NSS

Neutrosophic Sets and Systems, Vol. 99, 2026

University of New Mexico

Real-World Implementation of a Neutrosophic Logarithmic Estimator for Population Mean Estimation under Stratified Random Sampling

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Abstract. This paper addresses the problem of utilizing auxiliary information to estimate the population mean under neutrosophic stratified random sampling. Within the framework of neutrosophic statistics, we propose a neutrosophic combined logarithmic type estimator that effectively accounts for the uncertainty and indeterminacy inherent in survey sampling. The suggested estimator's bias and mean squared error (MSE) are expressed up to the first order of approximation, and ideal circumstances for reducing the MSE are determined. Neutroposophic combined mean, neutrosophic combined ratio, and neutrosophic combined difference estimators are theoretically compared with the suggested estimator.The analytical findings show that in some real-world situations, the suggested estimator performs better than its conventional competitors. Additionally, line and radar chart representations and empirical data analysis are used to assess the performance of the suggested estimator. The results provide useful information for practitioners and survey statisticians who use neutrosophic stratified random sampling when uncertainty is present.

Keywords: Bias, Neutrosophic Estimator, Mean Squared Error, Percent Relative Efficiency.

1. **Introduction**

Stratified random sampling is an essential component of modern survey techniques due to its ability to enhance accuracy through effective population stratification. However, real-world observations are often imprecise or uncertain, whereas traditional sample surveys typically assume precisely measured data. Smarandache [9] introduced neutrosophic statistics, which explicitly incorporate indeterminacy and represent data as intervals, providing a powerful framework for addressing problem of uncertainty. To specifically handle the uncertainty in survey sampling, a great deal of work has been done on the combination of neutrosophic theory and survey sampling. Tahir et al. [12], Sherwani and Saleem [5], Kumar et al. [2], Yadav and Prasad ([16], [17]), Singh et al. [6], Singh et al. [7], and Kumar and Tripathi [1] are notable authors in neutrosophic stratified sampling. These investigations have shown that neutrosophic stratified estimators are a good way to deal with sample survey uncertainty. Shabbir and Gupta [4] and Taneja et al. [13] draw attention to ongoing difficulties in enhancing estimator adaptability and efficiency at varying degrees of indeterminacy. The methodological foundation for handling ambiguous survey sampling data is strengthened by Smarandache's recent theoretical contributions [11], which also offer up exciting new research directions in neutrosophic sampling theory. When dealing with uncertainty and indeterminacy in real-world data, neutrophilic statistics provide a methodological framework, especially for imprecise or interval-valued observations ([9]; [10]). Stratified and ranked set sampling estimators that use neutrosophic principles allow for more precise inference while clearly measuring uncertainty. Research by Priya and Kumar [3], Singh and Kumari [8], Vishwakarma and Singh [14], and Yadav et al. [18] shows how useful these techniques are in improving decision-making in the social sciences, climatology, and economics.

This study presents a neutrosophic combined logarithmic estimator for estimating the population mean in stratified random sampling, motivated by the growing need for techniques that handle uncertainty in survey sampling. Extending Taneja et al. [13], the estimator directly models data indeterminacy by including neutrosophic notions. We define optimum conditions for minimization of MSE and derive the formulas of bias and mean squared error (MSE) to the first order of approximation. Theoretical and empirical analysis evaluation using climate datasets reveal that the suggested estimator significantly improves precision compared with the other existing neutrosophic estimators in suvey sampling.

The paper is structured as follows: Section 2 presents the methodology and notation. Section 3 provides the literature on existing estimators. Section 4 describes the formulation and development of the suggested neutrosophic combined logarithmic estimator. Section 5 presents an efficiency comparison of the MSE values between the suggested and existing estimators. In Section 6, real climate datasets demonstrate the applicability of the suggested estimator. The results and discussion are presented in Section 7, Section 8 concludes the study by highlighting the key results and suggesting potential directions for future research of survey sampling.

2. Methodology and Notation

Measurements in real-world data collection often exhibit uncertainty or indeterminacy, which can be attributed to measurement limits or inherent variability. By representing each

