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Foundations of Type-1 Neutrosophic Topological Modules

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Abstract. This paper develops a topological framework based on type-1 neutrosophic sets, extending the study of uncertainty beyond classical and intuitionistic fuzzy structures. We investigate fundamental operators—interior, closure, and exterior—on type-1 neutrosophic sets $X[I]$, and establish separation axioms for type-1 topological spaces $T_i[I], i = 0, 1, \dots, 4)$. Furthermore, we analyze continuous and contra type-1 neutrosophic functions, exploring their structural properties within this framework. By isolating type-1 neutrosophic sets and formalizing their topological behavior, our results provide new insights into neutrosophic topology and lay the groundwork for broader applications in mathematical analysis, algebra, and related fields.

Keywords: Type-1 Nutrosophic Sets; Type-1 Nutrosophic Topology; Type-1 Nutrosophic Interior Sets;Type-1 Nutrosophic Closure Sets; Type-1 Nutrosophic Exterior Sets; Type-1 Nutrosophic Boundary Sets; Separation Axioms for $T_i[I], i = 0, 1, \dots, 4)$.; Continuity on Type-1 Nutrosophic Topological Spaces

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1. **Introduction**

Neutrosophy, introduced by Smarandache in the 1998 [39], is a branch of philosophy that investigates the origin and characteristics of neutrality in nature. The mathematical study of uncertainty has evolved through several foundational milestones. In 1965, Zadeh proposed fuzzy sets as an extension of classical crisp sets, establishing the framework for fuzzy mathematics [43]. This was followed in 1968 by Chang, who extended fuzzy concepts to topological spaces, exploring notions such as open and closed sets, neighborhoods, interiors, continuity, and compactness [17]. Lowen further advanced this structure in 1975 by studying fuzzy compactness [31]. In 1983, Atanassov enriched fuzzy theory with intuitionistic fuzzy sets, introducing degrees of membership and non-membership [12,13], and in 1997, Coker extended these ideas to intuitionistic topological spaces [18]. Building on these developments, Smarandache introduced neutrosophic sets [40] as a generalization of intuitionistic fuzzy sets [12,13], incorporating three components for each element in (X): the degree of membership, the degree of indeterminacy, and the degree of non-membership. Subsequent research investigated neutrosophic set operations and their topological properties. Lupianez [29,30], Salama [35,36] defined various topologies on neutrosophic sets, though some did not satisfy De Morgan's laws. Also, neutrosophic closed set "and "neutrosophic continuous function investigated in [37]. Karataş and Kuru [28] redefined neutrosophic operations and introduced neutrosophic topology, studying properties such as closure, interior, exterior, boundary, and subspace. In algebraic structures, groups play a central role [21, 23], and neutrosophic sets have been applied in diverse areas including analysis [16,34], neutrosophic crisp topology [11], algebra [1,9,10], uncertainty graph theory [24, 25, 40], and practical domains such as sentiment analysis and medical diagnostics [15]. Smarandache and Kandasamy [26] extended neutrosophic theory to algebraic structures by introducing an indeterminate element I. The concept of neutrosophic groups introduced in [27]. While other explored algebraic operations from different viewpoints [33]. Cetkin and Aygun [19] proposed an approach to neutrosophic subgroups, and Mohamed Abdalla and Saleem [22]. Recently, Al-Odhari [2–8] developed neutrosophic sets generated by classical set theory, identifying three types. In revised work, he isolated type-1 neutrosophic sets and investigated their properties to construct new mathematical structures [2]. In this article, we build a topological framework based on type-1 neutrosophic sets and examine properties such as interior, closure,exterior, and boundary operators on type-1 neutrosophic sets $X[I]$. Furthermore, we study separation axioms for type-1 topological spaces ($T_i[I], i = 0, 1, \dots, 4$), and analyse continuous and contra type-1 neutrosophic functions along with their properties.

2. **Type-1 Neutrosophic Topological Spaces**

In this section, we introduce the concept of type-1 topological spaces, which is based on type-1 neutrosophic sets in [2–8]. Perspective of a neutrosophic set of type-1 induced from a neutrosophic element $a + bI$ where I is called the neutrosophic element with property $I^2 = I$ and $0I = 0$ that appeared in the work of Kandasamy and Smarandache with neutrosophic algebraic structures in [26, 27]. An attempt to present an intuitive mathematical framework that includes type-1 neutrosophic set theory was presented in previous works and is still under development and mathematical construction.

Definition 2.1. Let $X[I]$ be a type-1 neutrosophic set and $\tau[I]$ be a collection of neutrosophic subsets of $X[I]$. Then $\tau[I]$ is a type-1 neutrosophic topology on $X[I]$(in short (T1NTS)), if and only if

(1) $\emptyset[I]$ and $X[I] \in \tau[I]$.
(2) If $A_1[I]$ and $A_2[I] \in \tau[I]$, then $A_1[I] \cap A_2[I] \in \tau[I]$.
(3) For each netrosophic union of type-1 neutrosophic members of $\tau[I]$ is also a type-1 neutrosophic member of $\tau[I]$.

The couple $(X[I], \tau[I])$ is called a type-1 topological space on $X[I]$. Moreover, the members of $\tau[I]$ are said to be type-1 neutrosophic open sets in $\tau[I]$. A subset $A[I] \subseteq X[I]$ is called a type-1 neutrosophic closed set in (T1NTS) $(X[I], \tau[I])$ if $X[I] - A[I] \in \tau[I]$.

Example 2.2. Let $X[I] = \{1 + I, 1 + 2I, 2 + I, 2 + 2I\}$ be a type-1 neutrosophic set and

$$\tau[I] = \{\emptyset[I], X[I], \{1 + I, 1 + 2I\}, \{1 + I, 2 + I\}, \{1 + I\}, \{1 + I, 1 + 2I, 2 + I\}\}$$

be a collection of neutrosophic subsets of $X[I]$. Then $(X[I], \tau[I])$ is a type-1 neutrosophic topological space.

Example 2.3. Let $\tau[I] = \{\emptyset[I], P(X[I])\}$, and $\tau[I] = \{\emptyset[I], X[I]\}$ is a called a type-1 neutrosophic discrete topological space and a type-1 neutrosophic indiscrete topological space over $X[I]$, respectively.

