



New Classes of $\delta - \beta^*$ -Continuous Mappings in m-Polar Neutrosophic Topological Spaces

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Abstract. We introduce and study two classes of neutrosophic continuous mappings: the neutrosophic irresolute $\delta - \beta^*$ -continuous mappings (*NIr $\delta - \beta^*$ CM*) and the neutrosophic contra $\delta - \beta^*$ -continuous mappings (*NC $\delta - \beta^*$ CM*). We establish their fundamental properties and provide characterizations in terms of preimages of $\delta - \beta^*$ -open and $\delta - \beta^*$ -closed sets. The interplay between the two notions is analyzed through implication chains, (non-)equivalences under mild hypotheses, and stability results under composition, subspaces, and products. We then extend the framework to the m-polar setting by defining m-polar neutrosophic irresolute $\delta - \beta^*$ -continuous mappings (*MPNIr $\delta - \beta^*$ CM*) and m-polar neutrosophic contra $\delta - \beta^*$ -continuous mappings (*MPNC $\delta - \beta^*$ CM*), showing how core properties lift to the m-polar case and where genuinely new phenomena arise. Throughout, examples and counterexamples are provided to separate the classes and to illustrate the sharpness of the obtained results.

Keywords: Contra m-polar $\delta - \beta^*$ mappings, Irresolute m-polar $\delta - \beta^*$ mappings, Generalized continuous neutrosophic mappings.

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1. Introduction

The concept of *intuitionistic fuzzy sets*, introduced by Atanassov [13] as a generalization of the classical fuzzy sets of Zadeh [36], opened the way to several extensions and refinements in the study of uncertainty. Building on these foundations, Smarandache [31] proposed in 1999 the theory of *neutrosophic sets*, which explicitly model truth, falsity and indeterminacy, and rapidly attracted the attention of many researchers worldwide.

Neutrosophic topology emerged soon after: Salama and Alblowi [29] introduced neutrosophic topological spaces and explored their basic properties, while Salama et al. [30] investigated continuous mappings in this new framework. Further applications and developments can be found for instance in [6, 11, 12]. The algebraic side of the theory was first outlined by Kandasamy and Smarandache [24], and later enriched by Abed et al. [1, 2] through the interplay between neutrosophic structures and module theory. Complex-valued neutrosophic sets were then studied and applied to real-world problems in [10, 27].

Topological investigations have also played a central role. Hatir and Noiri [17] examined the notion of δ - β -open sets, a novel type of generalized open sets providing a fertile ground for new separation axioms and continuity concepts. Al-Jumaili et al. [7, 23] analyzed new families of mappings with strongly closed networks in classical topological spaces, while Ishwarya and Bageerathi [21] focused on neutrosophic semi-open sets. Other important contributions on weakly neutrosophic crisp continuity and generalized separation axioms were given by Imran et al. [18–20], Abdulkadhim et al. [3, 4], and Al-Obaidi et al. [8], together with further recent studies such as [5, 16, 32–35].

In a parallel line of research, Chen et al. [14] introduced the notion of *m-polar fuzzy sets* (MPFS), capable of handling multi-criteria and multi-source information. To address simultaneously the multi-attribute setting and the intrinsic indeterminacy of neutrosophic data, the concept of *m-polar neutrosophic sets* (MPNS) has been proposed: it extends the bipolar neutrosophic set and allows each of the m attributes to carry independent degrees of truth, falsity and indeterminacy. On MPNSs one can define suitable operations and scoring functions for comparing m-polar neutrosophic values.

Recent works have further expanded the neutrosophic landscape. Nordo and collaborators, for instance, contributed to the development of soft and neutrosophic sets [25], to the design of computational tools for neutrosophic topologies in Python [26], to implication-based neutrosophic finite state machines [22], and to new classes of neutrosophic crisp generalized closed functions [9], showing the breadth of current applications and computational perspectives.

Within this rich framework, the present paper introduces two new classes of generalized continuous neutrosophic mappings: the *neutrosophic irresolute δ - β^* -continuous mappings* (NIr δ - β^* CM) and the *neutrosophic contra δ - β^* -continuous mappings* (NC δ - β^* CM). We

investigate their fundamental properties, analyze the relationships between these two notions, and finally extend the study to the m-polar setting by defining m-polar neutrosophic irresolute and contra δ - β^* -continuous mappings, highlighting the specific features that arise in the multi-attribute context.

2. Preliminaries

In this section we recall the basic notions on neutrosophic sets and neutrosophic topologies that will be used throughout the paper, together with their m-polar counterparts.

Definition 2.1 ([29]). Let \mathfrak{V} be a nonempty set. A *neutrosophic set* (NS) \mathcal{C} on \mathfrak{V} is a set of ordered quadruples

$$\mathcal{C} = \{ \langle \mathcal{J}, \mu_{\mathcal{C}}(\mathcal{J}), \sigma_{\mathcal{C}}(\mathcal{J}), v_{\mathcal{C}}(\mathcal{J}) \rangle : \mathcal{J} \in \mathfrak{V} \},$$

where, for every $\mathcal{J} \in \mathfrak{V}$, the functions $\mu_{\mathcal{C}}$, $\sigma_{\mathcal{C}}$, and $v_{\mathcal{C}}$ assign, respectively, the degrees of membership, indeterminacy, and non-membership in $[0, 1]$.