observation as an interval rather than a definite value, neutrosophic statistics overcomes this difficulty. A neutrosophic number Z_N is expressed as an interval $[Z_L, Z_U]$, where Z_L and Z_U represent the lower and upper bounds of possible values. It is given by $Z_N = Z_L + Z_U I_N$, with $I_N \in [I_L, I_U]$ denoting the indeterminacy component. This formulation captures the possible range of the true value and provides a robust framework for handling uncertainty in data, as introduced by Yadav and Smarandache [15]. In stratified sampling, let a finite population Ω of size P_N be divided into k homogeneous and non-overlapping strata. The h^{th} stratum consists of P_{hN} units such that $\sum_{h=1}^{k} P_{hN} = P_N$, and the corresponding stratum weight is defined as $W_{hN} = \frac{P_{hN}}{P_N}$ for $h = 1, 2, \ldots, k$. Within the neutrosophic framework, the population size P_N and the stratum sizes P_{hN} are not precisely known and are instead expressed as intervals to account for uncertainty: $P_N \in [P_L, P_U], \quad P_{hN} \in [P_{hL}, P_{hU}]$. From each stratum h, a neutrosophic random sample of size $p_{hN} \in [p_{hL}, p_{hU}]$ is selected using simple random sampling without replacement (SRSWOR). The total sample size is given by $p_N = \sum_{h=1}^{k} p_{hN}$. Let Y_{ihN} and X_{ihN} denote the neutrosophic observations of the study and auxiliary variables for the i^{th} unit in the h^{th} stratum, expressed as $Y_{ihN} \in [Y_{ihL}, Y_{ihU}], \quad X_{ihN} \in [X_{ihL}, X_{ihU}]$. The corresponding stratum means are:
$\bar{Y}_{hN} = \frac{1}{P_{hN}} \sum_{i=1}^{P_{hN}} Y_{ihN}, \quad \bar{X}_{hN} = \frac{1}{P_{hN}} \sum_{i=1}^{P_{hN}} X_{ihN}$, each representing an interval-valued mean. The overall population means are $\bar{Y}_N = \sum_{h=1}^{k} W_{hN} \bar{Y}_{hN}, \quad \bar{X}_N = \sum_{h=1}^{k} W_{hN} \bar{X}_{hN}$, where $\bar{X}_N$ is assumed known. For sampled units in stratum h, $y_{ihN} \in [y_{ihL}, y_{ihU}], \quad x_{ihN} \in [x_{ihL}, x_{ihU}]$. The objective is to estimate the population mean $\bar{Y}_{hN}$ using the known population mean $\bar{X}_{hN}$ of the auxiliary variable x. The sample means within the h^{th} stratum are given by $\bar{y}_{hN} = \frac{1}{p_{hN}} \sum_{i=1}^{p_{hN}} y_{ihN}, \quad \bar{x}_{hN} = \frac{1}{p_{hN}} \sum_{i=1}^{p_{hN}} x_{ihN}$. Within each stratum, the following neutrosophic descriptive statistics are defined: Coefficients of variation for the study and auxiliary variables: $C_{yhN} \in [C_{yhL}, C_{yhU}], \quad C_{xhN} \in [C_{xhL}, C_{xhU}]$. Correlation coefficient between the study and auxiliary variables: $\rho_{yxhN} \in [\rho_{yxhL}, \rho_{yxhU}]$. Population mean squares: $S_{yN}^2 = \frac{1}{P_N - 1} \sum_{h=1}^{k} \sum_{j=1}^{P_{hN}} (y_{jhN} - \bar{Y}_N)^2, \quad S_{xN}^2 = \frac{1}{P_N - 1} \sum_{h=1}^{k} \sum_{j=1}^{P_{hN}} (x_{jhN} - \bar{X}_N)^2$. Population mean squares within the h^{th} stratum: $S_{yhN}^2 = \frac{1}{P_{hN} - 1} \sum_{j=1}^{P_{hN}} (y_{jhN} - \bar{Y}_{hN})^2, \quad S_{xhN}^2 = \frac{1}{P_{hN} - 1} \sum_{j=1}^{P_{hN}} (x_{jhN} - \bar{X}_{hN})^2$. Population covariance within the h^{th} stratum: $S_{yxhN} = \frac{1}{P_{hN} - 1} \sum_{j=1}^{P_{hN}} (y_{jhN} - \bar{Y}_{hN})(x_{jhN} - \bar{X}_{hN})$. These measures reflect the variability, association, and shape characteristics of the neutrosophic data within each stratum and play crucial roles in designing efficient estimators. This stratified sampling scheme with neutrosophic interval-valued data provides a flexible and realistic framework for survey sampling under uncertainty and indeterminacy. It allows modeling of imprecise population parameters and sample observations, enabling the development of robust estimators that effectively integrate auxiliary information while accounting for the inherent data uncertainty.