Theorem 2.4. *Let $(X[I], \tau[I])$ be a type-1 neutrosophic topological space. Then:*

(1) *$\emptyset[I]$ and $X[I]$, are type-1 neutrosophic open(or closed) sets over $X[I]$.*
(2) *The intersection of any finite number of type-1 neutrosophic open (or closed) sets is a type-1 neutrosophic open (or closed) set over $X[I]$.*
(3) *The union of any two type-1 neutrosophic open (or closed) sets is a type-1 neutrosophic open (or closed) set over $X[I]$.*

Proof. Straightforward. □

Theorem 2.5. Let $(X[I], \tau_1[I])$ and $(X[I], \tau_2[I])$ be two type-1 neutrosophic topological spaces over $X[I]$, then $(X[I], \tau_1[I] \cap \tau_2[I])$ is a type-1 neutrosophic topological space over $X[I]$.

Proof. (1) Let $(X[I], \tau_1[I])$ and $(X[I], \tau_2[I])$ be two type-1 neutrosophic topological spaces over $X[I]$. Since $\emptyset[I]$ and $X[I] \in \tau_1[I]$ and $\tau_2[I]$. Then $\emptyset[I]$ and $X[I] \in \tau_1[I] \cap \tau_2[I]$.

(2) Suppose that $A_1[I]$, $A_2[I] \in \tau_1[I] \cap \tau_2[I]$ are two type-1 neutrosophic open sets. Then $A_1[I]$, $A_2[I] \in \tau_1[I]$ and $A_1[I]$, $A_2[I] \in \tau_2[I]$. Implies that $A_1[I] \cap A_2[I] \in \tau_1[I]$ and $A_1[I] \cap A_2[I] \in \tau_2[I]$. Therefore, $A_1[I] \cap A_2[I] \in \tau_1[I] \cap \tau_2[I]$.

(3) Assume that $\{A_i[I] : i \in J\} \in \tau_1[I] \cap \tau_2[I]$. Then $A_i[I] \in \tau_1[I]$ and $A_i[I] \in \tau_2[I]$, for all $i \in J$. Therefore, $\bigcup_{i\in J} A_i[I] \in \tau_1[I]$ and $\bigcup_{i\in J} A_i[I] \in \tau_2[I]$, for all $i \in J$. So, we deduced that $\bigcup_{i\in J} A_i[I] \in \tau_1[I] \cap \tau_2[I]$. And Consequently, $\big(X[I], \tau_1[I] \cap \tau_2[I]\big)$ is a type-1 neutrosophic topological space.

□

Corollary 2.6. Let $\{(X[I], \tau_i[I]) : i \in J\}$ be a family of type-1 neutrosophic topology spaces over $X[I]$. Then $(X[I], \cap_{i\in J}\tau_i[I])$ is a type-1 neutrosophic topological space over $X[I]$.

Proof. By the similar method of the Theorem 2.5. □

Remark 2.1. The union of type-1 neutrosophic topological spaces may not be a type-1 neutrosophic topological space.

Example 2.7. Let $X[I] = \{a + aI, a + bI, b + aI, b + bI\}$ be a type-1 neutrosophic set and let $\tau_1[I] = \{\emptyset[I], X[I], a+aI\}$, $\tau_2[I] = \{\emptyset[I], X[I], \{b+aI\}\}$ be two type-1 neutrosophic topology on $X[I]$. Then $\tau_1[I] \cup \tau_2[I]$ is not a type-1 neutrosophic topology space.

Definition 2.8. *Let $(X[I], \tau[I])$ be a type-1 neutrosophic topological space, $N[I] \subseteq X[I]$, and $x \in X[I]$. we said that $N[I]$ is a neighborhood (in short nbd) of a type-1 neutrosophic ponit $x \in X[I]$, if there exists any $O[I] \in \tau[I]$ such that $x \in O[I] \subseteq N[I]$. And denoted by $N_{nbd}[I](x)$. The set which contains all sets of $N_{nbd}[I](x)$. is called neighborhood type-1 neutrosophic system and denoted by:*

$$N_{nbd}[I](x) = \{N[I] \subseteq X[I] : \exists\, O[I] \in \tau[I] \textit{ such that } x \in O[I] \subseteq N[I]\}.$$

Example 2.9. Let $X[I] = \{0, 1, 2, I, 2I, 1 + I, 1 + 2I, 2 + I, 2 + 2I\}$ be a type-1 neutrsophic set and consider the type-1 neutrosophic sub-collection $\tau[I]$ of $X[I]$ is given by: $\tau[I] = \{\emptyset[I], X[I], \{0\}, \{0, 1, I, 2I, 1 + I, 1 + 2I\}\}$, where $(X[I], \tau[I])$ is a type-1 neutrosophic topology space.

Note $X[I]$ neighborhood ponit 0, $0 \in X[I] \subset X[I]$.

$\{0\}$ neighborhood ponit 0, beacuse $0 \in \{0\} \subset \{0\}$.

$\{0, 1\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 1\}$.

$\{0, 2\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 2\}$.

$\{0, I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, I\}$.

$\{0, 2I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 2I\}$.

$\{0, 1 + I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 1 + I\}$.

$\{0, 1 + 2I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 1 + 2I\}$.

$\{0, 2 + I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 2 + I\}$.

$\{0, 2+2I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 2+2I\}$.
$\{0, 1, 2\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 1, 2\}$.
$\{0, 1, 2, I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 1, 2, I\}$.
$\{0, 1, 2, I, 2I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 1, 2, I, 2I\}$.
$\{0, 1, 2, I, 2I, 1+I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 1, 2, I, 2I, 1+I\}$.
$\{0, 1, 2, I, 2I, 1+I, 1+2I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 1, 2, I, 2I, 1+I, 1+2I\}$.
$\{0, 1, 2, I, 2I, 1+I, 1+2I, 2+I\}$ neighborhood ponit 0 beacuse $0 \in \{0\} \subset \{0, 1, 2, I, 2I, 1+I, 1+2I, 2+I\}$.

3. Properties of Type-1 Neutrosophic Subsets on Type-1 Topological Spaces

In this section, we investigate the main properties of the type-1 neutrosophic subspace of type-1 topological spaces. We studied Types-1 neutrosophic Interior, closure, and exterior operators via the $X[I]$ set and their properties.

3.1. *Type-1 Neutrosophic Interior points*

This subsection is devoted to studying the interior operator via $X[I]$ sets and their properties.

Definition 3.1. *Let $(X[I], \tau[I])$ be a type-1 neutrosophic topological space and $A[I] \subseteq X[I]$. The Interior functor of $A[I]$ is denoted by $Int_n(A[I])$ and defined by*

$$Int_n(A[I]) = \cup\{O[I] \subseteq X[I] : O[I] \subseteq A[I] and O[I] \in \tau[I]\}.$$

Remark 3.1. $Int_n(A[I])$ is the largest type-1 neutrosophic open set contained in $A[I]$. So that $Int_n(A[I]) \subseteq A[I]$.

