(a)}(w,u),I_{\nabla^Q(b)}(f,u)\},\\F_{\Pi^Q(c)}(w\backslash f,u)&\leq\min\{F_{\Psi^Q(a)}(w,u),F_{\nabla^Q(b)}(f,u)\}.\end{aligned}$$

Theorem 3.18. *Let $(\Phi^Q,\mathfrak{A})$ and $(\Psi^Q,\mathfrak{B})$ be two Q-neutrosophic soft quasigroups over entropic quasigroup $(\hat{G},\odot,/,\backslash)$. Then*

(1) $(\Phi^Q,\mathfrak{A})\odot(\Psi^Q,\mathfrak{B})$ *is a Q-neutrosophic soft subquasigroup over* $(\hat{G},\odot)$.

(2) $(\Phi^Q, \mathfrak{A})/(\Psi, \mathfrak{B})$ *is a Q-neutrosophic soft subquasigroup over* $(\hat{G}, /)$.

(3) $(\Phi^Q, \mathfrak{A})\backslash(\Psi, \mathfrak{B})$ *is a Q-neutrosophic soft subquasigroup over* $(\hat{G}, \backslash)$.

Proof. (1) Let $(\Phi^Q, \mathfrak{A}) \odot (\Psi_Q, \mathfrak{B}) = (\Lambda, \mathfrak{C})$ and $\Phi^Q(a) \odot \Psi_Q(b) = \Pi^Q(c)$, such that $\mathfrak{C} = \mathfrak{A} \cap \mathfrak{B}$. Let $(z_1, z_2) \in \hat{G}$ such that $z_1 = w_1 \odot t_1$ and $z_2 = w_2 \odot t_2$ for all $w_1, w_2, t_1, t_2 \in \hat{G}$. Then,

$$T_{\Lambda^Q(c)}(z_1 \odot z_2, u) = T_{\Lambda^Q(c)}\big((w_1t_1) \odot (w_2t_2), u\big)$$
$$= T_{\Lambda^Q(c)}[(w_1t_1, u) \odot (w_2t_2, u)] = T_{\Lambda^Q(c)}[(w_1w_2, u) \odot (t_1t_2, u)]$$
$$\geq \min\{T_{\Phi^Q(a)}(w_1w_2, u), T_{\Psi_Q(b)}(t_1t_2, u)\}$$
$$\geq \min\Big\{ \min\big\{T_{\Phi^Q(a)}(w_1, u), T_{\Phi^Q(a)}(w_2, u)\}, \min\{T_{\Psi_Q(b)}(t_1, u), T_{\Psi_Q(b)}(t_2, u)\}\Big\}$$
$$= \min\Big\{\Big[\min\big\{T_{\Phi^Q(a)}(w_2, u), T_{\Phi^Q(a)}(w_1, u)\}, T_{\Psi_Q(b)}(t_1, u)\Big], T_{\Psi_Q(b)}(t_2, u)\}\Big\}$$
$$= \min\Big\{\Big[T_{\Phi^Q(a)}(w_2, u), \min\big\{T_{\Phi^Q(a)}(w_1, u), T_{\Psi_Q(b)}(t_1, u)\}\Big], T_{\Psi_Q(b)}(t_2, u)\}\Big\}$$
$$= \min\Big\{ \min\big\{T_{\Phi^Q(a)}(w_1, u), T_{\Psi_Q(b)}(t_1, u)\}, \min\{T_{\Phi^Q(a)}(w_2, u), T_{\Psi_Q(b)}(t_2, u)\}\Big\}$$
$$= \min\{T_{\Lambda^Q(c)}(w_1 \odot t_1, u), T_{\Lambda^Q(c)}(w_2 \odot t_2, u)$$
$$= \min\{T_{\Lambda^Q(c)}(z_1, u), T_{\Lambda^Q(c)}(z_2, u).$$

The indeterminate menbership degree,

$$I_{\Lambda^Q(c)}(z_1 \odot z_2, u) = I_{\Lambda^Q(c)}\big((w_1t_1) \odot (w_2t_2), u\big)$$
$$= I_{\Lambda^Q(c)}[(w_1t_1, u)(w_2t_2, u)] = I_{\Lambda^Q(c)}[(w_1w_2, u)(t_1t_2, u)]$$
$$\leq \max\{I_{\Phi^Q(a)}(w_1w_2, u), I_{\Psi_Q(b)}(t_1t_2, u)\}$$
$$\leq \max\Big\{ \max\big\{I_{\Phi^Q(a)}(w_1, u), I_{\Phi^Q(a)}(w_2, u)\}, \max\{I_{\Psi_Q(b)}(t_1, u), I_{\Psi_Q(b)}(t_2, u)\}\Big\}$$
$$= \max\Big\{\Big[\max\big\{I_{\Phi^Q(a)}(w_2, u), I_{\Phi^Q(a)}(w_1, u)\}, I_{\Psi_Q(b)}(t_1, u)\Big], I_{\Psi_Q(b)}(t_2, u)\}\Big\}$$
$$= \max\Big\{\Big[I_{\Phi^Q(a)}(w_2, u), \max\big\{I_{\Phi^Q(a)}(w_1, u)\}, I_{\Psi_Q(b)}(t_1, u)\Big], I_{\Psi_Q(b)}(t_2, u)\}\Big\}$$
$$= \max\Big\{ \max\big\{I_{\Phi^Q(a)}(w_1, u), I_{\Psi_Q(b)}(t_1, u)\}, \max\{I_{\Phi^Q(a)}(w_2, u), I_{\Psi_Q(b)}(t_2, u)\}\Big\}$$
$$= \max\{I_{\Lambda^Q(c)}(w_1 \odot t_1, u), I_{\Lambda^Q(c)}(w_2 \odot t_2, u)$$
$$= \max\{I_{\Lambda^Q(c)}(z_1, u), I_{\Lambda^Q(c)}(z_2, u).$$

The falsity membership degree,

(2) Let $(\Phi^Q, A)/(\Psi, B) = (\Lambda, C)$ such that $\Phi^Q(a)/\Psi_Q(b) = \Lambda^Q(c)$.