Definition 2.2 ([29]). Let $\mathcal{H} = \{ \langle \mathcal{J}, \mu_{\mathcal{H}}(\mathcal{J}), \sigma_{\mathcal{H}}(\mathcal{J}), v_{\mathcal{H}}(\mathcal{J}) \rangle : \mathcal{J} \in \mathfrak{V} \}$ and $\mathcal{K} = \{ \langle \mathcal{J}, \mu_{\mathcal{K}}(\mathcal{J}), \sigma_{\mathcal{K}}(\mathcal{J}), v_{\mathcal{K}}(\mathcal{J}) \rangle : \mathcal{J} \in \mathfrak{V} \}$. Then:

- (1) $0_{\mathcal{N}} = \{ \langle \mathcal{J}, 0, 0, 1 \rangle : \mathcal{J} \in \mathfrak{V} \}$ and $1_{\mathcal{N}} = \{ \langle \mathcal{J}, 1, 0, 0 \rangle : \mathcal{J} \in \mathfrak{V} \}$;
- (2) $\mathcal{H} \subseteq \mathcal{K}$ iff, for all $\mathcal{J} \in \mathfrak{V}$, $\mu_{\mathcal{H}}(\mathcal{J}) \leq \mu_{\mathcal{K}}(\mathcal{J})$, $\sigma_{\mathcal{H}}(\mathcal{J}) \leq \sigma_{\mathcal{K}}(\mathcal{J})$, and $v_{\mathcal{H}}(\mathcal{J}) \geq v_{\mathcal{K}}(\mathcal{J})$;
- (3) $\mathcal{H} = \mathcal{K}$ iff $\mathcal{H} \subseteq \mathcal{K}$ and $\mathcal{K} \subseteq \mathcal{H}$;
- (4) $\mathcal{H}^c = \{ \langle \mathcal{J}, v_{\mathcal{H}}(\mathcal{J}), 1 - \sigma_{\mathcal{H}}(\mathcal{J}), \mu_{\mathcal{H}}(\mathcal{J}) \rangle : \mathcal{J} \in \mathfrak{V} \}$;
- (5) $\mathcal{H} \cup \mathcal{K} = \{ \langle \mathcal{J}, \mu_{\mathcal{H}}(\mathcal{J}) \vee \mu_{\mathcal{K}}(\mathcal{J}), \sigma_{\mathcal{H}}(\mathcal{J}) \vee \sigma_{\mathcal{K}}(\mathcal{J}), v_{\mathcal{H}}(\mathcal{J}) \wedge v_{\mathcal{K}}(\mathcal{J}) \rangle : \mathcal{J} \in \mathfrak{V} \}$.

Definition 2.3 ([29]). A *neutrosophic topology* (NT) on a nonempty set \mathcal{C} is a collection $\tau_{\mathcal{N}}$ of neutrosophic sets on \mathcal{C} such that:

- (i) $0_{\mathcal{N}}, 1_{\mathcal{N}} \in \tau_{\mathcal{N}}$;
- (ii) if $\mathcal{K}_1, \mathcal{K}_2 \in \tau_{\mathcal{N}}$, then $\mathcal{K}_1 \cap \mathcal{K}_2 \in \tau_{\mathcal{N}}$;
- (iii) if $\{ \mathcal{K}_j \}_{j \in I} \subseteq \tau_{\mathcal{N}}$, then $\bigcup_{j \in I} \mathcal{K}_j \in \tau_{\mathcal{N}}$.

Members of $\tau_{\mathcal{N}}$ are called *neutrosophic open sets* (NOS). A neutrosophic set \mathcal{K} is *neutrosophic closed* (NCS) iff its complement \mathcal{K}^c is NOS. The pair $(\mathcal{C}, \tau_{\mathcal{N}})$ is a *neutrosophic topological space* (NTS).

Definition 2.4 ([29]). Let \mathcal{H} be an NS on the NTS $(\mathcal{C}, \tau_{\mathcal{N}})$. The *neutrosophic closure* and *neutrosophic interior* of \mathcal{H} are

$$NCl(\mathcal{H}) = \bigcap \{ \mathcal{D} : \mathcal{D} \text{ is NCS in } \mathcal{C} \text{ and } \mathcal{H} \subseteq \mathcal{D} \},$$

$$NInt(\mathcal{H}) = \bigcup \{ \mathcal{P} : \mathcal{P} \text{ is NOS in } \mathcal{C} \text{ and } \mathcal{P} \subseteq \mathcal{H} \}.$$

They satisfy $NCl(\mathcal{H}^c) = (NInt(\mathcal{H}))^c$ and $NInt(\mathcal{H}^c) = (NCl(\mathcal{H}))^c$.

Proposition 2.5 ([15]). *Let (\mathcal{C}, τ_N) be an NTS. Then:*

- (1) every NOS is an $N\beta$ -open set;
- (2) every $N\alpha$ -open set is $N\beta$ -open.

Definition 2.6 ([34]). Let (\mathcal{C}, τ_N) be an NTS. A neutrosophic set \mathcal{H} is *neutrosophic δ -open* (briefly, $N\delta$ OS) if

$$\mathcal{H} = N\delta Int(\mathcal{H}),$$

where $N\delta Int$ denotes the δ -interior operator associated with the $N\delta$ -open sets.

Definition 2.7 ([28]). Let \mathfrak{G} be a nonempty reference set and $m \in \mathbb{N}$. An *m -polar neutrosophic set* (MPNS) on \mathfrak{G} is

$$\mathbb{M}_{\mathfrak{G}} = \left\{ (\mathcal{J}, \mu_{\alpha}(\mathcal{J}), \sigma_{\alpha}(\mathcal{J}), \nu_{\alpha}(\mathcal{J})) : \mathcal{J} \in \mathfrak{G}, \alpha = 1, 2, \dots, m \right\},$$

with $\mu_{\alpha}, \sigma_{\alpha}, \nu_{\alpha} : \mathfrak{G} \rightarrow [0, 1]$ and $0 < \mu_{\alpha}(\mathcal{J}) + \sigma_{\alpha}(\mathcal{J}) + \nu_{\alpha}(\mathcal{J}) \leq 3$ for each α and \mathcal{J} . The triple $\mathfrak{h}_{\alpha}(\mathcal{J}) = (\mu_{\alpha}(\mathcal{J}), \sigma_{\alpha}(\mathcal{J}), \nu_{\alpha}(\mathcal{J}))$ is an *m -polar neutrosophic numeral* (MPNN).