Example 3.2. *Consider the type-1 neutrosophic topological space in the Example 2.2. Let $A[I] = \{1+I, 1+2I, 2+2I\}$ and $B[I] = \{1+I, 1+2I, 2+I\}$ be two type-1 neutrosophic subsets of $X[I]$. Then $Int_n(A[I]) = \{1+I, 1+2I\} \cup \{1+I\} = \{1+I, 1+2I\}$, and*
$Int_n(B[I]) = \{1+I, 1+2I\} \cup \{1+I, 2+I\} \cup \{1+I\} \cup \{1+I, 1+2I, 2+I\} = \{1+I, 1+2I, 2+I\}$.

Theorem 3.3. Let $(X[I], \tau[I])$ be a type-1 neutrosophic topological space and $A[I] \subseteq X[I]$. Then $Int_n(A[I]) = A[I]$ if and only if $A[I]$ is a type-1 neutrosophic open set.

Proof. If $A[I]$ is a type-1 neutrosophic open set over $X[I]$, then $A[I]$ is itself a type-1 neutrosophic open set over $X[I]$ which contains $A[I]$. So, $A[I]$ is the largest a type-1 neutrosophic open set contained in $A[I]$ and $Int_n(A[I]) = A[I]$. Conversely, suppose that $Int_n(A[I]) = A[I]$. Then $Int_n(A[I]) = A[I] \in \tau[I]$. □

Theorem 3.4. Let $(X[I], \tau[I])$ be a type-1 neutrosophic topological space and $A[I] \subseteq X[I]$. If $x \in Int_n(A[I])$ if and only if there is a type-1 neutrosophic open set $B[I]$ such that $x \in B[I] \subseteq A[I]$.

Proof. Let $x \in Int_n(A[I])$ and take $B[I] = Int_n(A[I]$. Then by definition of $Int_n(A[I])$ we get that $B[I]$ is a type-1 neutrosophic open set and by Remark 3.1, $x \in B[I] \subseteq A[I]$.
Conversely, let there a type-1 neutrosophic open set $B[I]$ such that $x \in B[I] \subseteq A[I]$. Then by definition of $Int_n(A[I])$, $x \in B[I] \subseteq Int_n(A[I])$. □

Theorem 3.5. *Let $(X[I], \tau[I])$ be a type-1 neutrosophic topological space and and $A[I], B[I] \subseteq X[I]$. Then the following axioms hold:*

(1) $Int_n(\emptyset[I]) = \emptyset[I]$ *and* $Int_n(X[I]) = X[I]$.
(2) $Int_n(Int_n(A[I])) = Int_n(A[I])$
(3) *If* $A[I] \subseteq B[I]$ *then* $Int_n(A[I]) \subseteq Int_n(B[I])$.
(4) $Int_n(A[I]) \cup Int_n(B[I]) \subseteq Int_n(A[I] \cup B[I])$.
(5) $Int_n(A[I] \cap B[I]) = Int_n(A[I]) \cap Int_n(B[I])$.

Proof.

(1) is obvious.
(2) Let $Int_n(A[I]) = B[I]$. Then, $Int_n(B[I]) = B[I]$ by Theorem 3.3. and then, $Int_n(Int_n(A[I])) = Int_n(A[I])$.
(3) Let $x \in Int_n(A[I])$. Then by Theorem 3.4, there is a type-1 neutrosophic open set $O[I]$ such that $x \in O[I] \subseteq A[I]$ Since $A[I] \subseteq B[I]$ then $x \in O[I] \subseteq B[I]$. Hence $x \in Int_n(B[I])$. That is, $Int_n(A[I]) \subseteq Int_n(B[I])$.
(4) Since $A[I] \subseteq A[I] \cup B[I]$ and $B[I] \subseteq A[I] \cup B[I]$, then by part(3),
$Int_n(A[I]) \subseteq Int_n(A[I] \cup B[I])$ and $Int_n(B[I]) \subseteq Int_n(A[I] \cup B[I])$.
Hence $Int_n(A[I]) \cup Int_n(B[I]) \subseteq Int_n(A[I] \cup B[I])$.
(5) Since $A[I] \cap B[I] \subseteq A[I]$ and $A[I] \cap B[I] \subseteq B[I]$, then by part(3),
$Int_n(A[I] \cap B[I]) \subseteq Int_n(A[I])$ and $Int_n(A[I] \cap B[I]) \subseteq Int_n(B[I])$.
Hence $Int_n(A[I] \cap B[I]) \subseteq Int_n(A[I]) \cap Int_n(B[I])$. (1)
Also, let $x \in Int_n(A[I]) \cap Int_n(B[I])$, then $x \in Int_n(A[I])$, $x \in Int_n(B[I])$, there two type-1 neutrosophic open sets $M[I]$ and $N[I]$ such that $x \in M[I] \subseteq A[I]$, $x \in N[I] \subseteq B[I]$, then $x \in (M[I] \cap X[I]) \subseteq (A[I] \cap B[I])$ implies that $x \in Int_n(A[I] \cap B[I])$ by Theorem 3.4, then $Int_n(A[I]) \cap Int_n(B[I]) \subseteq Int_n(A[I] \cap B[I])$. (2)
From (1) and (2). We deduce that $Int_n(A[I] \cap B[I]) = Int_n(A[I]) \cap Int_n(B[I])$.

□

Example 3.6. Consider the type-1 neutrosophic topological space in Example 3.2.
Let $A[I] = \{1+I, 1+2I, 2+2I\}$*, and* $B[I] = \{1+I, 1+2I, 2+I\}$ *be two type-1 neutrosophic subset of* $X[I]$*. Then* $Int_n(A[I]) = \{1+I, 1+2I\}$*, and* $Int_n(B[I]) = \{1+I, 1+2I, 2+I\}$*. Also,* $(A[I] \cup B[I]) = X[I]$*, while,* $Int_n(A[I] \cup B[I]) = X[I]$*, therefore,*

$$Int_n(A[I]) \cup Int_n(B[I]) \subseteq Int_n(A[I] \cup B[I]).$$

3.2. *Type-1 Neutrosophic Cluster points*

This subsection is addressed to studying the closure operator via $X[I]$ sets and their properties.