3. Theoretical Framework of Existing Estimators

3.1. *Neutrosophic combined mean estimator*

The neutrosophic combined mean estimator for population mean $\bar{Y}_N$ is provide as:

$$\bar{y}_{stN} = \sum_{h=1}^{k} W_{hN}\bar{y}_{hN} \tag{1}$$

The variance of $\bar{y}_{stN}$ is given by

$$Var(\bar{y}_{stN}) = \bar{Y}_N^2 \sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} C_{yhN}^2, \tag{2}$$

where $\gamma_{hN} = \left(\frac{1}{p_{hN}} - \frac{1}{P_{hN}}\right)$, $C_{yhN}^2 = S_{yhN}^2/\bar{Y}_N^2$.
Similarly, we define

$$\bar{x}_{stN} = \sum_{h=1}^{k} W_{hN}\bar{x}_{hN} \tag{3}$$

The variance of $\bar{x}_{stN}$ is given by

$$Var(\bar{x}_{stN}) = \bar{X}_N^2 \sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} C_{xhN}^2, \tag{4}$$

where $C_{xhN}^2 = S_{xhN}^2/\bar{X}_N^2$.

3.2. *Neutrosophic combined ratio estimator*

The neutrosophic combined ratio estimator for $\bar{Y}_N$ is defined by

$$\bar{Y}_{NCR} = \bar{y}_{stN}\left(\frac{\bar{X}_N}{\bar{x}_{stN}}\right) \tag{5}$$

To the first degree of approximation, the MSE of $\bar{Y}_{NCR}$, is given by

$$MSE(\bar{Y}_{NCR}) = \bar{Y}_N^2 \sum_{h=1}^{k} W_{hN}^2 \gamma_{hN}\left(C_{yhN}^2 + C_{xhN}^2(1 - 2C_{hN})\right), \tag{6}$$

where $C_{hN} = \rho_{yxhN}(\frac{C_{yhN}}{C_{xhN}})$, $\rho_{yxhN} = \frac{S_{yxhN}}{S_{yhN}S_{xhN}}$.

3.3. *Neutrosophic combined difference estimator*

The neutrosophic combined difference estimator for population mean $\bar{Y}_N$ is given by

$$\bar{Y}_{NCD} = \bar{y}_{stN} + D_N(\bar{X}_N - \bar{x}_{stN}). \tag{7}$$

where ($D_N \in [D_L, D_U]$) is suitable chosen scalar.
The MSE of $\bar{Y}_{NCD}$ is provide as

$$MSE(\bar{Y}_{NCD}) = \sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} S_{yhN}^2 + D_N^2 \sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} S_{xhN}^2 - 2D_N \sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} S_{yxhN} \tag{8}$$

which is minimum when

$$D_N = \frac{\sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} S_{yxhN}}{\sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} S_{xhN}^2} = D_N^*(say). \tag{9}$$

Thus the resulting minimum MSE of $\bar{Y}_{NCD}$ is given by

$$\begin{aligned} MSE(\bar{Y}_{NCD}) &= \sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} S_{yhN}^2 + D_N^{*^2} \sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} S_{xhN}^2 - 2D_N^* \sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} S_{yxhN} \\ &= (1-\rho_{yxhN}^{*^2}) \sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 S_{yhN}^2 \\ &= \bar{Y}_N^2 \left[\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{yhN}^2 - \frac{\{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{hN} C_{xhN}^2\}^2}{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{xhN}^2} \right], \end{aligned} \tag{10}$$

where $\rho_{yxhN}^* = \frac{Cov(\bar{y}_{stN}, \bar{x}_{stN})}{\sqrt{var(\bar{y}_{stN}) var(\bar{x}_{stN})}} = \frac{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 S_{yxhN}}{\sqrt{(\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 S_{yhN}^2)(\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 S_{xhN}^2)}}$.

4. Formulation and Development of the Proposed Estimator

Motivated by the work of Taneja et al. [13], we have considered a neutrosophic combined logarithmic type estimator for estimating population mean $\bar{Y}_N$ of the study variable y_N in neutrosophic stratified random sampling as

$$\bar{Y}_{NCL} = \beta_{1N} \bar{y}_{stN} \left[1 + \beta_{2N} log \left(\frac{\bar{x}_{stN}}{\bar{X}_N} \right) \right] + \beta_{3N} \left(\bar{X}_N - \bar{x}_{stN} \right), \tag{11}$$

where ($\beta_{1N} \in [\beta_{1L}, \beta_{1U}]$, $\beta_{2N} \in [\beta_{2L}, \beta_{2U}]$, $\beta_{3N} \in [\beta_{3L}, \beta_{3U}]$) are suitable chosen scalars.
To obtain the bias and MSE of suggested estimator $\bar{Y}_{NCL}$, we write $\bar{y}_{stN} = \bar{Y}_N(1+e_{yN})$, $\bar{x}_{stN} = \bar{X}_N(1+e_{xN})$
such that $E(e_{yN}) = E(e_{xN}) = 0$ and $E(e_{yN}^2) = \sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{yhN}^2$,
$E(e_{xN}^2) = \sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{xhN}^2$, $E(e_{yN} e_{xN}) = \sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 \rho_{yxhN} C_{yhN} C_{xhN}$.
Expressing $\bar{Y}_{NCL}$ in terms of e_{yN} and e_{xN}, we have,

$$\bar{Y}_{NCL} = \beta_{1N} \bar{Y}_N (1+e_{yN}) \left[1 + \beta_{2N} log\left(1+e_{xN}\right) \right] - \beta_{3N} \bar{X}_N e_{xN} \tag{12}$$

