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Neutrosophication as a General Framework for Regret Theory, Grey System Theory, and Three-Way Decision Models

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Abstract: Neutrosophy represents a comprehensive philosophical and mathematical framework for the study of neutrality, indeterminacy, and the interaction between opposites. The subsequent development of Refined Neutrosophy and Refined Neutrosophication significantly expanded the applicability of neutrosophic concepts to decision sciences, artificial intelligence, engineering, economics, and uncertainty modeling. This paper demonstrates that several influential theories developed independently in decision sciences and systems analysis—namely Regret Theory, Grey System Theory, and Three-Way Decision Theory—can be interpreted as particular cases of Neutrosophication. A formal Neutrosophication Operator is introduced to show how binary frameworks are transformed into triadic neutrosophic structures. We further demonstrate that Three-Way Decision models arise naturally from Neutrosophic Probability and that n-Way Decision models emerge as particular cases of Refined Neutrosophy. The proposed framework establishes Neutrosophy as a unifying meta-theory capable of integrating multiple approaches to uncertainty, partial knowledge, and decision-making under indeterminate conditions.

Keywords: Neutrosophy, Neutrosophication, Refined Neutrosophy, Regret Theory, Grey System Theory, Three-Way Decision, n-Way Decision, Neutrosophic Probability, Uncertainty Modeling.

1. Introduction

Classical scientific reasoning has traditionally relied upon binary distinctions such as true/false, success/failure, accepted/rejected, and white/black. Such dichotomous frameworks have been extraordinarily successful in mathematics, formal logic, and engineering. Nevertheless, many real-world phenomena cannot be adequately represented through purely binary structures. Decision-makers frequently encounter incomplete information, contradictory evidence, uncertain forecasts, emotional biases, and partially observable systems. Under such conditions, binary classifications often oversimplify reality. To address these limitations, I introduced Neutrosophy as a new branch of philosophy devoted to studying neutralities and their interactions with opposing concepts. According to Neutrosophy, every concept is associated with its opposite and a spectrum of neutralities situated between them.

The neutrosophic triad *A, neutA, antiA* provides a natural representation of certainty, indeterminacy, and opposition. Over time, Neutrosophy evolved into several mathematical theories including Neutrosophic Logic, Neutrosophic Sets, Neutrosophic Probability,Neutrosophic Statistics, Refined Neutrosophy, and Neutrosophication.

This paper investigates how Regret Theory, Grey System Theory, and Three-Way Decision Theory can be interpreted as particular manifestations of the neutrosophic paradigm.

2. Neutrosophy and Neutrosophication

2.1. Fundamental Neutrosophic Structure

The fundamental neutrosophic structure consists of:

A = affirmation, truth, acceptance

$neutA$ = neutrality, indeterminacy

$antiA$ = negation, falsity, rejection

A neutrosophic set is characterized by three independent membership functions:

$T(x)$ = truth-membership

$I(x)$ = indeterminacy-membership

$F(x)$ = falsity-membership

for every element x belonging to universe U.

The neutrosophic representation is:

$$N(x) = (T(x), I(x), F(x))$$

with

$$0 \leq T(x) \leq 1$$
$$0 \leq I(x) \leq 1$$
$$0 \leq F(x) \leq 1$$

and

$$0 \leq T(x) + I(x) + F(x) \leq 3$$

Unlike fuzzy systems, the three components are independent. This allows simultaneous representation of certainty, uncertainty, contradiction, and ignorance.

2.2. Neutrosophication

Neutrosophication is the process of transforming a binary system into a neutrosophic triadic system. Consider a binary partition:

$$P = \{A, antiA\}$$

The neutrosophication operator N transforms this partition into:

$$N(P) = \{A, neutA, antiA\}$$

The intermediate state *neutA* explicitly captures uncertainty, ambiguity, hesitation, ignorance, conflict, or neutrality.

Graphically:

Binary Structure

A ____________________ $antiA$

↓

Neutrosophication

A__________ $neutA$ __________ $antiA$

The introduction of *neutA* converts a rigid binary framework into a more realistic model capable of handling incomplete information.

