



Novel Similarity Technique of Neutrosophic Sets and Application in Higher Education Context

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Abstract. Single-value neutrosophic set (SVNS), which is characterized by a degree of membership degree, of non-membership and indeterminacy degree, is a very powerful tool to process vague information. After the pioneering study of the intuitionistic fuzzy sets (IFS), single value of neutrosophic set (SVNS) has captured much attention from researchers in various fields and many achievements have been made. This paper proposes a new similarity technique between SVNS and applies to determine the adaptiveness of higher education institution standard in real-world context.

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1. Introduction

1.1. Preliminaries

Today, Vietnamese industrialization, modernization and international economic integration process make everything "flat" and provide educational organizations many opportunities and challenges in which every institution needs self-improvement, innovation, and enhancing the capability of providing prospective employers with quality people. Higher education institution today must regard their students as "customers", who should be provided a specific type of service; it is training service.

To attract good learners and offer sustainable development strategies, the university leaders should have a practical point of view and really listen to the "voice" of the students. This is achieved by monitoring and evaluating students' perceived value about the training service

that the universities are providing students, to see such assessments about the actual value they receive through the education process.

To simulate this content, we chose to use the fuzzy integral theory to have a more general and comprehensive view. According to previous studies such as: **Fuzzy set theory (FS)** - first given by Zadeh(1965) [11], **Intuitionistic fuzzy set (IFS)** - prompted by Atanassov(1987) [2] and **Neutrosophic fuzzy set (NFS)** - founded by F.Smarandache(1998) [8], which captures membership degree T , non-membership degree F and indeterminacy-membership degree I , proffered solution to handle vague, subjective, or conflicting evaluations in many decision making situations.

In addition, F. Smarandache (1998) introduced the Neutrosophic Set (NS), which includes the degree of indeterminacy (i) as an independent component, first in his manuscript and subsequently published in his work - Neutrosophic Probability, Set, and Logic (2000). Moreover, Yameng Wang & Keyun Qin (2019) mentioned new measures that modify the existing cross-entropy to strictly enforce the priority structure of Type 3 inclusion, in which the Truth Order and False Order are considered first, overcoming the assumption that all three membership orders are equally important in the previous inclusion relationships. Although the paper uses a well-structured example to demonstrate the validity and effectiveness of the new measure, the authors themselves have not yet studied the application of information measures in information processing from the perspective of future practical problems.

1.2. *The objective of our work*

This paper presents new similarity-distance technique based on NFS and applied in analyzing student's perceptions from Business, Economics, Management and Law majors in University of Economics and Law. This is the first time this theory has been applied to the field of education for consideration. We hope that with this contribution, it will help education policy makers have a broader and more correct perspective in the following issues:

- (1) Introduce novel similarity measure between single value neutrosophic sets
- (2) Apply it for identifying adaptiveness of real student perception with Higher education institution standard based on the regulation of VietNam Ministry of education and training (MOET)
- (3) Propose an analysis for qualitative survey and mapping the result with a predetermined target in decision-making context

2. Theory background

2.1. Neutrosophic fuzzy set

Definition 1. Let X be a universal set, a neutrosophic set A in X is characterized by: Truth-membership function $T_A(x)$; Indeterminacy-membership function $I_A(x)$ and non-membership function (Falsity-membership function) $F_A(x)$, in which $T_A(x), I_A(x), F_A(x) : X \rightarrow]-0, 1^+[$

Wang et al.(2010) [10] introduced the definition of single value neutrosophic set (SVNS) for the better application in the engineering field. SVNS is an extension of the IFS, and also provides another way in which to express and process uncertainty, incomplete, and inconsistent information in the real world.