Expanding the right hand side of (12), multiplying out and negelecting terms of e's having power greater than two, we have

$$\bar{Y}_{NCL} \cong \beta_{1N} \bar{Y}_N \left[1 + e_{yN} + \beta_{2N} e_{xN} + \beta_{2N} e_{yN} e_{xN} - \frac{1}{2} \beta_{2N} e_{xN}^2 \right] - \beta_{3N} \bar{X}_N e_{xN} \tag{13}$$

Subtracting $\bar{Y}_N$ from both sides of (13), we have

$$(\bar{Y}_{NCL} - \bar{Y}_N) \cong \bar{Y}_N \left[\beta_{1N} \left\{ 1 + e_{yN} + \beta_{2N} e_{xN} + \beta_{2N} e_{yN} e_{xN} - \frac{1}{2} \beta_{2N} e_{xN}^2 \right\} - \beta_{3N} \frac{1}{R_N} e_{xN} - 1 \right] \tag{14}$$

where $R_N = \frac{\bar{Y}_N}{\bar{X}_N}$.
Taking expectation of both sides of (14), we get the bias of $\bar{Y}_{NCL}$ to the first degree of

approximation as

$$B\left(\bar{Y}_{NCL}\right)=E\left(\bar{Y}_{NCL}-\bar{Y}_{N}\right)=\bar{Y}_{N}\left[\beta_{1N}\left\{1+\frac{\beta_{2N}}{2}\sum_{h=1}^{k}\gamma_{hN}W_{hN}^{2}C_{xhN}^{2}(2C_{hN}-1)\right\}-1\right] \quad (15)$$

Squaring both sides of (14) and neglecting terms of e's having power greater than two, we have

$$\begin{aligned}\left(\bar{Y}_{NCL}-\bar{Y}_{N}\right)^{2}=\bar{Y}_{N}^{2}\Big[1+\beta_{1N}^{2}\{1+2e_{yN}+2\beta_{2N}e_{xN}+e_{yN}^{2}+4\beta_{2N}e_{yN}e_{xN}+\beta_{2N}(\beta_{2N}-1)e_{xN}^{2}\}\\+\beta_{3N}^{2}(1/R_{N}^{2})e_{xN}^{2}-2\beta_{1N}\beta_{3N}(1/R_{N})\{\beta_{2N}e_{xN}^{2}+e_{yN}e_{xN}+e_{xN}\}\\-2\beta_{1N}\{1+e_{yN}+\beta_{2N}e_{xN}+\beta_{2N}e_{yN}e_{xN}-\frac{1}{2}\beta_{2N}e_{xN}^{2}\}+2\beta_{3N}(1/R_{N})e_{xN}\Big]\end{aligned} \quad (16)$$

Taking expectation of both sides of (16), we found the MSE of $\bar{Y}_{NCL}$ as

$$\begin{aligned}MSE\left(\bar{Y}_{NCL}\right)=E\left(\bar{Y}_{NCL}-\bar{Y}_{N}\right)^{2}=\Bigg[1+\beta_{1N}^{2}\Bigg\{1+\sum_{h=1}^{k}\gamma_{Nh}W_{hN}^{2}\Big(C_{yhN}^{2}\\+2\beta_{2N}C_{xhN}^{2}(\beta_{2N}+4C_{hN}-1)\Big)\Bigg\}+\beta_{3N}^{2}\left(\frac{1}{R_{N}^{2}}\right)\sum_{h=1}^{k}\gamma_{Nh}W_{hN}^{2}C_{xhN}^{2}\\-2\beta_{1N}\beta_{3N}\left(\frac{1}{R_{N}}\right)\sum_{h=1}^{k}\gamma_{hN}W_{hN}^{2}C_{xhN}^{2}(\beta_{2N}+C_{hN})\\-2\beta_{1N}\left\{1+\frac{\beta_{2N}}{2}\sum_{h=1}^{k}\gamma_{hN}W_{hN}^{2}C_{xhN}^{2}(2C_{hN}-1)\right\}\Bigg]\end{aligned} \quad (17)$$

$$\begin{aligned}MSE\left(\bar{Y}_{NCL}\right)&=E\left(\bar{Y}_{NCL}-\bar{Y}_{N}\right)^{2}\\&=\bar{Y}_{N}^{2}\left[1+\beta_{1N}^{2}A_{2hN}+\beta_{3N}^{2}C_{1hN}-2\beta_{1N}\beta_{3N}D_{1hN}-2\beta_{1N}B_{2hN}\right],\end{aligned} \quad (18)$$