Definition 3.7. Let $(X[I], \tau[I])$ be a type-1 neutrosophic topology space and $H[I] \subseteq X[I]$. The closure operator of $H[I]$ is denoted by $Cl_n(H[I])$ and defined by

$$Cl_n(H[I]) = \cap\{B[I] \subseteq X[I] : H[I] \subseteq B[I] \text{ and } B[I] \text{ is a type-1 neutrosophic closed set}\}.$$

Example 3.8. Consider the type-1 neutrosophic topological space in Example 2.2 and $H[I] = \{1+2I, 2+I\} \subseteq X[I]$. A type-1 neutrosophic closed set:

$$F = \{\emptyset[I], X[I], \{2+2I\}, \{2+I, 2+2I\}, \{1+2I, 2+2I\}, \{1+2I, 2+I, 2+2I\}\}.$$

Then $Cl_n(H[I]) = X[I] \cap \{1+2I, 2+I, 2+2I\} = \{1+2I, 2+I, 2+2I\}$.

Remark 3.2. $Cl_n(H[I])$ is the smallest type-1 neutrosophic closed set containing $H[I]$, therefore $H[I] \subseteq Cl_n(H[I])$. And consequently, for any $H[I] \subseteq X[I]$, we have
$Int_n(H[I]) \subseteq H[I] \subseteq Cl_n(H[I])$ by Remarks 3.1 and 3.2, respectively.

Theorem 3.9. For a type-1 neutrosophic topology space $(X[I], \tau[I])$ and $H[I] \subseteq X[I]$, $Cl_n(H[I]) = H[I]$ if and only if $H[I]$ is a type-1 neutrosophic closed set.

Proof. Suppose that $H[I]$ is a type-1 neutrosophic closed set. Since $H[I] \subseteq H[I]$, therefore $Cl_n(H[I])$ is a type-1 neutrosophic closed set. But $H[I] \subseteq Cl_n(H[I])$. Therefore,
$Cl_n(H[I]) = H[I]$. Conversely, assume that $Cl_n(H[I]) = H[I]$, implies that $Cl_n(H[I])$ is a type-1 neutrosophic closed set, and consequently, $H[I]$ is a type-1 neutrosophic closed set. □

Theorem 3.10. For a type-1 neutrosophic topology space $(X[I], \tau[I])$ and $H[I] \subseteq X[I]$, $x \in Cl_n H[I]$ if and only if for all a type-1 neutrosophic open set $B[I]$ containing x, $B[I] \cap H[I] \neq \emptyset[I]$.

Proof. Let $x \in Cl_n(H[I])$ and $B[I]$ be any a type-1 neutrosophic open set containing x. If $B[I] \cap H[I] = \emptyset[I]$ then $H[I] \subseteq X[I] - B[I]$. Since $X[I] - B[I]$ is a type-1 neutrosophic closed set containing $H[I]$, we get $Cl_n(H[I]) \subseteq X[I] - B[I]$ and so $x \in Cl_n(H[I]) \subseteq X[I] - B[I]$. Hence this is contradiction, because $x \in B[I]$. Therefore $B[I] \cap H[I] \neq \emptyset[I]$.

Conversely, Let $x \notin Cl_n(H[I])$. Then $X[I] - Cl_n(H[I])$ is a type-1 neutrosophic open set containing x. Hence by hypothesis, $[X[I] - Cl_n(H[I])] \cap H[I] \neq \emptyset[I]$. But this is contradiction, because $[X[I] - Cl_n(H[I])] \subseteq [X[I] - H[I]]$. □

Theorem 3.11. For a type-1 neutrosophic topology space $(X[I], \tau[I])$ and $H[I], B[I] \subseteq X[I]$. The following axioms hold:

(1) $Cl_n(\emptyset[I]) = \emptyset[I]$ and $Cl_n(X[I]) = X[I]$.
(2) $Cl_n(Cl_n(H[I])) = Cl_n(H[I])$.
(3) If $H[I] \subseteq B[I]$ then $Cl_n(H[I]) \subseteq Cl_n(B[I])$.
(4) $Cl_n(H[I]) \cup Cl_n(B[I]) = Cl_n(H[I] \cup B[I])$.
(5) $Cl_n(H[I] \cap B[I]) \subseteq Cl_n(H[I]) \cap Cl_n(B[I])$.

Proof. (1) is an obvious.

(2) The proof results directly from the Theorem 3.9.

(3) Let $x \in Cl_n(H[I])$. Then by Theorem 3.10, for all a type-1 neutrosophic open set $A[I]$ containing x, $A[I] \cap H[I] \neq \emptyset[I]$. Since $H[I] \subseteq B[I]$ then $A[I] \cap B[I] \neq \emptyset[I]$. Hence $x \in Cl_n(B[I])$. That is, $Cl_n(H[I]) \subseteq Cl_n(B[I])$.

(4) Since $H[I] \subseteq H[I] \cup B[I]$ and $B[I] \subseteq H[I] \cup B[I]$, then by part(3), $Cl_n(H[I]) \subseteq Cl_n(H[I] \cup B[I])$ and $Cl_n(B[I]) \subseteq Cl_n(H[I] \cup B[I])$. Hence $Cl_n(H[I]) \cup Cl_n(B[I]) \subseteq Cl_n(H[I] \cup B[I])$. Since $H[I]$ and $B[I]$ a type-1 neutrosophic closed sets then $Cl_n(H[I]) \cup Cl_n(B[I])$ a type-1 neutrosophic closed set also $H[I] \subseteq Cl_n(H[I])$,$B[I] \subseteq Cl_n(B[I])$ then $H[I] \cup B[I] \subseteq Cl_n(H[I]) \cup Cl_n(B[I])$ then $Cl_n(H[I]) \cup Cl_n(B[I])$ is the smallest type-1 neutrosophic closed set containing $H[I] \cup B[I]$ then $Cl_n(H[I] \cup B[I]) \subseteq Cl_n(H[I]) \cup Cl_n(B[I])$ then $Cl_n(H[I]) \cup Cl_n(B[I]) = Cl_n(H[I] \cup B[I])$.

(5) Since $H[I] \cap B[I] \subseteq H[I]$ and $H[I] \cap B[I] \subseteq B[I]$, then by part(3), $Cl_n(H[I] \cap B[I]) \subseteq Cl_n(H[I])$ and $Cl_n(H[I] \cap B[I]) \subseteq Cl_n(B[I])$. Hence $Cl_n(H[I] \cap B[I]) \subseteq Cl_n(H[I]) \cap Cl_n(B[I])$.

□

Example 3.12. Consider the type-1 neutrosophic topological space in Example 2.2. Let $H[I] = \{1 + I\}$, and $B[I] = \{2 + I\}$ be two type-1 neutrosophic subset of $X[I]$. Then $Cl_n(H[I]) = X[I]$, and $Cl_n(B[I]) = \{2 + I, 2 + 2I\}$. Therefore, $(H[I] \cap B[I]) = \{1 + I, 2 + I\}$, $Cl_n(H[I] \cap B[I]) = \emptyset[I]$, $Cl_n(H[I]) \cap Cl_n(B[I]) = \{2 + I, 2 + 2I\}$, we deduce that;
$Cl_n(H[I] \cap B[I]) \subseteq Cl_n(H[I]) \cap Cl_n(B[I])$.