2.3 General Neutrosophication Operator

Definition 1

Let

$$P = \{A, antiA\}$$

be a binary partition.

The Neutrosophication Operator is defined as

$$N : \{A, antiA\} \rightarrow \{A, neutA, antiA\}$$

where *neutA* represents the set of all neutral, uncertain, or indeterminate states between A and $antiA$.

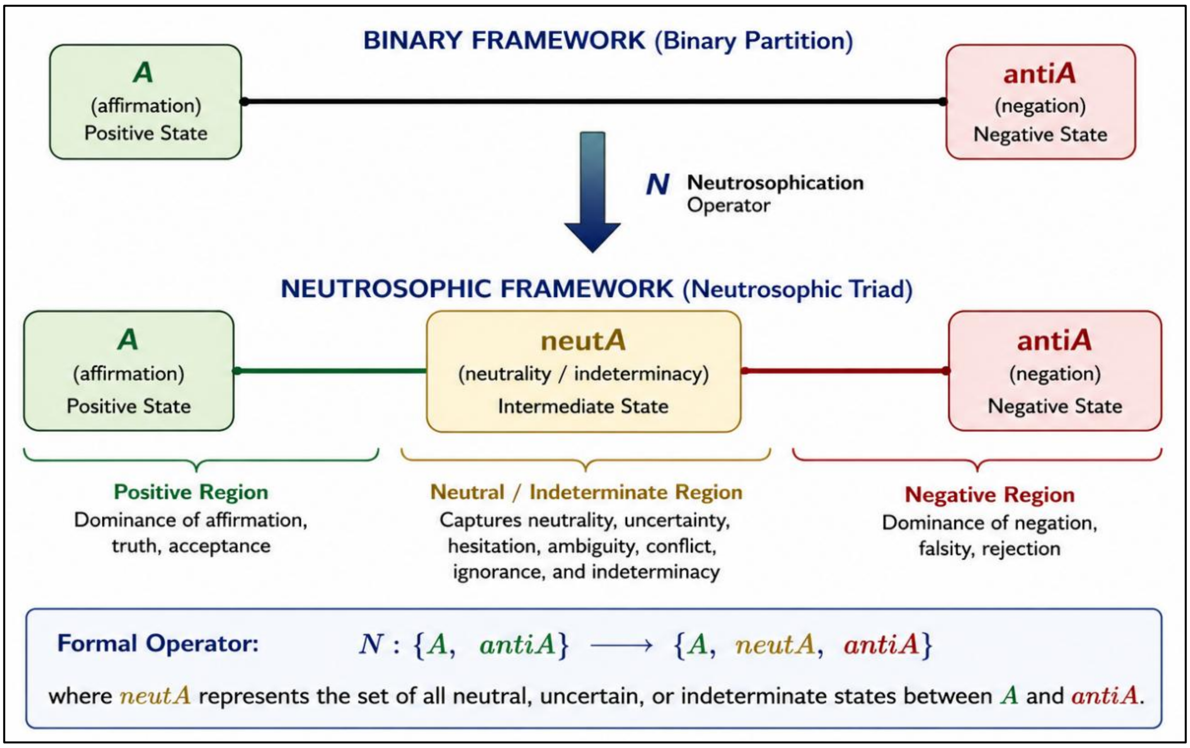

Figure 1. General Neutrosophication Operator transforming a binary partition {*A*, *antiA*} into the neutrosophic triad {*A*, *neutA*, *antiA*}. The intermediate state explicitly captures neutrality, uncertainty, hesitation, ambiguity, and indeterminacy.

Proposition 1

Any system containing:

- a positive state,
- a negative state,
- an intermediate state,

can be represented as a neutrosophic structure.

Proof.

Let

$$S = \{S_+, S_0, S_-\}$$

where:

S_+ = positive state

S_0 = intermediate state

S_- = negative state

Define the mapping:

$$\varphi(S_+) = A$$
$$\varphi(S_0) = neutA$$
$$\varphi(S_-) = antiA$$

The mapping is one-to-one and onto.