where $A_{2hN}=1+\sum_{h=1}^{k}\gamma_{hN}W_{hN}^{2}\left(C_{yhN}^{2}+\beta_{2N}C_{xhN}^{2}\left(\beta_{2N}+4C_{hN}-1\right)\right)$,
$B_{2nh}=1+\frac{\beta_{2N}}{2}\sum_{h=1}^{k}\gamma_{hN}W_{hN}^{2}C_{xhN}^{2}\left(2C_{hN}-1\right)$, $C_{1hN}=\frac{1}{R_{N}^{2}}\sum_{h=1}^{k}\gamma_{hN}W_{hN}^{2}C_{xhN}^{2}$,
$D_{1hN}=\frac{1}{R_{N}}\sum_{h=1}^{k}\gamma_{hN}W_{hN}^{2}C_{xhN}^{2}\left(\beta_{2N}+C_{hN}\right)$, $C_{hN}=\rho_{yxhN}(\frac{C_{yhN}}{C_{xhN}})$, $\rho_{yxhN}=\frac{S_{yxhN}}{S_{yhN}S_{xhN}}$.

It is difficult to obtain the optimum values of $(\beta_{1N},\beta_{2N},\beta_{3N})$ simultaneously. Therefore, we fixed the value of β_{2N} and minimize the MSE of $\bar{Y}_{NCL}$ by partially differentiating Equation (18) with respect to β_{1N} and β_{3N}, and equating the derivatives to zero, we obtain the optimum values as follows:

$$\beta_{1N}=\frac{B_{2hN}C_{1hN}}{A_{2hN}C_{1hN}-D_{1hN}^{2}}=\beta_{1N(opt)},(say), \quad (19)$$

$$\beta_{3N}=\frac{B_{2hN}D_{1hN}}{A_{2hN}C_{1hN}-D_{1hN}^{2}}=\beta_{3N(opt)},(say), \quad (20)$$

$$\beta_{2N} = -\frac{\sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} C_{hN} C_{xhN}^2}{\sum_{h=1}^{k} W_{hN}^2 \gamma_{hN} C_{xhN}^2} = \beta_{2N(opt)}, (fixed) \tag{21}$$

Now, substituting the optimum values of β_{1N}, β_{2N} and β_{3N} into equation (18), we obtain the resulting minimum MSE of $\bar{Y}_{NCL}$ as:

$$MSE_{min}(\bar{Y}_{NCL}) = \bar{Y}_N^2 \left[1 - \frac{B_{2hN}^2 C_{1hN}}{A_{2hN} C_{1hN} - D_{1hN}^2}\right]. \tag{22}$$

which is non-negative, if $0 < \frac{B_{2hN}^2 C_{1hN}}{A_{2hN} C_{1hN} - D_{1hN}^2} < 1$ and $(A_{2hN} C_{1hN} - D_{1hN}^2) > 0$.
Remark: To determine the value of β_{2N}, we set $(\beta_{1N}, \beta_{3N}) = (1, 0)$ without loss of generality for analytical convenience and substitute these into Equation (18). This reduces the MSE of $\bar{Y}_{NCL}$ to a function of β_{2N} only. Differentiating with respect to β_{2N} and equating to zero, we obtain the optimum value of β_{2N} which is given in Equation 21.

5. Efficiency Comparison

From (2), (6) and (10), we have

$$MSE(\bar{y}_{stN}) - MSE_{min}(\bar{Y}_{NCD}) = \bar{Y}_N^2 \frac{\{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{hN} C_{xhN}^2\}^2}{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{xhN}^2} \geq 0, \tag{23}$$

$$MSE(\bar{Y}_{NCR}) - MSE_{min}(\bar{Y}_{NCD}) = \bar{Y}_N^2 \frac{\left[(\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{xhN}^2) - (\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{hN} C_{xhN}^2)\right]^2}{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{xhN}^2} \geq 0 \tag{24}$$

From (23) and (24), we have the inequalities:

$$MSE_{min}(\bar{Y}_{NCD}) \leq MSE(\bar{y}_{stN}) \tag{25}$$

$$MSE_{min}(\bar{Y}_{NCD}) \leq MSE(\bar{Y}_{NCR}) \tag{26}$$

Thus the estimators $\bar{Y}_{NCD}$ is more efficient than the neutrosophic combined mean estimator $\bar{y}_{stN}$.
From (2) and (6), we note that

$$MSE(\bar{Y}_{NCR}) \leq MSE(\bar{y}_{stN}) \quad if \quad \frac{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{hN} C_{xhN}^2}{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{xhN}^2} > \frac{1}{2} \tag{27}$$