Let $z_1, z_2 \in \hat{G}$, for all $w_1, w_2, t_1, t_2 \in \hat{G}, z_1 = w_1/t_1$ and $z_2 = w_2/t_2$.

$$T_{\Lambda^Q(c)}(z_1/z_2, u) = T_{\Lambda^Q(c)}\big((w_1/t_1)/(w_2/t_2), u\big)$$
$$= T_{\Lambda^Q(c)}\big((w_1/w_2, u)/(t_1/t_2, u)\big) \geq \max\{T_{\Phi^Q(a)}(w_1/w_2, u), T_{\Psi_Q(b)}(t_1/t_2, u)\}.$$

This follow from the last equality,

$$\begin{aligned}
\geq \quad & \max\Big\{\max\big\{T_{\Phi^Q(a)}(w_1, u), T_{\Phi^Q(a)}(w_2, u)\}, \max\{T_{\Psi_Q(b)}(t_1, u), T_{\Psi_Q(b)}(t_2, u)\}\Big\} \\
= \quad & \max\Big\{\Big[\max\big\{T_{\Phi^Q(a)}(w_1, u), T_{\Phi^Q(a)}(w_2, u)\}, T_{\Psi_Q(b)}(t_2, u)\Big], T_{\Psi_Q(b)}(t_1, u)\}\Big\} \\
= \quad & \max\Big\{\Big[T_{\Phi^Q(a)}(w_1, u), \max\big\{T_{\Phi^Q(a)}(w_2, u), T_{\Psi_Q(b)}(t_2, u)\big\}\Big], T_{\Psi_Q(b)}(t_1, u)\}\Big\} \\
= \quad & \max\Big\{\max\big\{T_{\Phi^Q(a)}(w_1, u), T_{\Psi_Q(b)}(t_1, u)\big\} \max\big\{T_{\Phi^Q(a)}(w_2, u), T_{\Psi_Q(b)}(t_2, u)\big\}\Big\} \\
= \quad & \max\Big\{\max\big\{T_{\Phi^Q(a)}(w_1, u), T_{\Psi_Q(b)}(t_1, u)\}, \max\{T_{\Phi^Q(a)}(w_2, u), T_{\Psi_Q(b)}(t_2, u)\}\Big\}. \\
= \quad & \max\big\{T_{\Lambda^Q(c)}((z_1, u), T_{\Lambda^Q(c)}((z_2, u)\}.
\end{aligned}$$

The indeterminate membership degree is given by

$$I_{\Lambda^Q(c)}(z_1/z_2, u) = I_{\Lambda^Q(c)}\big((w_1/t_1)/(w_2/t_2), u\big)$$
$$= I_{\Lambda^Q(c)}\big((w_1/w_2, u)/(t_1/t_2, u)\big) \leq \min\{I_{\Phi^Q(a)}(w_1/w_2, u), I_{\Psi_Q(b)}(t_1/t_2, u)\}.$$

From the last equality, we obtain

$$\begin{aligned}
\leq \quad & \min\Big\{\min\big\{I_{\Phi^Q(a)}(w_1, u), I_{\Phi^Q(a)}(w_2, u)\}, \min\{I_{\Psi_Q(b)}(t_1, u), I_{\Psi_Q(b)}(t_2, u)\}\Big\} \\
= \quad & \min\Big\{\Big[\min\big\{I_{\Phi^Q(a)}(w_1, u), I_{\Phi^Q(a)}(w_2, u)\}, I_{\Psi_Q(b)}(t_2, u)\Big], I_{\Psi_Q(b)}(t_1, u)\}\Big\} \\
= \quad & \min\Big\{\Big[I_{\Phi^Q(a)}(w_1, u), \min\big\{I_{\Phi^Q(a)}(w_2, u), I_{\Psi_Q(b)}(t_2, u)\big\}\Big], I_{\Psi_Q(b)}(t_1, u)\}\Big\} \\
= \quad & \min\Big\{\min\big\{I_{\Phi^Q(a)}(w_1, u), I_{\Psi_Q(b)}(t_1, u)\big\} \min\big\{I_{\Phi^Q(a)}(w_2, u), I_{\Psi_Q(b)}(t_2, u)\big\}\Big\} \\
= \quad & \min\Big\{\min\big\{I_{\Phi^Q(a)}(w_1, u), I_{\Psi_Q(b)}(t_1, u)\}, \min\{I_{\Phi^Q(a)}(w_1, u), I_{\Psi_Q(b)}(t_2, u)\}\Big\} \\
= \quad & \min\big\{I_{\Lambda^Q(c)}((z_1, u), I_{\Lambda^Q(c)}((z_2, u)\}.
\end{aligned}$$

The proof of falsity membership degree is similar to the evidence of indeterminate membership degree.

(3) Follow similar steps to the proof (2).

□

4. An Application of $Q-$neutrosophic Soft Quasigroups

Let

$$\begin{aligned}\Psi^Q(a) &= \text{Truth, indeterminate, and falsity membership degrees,}\\ \mathfrak{A} &= \text{Clinical parameters (e.g., symptoms, biomarkers, diagnostic indicators),}\\ Q &= \text{Treatment or diagnostic policy options,}\\ \hat{G} &= \text{Set of patients, medical cases, or clinical profiles.}\end{aligned}$$

In this context, the mappings

$$T_{\Phi Q},\ I_{\Phi Q},\ F_{\Phi Q} : X \times Q \longrightarrow [0,1]$$

represent the degrees to which each clinical parameter supports, contradicts, or remains uncertain regarding a particular treatment policy $q \in Q$ for a patient $x \in \hat{G}$. These degrees satisfy $0 \leq T_{\Phi Q} + I_{\Phi Q} + F_{\Phi Q} \leq 3^+$.

The quasigroup structure on $\hat{G}$ models the clinical interaction among patients or clinical profiles—for example, interaction of comorbidities, treatment compatibility, or patient-specific response patterns. Using this structure, the Q-neutrosophic soft set $\Psi^Q(a)$ captures uncertain and indeterminate clinical information for every parameter $a \in \mathfrak{A}$.

To compute the truth, indeterminate, and falsity membership degrees in the medical framework, we use Definition 3.1, Definition 3.17 and Theorem 3.18 to guide the construction of the following algorithm.