Definition 2.8 ([28]). Let $\text{mpn}({}^1\mathbb{M}_{\mathfrak{G}})$ denote the family of all MPNSs over \mathfrak{G} . A subfamily $T_{\mathbb{M}_{\mathfrak{G}}} \subseteq \text{mpn}({}^1\mathbb{M}_{\mathfrak{G}})$ is an *m -polar neutrosophic topology* on \mathfrak{G} if:

- (i) ${}^0\mathbb{M}_{\mathfrak{G}}, {}^1\mathbb{M}_{\mathfrak{G}} \in T_{\mathbb{M}_{\mathfrak{G}}}$ (the empty and whole MPNS);
- (ii) if $\{(MN)_{\gamma}\}_{\gamma \in \Delta} \subseteq T_{\mathbb{M}_{\mathfrak{G}}}$, then $\bigcup_{\gamma \in \Delta} (MN)_{\gamma} \in T_{\mathbb{M}_{\mathfrak{G}}}$;
- (iii) if $\mathbb{M}_{\mathfrak{G}_1}, \mathbb{M}_{\mathfrak{G}_2} \in T_{\mathbb{M}_{\mathfrak{G}}}$, then $\mathbb{M}_{\mathfrak{G}_1} \cap \mathbb{M}_{\mathfrak{G}_2} \in T_{\mathbb{M}_{\mathfrak{G}}}$.

Then $(\mathfrak{G}, T_{\mathbb{M}_{\mathfrak{G}}})$ is an *m -polar neutrosophic topological space* (MPNTS). Members of $T_{\mathbb{M}_{\mathfrak{G}}}$ are open MPNSs; their complements are closed MPNSs.

Example 2.9. Let $\mathfrak{G} = \{\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3, \mathcal{J}_4\}$ (e.g., four books) and consider two 3-polar NSs:

$$\begin{aligned} \mathbb{M}_{\mathfrak{G}_1} = & \left\{ (\mathcal{J}_1, \langle 0.853, 0.423, 0.401 \rangle, \langle 0.328, 0.416, 0.311 \rangle, \langle 0.191, 0.222, 0.111 \rangle), \right. \\ & (\mathcal{J}_2, \langle 0.551, 0.147, 0.411 \rangle, \langle 0.523, 0.149, 0.107 \rangle, \langle 0.624, 0.503, 0.331 \rangle), \\ & (\mathcal{J}_3, \langle 0.631, 0.352, 0.242 \rangle, \langle 0.826, 0.123, 0.532 \rangle, \langle 0.522, 0.442, 0.209 \rangle), \\ & \left. (\mathcal{J}_4, \langle 0.522, 0.388, 0.805 \rangle, \langle 0.827, 0.333, 0.407 \rangle, \langle 0.909, 0.319, 0.509 \rangle) \right\}, \end{aligned}$$

and

$$\begin{aligned} \mathbb{M}_{\mathfrak{G}_2} = & \left\{ (\mathcal{J}_1, \langle 0.623, 0.499, 0.623 \rangle, \langle 0.301, 0.545, 0.428 \rangle, \langle 0.147, 0.402, 0.304 \rangle), \right. \\ & (\mathcal{J}_2, \langle 0.246, 0.209, 0.521 \rangle, \langle 0.447, 0.226, 0.308 \rangle, \langle 0.542, 0.524, 0.447 \rangle), \\ & (\mathcal{J}_3, \langle 0.435, 0.483, 0.334 \rangle, \langle 0.651, 0.229, 0.363 \rangle, \langle 0.304, 0.593, 0.327 \rangle), \\ & \left. (\mathcal{J}_4, \langle 0.228, 0.501, 0.793 \rangle, \langle 0.734, 0.513, 0.529 \rangle, \langle 0.742, 0.442, 0.626 \rangle) \right\}. \end{aligned}$$

Then $T_{\mathbb{M}_{\mathfrak{G}}} = \{{}^0\mathbb{M}_{\mathfrak{G}}, {}^1\mathbb{M}_{\mathfrak{G}}, \mathbb{M}_{\mathfrak{G}_1}, \mathbb{M}_{\mathfrak{G}_2}\}$ is a 3-polar neutrosophic topology on \mathfrak{G} .

Theorem 2.10 ([28]). Let $(\mathfrak{G}, T_{\mathbb{M}_G})$ be an MPNTS. Then:

- (i) ${}^0\mathbb{M}_G$ and ${}^1\mathbb{M}_G$ are closed MPNSs;
- (ii) arbitrary unions of open MPNSs are open;
- (iii) finite intersections of closed MPNSs are closed.

Definition 2.11 ([28]). Let $\mathbb{M}_G \in \text{mpn}({}^1\mathbb{M}_G)$ and $(\mathfrak{G}, T_{\mathbb{M}_G})$ be an MPNTS. Then:

- (i) the *interior* of \mathbb{M}_G , denoted \mathbb{M}_G° , is the union of all open MPNSs contained in \mathbb{M}_G ; it is the largest open MPNS contained in \mathbb{M}_G ;
- (ii) the *closure* of \mathbb{M}_G , denoted $\overline{\mathbb{M}_G}$, is the intersection of all closed MPNSs containing \mathbb{M}_G ; it is the least closed MPNS containing \mathbb{M}_G .