Thus combining (25), (26) and (27), we have the inequality:

$$MSE_{min}(\bar{Y}_{NCD}) \leq MSE(\bar{Y}_{NCR}) \leq MSE(\bar{y}_{stN}) \quad if \quad \frac{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{hN} C_{xhN}^2}{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{xhN}^2} > \frac{1}{2} \tag{28}$$

From ((10)) and (22), we have

$$MSE_{min}(\bar{Y}_{NCL}) < MSE_{min}(\bar{Y}_{NCD}) \quad if \ \frac{B_{1hN}^2 C_{1hN}}{A_{1hN}C_{1hN} - C_{2hN}^2} < \left[\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{yhN}^2 - \frac{\{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{hN} C_{xhN}^2\}^2}{\sum_{h=1}^{k} \gamma_{hN} W_{hN}^2 C_{xhN}^2}\right]. \quad (29)$$

Thus the proposed estimator $\bar{Y}_{NCL}$ is more efficient than the estimator $\bar{Y}_{NCD}$ as long as the condition (29) is satisfied.

It follows from (28) and (29) that the suggested estimator $\bar{Y}_{NCL}$ is are more efficient than the existing estimators $\bar{y}_{stN}$, $\bar{Y}_{NCR}$ and $\bar{Y}_{NCD}$.

6. **Application**

To evaluate the efficiency of the proposed logarithmic estimators relative to existing estimation procedures, two real-life indeterminate climate data sets were analyzed. Data Set I consists of climatic observations from the U.S. states of Alabama and Georgia, representing two strata. In this data set, dew point temperature was considered as the auxiliary variable, $\bar{X}_{hN} \in [\bar{X}_{hL}, \bar{X}_{hU}]$, while relative humidity served as the study variable, expressed as $\bar{Y}_{hN} \in [\bar{Y}_{hL}, \bar{Y}_{hU}]$ [6].

Data Set II was constructed from indeterminate climatic data collected from four major Pakistani cities—Karachi, Lahore, Peshawar, and Multan—representing four distinct strata. Consistent with Data Set I, dew point temperature was taken as the auxiliary variable, expressed as $\bar{X}_{hN} \in [\bar{X}_{hL}, \bar{X}_{hU}]$, and relative humidity was considered as the study variable, denoted by $\bar{Y}_{hN} \in [\bar{Y}_{hL}, \bar{Y}_{hU}]$ [18]. The parameters and their corresponding details are summarized in Table 1. The PRE of an estimator ζ w. r. to neutrosophic combined mean estimator ($\bar{y}_{stN}$) is defined as:

$$PRE(\zeta, \bar{y}_{stN}) = \frac{Var(\bar{y}_{stN})}{MSE(\zeta)} \times 100,$$

where $\zeta = \bar{y}_{stN}, \bar{Y}_{NCR}, \bar{Y}_{NCD}$ and $\bar{Y}_{NCL}$.

The Variance (or MSE) and PRE of the neutrosophic combined mean estimator ($\bar{y}_{stN}$), neutrosophic combined ratio estimator ($\bar{Y}_{NCR}$) and neutrosophic combined difference estimator ($\bar{Y}_{NCD}$), and the proposed neutrosophic combined logarithmic estimator $\bar{Y}_{NCL}$ relative to neutrosophic combined mean estimator ($\bar{y}_{stN}$) are reported in Table 2. Higher PRE values correspond to greater estimator efficiency, indicating superior performance in terms of precision and reliability.

TABLE 1. Datasets I and II were obtained from the studies by [6] and [18], respectively.

Dataset I		
Parameter	Stratum 1	Stratum 2
N_{hN}	[19, 19]	[22, 22]
n_{hN}	[6, 6]	[7, 7]
$\bar{X}_{hN}$	[19.58, 61.95]	[22.55, 62.23]
$\bar{Y}_{hN}$	[28.21, 96.47]	[31.77, 93.86]
S_{xhN}	[10.86, 14.22]	[8.67, 13.06]
S_{yhN}	[13.55, 23.30]	[11.78, 21.88]
ρ_{yxhN}	[0.966, 0.941]	[0.8854, 0.9481]

Dataset II				
Parameter	Stratum 1	Stratum 2	Stratum 3	Stratum 4
N_{hN}	[1460,1460]	[1460,1460]	[1460,1460]	[1460,1460]
n_{hN}	[365,365]	[365,365]	[365,365]	[365,365]
$\bar{X}_{hN}$	[74.78356,99.88767]	[73.59178,96.74247]	[73.59452,74.72055]	[75.45753,81.88767]
$\bar{Y}_{hN}$	[76.03014,102.44660]	[78.46849,96.77534]	[70.90137,77.46301]	[78.80000,81.94521]
S_{xhN}	[201.4727,253.2893]	[194.2290,252.9515]	[200.5353,125.8221]	[194.6683,126.1716]
S_{yhN}	[200.6141,252.8721]	[200.6148,253.2312]	[200.4628,125.4872]	[200.5880,126.1716]
ρ_{yxhN}	[0.9944705,0.9993912]	[0.9657781,0.9997780]	[0.9941006,0.9986742]	[0.9649780,0.9984879]

TABLE 2. MSE and PREs of the suggested estimator $\bar{Y}_{NCL}$ over other existing estimators for Datasets I and II.