Definition 2. Let X be a space of points, where a **single-value neutrosophic set (SVNS)** A in X is comprised of the truth-membership function $T_A(x)$; indeterminacy-membership function $I_A(x)$ and non-membership function (falsity-membership function) $F_A(x)$. For each point x in X , we have that $T_A(x), I_A(x), F_A(x) \in [0, 1]$. Then a SVNS A can be denoted by:

$$A = \{(x, T_A(x), I_A(x), F_A(x)) \mid x \in X\}$$

There is no restriction on the sum of $T_A(x), I_A(x), F_A(x)$, Thus

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$$

2.2. Similarity measure between SVNS

Definition 3. A function $S: SVNS(X) \times SVNS(X) \rightarrow (0, 1)$ is called a similarity measure for SVNSs, if the following conditions are satisfied: $A, B, C \in SVNS(X)$

- (1) (**Boundedness**) $0 \leq S(A, B) \leq 1$
- (2) (**Identity**) $S(A, B) = 1$ iff $A = B$
- (3) (**Symmetry**) $S(A, B) = S(B, A)$
- (4) (**Monotonicity**) If $A \subseteq B \subseteq C$ then $S(A, C) \leq S(A, B), S(A, C) \leq S(B, C)$

When $S(A, B)$ approaches 1, it implies A and B are more close (i.e., high similarity rate), and if $S(A, B)$ approaches 0, then the similarity rate between A and B are low.

3. New Similarity Technique of SVNS

Theorem 3.1. Let A and B be two single-valued neutrosophic fuzzy sets (SVNFSs) defined on the discrete universe $X = \{x_1, x_2, \dots, x_n\}$, where the membership functions $T_A(x_i), I_A(x_i), F_A(x_i) \in [0, 1]$ satisfy the normalization condition

$$T_A(x_i) + F_A(x_i) = 1, \quad \forall x_i \in X.$$

Then, the similarity measure between A and B is defined as follows:

$$S(A, B) = \frac{1}{2n} \sum_{i=1}^n \left[\sqrt{T_A(x_i) T_B(x_i)} + \sqrt{F_A(x_i) F_B(x_i)} + \sqrt{I_A(x_i) I_B(x_i)} + \sqrt{(1 - I_A(x_i))(1 - I_B(x_i))} \right] \tag{1}$$

it satisfies all the axioms of a similarity measure as presented in **Definition 4**

Proof. We verify each property in turn.

(1) Non-negativity. Since $T(x_i), I(x_i), F(x_i) \in [0, 1]$, each term under the square root is non-negative. Hence $S(A, B) \geq 0$.

(2) Symmetry. Interchanging A and B does not affect the expression, so $S(A, B) = S(B, A)$.

(3) Upper bound. By the arithmetic–geometric mean inequality (AM–GM), for all $u, v \geq 0$ we have $\sqrt{uv} \leq \frac{u+v}{2}$. Applying this inequality to each pair of terms gives

$$\begin{aligned} \sqrt{T_A T_B} &\leq \frac{T_A + T_B}{2}, & \sqrt{F_A F_B} &\leq \frac{F_A + F_B}{2}, \\ \sqrt{I_A I_B} &\leq \frac{I_A + I_B}{2}, & \sqrt{(1 - I_A)(1 - I_B)} &\leq \frac{(1 - I_A) + (1 - I_B)}{2}. \end{aligned}$$

Summing these inequalities for each x_i yields

$$\begin{aligned} S(A, B) &:= \sqrt{T_A T_B} + \sqrt{F_A F_B} + \sqrt{I_A I_B} + \sqrt{(1 - I_A)(1 - I_B)} \\ &\leq \frac{T_A + T_B + F_A + F_B + I_A + I_B + (1 - I_A) + (1 - I_B)}{2} \\ &= \frac{(T_A + F_A + 1) + (T_B + F_B + 1)}{2}. \end{aligned}$$

Using the normalization condition $T_A + F_A = 1$ and $T_B + F_B = 1$, we obtain $N_i \leq 2$. Therefore,

$$S(A, B) = \frac{1}{2n} \sum_{i=1}^n N_i \leq \frac{1}{2n} \sum_{i=1}^n 2 = 1.$$

Combining with non-negativity, we have $0 \leq S(A, B) \leq 1$.

(4) Identity. If $A = B$, then

$$\begin{aligned} S(A, A) &= \frac{1}{2n} \sum_{i=1}^n (\sqrt{T_A^2} + \sqrt{F_A^2} + \sqrt{I_A^2} + \sqrt{(1 - I_A)^2}) \\ &= \frac{1}{2n} \sum_{i=1}^n (T_A + F_A + I_A + (1 - I_A)) = \frac{1}{2n} \sum_{i=1}^n (T_A + F_A + 1). \end{aligned}$$

Since $T_A + F_A = 1$, we get $T_A + F_A + 1 = 2$ for all i , and thus $S(A, A) = 1$. \square