Algorithm for Medical Decision-Making Under Uncertainty

Step 1: Construct a groupoid of finite order representing basic clinical interactions or patient-state transformations.

Step 2: Construct a quasigroup of finite order to model structured clinical operations such as treatment interaction, comorbidity influence, or diagnostic pathways.

Step 3: Construct two Q-neutrosophic soft quasigroups $(\Phi^Q, \mathfrak{A})$ and $(\Psi^Q, \mathfrak{B})$ over the finite quasigroup $\hat{G}$, representing two independent medical information sources such as diagnostic findings vs. treatment guidelines.

Step 4: Apply Theorem 3.18 to compute the combined structure

$$(\Pi, \mathfrak{C}) = (\Phi^Q, \mathfrak{A}) \odot (\Psi^Q, \mathfrak{B}),$$

where $a_i \odot b_j = c_k$ denotes the medically meaningful fusion (e.g., diagnostic-treatment interaction) producing a unique clinical state $c_k \in \hat{G}$.

Step 5: Using Step 4, compute $\Pi_{Q(e_i e_j)}$ for all $i \neq j$, describing how combined clinical states influence the neutrosophic membership values.

Step 6: Construct the medical comparison table with the evaluation score $T_{\Pi^Q(c)} + I_{\Pi^Q(c)} - F_{\Pi^Q(c)}$, which quantifies the net clinical suitability of each treatment policy for each patient profile.

Step 7: Compute the total clinical score $S_{(x,q)} \in (\hat{G} \times Q)$ for every patient-treatment pair by summing the numerical grades.

Step 8: Identify the highest numerical score associated with each clinical parameter pair, representing the best-supported treatment option for each diagnostic configuration.

Step 9: The recommended medical decision (optimal treatment or diagnostic policy) corresponds to any element of $M = \max_{(x,q)\in \hat{G}\times Q} S(x,q)$, which gives the most suitable treatment option under uncertainty and indeterminacy.

Example 4.1. Two physicians collaborate to determine the most suitable treatment for a newly diagnosed patient by combining uncertain, incomplete, and indeterminate clinical information.The set of possible treatment options is the quasigroup $\widehat{G} = \{a, b, c, d, e, f, g, h\}$, equipped with a binary operation $\odot$ whose Cayley table defines the quasigroup structure. Suppose that two consulting physicians $\mathfrak{A}$ and $\mathfrak{B}$ are collaborating to select the most suitable treatment protocol for a newly diagnosed patient. Let $Q = \{q_1, q_2, q_3, q_4, q_5\}$ the set of clinical guidelines governing treatment selection, and $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ be the set of clinical parameters, where

e_1 = expected improvement in patient symptoms,

e_2 = treatment safety profile compared to standard care,

e_3 = physician expertise required for administration,

e_4 = compatibility with patient comorbidities,

e_5 = long-term efficacy and stability,

e_6 = cost-effectiveness of the treatment.

The physicians evaluate treatments according to their prioritized parameter sets:

$$\mathfrak{A} = \{e_1, e_2, e_5\}, \qquad \mathfrak{B} = \{e_3, e_4, e_6\}.$$

Table 2. Given a quasigroup $(\hat{G}) = \{a, b, c, d, e, f, g, h, \}$ of order 8 with the relative operation $(\hat{G}, \odot)$ shown in the Cayley table below such that $(ge)h \neq g(eh)$.

⊙	a	b	c	d	e	f	g	h
a	a	b	c	d	e	f	g	h
b	b	a	d	c	f	e	h	g
c	c	d	a	b	h	g	e	f
d	d	c	b	a	g	h	f	e
e	e	f	g	h	a	b	c	d
f	f	e	h	g	b	a	d	c
g	g	h	e	f	d	c	a	b
h	h	g	f	e	c	d	b	a

Table 3. Given a groupoid $(Q) = \{q_1, q_2, q_3, q_4, q_5\}$ of order 5 representing 5 different policies with their mode of operations $(Q, +)$ is shown in Cayley table below.

$+$	q_1	q_2	q_3	q_4	q_5
q_1	q_1	q_2	q_3	q_4	q_5
q_2	q_2	q_1	q_3	q_5	q_4
q_3	q_3	q_5	q_4	q_2	q_1
q_4	q_4	q_2	q_1	q_3	q_5
q_5	q_5	q_2	q_3	q_1	q_4

$$(\Psi^Q, \mathfrak{A}) = \left\{ \begin{array}{lcl} \Phi^Q(e_1) & = & \{[(a, q_1)\frac{9}{10}, \frac{7}{10}, \frac{2}{5}],\ [(b, q_2)\frac{4}{5}, \frac{1}{2}, \frac{7}{10}],\ [(e, q_3)\frac{9}{10}, \frac{7}{10}, \frac{1}{5}],\ [(h, q_4)\frac{9}{10}, \frac{7}{10}, \frac{1}{5}]\} \\ \Phi^Q(e_2) & = & \{[(a, q_2)\frac{3}{10}, \frac{1}{2}, \frac{1}{5}],\ [(b, q_3)\frac{9}{10}, \frac{3}{5}, \frac{2}{5}],\ [(f, q_4)\frac{1}{10}, \frac{1}{2}, \frac{7}{10}],\ [(g, q_5)\frac{3}{5}, \frac{7}{10}, \frac{9}{10}]\} \\ \Phi^Q(e_5) & = & \{[(b, q_1)\frac{9}{10}, \frac{3}{5}, \frac{3}{10}],\ [(e, q_3)\frac{3}{5}, \frac{3}{10}, \frac{7}{10}],\ [(f, q_4)\frac{1}{2}, \frac{2}{5}, \frac{9}{10}],\ [(h, q_4)\frac{1}{2}, \frac{2}{5}, \frac{9}{10}]\} \end{array} \right\}.$$