Dataset I and Dataset II Comparison				
Estimator	Dataset I		Dataset II	
	MSE	PRE	MSE	PRE
Existing Estimators				
$\bar{y}_{stN}$	[8.388008, 26.72093]	[100.000, 100.000]	[20.66522, 20.51413]	[100.000, 100.000]
$\bar{Y}_{NCR}$	[1.584400, 2.896382]	[529.412, 922.562]	[0.837912, 0.030745]	[2466.276, 66722.350]
$\bar{Y}_{NCD}$	[1.313984, 2.885246]	[638.365, 926.123]	[0.821150, 0.025249]	[2516.618, 81245.770]
Proposed Estimator				
$\bar{Y}_{NCL}$	[**1.311386**, **2.875674**]	[**639.629**, **929.206**]	[**0.807679**, **0.012649**]	[**2558.591**, **162173.100**]

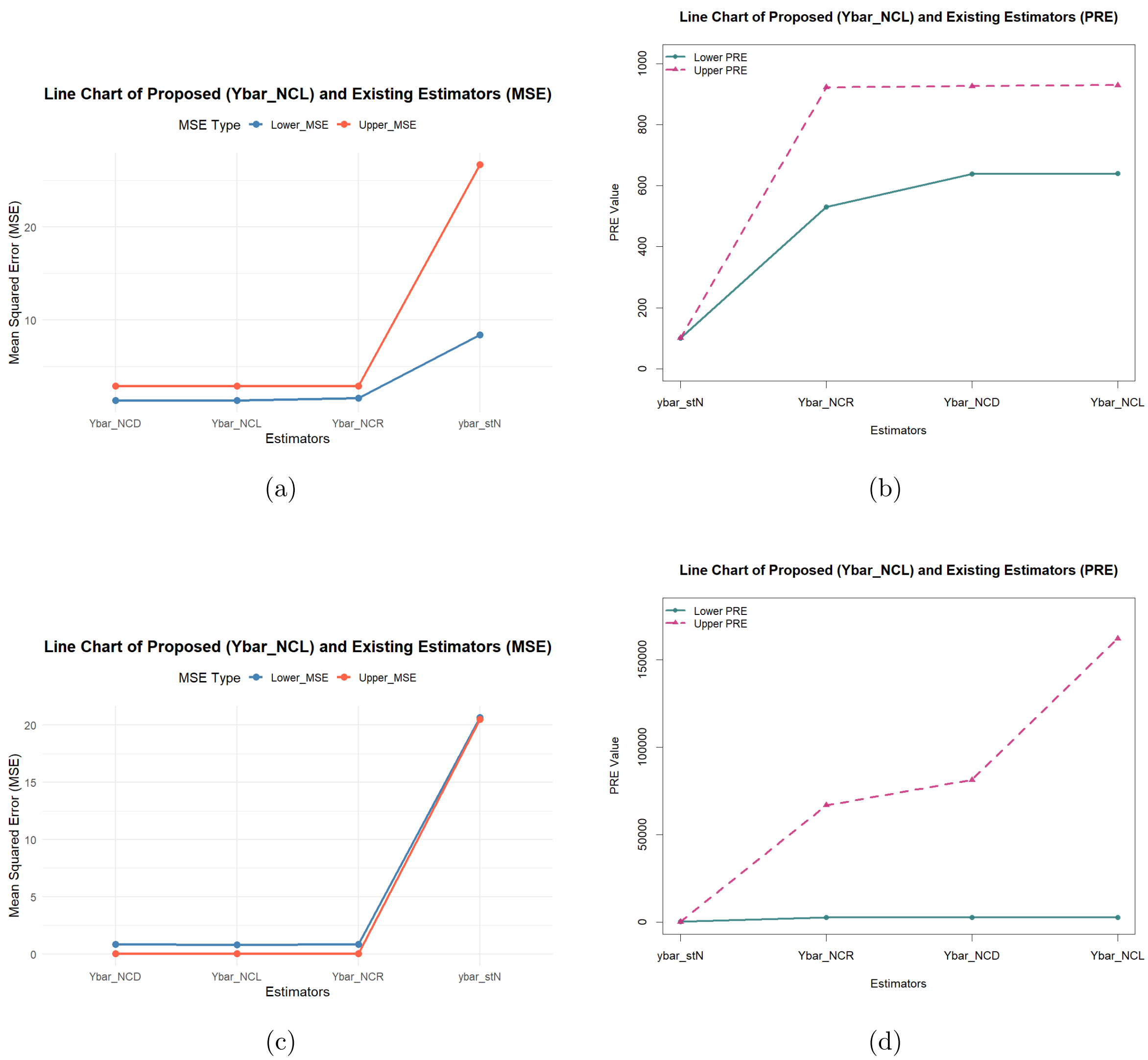

FIGURE 1. Line charts comparing estimator performance based on MSE and PRE. (a) MSE for Dataset I; (b) PRE for Dataset I; (c) MSE for Dataset II; (d) PRE for Dataset II. The proposed estimator $\bar{Y}_{NCL}$ is compared with other existing estimators.

FIGURE 2. Radar charts comparing estimator performance based on MSE and PRE. (a) MSE for Dataset I; (b) PRE for Dataset I; (c) MSE for Dataset II; (d) PRE for Dataset II. The proposed estimator $\bar{Y}_{NCL}$ is compared with other existing estimators.

7. Result and Discussion

As shown in Table 2 and Figures 1 and 2 , the proposed estimator $\bar{Y}_{NCL}$ demonstrates superior performance compared to the existing estimators neutrosophic combined mean $\bar{y}_{stN}$, neutrosophic combined ratio estimator $\bar{Y}_{NCR}$ and neutrosophic combined difference estimator $\bar{Y}_{NCD}$ for both Data Sets I and II in terms of PRE.

For Data Set I, The proposed estimator $\bar{Y}_{NCL}$ achieves higher PRE values, ranging from **639.629%** to **929.206%**, the neutrosophic combined ratio estimator $\bar{Y}_{NCR}$ exhibits PRE values between **529.412%** and **922.562%** and the neutrosophic combined difference estimator $\bar{Y}_{NCD}$ attains PRE values ranging from **638.365%** to **926.123%**, relative to the neutrosophic combined mean estimator $\bar{y}_{stN}$ respectively. The above results clearly indicate that the proposed neutrosophic combined logarithmic estimator $\bar{Y}_{NCL}$ consistently yields the best efficiency, making it the most successful estimator considered for Data Set I.

For Data Set II, the suggested estimator $\bar{Y}_{NCL}$ again demonstrates outstanding performance, with PRE values ranging from **2558.591%** to **162173.100%**, the neutrosophic combined ratio estimator $\bar{Y}_{NCR}$ exhibits PRE values between **2466.276%** and **66722.350%**, and the neutrosophic combined difference estimator $\bar{Y}_{NCD}$ achieves PRE values ranging from **2516.618%** to **81245.770%**, surpassing those of the neutrosophic combined mean estimator $\bar{y}_{stN}$ respectively.