Therefore S is structurally equivalent to the neutrosophic triad. □

3. Regret Theory as a Neutrosophication Model

3.1 Overview of Regret Theory

Regret Theory was proposed as an alternative to Expected Utility Theory. According to Regret Theory, decision-makers compare obtained outcomes with outcomes that could have been achieved through alternative choices.

The emotional consequences of such comparisons produce:

- Rejoicing
- Regret

The overall evaluation may be represented by:

$$U = v(x) + R(x, y)$$

where

$v(x)$ = traditional utility
$R(x, y)$ = regret-rejoicing function
x = chosen outcome
y = foregone alternative.

3.2 Neutrosophic Interpretation

Regret Theory naturally partitions the cognitive space into three regions.

Regret Theory	*Neutrosophic Equivalent*
Rejoicing	A
Regret Zone	neutA
Severe Loss	antiA

Thus, (Rejoicing, Regret, Loss) corresponds to (*A, neutA, antiA*). The regret region is neither fully positive nor fully negative. Instead, it reflects uncertainty regarding the optimality of the chosen action.

3.3 Regret as Indeterminacy

Regret arises because decision-makers cannot know with certainty whether an alternative choice would have produced a superior outcome.

Hence:

$$\text{Regret} \subseteq \text{Indeterminacy},$$

or equivalently:

$$\text{Regret} \subseteq \text{neutA}.$$

Therefore Regret Theory can be interpreted as a behavioral implementation of Neutrosophication.

4. Grey System Theory as a Neutrosophication

4.1 Fundamentals of Grey Systems

Grey System Theory was introduced by Deng to model systems characterized by incomplete information.

The theory distinguishes:

- White Systems = complete information,
- Black Systems = no information,
- Grey Systems = partial information.

4.2 Neutrosophic Correspondence

Grey System Theory	*Neutrosophy*
White System	*A*
Grey System	*neutA*
Black System	*antiA*

This correspondence is exact.

The grey region functions as the neutrosophic indeterminate zone.

4.3 Greyness as Indeterminacy

Let G denote the degree of greyness. A neutrosophic representation may be written as:

$$N = (T, I, F)$$

where

$$I = G$$

Thus:

Greyness ⊆ Indeterminacy,

or

$$G \subseteq I.$$

Consequently, Grey System Theory represents a specialized application of Neutrosophic modeling.

5. Three-Way Decision as a Particular Case of Neutrosophication

5.1 Three-Way Decision Theory

Classical decisions are binary:

Decision = {Accept, Reject}

Three-Way Decision introduces a third region:

Decision = {Accept, Defer, Reject}

The deferment region allows the decision-maker to postpone commitment until additional evidence becomes available.

5.2 Neutrosophic Equivalence

Three-Way Decision	***Neutrosophy***
Accept	*A*
Defer	*neutA*
Reject	*antiA*

The structural equivalence is immediate. The deferment region is the neutrosophic neutrality zone.

5.3 Three-Way Decision and Neutrosophic Probability

Neutrosophic probability characterizes events using:

$$NP(E) = (T, I, F)$$

where:

T = truth probability
I = indeterminate probability
F = falsity probability

Define thresholds:

α = acceptance threshold
β = rejection threshold

with

$$0 \leq \beta < \alpha \leq 1$$

Decision rules become:

Accept if $T \geq \alpha$
Defer if $\beta < T < \alpha$ or I is dominant
Reject if $T \leq \beta$

These rules reproduce exactly the Three-Way Decision framework.