Theorem 3.13. For a type-1 neutrosophic topology space $(X[I], \tau[I])$ and $H[I] \subseteq X[I]$, the following hold:

(1) $Int_n(X[I] - H[I]) = X[I] - Cl_n(H[I])$.
(2) $Cl_n(X[I] - H[I]) = X[I] - Int_n(H[I])$.

Proof. (1) Since $H[I] \subseteq Cl_n(H[I])$, then $X[I] - Cl_n(H[I]) \subseteq X[I] - H[I]$. Since $Cl_n(H[I])$ is a type-1 neutrosophic closed set, we get $X[I] - Cl_n(H[I])$ is a type-1 neutrosophic open set. Then

$$X[I] - Cl_n(H[I]) = Int_n[X[I] - Cl_n(H[I])] \subseteq Int_n(X[I] - H[I]).$$

On the other side, let $x \in Int_n(X[I] - H[I])$. Then there is a type-1 neutrosophic open set $A[I]$ such that $x \in A[I] \subseteq X[I] - H[I]$. We have $X[I] - A[I]$ is a type-1 neutrosophic closed set containing $H[I]$ and $x \notin X[I] - A[I]$. Hence $x \notin Cl_n(H[I])$, that is, $x \in X[I] - Cl_n(H[I])$.

(2) Since $Int_n(H[I]) \subseteq H[I]$, then $X[I] - H[I] \subseteq X[I] - Int_n(H[I])$. Since $Int_n(H[I])$ is a type-1 neutrosophic open set then $X[I] - Int_n(H[I]))$ is a type-1 neutrosophic closed set. we have

$$Cl_n(X[I] - H[I]) = Cl_n[X[I] - Int_n(H[I])] = X[I] - Int_n(H[I])$$

. On the other side, let $x \notin Cl_n(X[I] - H[I])$. Then by Theorem 3.9, there is a type-1 neutrosophic open set $A[I]$ containing x such that $A[I] \cap (X[I] - H[I]) = \emptyset[I]$. Then $x \in A[I] \subseteq H[I]$, that is, $x \in Int_n(H[I])$. Hence $x \notin X[I] - Int_n(H[I])$. Therefore $X[I] - Int_n(H[I]) \subseteq Cl_n(X[I] - H[I])$.

□

Theorem 3.14. For a type-1 neutrosophic subset $H[I] \subseteq X[I]$ of a type-1 neutrosophic topology space $(X[I], \tau[I])$. The following holds:

(1) *If $B[I]$ is a type-1 neutrosophic open set in $X[I]$, then:*

$$Cl_n(H[I]) \cap B[I] \subseteq Cl_n(H[I] \cap B[I]).$$

(2) *If $B[I]$ is a type-1 neutrosophic closed set in $X[I]$., then:*

$$Int_n(H[I] \cup B[I]) \subseteq Int_n(H[I]) \cup B[I].$$

Proof. (1) Let $x \in Cl_n(H[I]) \cap B[I]$. Then $x \in Cl_n(H[I])$ and $x \in B[I]$. Let $C[I]$ be any type-1 neutrosophic open set in $(X[I], \tau[I])$ containing x. By Theorem 3.10, $C[I] \cap B[I]$ is a type-1 neutrosophic open set containing x. Since $x \in Cl_n(H[I])$ then by Theorem 3.10, $(C[I] \cap B[I]) \cap H[I] \neq \emptyset$. This implies, $C[I] \cap (B[I] \cap H[I]) \neq \emptyset[I]$. Hence by Theorem(4.5), $x \in Cl_n(H[I] \cap B[I])$. That is, $Cl_n(H[I]) \cap B[I] \subseteq Cl_n(H[I] \cap B[I])$.

(2) Since $B[I]$ is a type-1 neutrosophic closed set in $X[I]$ then by the part(1) and Theorem 3.13,

$$\begin{aligned} X[I]-[Int_n(H[I])\cup B[I]] &= [X[I]-Int_n(H[I])]\cap[X[I]-B[I]] \\ &= [Cl_n(X[I]-H[I])]\cap[X[I]-B[I]] \\ &\subseteq Cl_n[(X[I]-H[I])\cap(X[I]-B[I])] \\ &= Cl_n(X[I]-(H[I]\cup B[I])) \\ &= X[I]-(Int_n(H[I]\cup B[I])). \end{aligned}$$

Hence $Int_n(H[I]\cup B[I])\subseteq Int_n(H[I])\cup B[I]$.

□

Theorem 3.15. Let $(X[I],\tau[I])$ be a type-1 neutrosophic topological space. Consider $H[I];B[I]\in X[I]$. Then

(1) $Int_n(H[I]^c)=(Cl_n(H[I]))^c$,
(2) $Cl_n(H[I]^c)=(Int_n(H[I]))^c$.

Proof. Let $H[I];B[I]\in X[I]$. Then,

(1) It is known that

$$Cl_n(H[I])=\cap\{B[I]\subseteq X[I]:H[I]\subseteq B[I] \text{ and } B[I] \text{ is a type-1 neutrosophic closed set}\}.$$

Therefore,

$$(Cl_n(H[I]))^c=\cup\{B[I]^c\subseteq X[I]:B[I]^c\subseteq H[I]^c \text{ and } B[I] \text{ is a type-1 neutrosophic closed set}\}.$$

Right hand of above equality is $Int_n(H[I]^c)$, thus $Int_n(H[I]^c)=(Cl_n(H[I]))^c$.

(2) Take $H[I]^c$ instead of $H[I]$ in 1, then $Cl_n(H[I])$, and it is clear that

$$Cl_n(H[I]^c)^c=(Int_n(H[I]^c))^c)=Int_n(H[I])$$

So, $Cl_n(H[I]^c)=(Int_n(H[I]))^c$.

□

3.3. *Type-1 Neutrosophic Exterior and Boundary Points*

This subsection is exclusive to studying the exterior and boundary operator via $X[I]$ sets and their properties.

Definition 3.16. Let $(X[I],\tau[I])$ be a type-1 neutrosophic topological space and $H[I]\subseteq X[I]$. The exterior of a type-1 neutrosophic set $H[I]$ over $X[I]$ is denoted by $Ext(H[I])$ and is defined as $Ext_n(H[I])=Int_n(H[I]^c)$.