$$(\Psi^Q, \mathfrak{B}) = \left\{ \begin{array}{lcl} \Psi^Q(e_3) & = & \{[(c, q_1)\frac{1}{10}, \frac{2}{5}, \frac{1}{2}],\ [(e, q_3)\frac{7}{10}, \frac{3}{10}, \frac{1}{5}],\ [(f, q_4)\frac{4}{5}, \frac{3}{10}, \frac{3}{5}],\ [(g, q_5)\frac{4}{5}, \frac{3}{10}, \frac{3}{5}]\} \\ \Psi^Q(e_4) & = & \{[(a, q_2)\frac{1}{10}, \frac{3}{5}, \frac{3}{10}],\ [(b, q_3)\frac{1}{10}, \frac{3}{10}, \frac{1}{2}],\ [(c, q_4)\frac{7}{10}, \frac{1}{10}, \frac{9}{10}],\ [(d, q_5)\frac{7}{10}, \frac{1}{2}, \frac{9}{10}]\} \\ \Psi^Q(e_6) & = & \{[(c, q_1)\frac{3}{10}, \frac{2}{5}, \frac{3}{5}],\ [(d, q_2)\frac{9}{10}, \frac{1}{10}, \frac{3}{5}],\ [(f, q_4)\frac{3}{5}, \frac{1}{5}, \frac{3}{10}],\ [(g, q_5)\frac{1}{2}, \frac{1}{5}, \frac{3}{5}]\}. \end{array} \right\}$$

Next, compute step 2 to get a $Q-$neutrosophic soft quasigroup $(\Pi^Q, \mathfrak{C})$ such that $(\Psi^Q, \mathfrak{A}) \odot (\Psi^Q, \mathfrak{B}) = (\Pi^Q, \mathfrak{C})$ under a quasigroup of order 8. Then, $\Pi^Q(e_ie_j) = \Psi^Q(e_i) \odot \Psi^Q(e_j)$, for all $e_i \in \mathfrak{A}, e_j \in \mathfrak{B}$.

$$\begin{aligned}
\Pi^Q(e_1e_3) \quad = \quad & \Big\{\langle[(c,q_1)\tfrac{1}{10},\tfrac{7}{10},\tfrac{1}{2}],[(e,q_3)\tfrac{7}{10},\tfrac{7}{10},\tfrac{2}{5}],[(f,q_4)\tfrac{4}{5},\tfrac{7}{10},\tfrac{3}{5}],[(g,q_5)\tfrac{9}{10},\tfrac{7}{10},\tfrac{3}{5}],\\
& [(d,q_2)\tfrac{1}{10},\tfrac{1}{2},\tfrac{7}{10}],[(f,q_3)\tfrac{7}{10},\tfrac{1}{2},\tfrac{7}{10}],[(e,q_5)\tfrac{4}{5},\tfrac{1}{2},\tfrac{7}{10}],[(h,q_2)\tfrac{4}{5},\tfrac{1}{2},\tfrac{7}{10}],\\
& [(g,q_3)\tfrac{1}{10},\tfrac{7}{10},\tfrac{2}{5}],[(a,q_4)\tfrac{9}{10},\tfrac{7}{10},\tfrac{1}{5}],[(b,q_2)\tfrac{4}{5},\tfrac{7}{10},\tfrac{7}{10}],[(c,q_1)\tfrac{4}{5},\tfrac{7}{10},\tfrac{3}{5}],\\
& [(f,q_4)\tfrac{1}{10},\tfrac{7}{10},\tfrac{1}{2}],[(c,q_1)\tfrac{9}{10},\tfrac{7}{10},\tfrac{1}{5}],[(d,q_3)\tfrac{9}{10},\tfrac{7}{10},\tfrac{3}{5}],[(b,q_5)\tfrac{4}{5},\tfrac{7}{10},\tfrac{3}{5}]\rangle\Big\}
\end{aligned}$$

$$\begin{aligned}
\Pi^Q(e_1e_4) \quad = \quad & \Big\{\langle[(a,q_5)\tfrac{1}{10},\tfrac{7}{10},\tfrac{2}{5}],[(b,q_3)\tfrac{1}{10},\tfrac{7}{10},\tfrac{1}{2}],[(c,q_4)\tfrac{7}{10},\tfrac{7}{10},\tfrac{9}{10}],[(d,q_1)\tfrac{7}{10},\tfrac{7}{10},\tfrac{2}{5}],\\
& [(b,q_1)\tfrac{1}{10},\tfrac{3}{5},\tfrac{7}{10}],[(a,q_3)\tfrac{1}{10},\tfrac{1}{2},\tfrac{7}{10}],[(d,q_5)\tfrac{7}{10},\tfrac{1}{2},\tfrac{9}{10}],[(c,q_4)\tfrac{7}{10},\tfrac{1}{2},\tfrac{9}{10}],\\
& [(e,q_5)\tfrac{1}{10},\tfrac{7}{10},\tfrac{3}{10}],[(f,q_4)\tfrac{1}{10},\tfrac{7}{10},\tfrac{1}{2}],[(g,q_2)\tfrac{7}{10},\tfrac{7}{10},\tfrac{9}{10}],[(h,q_1)\tfrac{7}{10},\tfrac{7}{10},\tfrac{9}{10}],\\
& [(h,q_2)\tfrac{1}{10},\tfrac{7}{10},\tfrac{3}{10}],[(g,q_1)\tfrac{1}{10},\tfrac{7}{10},\tfrac{1}{2}],[(f,q_3)\tfrac{7}{10},\tfrac{7}{10},\tfrac{9}{10}],[(e,q_5)\tfrac{7}{10},\tfrac{7}{10},\tfrac{9}{10}]\rangle\Big\}.
\end{aligned}$$