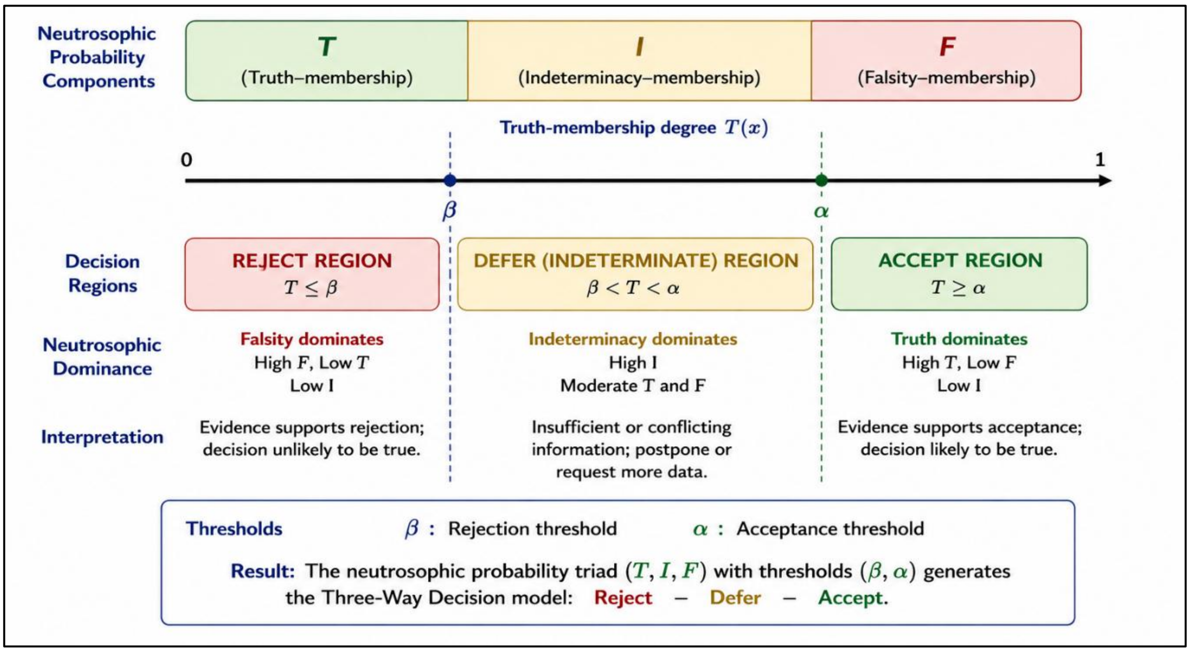

Figure 2. Generation of Three-Way Decision regions from neutrosophic probability through acceptance and rejection thresholds.

Theorem 1

Three-Way Decision is a particular case of Neutrosophic Probability.

Proof.

The decision regions {Accept, Defer, Reject} correspond respectively to {*T*, *I*, *F*} under threshold partitioning.

Therefore:

$$\text{Three-Way Decision} \subset \text{Neutrosophic Probability}. \ \square$$

6. From Three-Way Decision to n-Way Decision through Refined Neutrosophy

6.1 Refined Neutrosophy

Refined Neutrosophy decomposes the classical components *T*, *I*, and *F* into multiple subcomponents:

$$T \to T_1, T_2, \ldots, T_p$$
$$I \to I_1, I_2, \ldots, I_r$$
$$F \to F_1, F_2, \ldots, F_s$$

subject to

$$p + r + s = n$$

This refinement enables the representation of multiple forms of truth, falsity, and indeterminacy.

6.2 n-Way Decision

The refinement process naturally produces multiple decision actions. Instead of:

Accept
Defer
Reject

one obtains:

Absolute Accept
Conditional Accept
Accept with Monitoring

Request Additional Data
Expert Review
Conditional Reject
Absolute Reject

Hence: $3WD \rightarrow nWD,$ where $n > 3.$

6.3 Refined Neutrosophic Decision Spaces

Suppose:

$$I = \{I_1, I_2\}$$

where

I_1 = uncertainty due to missing data
I_2 = uncertainty due to conflicting data

Then:

I_1 → Wait for additional measurements
I_2 → Request expert validation

The single deferment region splits into multiple decision pathways.

Theorem 2

n-Way Decision is a particular case of Refined Neutrosophy.

Proof.

Let

$$RN = \{T1, \dots, Tp, I1, \dots, Ir, F1, \dots, Fs\}$$

with

$$p + r + s = n$$

Assign a unique action to each refined component. The resulting decision space contains n distinct actions. Therefore:

n-Way Decision ⊂ Refined Neutrosophy □

7. Comparative Analysis

Table 1. Structural Correspondence.

Generic Structure	*Neutrosophy*	*Regret Theory*	*Grey Systems*	*Three-Way Decision*
Positive Region	*A*	Rejoicing	White	Accept
Intermediate Region	*neutA*	Regret	Grey	Defer
Negative Region	*antiA*	Loss	Black	Reject

Table 2. Information Representation.