Example 3.17. Consider the type-1 neutrosophic topological space in the Example 2.2. Consider $H[I] = \{1+I, 1+2I, 2+2I\}$, $H[I]^c = \{2+I\}$,then $Int_n(H[I]) = \{1+I, 1+2I\}$. Then $Ext(H[I] = Int_n(H[I]^c) = \emptyset[I]$.

Theorem 3.18. Let $(X[I], \tau[I])$ be a type-1 neutrosophic topological space. and $A[I]$; $B[I] \in X[I]$. Then

(1) $Ext_n(A[I] \cup B[I]) = Ext_n(A[I]) \cap Ext_n(B[I])$.
(2) $Ext_n(A[I]) \cup Ext_n(B[I]) \subseteq Ext_n(A[I] \cap B[I])$.

Proof. By Definition 3.16 and Theorem 3.5 part(2).

(1)

$$\begin{aligned} Ext_n(A[I] \cup B[I]) &= Int_n((A[I] \cup B[I])^c) \\ &= Int_n(A[I]^c \cap B[I]^c) \\ &= Int_n(A[I]^c) \cap Int_n(B[I]^c)) \\ &= Ext(A[I]) \cap Ext(B[I]) \end{aligned}$$

(2) It is similar to 1.

□

Definition 3.19. Let $(X[I], \tau[I])$ be a type-1 neutrosophic topological space and $B[I] \in X[I]$. Then, the type-1 neutrosophic boundary of a type-1 neutrosophic set $B[I]$ over $X[I]$ is denoted by $\Gamma_n(B[I])$ and is defined by $\Gamma_n(B[I]) = Cl_n(B[I]) \cap Cl_n(B[I]^c)$. It must be noted that $\Gamma_n(B[I]) = \Gamma_n(B[I]^c)$.

Example 3.20. Consider the type-1 neutrosophic topological space in Example 2.2.
If $H[I] = \{1+2I, 2+I\}$, then $H[I]^c = \{1+I, 2+2I\}$, and $Cl_n(H[I]) = \{1+2I, 2+I, 2+2I\}$, $Cl_n(H[I]^c) = X[I]$. Therefore, $\Gamma_n(H[I]) = Cl_n(H[I]) \cap Cl_n(H[I]^c) = \{1+2I, 2+I, 2+2I\}$.

Theorem 3.21. Let $(X[I], \tau[I])$ be a type-1 neutrosophic topological space and $A[I] \in X[I]$. Then $(\Gamma_n(A[I]))^c = Ext_n(A[I] \cup Int_n(A[I])$.

Proof. Let $A[I] \in X[I]$. Since $Int_n(A[I]^c) = (Cl_n(A[I]))^c$ by theorem 3.15, we have

$$\begin{aligned} (\Gamma_n(A[I])^c &= (Cl_n(A[I])) \cap \Gamma_n(A[I]^c)) \\ &= (Cl_n(A[I]))^c \cup (\Gamma_n(A[I]^c))^c \\ &= (Cl_n(A[I]))^c \cup ((Int_n(A[I]))^c)^c \\ &= Ext_n(A[I] \cup Int_n(A[I]). \end{aligned}$$

□

Theorem 3.22. *For a type-1 neutrosophic topological space* $(X[I], \tau[I])$ *and* $H[I] \subseteq X[I]$. *Then* $\Gamma(Int_n(H[I])) \subseteq \Gamma(H[I])$.

Proof. Let $H[I] \in X[I]$. Since $Int_n(H[I]^c) = (Cl_n(H[I]))^c$, then

$$\begin{aligned}
\Gamma_n(Int_n(H[I])) &= Cl_n(Int_n(H[I])) \cap Cl_n(Int_n(H[I]))^c \\
&= Cl_n(Int_n(H[I])) \cap \Gamma_n(H[I]^c) \\
&\subseteq Cl_n(H[I]) \cap \Gamma_n(H[I]^c) \\
&= \Gamma_n(H[I]).
\end{aligned}$$

□

Theorem 3.23. For a subset $H[I] \subseteq X[I]$ of type-1 neutrosophic topological space $(X[I], \tau[I])$, the following holds:

(1) $Cl_n(H[I]) = \Gamma_n(H[I]) \cup Int_n(H[I])$.
(2) $\Gamma_n(H[I]) \cap Int_n(H[I]) = \emptyset[I]$.
(3) $\Gamma_n(H[I]) = Cl_n(H[I]) \cap Cl_n(X[I] - H[I])$.

Proof. (1) Note that

$$\begin{aligned}
\Gamma_n(H[I]) \cup Int_n(H[I]) &= (Cl_n(H[I]) - Int_n(H[I])) \cup Int_n(H[I]) \\
&= [Cl_n(H[I]) \cap (X[I] - Int_n(H[I]))] \cup Int_n(H[I]) \\
&= [Cl_n(H[I]) \cup Int_n(H[I])] \cap [(X[I] - Int_n(H[I])) \cup Int_n(H[I])] \\
&= Cl_n(H[I]) \cap X[I] = Cl_n(H[I]).
\end{aligned}$$

(2) It is clear from the definition of $\Gamma_n(H[I])$.
(3) By Theorem 3.22,

$$\begin{aligned}
\Gamma(H[I]) &= Cl_n(H[I]) - Int_n(H[I]) = Cl_n(H[I]) \cap (X[I] - Int_n(H[I])) \\
&= Cl_n(H[I]) \cap Cl_n(X[I] - H[I]).
\end{aligned}$$

This is desird. □

Corollary 3.24. For a type-1 neutrosophic subset $H[I] \subseteq X[I]$ of type-1 neutrosophic topological space $(X[I], \tau[I])$, $\Gamma_n(H[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$.

Proof. By Theorems 3.22 and 3.23. □

Theorem 3.25. For a subset $H[I] \subseteq X[I]$ of type-1 neutrosophic topological space $(X[I], \tau[I])$, the following holds:

(1) $H[I]$ is a type-1 neutrosophic open set if and only if $\Gamma_n(H[I]) \cap H[I] = \emptyset[I]$.

(2) $H[I]$ is a type-1 neutrosophic closed set if and only if $\Gamma_n(H[I]) \subseteq H[I]$.

(3) $H[I]$ is both a type-1 neutrosophic open set and closed set if and only if $\Gamma_n(H[I]) = \emptyset[I]$.