3. Main results and extensions

We introduce two notions of neutrosophic $\delta-\beta^*$ -continuity: the *neutrosophic irresolute $\delta-\beta^*$ -continuous mappings* ($NIr \delta-\beta^*$ CM) and the *neutrosophic contra $\delta-\beta^*$ -continuous mappings* ($NC \delta-\beta^*$ CM). We establish basic properties, relationships, and stability under composition, and we indicate the m-polar counterparts.

Definition 3.1. Let (\mathcal{C}, τ_N) and (\mathcal{O}, ϱ_N) be NTSs and $h : (\mathcal{C}, \tau_N) \rightarrow (\mathcal{O}, \varrho_N)$ a mapping. We say that h is *neutrosophic irresolute $\delta-\beta^*$ -continuous* ($NIr \delta-\beta^*$ CM) if for every neutrosophic $\delta-\beta^*$ -open set \mathcal{H} in (\mathcal{O}, ϱ_N) , the preimage $h^{-1}(\mathcal{H})$ is $\delta-\beta^*$ -open in (\mathcal{C}, τ_N) .

Example 3.2. Let $\mathcal{C} = \{\mathfrak{p}, \mathfrak{q}\}$ and $\mathcal{O} = \{\mathfrak{s}, \mathfrak{r}\}$. Let $\tau_N = \{0_N, 1_N, \mathcal{Q}, \mathcal{H}\}$ and $\varrho_N = \{0_N, 1_N, \mathcal{P}, \mathcal{K}\}$ be neutrosophic topologies on \mathcal{C} and \mathcal{O} , respectively, where

$$\begin{aligned} \mathcal{Q} &= \langle \mathfrak{p}; (0.3, 0.4, 0.3), (0.2, 0.2, 0.2), (0.7, 0.8, 0.7) \rangle, \\ \mathcal{H} &= \langle \mathfrak{q}; (0.3, 0.5, 0.3), (0.3, 0.5, 0.3), (0.7, 0.7, 0.7) \rangle, \\ \mathcal{P} &= \langle \mathfrak{s}; (0.6, 0.6, 0.6), (0.5, 0.5, 0.5), (0.4, 0.4, 0.4) \rangle, \\ \mathcal{K} &= \langle \mathfrak{r}; (0.2, 0.4, 0.3), (0.4, 0.6, 0.5), (0.7, 0.7, 0.6) \rangle. \end{aligned}$$

Define $h : (\mathcal{C}, \tau_N) \rightarrow (\mathcal{O}, \varrho_N)$ by $h(\mathfrak{p}) = \mathfrak{s}$ and $h(\mathfrak{q}) = \mathfrak{r}$. If \mathcal{W} is $\delta-\beta^*$ -open in \mathcal{O} , then $h^{-1}(\mathcal{W})$ is $\delta-\beta^*$ -open in \mathcal{C} (for instance via $NCl(NInt(NCl(h^{-1}(\mathcal{P})))) = 1_N$ when $h^{-1}(\mathcal{P}) \subseteq \mathcal{H}$). Hence h is $NIr \delta-\beta^*$ CM.

Theorem 3.3. Every $NIr \beta^*$ CM is $NIr \delta-\beta^*$ CM.

Proof. Let $h : (\mathcal{C}, \tau_N) \rightarrow (\mathcal{O}, \varrho_N)$ be $NIr \beta^*$ CM. If \mathcal{P} is $\delta-\beta^*$ -open in \mathcal{O} , then it is in particular β^* -open. By irresoluteness, $h^{-1}(\mathcal{P})$ is β^* -open in \mathcal{C} ; since $\delta-\beta^*$ -openness is weaker, $h^{-1}(\mathcal{P})$ is $\delta-\beta^*$ -open as well. Hence h is $NIr \delta-\beta^*$ CM. \square

Remark 3.4. The converse of Theorem 3.3 does not hold in general (see Example 3.5).

Example 3.5. In the setting of Example 3.2, one may arrange ϱ_N so that some β^* -open set in \mathcal{O} fails to be $\delta-\beta^*$ -open; then h is $NIr \delta-\beta^*$ CM but not $NIr \beta^*$ CM, because the preimage of that β^* -open set need not be β^* -open in \mathcal{C} .

Theorem 3.6. *If $h : (\mathcal{C}, \tau_N) \rightarrow (\mathcal{O}, \varrho_N)$ and $\mathcal{S} : (\mathcal{O}, \varrho_N) \rightarrow (\mathcal{G}, \rho_N)$ are both $NIr \beta^*$ CM, then $\mathcal{S} \circ h$ is $NIr \delta-\beta^*$ CM.*

Proof. Let \mathcal{H} be $\delta-\beta^*$ -open in (\mathcal{G}, ρ_N) . Since \mathcal{S} is $NIr \beta^*$ CM, $\mathcal{S}^{-1}(\mathcal{H})$ is β^* -open in (\mathcal{O}, ϱ_N) ; applying h , we get $h^{-1}(\mathcal{S}^{-1}(\mathcal{H}))$ β^* -open in (\mathcal{C}, τ_N) . Therefore it is $\delta-\beta^*$ -open in (\mathcal{C}, τ_N) , showing that $\mathcal{S} \circ h$ is $NIr \delta-\beta^*$ CM. \square

Theorem 3.7. *Let $h : (\mathcal{C}, \tau_N) \rightarrow (\mathcal{O}, \varrho_N)$ be $NIr \delta-\beta^*$ CM and $\mathcal{S} : (\mathcal{O}, \varrho_N) \rightarrow (\mathcal{G}, \rho_N)$ be $NIr \delta-\beta^*$ CM. Then $\mathcal{S} \circ h$ is $NIr \delta-\beta^*$ CM.*