(5.1) Monotonicity of Type-1 inclusion relation:

$$A \subseteq_1 B \subseteq_1 C \iff T_A \leq T_B \leq T_C, \quad I_A \geq I_B \geq I_C, \quad F_A \geq F_B \geq F_C.$$

Then the following inequalities hold:

$$S(A, C) \leq S(A, B), \quad S(A, C) \leq S(B, C)$$

Proof. Fix an index i and denote

$$a = T_A(x_i), \quad b = T_B(x_i), \quad c = T_C(x_i),$$

$$\alpha = I_A(x_i), \quad \beta = I_B(x_i), \quad \gamma = I_C(x_i).$$

From the Type-1 inclusion assumptions:

$$a \leq b \leq c, \quad \alpha \geq \beta \geq \gamma, \quad 1 - a \geq 1 - b \geq 1 - c.$$

Truth–falsity part: Define

$$g_a(x) = \sqrt{ax} + \sqrt{(1-a)(1-x)}, \quad x \in [0, 1].$$

A direct derivative calculation gives

$$g'_a(x) = \frac{1}{2} \left(\frac{\sqrt{a}}{\sqrt{x}} - \frac{\sqrt{1-a}}{\sqrt{1-x}} \right).$$

The function g_a increases on $(0, a]$, attains its maximum at $x = a$, and decreases on $[a, 1)$.

Since $a \leq b \leq c$, monotonicity yields

$$g_a(c) \leq g_a(b).$$

Indeterminacy part: Similarly define

$$h_\alpha(x) = \sqrt{\alpha x} + \sqrt{(1-\alpha)(1-x)}.$$

The same monotonicity analysis shows h_α achieves its maximum at $x = \alpha$. Since $\alpha \geq \beta \geq \gamma$, and h_α increases on $[0, \alpha]$ and decreases for $x > \alpha$, we have

$$h_\alpha(\gamma) \leq h_\alpha(\beta).$$

Combine results: The similarity summand for x_i is

$$S_i(A, B) = g_{T_A}(T_B) + h_{I_A}(I_B).$$

Thus

$$S_i(A, C) = g_a(c) + h_\alpha(\gamma) \leq g_a(b) + h_\alpha(\beta) = S_i(A, B).$$

Summing over $i = 1, \dots, n$ and applying the positive factor $1/(2n)$ yields

$$S(A, C) \leq S(A, B).$$

Symmetric argument: Repeating the same argument with B as the fixed set gives

$$S_i(A, C) \leq S_i(B, C) \quad \Rightarrow \quad S(A, C) \leq S(B, C).$$

Hence both inequalities are proved. \square

(5.2) Monotonicity of Type-2 inclusion relation:

$$A \subseteq_2 B \subseteq_2 C \iff T_A \leq T_B \leq T_C, \quad I_A \leq I_B \leq I_C, \quad F_A \geq F_B \geq F_C.$$

Then the following inequalities (**Monotonicity**) hold:

$$S(A, C) \leq S(A, B), \quad S(A, C) \leq S(B, C).$$

Proof. Fix an index i and set, for brevity,

$$a := T_A(x_i), \quad b := T_B(x_i), \quad c := T_C(x_i),$$

$$\alpha := I_A(x_i), \quad \beta := I_B(x_i), \quad \gamma := I_C(x_i).$$

By the Type-2 inclusion assumptions we have

$$a \leq b \leq c, \quad \alpha \leq \beta \leq \gamma, \quad \text{and} \quad F_U(x_i) = 1 - T_U(x_i), \text{ so } 1 - a \geq 1 - b \geq 1 - c.$$

Write the summand corresponding to x_i as

$$S_i(A, B) = \sqrt{T_A T_B} + \sqrt{F_A F_B} + \sqrt{I_A I_B} + \sqrt{(1 - I_A)(1 - I_B)}.$$

Group the truth/falsity terms and the indeterminacy terms via the one-variable functions

$$g_a(x) = \sqrt{ax} + \sqrt{(1-a)(1-x)}, \quad h_\alpha(x) = \sqrt{\alpha x} + \sqrt{(1-\alpha)(1-x)},$$

so that

$$\sqrt{T_A T_B} + \sqrt{F_A F_B} = g_{T_A}(T_B), \quad \sqrt{I_A I_B} + \sqrt{(1 - I_A)(1 - I_B)} = h_{I_A}(I_B).$$