$$\begin{aligned}
\Pi^Q(e_1e_6) \quad = \quad & \Big\{\langle[(c,q_1)\tfrac{3}{10},\tfrac{7}{10},\tfrac{1}{2}],[(d,q_2)\tfrac{9}{10},\tfrac{7}{10},\tfrac{3}{5}],[(f,q_4)\tfrac{3}{5},\tfrac{7}{10},\tfrac{2}{5}],[(g,q_5)\tfrac{1}{2},\tfrac{7}{10},\tfrac{3}{5}],\\
& [(d,q_2)\tfrac{3}{10},\tfrac{1}{2},\tfrac{7}{10}],[(e,q_1)\tfrac{4}{5},\tfrac{1}{2},\tfrac{7}{10}],[(e,q_5)\tfrac{3}{5},\tfrac{1}{2},\tfrac{7}{10}],[(h,q_2)\tfrac{1}{2},\tfrac{1}{2},\tfrac{7}{10}],\\
& [(g,q_3)\tfrac{3}{10},\tfrac{7}{10},\tfrac{3}{5}],[(h,q_5)\tfrac{9}{10},\tfrac{7}{10},\tfrac{3}{5}],[(b,q_1)\tfrac{3}{5},\tfrac{7}{10},\tfrac{3}{10}],[(c,q_3)\tfrac{1}{2},\tfrac{7}{10},\tfrac{3}{5}],\\
& [(f,q_4)\tfrac{3}{10},\tfrac{7}{10},\tfrac{3}{5}],[(e,q_2)\tfrac{9}{10},\tfrac{7}{10},\tfrac{3}{5}],[(d,q_3)\tfrac{3}{5},\tfrac{7}{10},\tfrac{3}{10}],[(b,q_5)\tfrac{1}{2},\tfrac{7}{10},\tfrac{3}{5}]\rangle\Big\}.
\end{aligned}$$

$$\begin{aligned}
\Pi^Q(e_2e_3) \quad = \quad & \Big\{\langle[(c,q_2)\tfrac{1}{10},\tfrac{1}{2},\tfrac{1}{2}],[(e,q_3)\tfrac{3}{10},\tfrac{1}{2},\tfrac{1}{5}],[(f,q_4)\tfrac{4}{5},\tfrac{7}{10},\tfrac{3}{5}],[(g,q_5)\tfrac{9}{10},\tfrac{7}{10},\tfrac{3}{5}],\\
& [(d,q_2)\tfrac{1}{10},\tfrac{1}{2},\tfrac{7}{10}],[(f,q_3)\tfrac{7}{10},\tfrac{1}{2},\tfrac{7}{10}],[(e,q_5)\tfrac{4}{5},\tfrac{1}{2},\tfrac{7}{10}],[(h,q_2)\tfrac{4}{5},\tfrac{1}{2},\tfrac{7}{10}],\\
& [(g,q_3)\tfrac{1}{10},\tfrac{7}{10},\tfrac{2}{5}],[(a,q_4)\tfrac{9}{10},\tfrac{7}{10},\tfrac{1}{5}],[(b,q_2)\tfrac{4}{5},\tfrac{7}{10},\tfrac{7}{10}],[(c,q_1)\tfrac{4}{5},\tfrac{7}{10},\tfrac{3}{5}],\\
& [(f,q_4)\tfrac{1}{10},\tfrac{7}{10},\tfrac{1}{2}],[(c,q_1)\tfrac{9}{10},\tfrac{7}{10},\tfrac{1}{5}],[(d,q_3)\tfrac{9}{10},\tfrac{7}{10},\tfrac{3}{5}],[(b,q_5)\tfrac{4}{5},\tfrac{7}{10},\tfrac{3}{5}]\rangle\Big\}.
\end{aligned}$$

$$\begin{aligned}
\Pi^Q(e_2e_4) \quad = \quad & \Big\{\langle[(a,q_1)\tfrac{1}{10},\tfrac{3}{5},\tfrac{3}{10}],[(b,q_3)\tfrac{1}{10},\tfrac{1}{2},\tfrac{1}{2}],[(c,q_5)\tfrac{3}{10},\tfrac{1}{2},\tfrac{9}{10}],[(d,q_4)\tfrac{3}{10},\tfrac{1}{2},\tfrac{9}{10}],\\
& [(b,q_5)\tfrac{1}{10},\tfrac{3}{5},\tfrac{2}{5}],[(a,q_4)\tfrac{1}{10},\tfrac{3}{10},\tfrac{1}{2}],[(d,q_2)\tfrac{7}{10},\tfrac{3}{5},\tfrac{9}{10}],[(c,q_1)\tfrac{7}{10},\tfrac{3}{5},\tfrac{9}{10}],\\
& [(f,q_2)\tfrac{1}{10},\tfrac{3}{5},\tfrac{7}{10}],[(e,q_1)\tfrac{1}{10},\tfrac{1}{2},\tfrac{7}{10}],[(h,q_3)\tfrac{1}{10},\tfrac{1}{2},\tfrac{9}{10}],[(g,q_5)\tfrac{1}{10},\tfrac{1}{2},\tfrac{9}{10}],\\
& [(g,q_2)\tfrac{1}{10},\tfrac{7}{10},\tfrac{9}{10}],[(h,q_1)\tfrac{1}{10},\tfrac{7}{10},\tfrac{9}{10}],[(e,q_3)\tfrac{3}{5},\tfrac{7}{10},\tfrac{9}{10}],[(f,q_5)\tfrac{3}{5},\tfrac{7}{10},\tfrac{9}{10}]\rangle\Big\}.
\end{aligned}$$

$$\begin{aligned}\Pi^{Q}(e_1e_6) \;=\; & \Big\{\langle[(c,q_1)\tfrac{3}{10},\tfrac{1}{2},\tfrac{3}{5}],[(d,q_2)\tfrac{3}{10},\tfrac{1}{2},\tfrac{3}{5}],[(f,q_5)\tfrac{3}{10},\tfrac{1}{2},\tfrac{3}{10}],[(g,q_4)\tfrac{3}{10},\tfrac{1}{2},\tfrac{3}{5}],\\ & [(d,q_1)\tfrac{3}{10},\tfrac{3}{5},\tfrac{3}{5}],[(c,q_5)\tfrac{9}{10},\tfrac{3}{5},\tfrac{3}{5}],[(e,q_2)\tfrac{3}{5},\tfrac{3}{5},\tfrac{2}{5}],[(h,q_1)\tfrac{1}{2},\tfrac{3}{5},\tfrac{3}{5}],\\ & [(h,q_1)\tfrac{1}{10},\tfrac{1}{2},\tfrac{7}{10}],[(g,q_2)\tfrac{1}{10},\tfrac{1}{2},\tfrac{7}{10}],[(a,q_3)\tfrac{1}{10},\tfrac{1}{2},\tfrac{7}{10}],[(d,q_5)\tfrac{1}{10},\tfrac{1}{2},\tfrac{7}{10}],\\ & [(e,q_5)\tfrac{3}{10},\tfrac{7}{10},\tfrac{9}{10}],[(f,q_2)\tfrac{3}{5},\tfrac{7}{10},\tfrac{9}{10}],[(c,q_1)\tfrac{3}{5},\tfrac{7}{10},\tfrac{9}{10}],[(a,q_4)\tfrac{1}{2},\tfrac{7}{10},\tfrac{9}{10}]\rangle\Big\}.\end{aligned}$$