Proof. If \mathcal{H} is $\delta-\beta^*$ -open in (\mathcal{G}, ρ_N) , then $\mathcal{S}^{-1}(\mathcal{H})$ is $\delta-\beta^*$ -open in (\mathcal{O}, ϱ_N) and $h^{-1}(\mathcal{S}^{-1}(\mathcal{H}))$ is $\delta-\beta^*$ -open in (\mathcal{C}, τ_N) . Hence $\mathcal{S} \circ h$ is $NIr \delta-\beta^*$ CM. \square

Definition 3.8. Let (\mathcal{C}, τ_N) and (\mathcal{O}, ϱ_N) be NTSs. A mapping $h : (\mathcal{C}, \tau_N) \rightarrow (\mathcal{O}, \varrho_N)$ is *neutrosophic contra $\delta-\beta^*$ -continuous* ($NC \delta-\beta^*$ CM) if for every $\delta-\beta^*$ -open set \mathcal{H} in (\mathcal{O}, ϱ_N) the preimage $h^{-1}(\mathcal{H})$ is $\delta-\beta^*$ -closed in (\mathcal{C}, τ_N) .

Remark 3.9. Every $NC \beta^*$ CM is $NC \delta-\beta^*$ CM. The converse is, in general, false (see Example 3.10).

Example 3.10. Let $\mathcal{C} = \{\mathfrak{p}, \mathfrak{q}, \mathfrak{g}\}$ and $\mathcal{O} = \{\mathfrak{s}, \mathfrak{r}\}$. Consider neutrosophic topologies $\tau_N = \{0_N, 1_N, \mathcal{Q}, \mathcal{H}, \mathcal{F}\}$ on \mathcal{C} and $\varrho_N = \{0_N, 1_N, \mathcal{P}, \mathcal{K}\}$ on \mathcal{O} , where (triples are $\langle \mu, \sigma, \nu \rangle$)

$$\mathcal{Q} = \langle \mathfrak{s}; (0.3, 0.5, 0.3), (0.5, 0.5, 0.5), (0.3, 0.4, 0.3) \rangle,$$

$$\mathcal{H} = \langle \mathfrak{r}; (0.6, 0.4, 0.4), (0.4, 0.5, 0.4), (0.6, 0.5, 0.4) \rangle,$$

$$\mathcal{F} = \langle \mathfrak{g}; (0.8, 0.3, 0.5), (0.5, 0.4, 0.5), (0.7, 0.2, 0.3) \rangle,$$

$$\mathcal{P} = \langle \mathfrak{g}; (0.3, 0.4, 0.3), (0.5, 0.4, 0.4), (0.3, 0.7, 0.5) \rangle,$$

$$\mathcal{K} = \langle \mathfrak{g}; (0.6, 0.4, 0.3), (0.3, 0.4, 0.5), (0.5, 0.5, 0.4) \rangle.$$

Define $h : (\mathcal{C}, \tau_N) \rightarrow (\mathcal{O}, \varrho_N)$ by $h(\mathfrak{p}) = \mathfrak{s} = h(\mathfrak{g})$ and $h(\mathfrak{q}) = \mathfrak{r}$. If \mathcal{W} is $\delta-\beta^*$ -open in \mathcal{O} , one checks that $h^{-1}(\mathcal{W})$ is $\delta-\beta^*$ -closed in \mathcal{C} (for instance $NCl(NInt(NCl(h^{-1}(\mathcal{P})))) = 1_N \subseteq \mathcal{Q}$), hence h is $NC \delta-\beta^*$ CM.

Theorem 3.11. *If $h : (\mathcal{C}, \tau_N) \rightarrow (\mathcal{O}, \varrho_N)$ and $\mathcal{S} : (\mathcal{O}, \varrho_N) \rightarrow (\mathcal{G}, \rho_N)$ are both $NC \beta^*$ CM, then $\mathcal{S} \circ h$ is $NC \delta-\beta^*$ CM.*

Proof. Let \mathcal{H} be $\delta-\beta^*$ -open in (\mathcal{G}, ρ_N) . Then $\mathcal{S}^{-1}(\mathcal{H})$ is β^* -open in (\mathcal{O}, ϱ_N) and, since h is $NC \beta^*$ CM, $h^{-1}(\mathcal{S}^{-1}(\mathcal{H}))$ is β^* -closed in (\mathcal{C}, τ_N) , hence $\delta-\beta^*$ -closed. Thus $\mathcal{S} \circ h$ is $NC \delta-\beta^*$ CM. \square

Theorem 3.12. Let $h : (\mathcal{C}, \tau_N) \rightarrow (\mathcal{O}, \varrho_N)$ be $NC \delta-\beta^*$ CM and $\mathcal{S} : (\mathcal{O}, \varrho_N) \rightarrow (\mathcal{G}, \rho_N)$ be $NC \delta-\beta^*$ CM. Then $\mathcal{S} \circ h$ is $NC \delta-\beta^*$ CM.

Proof. As in Theorem 3.11, using $\delta-\beta^*$ -open preimages and contra continuity. \square

Theorem 3.13. Let $h : (\mathcal{C}, \tau_N) \rightarrow (\mathcal{O}, \varrho_N)$ be $NIr \delta-\beta^*$ CM and $\mathcal{S} : (\mathcal{O}, \varrho_N) \rightarrow (\mathcal{G}, \rho_N)$ be $NC \delta-\beta^*$ CM. Then $\mathcal{S} \circ h$ is $NC \delta-\beta^*$ CM.