We prove monotonicity properties of g_a and h_α . Compute derivatives (for $x \in (0, 1)$):

$$g'_a(x) = \frac{1}{2} \left(\frac{\sqrt{a}}{\sqrt{x}} - \frac{\sqrt{1-a}}{\sqrt{1-x}} \right), \quad h'_\alpha(x) = \frac{1}{2} \left(\frac{\sqrt{\alpha}}{\sqrt{x}} - \frac{\sqrt{1-\alpha}}{\sqrt{1-x}} \right).$$

Each function attains its unique maximum at $x = a$ (resp. $x = \alpha$): indeed $g'_a(a) = 0$ and g'_a is positive for $x < a$ and negative for $x > a$; the same sign behavior holds for h_α about $x = \alpha$.

Thus each function increases on $(0, \cdot]$ up to its parameter and then decreases on $[\cdot, 1)$.

Now use these monotonicity properties together with the Type-2 inequalities:

1. Since $a \leq b \leq c$ and g_a decreases on $[a, 1)$, we get

$$g_a(c) \leq g_a(b).$$

2. Since $\alpha \leq \beta \leq \gamma$ and h_α decreases on $[\alpha, 1)$, we get

$$h_\alpha(\gamma) \leq h_\alpha(\beta).$$

Add the two inequalities to obtain, for the fixed index i ,

$$S_i(A, C) = g_a(c) + h_\alpha(\gamma) \leq g_a(b) + h_\alpha(\beta) = S_i(A, B).$$

Summing over $i = 1, \dots, n$ and multiplying by the positive factor $1/(2n)$ yields

$$S(A, C) \leq S(A, B)$$

A symmetric argument (viewing B as the fixed first argument and applying the same monotonicity facts with b or β as the parameter) produces

$$S_i(A, C) \leq S_i(B, C) \Rightarrow S(A, C) \leq S(B, C).$$

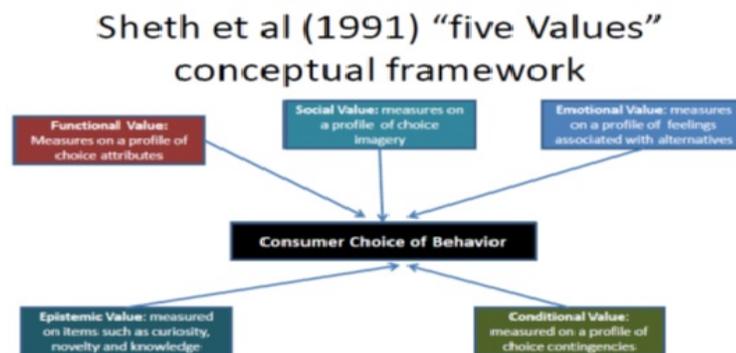
Therefore both claimed inequalities hold. \square

Remark 3.2. From (1)–(4) and (5.1), (5.2), it follows that $S(A, B)$ satisfies all the properties of a valid similarity measure.

4. Application in higher education

4.1. Perceived value in higher education

Consistently with marketing literature, value is something that is perceived by customers Zeithaml (1988) [12]) rather than objectively determined by the seller (Day and Crask (2000) [4]). Theory of Consumption Values proposed by Sheth et al.,(1991) [7] explains the basic consumption values that guide consumers when they make the choices that they do. The authors view choice as a function of multiple independent consumption values that can vary in importance in various situations. Five consumption values - namely, Functional value, Social value, Emotional value, Epistemic value, and Condition value as below



Adapted from: Sheth, Newman, and Gross (1991, p.160)

In the higher educational context,

- (1) **Functional value** is related to the perceived performance or utility of the service. These aspects are related to both the so-called tangibles and intangibles. The former refer to facilities or infrastructures such as library, computer equipment, canteens, student housing, which are potential determinant dimensions of student satisfaction

since they are easily recognizable by students. The latter are related to the process of service assembly that is particularly important with regard to student satisfaction level and service expectation Appleton-Knapp and Krentler (2006) [1]. They are closely related to the essential service provided by universities – therefore; they are factors that can shape student satisfaction.