Framework	*Complete Information*	*Partial Information*	*No Information*
Classical Logic	True	Not Represented	False
Grey Systems	White	Grey	Black
Three-Way Decision	Accept	Defer	Reject
Neutrosophy	*T*	*I*	*F*

Table 3. Evolution of Decision Models.

Model	*Number of States*
Binary Decision	2
Three-Way Decision	3
Refined Three-Way Decision	4–10
n-Way Decision	*n*
Refined Neutrosophic Decision	*p+r+s*

8. Mathematical Generalization Theorem: Every Triadic Decision System is Neutrosophic

8.1 Motivation

The previous sections demonstrated that Regret Theory, Grey System Theory, and Three-Way Decision Theory can be represented through the neutrosophic triad (A, neutA, antiA). An important question naturally arises: *Is this correspondence accidental, or does it reveal a deeper mathematical principle?*

We argue that the latter is true. The neutrosophic structure is not merely one example among many triadic structures; rather, it represents a universal mathematical form underlying all triadic decision systems. This section formalizes this claim.

8.2 Definition of a Triadic Decision System

A triadic decision system is a system whose state space is partitioned into three mutually distinguishable regions:

$$D = \{D_+, D_0, D_-\}$$

where:

D_+ = positive region,
D_0 = intermediate region,
D_- = negative region.

The three regions need not be equally sized, symmetric, or probabilistically balanced. The only requirement is that D_0 represents an intermediate state separating D_+ from D_-. Examples include:

Accept / Defer / Reject,
White / Grey / Black,
Rejoicing / Regret / Loss,
Safe / Uncertain / Dangerous,
Success / Pending / Failure.

8.3 General Representation Theorem

Theorem 3

Every triadic decision system is representable as a neutrosophic structure.

Proof.

Let

$$D = \{D_+, D_0, D_-\}$$

be an arbitrary triadic decision system.

Construct the mapping

$$\varphi : D \to N$$

where N denotes the neutrosophic triad N = {A, neutA, antiA}, such that

$$\varphi(D_+) = \text{A}$$
$$\varphi(D_0) = \text{neutA}$$
$$\varphi(D_-) = \text{antiA}$$

Since every element of D is uniquely assigned to one element of N and every element of N receives one corresponding image, φ is a bijection. Therefore D and N are structurally isomorphic.

Hence every triadic decision system can be represented as a neutrosophic structure. □

8.4 Corollary

Since Regret Theory, Grey System Theory, and Three-Way Decision Theory are triadic systems, they are all particular instances of the general neutrosophic structure. Therefore:

Regret Theory ⊂ Neutrosophy
Grey System Theory ⊂ Neutrosophy
Three-Way Decision ⊂ Neutrosophy □

8.5 Consequences

The theorem has important implications.

First, it demonstrates that neutrosophy is not merely another uncertainty framework competing with existing approaches. Instead, neutrosophy functions as a higher-order conceptual framework capable of embedding a broad family of triadic systems.

Second, the theorem suggests that future triadic decision architectures developed in artificial intelligence, economics, medicine, and engineering can be analyzed through neutrosophic methods.

Third, it establishes a mathematical justification for extending neutrosophication beyond the theories analyzed in this paper.

Consequently, neutrosophy may be viewed as a universal language for triadic reasoning under uncertainty.

9. Neutrosophic Connections with Rough Sets, Dempster–Shafer Theory, and Belief Functions

9.1 Extending the Scope of Neutrosophication

The previous sections focused on three theories that can be directly interpreted as particular cases of neutrosophication. However, the neutrosophic framework extends much further and exhibits deep structural relationships with other major uncertainty theories.

Among the most important are:

Rough Set Theory,
Dempster–Shafer Evidence Theory,
Belief Function Theory.

These theories emerged independently, yet each contains a fundamental intermediate region analogous to neutrosophic indeterminacy.

9.2 Rough Sets and Neutrosophic Boundary Regions

Rough Set Theory, introduced by Pawlak, represents a concept through:

Lower Approximation,
Boundary Region,
Upper Approximation.