Proof. Let H be $\delta-\beta^*$ -open in (\mathcal{G}, ρ_N) . Since \mathcal{S} is $NC \delta-\beta^*$ CM, $\mathcal{S}^{-1}(H)$ is $\delta-\beta^*$ -closed in (\mathcal{O}, ϱ_N) . Because h is $NIr \delta-\beta^*$ CM, h^{-1} preserves $\delta-\beta^*$ -open sets and hence also $\delta-\beta^*$ -closed sets (preimage commutes with complements). Thus $h^{-1}(\mathcal{S}^{-1}(H))$ is $\delta-\beta^*$ -closed in (\mathcal{C}, τ_N) , i.e., $\mathcal{S} \circ h$ is $NC \delta-\beta^*$ CM. \square

Remark 3.14. The implications among the classes of mappings follow the arrows $NIr \beta^* \Rightarrow NIr \delta-\beta^*$ and $NC \beta^* \Rightarrow NC \delta-\beta^*$; converses generally fail.

Definition 3.15. Let $(\mathfrak{G}, T_{\mathfrak{M}_G})$ and $(\mathfrak{L}, T_{\mathfrak{N}_G})$ be two MPNTSs. For any MPN $\delta-\beta^*$ -open set \mathcal{H} of $(\mathfrak{L}, T_{\mathfrak{N}_G})$, if $h^{-1}(\mathcal{H})$ is $\delta-\beta$ -open in $(\mathfrak{G}, T_{\mathfrak{M}_G})$ with respect to its m-polar neutrosophic topology, then

$$h : (\mathfrak{G}, T_{\mathfrak{M}_G}) \rightarrow (\mathfrak{L}, T_{\mathfrak{N}_G})$$

is an $MPNIr \delta-\beta^*$ CM.

Example 3.16. Let $\mathfrak{G} = \{t_1, t_2, t_3, t_4\}$ and $\mathfrak{L} = \{r_1, r_2, r_3\}$. Assume $T_{\mathfrak{M}_G}$ and $T_{\mathfrak{N}_G}$ are MPNTs on these sets. Let $\text{mpn}(\mathfrak{G}^1)$ and $\text{mpn}(\mathfrak{L}^1)$ denote the collections of all MPNSs over \mathfrak{G} and \mathfrak{L} .

Define the following two 3-polar neutrosophic sets on \mathfrak{G} :

$$\begin{aligned} \mathfrak{G}_1 = & \left\{ (t_1, \langle 0.313, 0.368, 0.401 \rangle, \langle 0.303, 0.288, 0.333 \rangle, \langle 0.361, 0.426, 0.406 \rangle), \right. \\ & (t_2, \langle 0.482, 0.392, 0.299 \rangle, \langle 0.592, 0.418, 0.571 \rangle, \langle 0.308, 0.516, 0.428 \rangle), \\ & (t_3, \langle 0.319, 0.455, 0.518 \rangle, \langle 0.529, 0.309, 0.419 \rangle, \langle 0.391, 0.409, 0.414 \rangle), \\ & \left. (t_4, \langle 0.472, 0.402, 0.317 \rangle, \langle 0.428, 0.319, 0.466 \rangle, \langle 0.327, 0.316, 0.345 \rangle) \right\}, \\ \mathfrak{G}_2 = & \left\{ (t_1, \langle 0.313, 0.345, 0.442 \rangle, \langle 0.319, 0.328, 0.311 \rangle, \langle 0.401, 0.424, 0.511 \rangle), \right. \\ & (t_2, \langle 0.523, 0.417, 0.431 \rangle, \langle 0.427, 0.518, 0.526 \rangle, \langle 0.518, 0.477, 0.317 \rangle), \\ & (t_3, \langle 0.409, 0.426, 0.481 \rangle, \langle 0.318, 0.369, 0.383 \rangle, \langle 0.407, 0.456, 0.515 \rangle), \\ & \left. (t_4, \langle 0.527, 0.408, 0.472 \rangle, \langle 0.517, 0.427, 0.513 \rangle, \langle 0.491, 0.333, 0.382 \rangle) \right\}. \end{aligned}$$

Define on \mathfrak{L} :

$$\begin{aligned} \mathfrak{L}_1 = & \left\{ (r_1, \langle 0.313, 0.368, 0.401 \rangle, \langle 0.327, 0.311, 0.308 \rangle, \langle 0.271, 0.272, 0.295 \rangle), \right. \\ & (r_2, \langle 0.307, 0.382, 0.437 \rangle, \langle 0.371, 0.387, 0.392 \rangle, \langle 0.427, 0.391, 0.403 \rangle), \\ & \left. (r_3, \langle 0.527, 0.418, 0.428 \rangle, \langle 0.417, 0.369, 0.383 \rangle, \langle 0.491, 0.333, 0.391 \rangle) \right\}, \\ \mathfrak{L}_2 = & \left\{ (r_1, \langle 0.402, 0.317, 0.308 \rangle, \langle 0.427, 0.361, 0.419 \rangle, \langle 0.518, 0.477, 0.317 \rangle), \right. \\ & (r_2, \langle 0.319, 0.455, 0.444 \rangle, \langle 0.333, 0.392, 0.299 \rangle, \langle 0.319, 0.444, 0.520 \rangle), \\ & \left. (r_3, \langle 0.417, 0.451, 0.326 \rangle, \langle 0.429, 0.318, 0.381 \rangle, \langle 0.482, 0.326, 0.377 \rangle) \right\}. \end{aligned}$$

Define $h : (\mathfrak{G}, T_{\mathfrak{M}_G}) \rightarrow (\mathfrak{L}, T_{\mathfrak{N}_G})$ by

$$h(t_1) = r_1 = h(t_2), \quad h(t_3) = r_3, \quad h(t_4) = r_2.$$

Let $T_{\mathfrak{M}_G} = \{\mathfrak{G}^0, \mathfrak{G}^1, \mathfrak{G}_1, \mathfrak{G}_2\}$ and $T_{\mathfrak{N}_G} = \{\mathfrak{L}^0, \mathfrak{L}^1, \mathfrak{L}_1, \mathfrak{L}_2\}$. Then

$$h^{-1}(\mathfrak{L}_1) = \mathfrak{G}_1, \quad h^{-1}(\mathfrak{L}_2) = \mathfrak{G}_2, \quad h^{-1}(\mathfrak{L}^0) = \mathfrak{G}^0, \quad h^{-1}(\mathfrak{L}^1) = \mathfrak{G}^1,$$

and each preimage shown is $\delta - \beta$ -open in $T_{\mathfrak{M}_G}$. Hence h is $MPN Ir \delta - \beta^*$ CM.