- (2) **Epistemic value** refers to benefits deriving from the university ability to improve the skills and knowledge base of students. The staff plays an important role in assuring the quality and the effectiveness of the instruction (e.g. the teaching methods used by the teaching staff for giving the classes). The elements concerning the core (teaching) and the additional services (administrative) are all related to the quality and effectiveness of the university.
- (3) **Emotional value** accounts for the affective commitment, which derives from the sense of belonging to an established institution. The aspects related to the surrounding environment such as the campus life or social life significantly affect the total university experience
- (4) **Social value** represents the relational benefits derived from being part of a social network
- (5) **Conditional value** is described as the set of situations that the customer faces when making a choice. In this context, situational variables are deemed to have an impact upon the customer's assessment of the utility of the product/service (Belk, (2007) [3]). For example, the size of university and the parents' views with regard to its programs are situational variables which have the potential to influence the value of the educational experience.

4.2. Background of the dataset

4.2.1. Designing Questionnaire

Creating the questionnaire followed a careful step- by- step process as below:

- (1) Propose first draft of the questionnaire based on background theory (Sheth et al 1991) and applied research in the same condition.
- (2) This was given to education management experts (such as head of academic affair office, deans of faculty) and stakeholders (such as heads of related office, employers, student's parent and student) for comments, edits and improvements.
- (3) The last and the most vital step for creating the questionnaire was the pilot test survey for the second draft. Through the results of detailed analysis, the questionnaire was finalized 32 items.

4.2.2. Sample and data presentation

The target sample size of this research is more than 400 students (**416**) with the population of 6.205 full - time undergraduate student in University of Economics and Law (UEL). Through Principal Component Analysis (PCA) and varimax rotation with Kaiser Normalization criteria results, we grouped thirty two (32) items in the questionnaire into 07 perfectly independent components that can be used to explain the the variation of students' evaluation about the quality of university activities, which is - Component 1: Price/Quality dimension; Component 2: Image dimension; Component 3: Functional dimension; Component 4: Social dimension; Component 5: Epistemic dimension; Component 6: Emotional dimension; Component 7: Conditional dimension

4.2.3. Higher education institution standard

Vietnamese Higher Education Institution Standard promulgated together with Circular 01/2024/TT-BGDDT February 05, 2024 which includes 6 standards with 20 criteria that is the basis for planning and arrangement of the higher education institution network; assessment and supervision of conditions for quality assurance and accountability of higher education institutions as prescribed by law. In which, It proposed a criteria in Standard 5 (Recruitment and training) for the ratio of graduates who are satisfied with the overall study process and experience is not lower than 70%.

Based on usual 5-level Likert scale for evaluation which 4 above 5 express for student satisfaction with overall education process, we infer that minimum mean of evaluation level could be at least $4 \times 0.7(70\%) = 2.8$ for all factors that comprise of student perception.

Linguistic terms proposed associated with SVN proposed neutrosophic index to evaluate the management of internal control in [6] as below table

TABLE 1. Linguistic terms proposed associated with SVN [6]

Evaluation	Truth (T)	Indeterminacy (I)	Falsity (F)	Linguistic scale
0.5 - 1	0.2	0.85	0.80	Very Bad
1 - 1.5	0.3	0.75	0.70	Bad
1.5 - 2	0.4	0.65	0.60	Medium Bad
2 - 2.5	0.5	0.5	0.50	Medium
2.5 - 3	0.6	0.35	0.40	Medium Good
3 - 3.5	0.7	0.25	0.30	Good
3.5 - 4	0.8	0.15	0.20	Very Good

From **Table 1**, we identify the pattern U that regularize student perceived value through education process as above:

$$U = ((p_R, 0.6, 0.35, 0.4) (i_R, 0.6, 0.35, 0.4) (f_R, 0.6, 0.35, 0.4) (e_R, 0.6, 0.35, 0.4) (em_R, 0.6, 0.35, 0.4) (c_R, 0.6, 0.35, 0.4))$$

In addition, descriptive statistics table of real data set is presented

TABLE 2. Descriptive statistics from the dataset

	Minimum	Maximum	Mean	Variance	SVNN (R)
PriceQuality	0.8	4.0	2.87	0.36	(0.60, 0.35, 0.40)
Image	1.48	3.69	2.96	2.96	(0.60, 0.35, 0.40)
Functional	0.68	3.38	2.54	0.17	(0.60, 0.35, 0.40)
Social	0.92	3.78	3.24	0.23	(0.70, 0.25, 0.30)
Epistemic	1.11	3.09	2.36	0.11	(0.70, 0.25, 0.30)
Emotional	0.75	3.77	2.59	0.33	(0.6, 0.35, 0.40)
Conditional	1.02	3.10	2.28	0.14	(0.50, 0.50, 0.50)