Proof. (1) Let $H[I]$ be a type-1 neutrosophic open set. Then $Int_n(H[I]) = H[I]$. Then by Theorem(3.23),

$$\Gamma_n(H[I]) \cap H[I] = \Gamma_n(H[I]) \cap Int_n(H[I]) = \emptyset[I]$$

Conversely, suppose that $\Gamma_n(H[I]) \cap H[I] = \emptyset[I]$. Then

$$\begin{aligned} H[I] - Int_n(H[I]) &= [H[I] \cap Cl_n(H[I])] - [H[I] \cap Int_n(H[I])] \\ &= H[I] \cap (Cl_n(H[I]) - Int_n(H[I])) = H[I] \cap \Gamma_n(H[I]) = \emptyset[I]. \end{aligned}$$

That is, $Int_n(H[I]) = H[I]$. Hence $H[I]$ is a type-1 neutrosophic open set.

(2) Let $H[I]$ be a type-1 neutrosophic closed set. Then $Cl_n(H[I]) = H[I]$. Then

$$\Gamma_n(H[I]) = Cl_n(H[I]) - Int_n(H[I]) = H[I] - Int_n(H[I]) \subseteq H[I].$$

Conversely, suppose that $\Gamma(H[I]) \subseteq H[I]$. Then by Theorem 3.23,

$$Cl_n(H[I]) = Int_n(H[I]) \cup \Gamma_n(H[I]) \subseteq Int_n(H[I]) \cup H[I] \subseteq H[I].$$

That is, $Cl_n(H[I]) = H[I]$. Hence $H[I]$ is a type-1 neutrosophic closed set.

(3) Let $H[I]$ be both type-1 neutrosophic closed set and a type-1 neutrosophic open set. Then $Cl_n(H[I]) = H[I] = Int_n(H[I])$. Then

$$\Gamma_n(H[I]) = Cl_n(H[I]) - Int_n(H[I]) = H[I] - H[I] = \emptyset[I].$$

Conversely, suppose that $\Gamma_n(H[I]) = \emptyset[I]$. Then $Cl_n(H[I]) - Int_n(H[I]) = \emptyset[I]$. Since $Int_n(H[I]) \subseteq Cl_n(H[I])$ we have $Cl_n(H[I]) = Int_n(H[I])$. Since $Int_n(H[I]) \subseteq H[I] \subseteq Cl_n(H[I])$ we have

$$Cl_n(H[I]) = H[I] = Int_n(H[I]).$$

That is, $Cl_n(H[I]) = H[I]$. Hence $H[I]$ is both type-1 neutrosophic closed set and type-1 neutrosophic open set.

□

Definition 3.26. For a type-1 neutrosophic topology space $(X[I], \tau[I])$ and $Y[I] \subseteq X[I]$, the relativization type-1 neutrosophic topology of $\tau[I]$ to $Y[I]$ is denoted by $\tau_Y[I]$ which defined by
$\tau_Y[I] = \{H[I] \cap Y[I]$ and $H[I]$ is a type-1 neutrosophic open set in X[I] $\}$. And we say the pair $(Y[I], \tau_Y[I])$ is a type-1 neutrosophic topology subspace of $(X[I], \tau[I])$.

Example 3.27. *Let $X = \{1, 2, 3\}$, $Y = \{1, 2\} \subset X$,*

$$X[I] = \{1 + 1I, 1 + 2I, 1 + 3I, 2 + 1I, 2 + 2I, 2 + 3I, 3 + 1I, 3 + 2I, 3 + 3I\}.$$

And $Y[I] = \{1 + 1I, 1 + 2I, 2 + 1I, 2 + 2I\}$.

$$\tau[I] = \{\emptyset[I], X[I], \{1 + 1I\}, \{1 + 3I\}, \{1 + 1I, 1 + 3I\}\}.$$

is a type-1 neutrosophic topology on X[I].
Therefore, $\tau_Y[I] = \{\emptyset[I], Y[I], \{1 + 1I\} \cap Y[I], \{1 + 3I\} \cap Y[I], \{1 + 1I, 1 + 3I\} \cap Y[I]\}$.
Then $\tau_Y[I] = \{\emptyset[I], Y[I], \{1 + 1I\}\}$ is a type-1 neutrosophic relative topology on $Y[I]$.

4. SEPARATION AXIOMS

Definition 4.1. *A type-1 neutrosophic topology space $(X[I], \tau[I])$ is called:*

(1) *$T_0[I]$ type-1 neutrosophic space if for two ponits $x \neq y \in X[I]$, there is type-1 neutrosophic open set $G[I]$ in $X[I]$ such that $x \in G[I]$ and $y \notin G[I]$.*
(2) *$T_1[I]$ type-1 neutrosophic space space if for two ponits $x \neq y \in X[I]$, there are two type-1 neutrosophic open set $G[I]$ and $U[I]$ in $X[I]$ such that $x \in G[I]$ and $y \notin G[I]$, $y \in U[I]$ and $x \notin U[I]$.*
(3) *$T_2[I]$ type-1 neutrosophic space or type-1 neutrosophic Hausdorff space if for two ponits $x \neq y \in X[I]$, there are two type-1 neutrosophicopen set $G[I]$ and $U[I]$ in $X[I]$ such that $x \in G[I]$ and $y \in U[I]$ and $U[I] \cap G[I] = \emptyset[I]$.*
(4) *Regular type-1 neutrosophic space if for each type-1 neutrosophic closed set $F[I]$ in $(X[I], \tau[I])$ and each $x \notin F[I]$, there are two type-1 neutrosophic open sets $G[I]$ and $U[I]$ in $(X[I], \tau[I])$ such that $F[I] \subseteq G[I]$, $x \in U[I]$ and $U[I] \cap G[I] = \emptyset[I]$.*
(5) *A type-1 neutrosophic topology space $(X[I], \tau[I])$ is called type-1 neutrosophic $T_3[I]$ space if it is regular type-1 neutrosophic space and $T_1[I]$ type-1 neutrosophic space.*
(6) *Normal type-1 neutrosophic space if for each two disjoint type-1 neutrosophic closed sets $F[I]$ and $M[I]$ in $(X, \tau[I])$, there are two type-1 neutrosophic open sets $G[I]$ and $U[I]$ in $(X[I], \tau[I])$ such that $F[I] \subseteq G[I]$, $M[I] \subseteq U[I]$ and $U[I] \cap G[I] = \emptyset[I]$. A type-1 neutrosophic topology space $(X[I], \tau[I])$ is called $T_4[I]$* type-1 neutrosophic space *if it is normal type-1 neutrosophic space and $T_1[I]$ type-1 neutrosophic space.*

Theorem 4.2. Let $T_1[I]$ type-1 neutrosophic space and x is a type-1 neutrosophic element. Then $\{x\}$ is a type-1 neutrosophic closed set.