$$\begin{aligned}\Pi^{Q}(e_5e_3) \;=\; & \Big\{\langle[(d,q_1)\tfrac{1}{10},\tfrac{2}{5},\tfrac{3}{10}],[(f,q_3)\tfrac{7}{10},\tfrac{3}{5},\tfrac{3}{10}],[(e,q_4)\tfrac{4}{5},\tfrac{3}{5},\tfrac{3}{5}],[(h,q_5)\tfrac{4}{5},\tfrac{1}{2},\tfrac{3}{5}],\\ & [(g,q_3)\tfrac{1}{10},\tfrac{2}{5},\tfrac{7}{10}],[(a,q_4)\tfrac{3}{5},\tfrac{3}{10},\tfrac{7}{10}],[(b,q_2)\tfrac{3}{5},\tfrac{3}{10},\tfrac{7}{10}],[(c,q_4)\tfrac{3}{5},\tfrac{3}{10},\tfrac{7}{10}],\\ & [(h,q_4)\tfrac{1}{10},\tfrac{2}{5},\tfrac{9}{10}],[(b,q_1)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}],[(a,q_3)\tfrac{1}{2},\tfrac{9}{10},\tfrac{2}{5}],[(c,q_5)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}],\\ & [(f,q_5)\tfrac{1}{10},\tfrac{2}{5},\tfrac{9}{10}],[(c,q_3)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}],[(d,q_1)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}],[(b,q_4)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}]\rangle\Big\}.\end{aligned}$$

$$\begin{aligned}\Pi^{Q}(e_5e_4) \;=\; & \Big\{\langle[(b,q_2)\tfrac{1}{10},\tfrac{3}{5},\tfrac{3}{5}],[(a,q_3)\tfrac{1}{10},\tfrac{3}{5},\tfrac{1}{2}],[(d,q_4)\tfrac{7}{10},\tfrac{3}{5},\tfrac{9}{10}],[(c,q_5)\tfrac{4}{5},\tfrac{3}{5},\tfrac{3}{5}],\\ & [(e,q_5)\tfrac{1}{10},\tfrac{3}{5},\tfrac{7}{10}],[(f,q_4)\tfrac{1}{10},\tfrac{3}{10},\tfrac{7}{10}],[(g,q_2)\tfrac{3}{5},\tfrac{3}{10},\tfrac{9}{10}],[(h,q_1)\tfrac{3}{5},\tfrac{1}{2},\tfrac{9}{10}],\\ & [(a,q_2)\tfrac{1}{10},\tfrac{3}{5},\tfrac{9}{10}],[(e,q_1)\tfrac{1}{10},\tfrac{2}{5},\tfrac{9}{10}],[(h,q_3)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}],[(g,q_5)\tfrac{1}{2},\tfrac{1}{2},\tfrac{9}{10}],\\ & [(h,q_2)\tfrac{1}{10},\tfrac{3}{5},\tfrac{9}{10}],[(g,q_1)\tfrac{1}{10},\tfrac{2}{5},\tfrac{9}{10}],[(f,q_3)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}],[(e,q_5)\tfrac{1}{2},\tfrac{1}{2},\tfrac{9}{10}]\rangle\Big\}.\end{aligned}$$

$$\begin{aligned}\Pi^{Q}(e_5e_6) \;=\; & \Big\{\langle[(d,q_3)\tfrac{3}{10},\tfrac{3}{5},\tfrac{3}{5}],[(c,q_2)\tfrac{9}{10},\tfrac{3}{5},\tfrac{9}{10}],[(e,q_4)\tfrac{3}{5},\tfrac{3}{5},\tfrac{3}{10}],[(h,q_5)\tfrac{1}{2},\tfrac{3}{5},\tfrac{3}{5}],\\ & [(g,q_3)\tfrac{3}{10},\tfrac{2}{5},\tfrac{7}{10}],[(h,q_5)\tfrac{3}{5},\tfrac{3}{10},\tfrac{7}{10}],[(b,q_2)\tfrac{3}{5},\tfrac{3}{10},\tfrac{3}{10}],[(c,q_1)\tfrac{1}{2},\tfrac{3}{10},\tfrac{7}{10}],\\ & [(h,q_4)\tfrac{3}{10},\tfrac{2}{5},\tfrac{9}{10}],[(g,q_2)\tfrac{3}{10},\tfrac{2}{5},\tfrac{9}{10}],[(a,q_3)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}],[(d,q_5)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}],\\ & [(f,q_5)\tfrac{3}{10},\tfrac{2}{5},\tfrac{9}{10}],[(e,q_2)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}],[(d,q_1)\tfrac{1}{2},\tfrac{2}{5},\tfrac{9}{10}],[(b,q_5)\tfrac{1}{2},\tfrac{1}{2},\tfrac{9}{10}]\rangle\Big\}.\end{aligned}$$

Nest, We compute the comparison table to obtain the highest numerical grade for each column.

Compute the sum of these numerical grades score $S_{(x,q)} \in (\hat{G} \times Q)$ of each object $\Pi_{Q(e_ie_j)}(x, q_k)$ for all $k = 1, ..., 5)$.