Using **(1)** and identified pattern U and evaluation R as in **Table 2**, we calculate the similarity between regulation and real student perception:

$$S(U, R) = \frac{1}{2n} \sum_{i=1}^n \left[\sqrt{T_U(x_i) T_R(x_i)} + \sqrt{F_U(x_i) F_R(x_i)} + \sqrt{I_U(x_i) I_R(x_i)} + \sqrt{(1 - I_U(x_i))(1 - I_R(x_i))} \right]$$

$$S(U, R) = 0.997$$

Remark 4.1. This result show that student perception of education quality for UEL are really compatible with criteria of Vietnamese Higher Education Institution Standard

5. Conclusion

In a way to fortify the capacity of similarity measure under SVNS as a key player in decision-making technique, we introduced a similarity technique in estimating relationship between SVNS. The neutrosophic similarity measure incorporates the three fundamental components of a neutrosophic fuzzy set—truth, indeterminacy, and falsity—allowing a comprehensive comparison between two evaluations. Compared with classical fuzzy or intuitionistic fuzzy similarity, the proposed measure includes both uncertainty and non-uncertainty information, making it suitable for complex decision-making environments where incomplete or ambiguous information frequently occurs. Finally, we have demonstrated how decision-making could be processed in higher education regulation context. However the novel similarity-distance technique need to be critical compared with other methods to demonstrate effectiveness and scope of use in the the real decision-making context.

References

- [1] S. L. Appleton–Knapp and K. A. Krentler, “Measuring student expectations and their effects on satisfaction: The importance of managing student expectations,” *Journal of Marketing Education*, vol. 28, no. 3, pp. 254–264, 2006.
- [2] K. T. Atanassov, “Intuitionistic fuzzy sets,” *Fuzzy Sets and Systems*, Springer-Verlag Berlin Heidelberg GmbH, 1987.
- [3] R. W. Belk, “Why Not Share Rather than Own?” *Annals of the American Academy of Political and Social Science*, vol. 611, pp. 126–140, May 2007.
- [4] E. Day and M. R. Crask, “Value Assessment: The Antecedent of Customer Satisfaction,” *Journal of Consumer Satisfaction, Dissatisfaction and Complaining Behavior*, vol. 13, pp. 52–60, 2000.
- [5] P. A. Ejegwa, “Novel correlation coefficient for intuitionistic fuzzy sets and its application to multi-criteria decision-making problems,” *International Journal of Fuzzy System Applications*, vol. 10, no. 2, pp. 39–58, 2021.
- [6] N. P. Lastra Calderón, D. L. Villarruel Delgado, D. S. Malquín, and E. A. Guerrón, “Proposal of a neutrosophic index to evaluate the management of internal control,” *Neutrosophic Sets and Systems*, vol. 34, no. 1, 2020.
- [7] J. Sheth, B. Newman, and B. Gross, “Why We Buy What We Buy: A Theory of Consumption Values,” *Journal of Business Research*, vol. 22, no. 2, pp. 159–170, 1991b.
- [8] F. Smarandache, *Neutrosophy: Neutrosophic Probability, Set, and Logic*. American Research Press, Rehoboth, DE, USA, 1998.
- [9] F. Smarandache, “Neutrosophic set — a generalization of the intuitionistic fuzzy sets,” *International Journal of Pure and Applied Mathematics*, vol. 24, pp. 287–297, 2005.
- [10] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman, “Single Valued Neutrosophic Sets,” *Multi-space and Multistructure*, vol. 4, pp. 410–413, 2010.
- [11] L. A. Zadeh, “Fuzzy sets,” *Information and Control*, vol. 8, pp. 338–353, 1965.
- [12] V. A. Zeithaml, “Consumer Perceptions of Price, Quality, and Value: A Means–End Model and Synthesis of Evidence,” *Journal of Marketing*, vol. 52, no. 3, pp. 2–22, 1988.

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