Formally,

$$R(X) = (\text{Lower}(X), \text{Boundary}(X), \text{Upper}(X)).$$

The boundary region contains objects that cannot be classified with certainty.

This corresponds directly to neutrosophic indeterminacy.

The mapping is:

Rough Sets	***Neutrosophy***
Lower Approximation	A
Boundary Region	$neutA$
Upper Complement Region	$antiA$

The Rough Set boundary region therefore functions as a neutrosophic neutral zone.

This observation explains why Three-Way Decision Theory, originally derived from rough sets, naturally aligns with neutrosophication.

9.3 Dempster–Shafer Theory and Neutrosophic Probability

Dempster–Shafer Theory characterizes uncertainty through:

Belief (*Bel*), Plausibility (*Pl*), Ignorance.

For a proposition A,

$$Bel(A) \leq Pl(A)$$

and

$$Ignorance(A) = Pl(A) - Bel(A)$$

The ignorance interval measures uncertainty that cannot be assigned either to acceptance or rejection. This structure mirrors neutrosophic probability:

$$NP(A) = (T, I, F)$$

where:

T corresponds to belief,
I corresponds to ignorance,
F corresponds to disbelief.

The correspondence is summarized below.

Dempster–Shafer	***Neutrosophic Probability***
Belief	T
Ignorance	I
Disbelief	F

Thus Dempster–Shafer Theory can be interpreted as a restricted form of neutrosophic probability.

9.4 Belief Functions and Refined Indeterminacy

Belief function theory extends classical probability by allowing masses to be assigned to sets of hypotheses rather than individual hypotheses.

This naturally generates multiple sources of uncertainty.

From a neutrosophic perspective, such uncertainties can be represented through refined indeterminacy components:

$$I = \{I_1, I_2, \ldots, I_r\},$$

where:

I_1 may represent measurement uncertainty,
I_2 may represent epistemic uncertainty,
I_3 may represent ambiguity,
I_4 may represent conflicting evidence.

The refined neutrosophic framework therefore offers a richer representation of belief functions than classical probability.

9.5 Unified View of Uncertainty Theories

The theories discussed throughout this paper can be organized into a single hierarchy.

Theory	***Positive Component***	***Indeterminate Component***	***Negative Component***
Regret Theory	Rejoicing	Regret	Loss
Grey Systems	White	Grey	Black
Rough Sets	Lower Approximation	Boundary	Outside Region
Three-Way Decision	Accept	Defer	Reject
Dempster–Shafer	Belief	Ignorance	Disbelief
Neutrosophic Probability	T	I	F

The repeated appearance of a positive state, an intermediate state, and a negative state suggests the existence of a common structural principle.

Neutrosophy provides a natural candidate for this unifying principle.