Figure 1 and Figure 2 display line and radar charts, respectively, comparing the MSE and PRE values of the estimators, offering a clear visual representation of their performances. The results confirm that the proposed neutrosophic combined logarithmic estimator $\bar{Y}_{NCL}$ consistently attains the highest PRE, making it the preferred choice for practical implementation in neutrosophic stratified random sampling scenarios.

8. Conclusion

In this paper, we have proposed a neutrosophic combined logarithmic estimator for estimating the population mean in stratified random sampling. The biases and mean squared errors (MSEs) of the suggested estimator have been derived and thoroughly analyzed. Empirical and Graphical results demonstrate that the MSEs of the proposed estimator are consistently lower than those of the neutrosophic combined mean estimator ($\bar{y}_{stN}$), the neutrosophic combined ratio estimator ($\bar{Y}_{NCR}$), and the neutrosophic combined difference estimator ($\bar{Y}_{NCD}$). These results show that the suggested estimator offers significant gains in accuracy and efficiency. Because it provides more precise, consistent, and dependable estimations of the population mean, the optimum neutrosophic combined logarithmic estimator is advised for real-world applications in stratified random sampling.

The suggested estimator may be extended to various sample designs, including systematic, cluster, and multistage sampling, in subsequent studies. Additionally, its performance can be examined with various types of supplementary information and under measurement errors. It would also be interesting to expand the method to other demographic parameters and examine the impact of changing the neutrosophic parameter.

Data Availability Statement: All the relevant data is available in the manuscript.

Funding: The authors received no financial support for the research, authorship, and/or publication of this article.

Acknowledgments: The authors would like to express their sincere gratitude to the Editor-in-Chief and the reviewers for their insightful comments, constructive suggestions, and valuable contributions to improving the manuscript.

Conflicts of Interest: The authors declare no conflicts of interest.

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Received: Dec 7, 2025. Accepted: May 30, 2026