Theorem 3.17. Let $h : (\mathfrak{G}, T_{\mathfrak{M}_G}) \rightarrow (\mathfrak{L}, T_{\mathfrak{N}_G})$ and $\mathcal{S} : (\mathfrak{L}, T_{\mathfrak{N}_G}) \rightarrow (\mathfrak{J}, T_{\mathfrak{D}_G})$ be $MPN Ir \delta - \beta^*$ CM. Then $\mathcal{S} \circ h : (\mathfrak{G}, T_{\mathfrak{M}_G}) \rightarrow (\mathfrak{J}, T_{\mathfrak{D}_G})$ is $MPN Ir \delta - \beta^*$ CM.

Proof. Immediate from Theorem 3.6. \square

Theorem 3.18. Let $h : (\mathfrak{G}, T_{\mathfrak{M}_G}) \rightarrow (\mathfrak{L}, T_{\mathfrak{N}_G})$ be $MPN \delta - \beta^*$ CM and $\mathcal{S} : (\mathfrak{L}, T_{\mathfrak{N}_G}) \rightarrow (\mathfrak{J}, T_{\mathfrak{D}_G})$ be $MPN Ir \delta - \beta^*$ CM. Then $\mathcal{S} \circ h : (\mathfrak{G}, T_{\mathfrak{M}_G}) \rightarrow (\mathfrak{J}, T_{\mathfrak{D}_G})$ is $MPN Ir \delta - \beta^*$ CM.

Proof. By Theorem 3.7. \square

Definition 3.19. Let $(\mathfrak{G}, T_{\mathfrak{M}_G})$ and $(\mathfrak{L}, T_{\mathfrak{N}_G})$ be two MPNTSs. For any MPN $\delta-\beta^*$ -open set \mathcal{H} of $(\mathfrak{L}, T_{\mathfrak{N}_G})$, if $h^{-1}(\mathcal{H})$ is $\delta-\beta$ -closed in $(\mathfrak{G}, T_{\mathfrak{M}_G})$ with respect to its m-polar neutrosophic topology, then

$$h : (\mathfrak{G}, T_{\mathfrak{M}_G}) \rightarrow (\mathfrak{L}, T_{\mathfrak{N}_G})$$

is an *MPNC $\delta-\beta^*$ CM*.

Theorem 3.20. Let $h : (\mathfrak{G}, T_{\mathfrak{M}_G}) \rightarrow (\mathfrak{L}, T_{\mathfrak{N}_G})$ and $\mathcal{S} : (\mathfrak{L}, T_{\mathfrak{N}_G}) \rightarrow (\mathfrak{J}, T_{\mathfrak{D}_G})$ be *MPNC $\delta-\beta^*$ CM*. Then $\mathcal{S} \circ h : (\mathfrak{G}, T_{\mathfrak{M}_G}) \rightarrow (\mathfrak{J}, T_{\mathfrak{D}_G})$ is *MPNC $\delta-\beta^*$ CM*.

Proof. By Theorem 3.11. \square

Example 3.21. Let $\mathfrak{G} = \{t_1, t_2, t_3\}$ and $\mathfrak{L} = \{r_1, r_2, r_3\}$. Let $T_{\mathfrak{M}_G}$ and $T_{\mathfrak{N}_G}$ be MPNTs, and $\text{mpn}(\mathfrak{G}^1), \text{mpn}(\mathfrak{L}^1)$ the corresponding families of MPNSs. Define:

$$\mathfrak{G}_1 = \left\{ (t_1, \langle 0.613, 0.627, 0.711 \rangle, \langle 0.501, 0.551, 0.524 \rangle, \langle 0.609, 0.617, 0.583 \rangle), \right. \\ (t_2, \langle 0.613, 0.627, 0.711 \rangle, \langle 0.627, 0.519, 0.618 \rangle, \langle 0.541, 0.617, 0.681 \rangle), \\ \left. (t_3, \langle 0.451, 0.525, 0.595 \rangle, \langle 0.725, 0.618, 0.723 \rangle, \langle 0.511, 0.619, 0.634 \rangle) \right\},$$

$$\mathfrak{G}_2 = \left\{ (t_1, \langle 0.613, 0.627, 0.711 \rangle, \langle 0.319, 0.328, 0.311 \rangle, \langle 0.401, 0.424, 0.511 \rangle), \right. \\ (t_2, \langle 0.523, 0.417, 0.431 \rangle, \langle 0.427, 0.518, 0.526 \rangle, \langle 0.518, 0.477, 0.317 \rangle), \\ \left. (t_3, \langle 0.409, 0.426, 0.481 \rangle, \langle 0.318, 0.369, 0.383 \rangle, \langle 0.407, 0.456, 0.515 \rangle) \right\},$$

and

$$\mathfrak{L}_1 = \left\{ (r_1, \langle 0.613, 0.627, 0.711 \rangle, \langle 0.547, 0.516, 0.563 \rangle, \langle 0.671, 0.634, 0.674 \rangle), \right. \\ (r_2, \langle 0.595, 0.561, 0.523 \rangle, \langle 0.482, 0.456, 0.490 \rangle, \langle 0.518, 0.477, 0.618 \rangle), \\ \left. (r_3, \langle 0.725, 0.618, 0.627 \rangle, \langle 0.723, 0.692, 0.614 \rangle, \langle 0.619, 0.612, 0.745 \rangle) \right\},$$