Table 4. Compute for treatment a

$(\hat{G}\times Q)$	(e_1e_3)	(e_1e_4)	(e_1e_6)	(e_2e_3)	(e_2e_4)	(e_2e_6)	(e_5e_3)	(e_5e_4)	(e_5e_6)
(a,q_1)	–	–	–	–	$\frac{2}{5}$	–	–	–	–
(a,q_2)	–	–	–	–	–	–	–	$-\frac{1}{5}$	–
(a,q_3)	–	–	–	–	–	$-\frac{1}{10}$	1	$\frac{1}{5}$	0
(a,q_4)	$\frac{7}{5}$	$-\frac{1}{10}$	–	$\frac{7}{5}$	$-\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{5}$	–	–
(a,q_5)	–	–	–	–	–	–	–	–	–

Table 5. Compute for treatment b

$(\hat{G}\times Q)$	(e_1e_3)	(e_1e_4)	(e_1e_6)	(e_2e_3)	(e_2e_4)	(e_2e_6)	(e_5e_3)	(e_5e_4)	(e_5e_6)
(b,q_1)	–	0	1	–	–	–	0	–	–
(b,q_2)	8	–	–	8	–	–	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{3}{5}$
(b,q_3)	–	–	–	–	$\frac{1}{10}$	–	–	–	–
(b,q_4)	–	–	–	–	–	–	0	–	–
(b,q_5)	$\frac{9}{10}$	–	$\frac{3}{5}$	$\frac{9}{10}$	$\frac{3}{10}$	–	–	–	$\frac{1}{10}$

Table 6. Compute for treatment c

$(\hat{G}\times Q)$	(e_1e_3)	(e_1e_4)	(e_1e_6)	(e_2e_3)	(e_2e_4)	(e_2e_6)	(e_5e_3)	(e_5e_4)	(e_5e_6)
(c,q_1)	$\frac{3}{10},\ \frac{9}{10},\ \frac{7}{5}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{9}{10}$	$\frac{2}{5}$	$\frac{1}{5},\ \frac{2}{5}$	–	–	$\frac{1}{10}$
(c,q_2)	–	–	–	$\frac{1}{10}$	–	–	–	–	$\frac{3}{5}$
(c,q_3)	–	–	$\frac{3}{5}$	$\frac{7}{5}$	–	–	0	–	–
(c,q_4)	–	$\frac{3}{10},\ \frac{1}{2}$	–	–	–	–	$\frac{1}{5}$	–	–
(c,q_5)	–	–	–	–	$-\frac{1}{10}$	$\frac{9}{10}$	0	$\frac{4}{5}$	–

Table 7. Compute for treatment d

$(\hat{G}\times Q)$	(e_1e_3)	(e_1e_4)	(e_1e_6)	(e_2e_3)	(e_2e_4)	(e_2e_6)	(e_5e_3)	(e_5e_4)	(e_5e_6)
(d,q_1)	–	1	–	–	–	$\frac{3}{10}$	$0,\ \frac{1}{5}$	–	0
(d,q_2)	$-\frac{1}{10}$	1	1	$-\frac{1}{10}$	$\frac{2}{5}$	$\frac{1}{5}$	–	–	–
(d,q_3)	$\frac{1}{10}$	$\frac{1}{10},\ 1$	1	–	–	–	–	–	$\frac{3}{10}$
(d,q_4)	–	–	–	–	$-\frac{1}{10}$	–	–	$\frac{2}{5}$	–
(d,q_5)	–	$\frac{3}{10}$	–	–	–	$-\frac{1}{10}$	–	–	0

Table 8. Compute for treatment e

$(\hat{G}\times Q)$	(e_1e_3)	(e_1e_4)	(e_1e_6)	(e_2e_3)	(e_2e_4)	(e_2e_6)	(e_5e_3)	(e_5e_4)	(e_5e_6)
(e,q_1)	–	–	$\frac{3}{5}$	–	$-\frac{1}{10}$	–	–	$-\frac{2}{5}$	0
(e,q_2)	–	–	1	–	–	$\frac{4}{5}$	–	–	0
(e,q_3)	1	–	–	$\frac{3}{10}$	–	–	–	–	–
(e,q_4)	–	–	–	–	–	–	$\frac{4}{5}$	–	$\frac{4}{5}$
(e,q_5)	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{1}{2}$	$\frac{1}{10}$	–	–	$\frac{1}{10},\ 0$

Table 9. Compute for treatment f

$(\hat{G} \times Q)$	(e_1e_3)	(e_1e_4)	(e_1e_6)	(e_2e_3)	(e_2e_4)	(e_2e_6)	(e_5e_3)	(e_5e_4)	(e_5e_6)
(f, q_1)	–	–	$\frac{3}{5}$	–	–	–	–	–	–
(f, q_2)	–	–	–	–	0	$\frac{2}{5}$	–	–	–
(f, q_3)	$\frac{1}{2}$	–	–	$\frac{1}{2}$	–	–	$\frac{2}{5}$	0	–
(f, q_4)	$\frac{9}{10}, \frac{3}{10}$	$\frac{3}{10}$	$\frac{9}{10}, \frac{2}{5}$	$\frac{9}{10}, \frac{3}{10}$	–	–	–	–	$-\frac{3}{10}$
(f, q_5)	–	–	–	–	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{2}{5}$	$\frac{1}{10}$	$-\frac{1}{5}$

Table 10. Compute for treatment g

$(\hat{G} \times Q)$	(e_1e_3)	(e_1e_4)	(e_1e_6)	(e_2e_3)	(e_2e_4)	(e_2e_6)	(e_5e_3)	(e_5e_4)	(e_5e_6)
(g, q_1)	–	–	–	–	–	–	–	$-\frac{7}{10}$	–
(g, q_2)	–	$\frac{1}{2}$	–	–	$-\frac{1}{10}$	$-\frac{1}{10}$	–	0	$-\frac{1}{5}$
(g, q_3)	$\frac{2}{5}$	–	$\frac{2}{5}$	$\frac{2}{5}$	–	–	$\frac{1}{5}$	–	0
(g, q_4)	–	–	–	–	–	$\frac{1}{5}$	–	–	–
(g, q_5)	1	–	$\frac{1}{2}$	1	$-\frac{3}{10}$	–	–	–	–