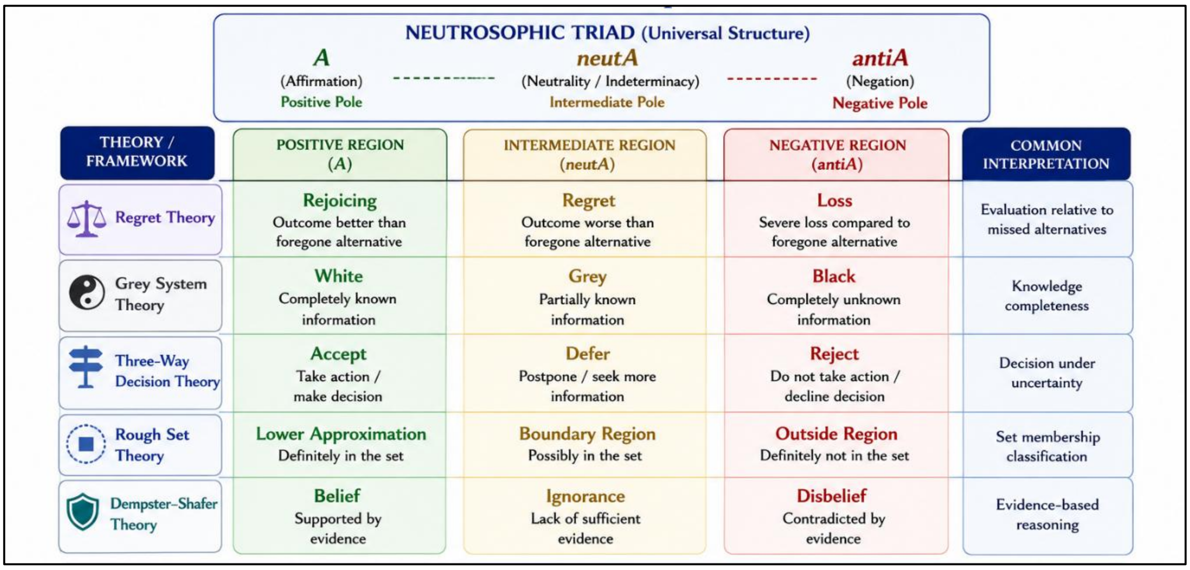

Figure 3. Unified Mapping of Theories into the Neutrosophic Triad.

9.6 Research Perspective

The structural correspondences identified in this paper suggest a new research direction *Neutrosophic Unification Theory*. Under this perspective, diverse uncertainty frameworks may be viewed as specialized realizations of a broader neutrosophic architecture.

Future research may investigate:

- Neutrosophic Rough Sets,
- Neutrosophic Evidence Theory,
- Refined Neutrosophic Belief Functions,
- Neutrosophic Artificial Intelligence.

Such developments may contribute toward a unified mathematical theory of uncertainty and decision-making.

10. Unified Inclusion Theorem

The relationships established throughout this paper can be summarized by:

Regret Theory ⊂ Neutrosophication

Grey System Theory ⊂ Neutrosophication

Three-Way Decision ⊂ Neutrosophication

Three-Way Decision ⊂ Neutrosophic Probability

n-Way Decision ⊂ Refined Neutrosophy.

These inclusions define a hierarchy:

Neutrosophy

↓

Neutrosophication

↓

{Regret Theory, Grey System Theory, Three-Way Decision}

↓

Neutrosophic Probability

↓

Refined Neutrosophy

↓

n-Way Decision

This hierarchy establishes Neutrosophy as a meta-theoretical framework encompassing several independent theories of uncertainty and decision-making.

11. Implications for Artificial Intelligence and Decision Sciences

The proposed unification has important implications.

11.1 Artificial Intelligence

Modern AI systems frequently encounter incomplete, contradictory, and uncertain information. Neutrosophic architectures provide richer representations than binary or probabilistic systems.

11.2 Multi-Criteria Decision Making (MCDM)

Refined neutrosophic decision spaces allow decision-makers to distinguish between different sources of uncertainty and apply tailored actions.

11.3 Socio-Ecological Systems

Complex socio-ecological systems often operate under partial information and conflicting stakeholder objectives. Neutrosophic decision frameworks provide a natural mechanism for representing these realities.

11.4 Grey-Neutrosophic Hybrid Models

Future research may combine Grey Relational Analysis, Neutrosophic MCDM, and Refined n-Way Decision to create next-generation decision-support systems.

12. Conclusions

This study demonstrates that Regret Theory, Grey System Theory, and Three-Way Decision Theory can be formally interpreted as particular cases of Neutrosophication. Each theory introduces an intermediate state situated between two opposing conditions, thereby reflecting the fundamental neutrosophic concept of neutrality and indeterminacy. The introduction of a General Neutrosophication Operator provides a formal mechanism for transforming binary systems into neutrosophic structures. Furthermore, Three-Way Decision Theory emerges naturally from Neutrosophic Probability, while n-Way Decision Theory emerges from Refined Neutrosophy. These results support the view that Neutrosophy functions as a comprehensive meta-theoretical framework capable of unifying diverse approaches to uncertainty, partial knowledge, behavioral evaluation, and complex decision-making. Future research should focus on the development of formal neutrosophic algorithms, grey-neutrosophic hybrid systems, and refined neutrosophic decision architectures for applications in artificial intelligence, economics, engineering, and socio-ecological management.

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