$$\mathfrak{L}_2 = \left\{ (r_1, \langle 0.501, 0.551, 0.534 \rangle, \langle 0.551, 0.524, 0.644 \rangle, \langle 0.608, 0.651, 0.811 \rangle), \right. \\ (r_2, \langle 0.627, 0.672, 0.781 \rangle, \langle 0.517, 0.519, 0.728 \rangle, \langle 0.681, 0.627, 0.617 \rangle), \\ \left. (r_3, \langle 0.614, 0.813, 0.725 \rangle, \langle 0.851, 0.735, 0.817 \rangle, \langle 0.689, 0.728, 0.744 \rangle) \right\}.$$

Define

$$h : (\mathfrak{G}, T_{\mathfrak{M}_G}) \rightarrow (\mathfrak{L}, T_{\mathfrak{N}_G}), \quad h(t_1) = r_3, \quad h(t_2) = r_1, \quad h(t_3) = r_2.$$

Let $T_{\mathfrak{M}_G} = \{\mathfrak{G}^0, \mathfrak{G}^1, \mathfrak{G}_1, \mathfrak{G}_2\}$ and $T_{\mathfrak{N}_G} = \{\mathfrak{L}^0, \mathfrak{L}^1, \mathfrak{L}_1, \mathfrak{L}_2\}$. Then

$$h^{-1}(\mathfrak{L}_1) = \mathfrak{G}_1, \quad h^{-1}(\mathfrak{L}_2) = \mathfrak{G}_2, \quad h^{-1}(\mathfrak{L}^0) = \mathfrak{G}^0, \quad h^{-1}(\mathfrak{L}^1) = \mathfrak{G}^1,$$

and the preimages asserted are $\delta-\beta$ -closed in $T_{\mathfrak{M}_G}$; hence h is *MPNC $\delta-\beta^*$ CM*.

Theorem 3.22. Let $h : (\mathfrak{G}, T_{\mathbb{M}_{\mathfrak{G}}}) \rightarrow (\mathfrak{L}, T_{\mathbb{N}_{\mathfrak{G}}})$ be MPN $\delta - \beta^*$ CM and $\mathcal{S} : (\mathfrak{L}, T_{\mathbb{N}_{\mathfrak{G}}}) \rightarrow (\mathfrak{J}, T_{\mathbb{D}_{\mathfrak{G}}})$ be MPNC $\delta - \beta^*$ CM. Then $\mathcal{S} \circ h : (\mathfrak{G}, T_{\mathbb{M}_{\mathfrak{G}}}) \rightarrow (\mathfrak{J}, T_{\mathbb{D}_{\mathfrak{G}}})$ is MPNC $\delta - \beta^*$ CM.

Proof. By Theorem 3.11. \square

Theorem 3.23. Let $\mathcal{S} : (\mathfrak{L}, T_{\mathbb{N}_{\mathfrak{G}}}) \rightarrow (\mathfrak{J}, T_{\mathbb{D}_{\mathfrak{G}}})$ be MPNC $\delta - \beta^*$ CM and $h : (\mathfrak{G}, T_{\mathbb{M}_{\mathfrak{G}}}) \rightarrow (\mathfrak{L}, T_{\mathbb{N}_{\mathfrak{G}}})$ be MPNIr $\delta - \beta^*$ CM. Then $\mathcal{S} \circ h : (\mathfrak{G}, T_{\mathbb{M}_{\mathfrak{G}}}) \rightarrow (\mathfrak{J}, T_{\mathbb{D}_{\mathfrak{G}}})$ is MPNC $\delta - \beta^*$ CM.

Proof. The claim follows from Theorem 3.13. \square

Remark 3.24. Figure 1 illustrates the relationships among the various classes for the map $h : (\mathfrak{G}, T_{\mathbb{M}_{\mathfrak{G}}}) \rightarrow (\mathfrak{L}, T_{\mathbb{N}_{\mathfrak{G}}})$.

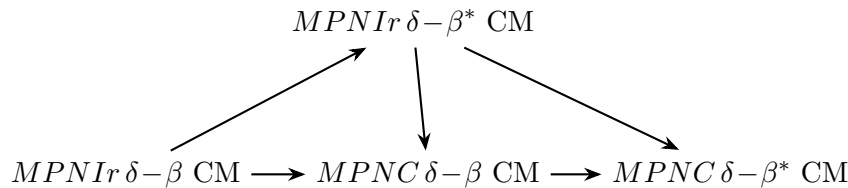


FIGURE 1. Relationships between different types of MPNTS continuous mappings.

4. Conclusions

We studied new classes of neutrosophic $\delta - \beta^*$ -continuous mappings, namely the irresolute and contra versions, and analyzed their fundamental properties and interrelations. We also discussed stability under composition and provided counterexamples showing that certain converses fail. Finally, we extended the framework to the m-polar setting, introducing MPNIr $\delta - \beta^*$ CM and MPNC $\delta - \beta^*$ CM and showing how the main results lift to MPNTSs.

The comparison with the $\delta - \beta$ (non-star) counterparts clarifies the hierarchy among the notions ($NIr \beta^* \Rightarrow NIr \delta - \beta^*$ and $NC \beta^* \Rightarrow NC \delta - \beta^*$, but not conversely) and highlights how irresolute and contra behaviors diverge. These findings contribute to a more refined picture of continuity in neutrosophic topology and suggest further applications in multi-criteria settings modeled via m-polar neutrosophic structures.

Conflict of Interest

No conflicts of interest are disclosed by the authors.

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