Table 11. Compute for treatment h

$(\hat{G} \times Q)$	(e_1e_3)	(e_1e_4)	(e_1e_6)	(e_2e_3)	(e_2e_4)	(e_2e_6)	(e_5e_3)	(e_5e_4)	(e_5e_6)
(h, q_1)	–	$\frac{1}{2}$	–	–	$-\frac{1}{10}$	$\frac{1}{2}, -\frac{1}{10}$	–	$\frac{1}{5}$	–
(h, q_2)	$\frac{3}{5}$	–	$\frac{3}{10}$	$\frac{3}{5}$	–	–	–	$-\frac{3}{10}$	–
(h, q_3)	–	–	–	–	$-\frac{3}{10}$	–	0	–	–
(h, q_4)	–	–	–	–	–	–	$\frac{2}{5}$	–	$-\frac{1}{5}$
(h, q_5)	–	–	1	–	–	–	$\frac{7}{10}$	–	$-\frac{1}{2}, \frac{1}{5}$

Table 12. Compute $\sum_{\Pi^Q(e_i \odot e_j)}(x, q_k)$ for all k=1,..., 5

$\hat{G} \times Q$	Total	$\hat{G} \times Q$	Total	$\hat{G} \times Q$	Total	(c, q_5)	$\frac{8}{5}$	$\hat{G} \times Q$	Total
(a, q_1)	$\frac{2}{5}$	(b, q_1)	1	(c, q_1)	$\frac{38}{5}$	(d, q_1)	$\frac{3}{2}$	(e, q_1)	$\frac{1}{10}$
(a, q_2)	$-\frac{1}{5}$	(b, q_2)	$\frac{5}{2}$	(c, q_2)	$\frac{1}{2}$	(d, q_2)	$\frac{7}{5}$	(e, q_2)	$\frac{9}{5}$
(a, q_3)	$\frac{11}{10}$	(b, q_3)	$\frac{1}{10}$	(c, q_3)	2	(d, q_3)	$\frac{5}{2}$	(e, q_3)	$\frac{13}{10}$
(a, q_4)	$\frac{31}{10}$	(b, q_4)	0	(c, q_4)	1	(d, q_4)	$\frac{3}{10}$	(e, q_4)	$\frac{8}{5}$
(a, q_5)	–	(b, q_5)	$\frac{13}{5}$	(c, q_5)	$\frac{8}{5}$	(d, q_5)	$\frac{1}{5}$	(e, q_5)	$\frac{14}{5}$

$\hat{G} \times Q$	Total	$\hat{G} \times Q$	Total	$\hat{G} \times Q$	Total
(f, q_1)	$\frac{3}{5}$	(g, q_1)	$-\frac{7}{10}$	(h, q_1)	$\frac{1}{1}$
(f, q_2)	$\frac{2}{5}$	(g, q_2)	$\frac{1}{10}$	(h, q_2)	$\frac{6}{5}$
(f, q_3)	$\frac{7}{5}$	(g, q_3)	$\frac{7}{5}$	(h, q_3)	$-\frac{3}{10}$
(f, q_4)	$\frac{37}{10}$	(g, q_4)	$\frac{1}{5}$	(h, q_4)	$\frac{2}{5}$
(f, q_5)	$\frac{1}{2}$	(g, q_5)	$\frac{11}{5}$	(h, q_5)	$\frac{2}{5}$

Now, selecting the highest numerical value from treatment a, b, c, d, e, f, g, h with their respective treatment guideline.

(1) $(a, q_4) = \frac{31}{10} > (a, q_3) = \frac{11}{10} > (a, q_1) = \frac{2}{5} > (a, q_2) = -\frac{1}{5} > (a, q_5)$ (invalid).
(2) $(b, q_5) = \frac{13}{5} > (b, q_2) = \frac{5}{2} > (b, q_1) = 1 > (b, q_3) = \frac{1}{10} > (b, q_4) = 0.$
(3) $(c, q_1) = \frac{38}{5} > (c, q_3) = 2 > (c, q_5) = \frac{8}{5} > (c, q_4) = 1 > (c, q_2) = \frac{1}{2}.$
(4) $(d, q_3) = \frac{5}{2} > (d, q_1) = \frac{3}{2} > (d, q_2) = \frac{7}{5} > (d, q_4) = \frac{3}{10} > (d, q_5) = \frac{1}{5}.$
(5) $(e, q_5) = \frac{14}{5} > (e, q_2) = \frac{9}{5} > (e, q_4) = \frac{8}{5} > (e, q_3) = \frac{13}{10} > (e, q_1) = \frac{1}{10}.$
(6) $(f, q_4) = \frac{37}{10} > (f, q_3) = \frac{7}{5} > (f, q_1) = \frac{3}{5} > (f, q_5) = \frac{1}{2} > (f, q_2) = \frac{2}{5}.$
(7) $(g, q_5) = \frac{11}{5} > (g, q_3) = \frac{7}{5} > (g, q_4) = \frac{1}{5} > (g, q_2) = \frac{1}{10} > (g, q_1) = -\frac{7}{10}.$
(8) $(h, q_2) = \frac{6}{5} > (h, q_1) = 1 > (h, q_4) = \frac{2}{5} = (h, q_5) = \frac{2}{5} > (h, q_3) = -\frac{3}{10}.$

The best possible treatment option with the treatment selection is $(c, q_1) = \frac{38}{5}$, follow by $(f, q_4) = \frac{37}{10}$.

5. Conclusion

This paper introduced the concept of Q-neutrosophic soft quasigroups as a new algebraic framework for modelling uncertainty, inconsistency, and indeterminacy on two universal sets connected by a quasigroup operation. We established several foundational properties, including the preservation of the structure under intersection and its non-preservation under union. We also showed that product, left division, and right division operations maintain the Q-neutrosophic soft quasigroup structure in entropic quasigroups, and we provided necessary and sufficient conditions for when a Q-neutrosophic soft groupoid becomes a Q-neutrosophic soft quasigroup.

In addition to these theoretical results, we proposed a decision-making algorithm based on Q-neutrosophic soft quasigroups and demonstrated its applicability to medical problems involving indeterminate or incomplete data. This highlights the practical value of the framework in contexts where classical, fuzzy, or soft approaches are insufficient. A promising direction for further research is the combination of Q-neutrosophic soft sets with n-ary quasigroup structures. Studying the resulting Q-neutrosophic soft n-ary quasigroups would allow for the analysis of more complex multi-argument operations and their associated structural properties.

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