Proof. We will prove that $\{x\}^c$ is a type-1 neutrosophic open set. Let y be a type-1 neutrosophic element and $y \in (X[I] - \{x\})$ this means $x \neq y$. By definition of a $T_1[I]$ for any two type-1 neutrosophic distinct elements x and y , there exists a type-1 neutrosophic open set $U[I]_y$, that is $y \in U[I]_y$ and $x \notin U[I]_y$. Since $x \notin U[I]_y$, type-1 neutrosophic open set $U[I]_y$ must lie within the complement of $\{x\}$. Therefore $U[I]_y \subseteq (X[I] - \{x\})$, we got $(X[I] - \{x\}) = \cup\{U[I]_y : x \neq y\}$. Since $(X[I] - \{x\})$ is a type-1 neutrosophic open set, then $\{x\}$ is a type-1 neutrosophic closed set. □

Theorem 4.3. Every $T_3[I]$ type-1 neutrosophic space is a $T_2[I]$ type-1 neutrosophic space.

Proof. Let $(X[I], \tau[I])$ be a $T_3[I]$ type-1 neutrosophic space and $x \neq y \in X[I]$ be any ponits in $(X[I], \tau[I])$. Since $(X[I], \tau[I])$ is a $T_1[I]$ type-1 neutrosophic space. Then by Theorem 4.2 , $\{x\}$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$ and $y \notin \{x\}$. Since $(X[I], \tau[I])$ is a regular type-1 neutrosophic space then there are two type-1 neutrosophic open sets $G[I]$ and U in $(X[I], \tau[I])$ such that $x \in \{x\} \subseteq G[I]$, $y \in U[I]$ and $U[I] \cap G[I] = \emptyset[I]$. Hence $(X[I], \tau[I])$ is a $T_2[I]$ type-1 neutrosophic space. □

Theorem 4.4. Every $T_4[I]$ type-1 neutrosophic space is a $T_3[I]$ type-1 neutrosophic space.

Proof. Let be a $T_4[I]$ type-1 neutrosophic space. Let $F[I]$ be any type-1 neutrosophic closed set in $(X[I], \tau[I])$ and $x \notin F[I]$ be any ponit in $(X[I], \tau[I])$. Since $(X[I], \tau[I])$ is a $T_1[I]$ type-1 neutrosophic space. Then by Theorem 4.2 , $\{x\}$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$ and $F[I] \cap \{x\} = \emptyset[I]$. Since $(X[I], \tau[I])$ is a normal type-1 neutrosophic space then there are two type-1 neutrosophic open sets $G[I]$ and $U[I]$ in $(X[I], \tau[I])$ such that $x \in \{x\} \subseteq G[I]$, $F[I] \subseteq U[I]$ and $U[I] \cap G[I] = \emptyset[I]$. Hence $(X[I], \tau[I])$ is a $T_3[I]$ type-1 neutrosophic space. □

Theorem 4.5. A type-1 neutrosophic topology space $(X[I], \tau[I])$ is a $T_0[I]$ type-1 neutrosophic space if and only if for two points $x \neq y \in X[I]$, $Cl_n(\{x\}) \neq Cl_n(\{y\})$.

Proof. Suppose that is a $T_0[I]$ type-1 neutrosophic space. Then for two points $x \neq y \in X[I]$, there is a type-1 neutrosophic open set $G[I]$ in such that $x \in G[I]$, $y \notin G[I]$. Since $X[I] - G[I]$ is a type-1 neutrosophic closed set and $y \in X[I] - G[I]$ then

$$Cl_n(\{y\}) \subseteq Cl_n(X[I] - G[I]) = X[I] - G[I].$$

Since $x \in Cl_n(\{x\})$ and $x \notin X[I] - G[I]$ then $x \notin Cl_n(\{y\})$. That is, $Cl_n(\{x\}) \neq Cl_n(\{y\})$.
Conversely, Let $x \neq y \in X[I]$ be any two points in $X[I]$. Then by hypothesis, $Cl_n(\{x\}) \neq Cl_n(\{y\})$. Then there is at least one point $z \in X[I]$ such that $z \in Cl_n(\{x\})$ and $z \notin Cl_n(\{y\})$. If $x \in Cl_n(\{y\})$ then $z \in Cl_n(\{x\}) \subseteq Cl_n(\{y\})$, and this is a contradiction. Hence $x \notin Cl_n(\{y\})$. Hence $G[I] = X[I] - Cl_n(\{y\})$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ such that $x \in G[I]$ and $y \notin G[I]$. Therefore is a $T_0[I]$ type-1 neutrosophic space. □

Theorem 4.6. A type-1 neutrosophic topology space $(X[I], \tau[I])$ is a $T_1[I]$ type-1 neutrosophic space if and only if $\{x\}$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$ for all $x \in X[I]$.

Proof. Suppose that $(X[I], \tau[I])$ is a $T_1[I]$ type-1 neutrosophic space. Let $x \in X[I]$ be any point in $X[I]$. Now we prove that $X[I] - \{x\}$ is type-1 neutrosophic open set in $(X[I], \tau[I])$. Let $y \in X[I] - \{x\}$. Then $x \neq y \in X[I]$. Since $(X[I], \tau[I])$ is a $T_1[I]$ type-1 neutrosophic space then there are two type-1 neutrosophic open sets $G[I]$ and $U[I]$ in $(X[I], \tau[I])$ such that $x \in G[I]$, $y \notin G[I]$, $y \in U[I]$ and $x \notin U[I]$. Then $y \in U[I] \subseteq X[I] - \{x\}$ Hence $X[I] - \{x\}$ is type-1 neutrosophic open set in $(X[I], \tau[I])$. That is, $\{x\}$ is type-1 neutrosophic closed set in $(X[I], \tau[I])$.
Conversely, Let $x \neq y \in X[I]$ be any two points in $X[I]$. Then by hypothesis, $\{x\}$ and $\{y\}$ are type-1 neutrosophic closed set in $(X[I], \tau[I])$. Then $X[I] - \{x\}$ and $X[I] - \{y\}$ are type-1 neutrosophic open set in $(X[I], \tau[I])$, $x \in X[I] - \{y\}$, $y \notin X[I] - \{y\}$, $x \notin X[I] - \{x\}$ and $y \in X[I] - \{x\}$. Therefore $(X[I], \tau[I])$ is a $T_1[I]$ type-1 neutrosophic space. □

Theorem 4.7. A type-1 neutrosophic topology space $(X[I], \tau[I])$ is $T_2[I]$ type-1 neutrosophic space if and only if for each $x \in X[I]$ and for $x \neq y \in X[I]$, there is a type-1 neutrosophic open set $H[I]$ in $(X[I], \tau[I])$ containing x such that $y \notin Cl_n(H[I])$.

Proof. Suppose that $(X[I], \tau[I])$ is $T_2[I]$ type-1 neutrosophic space. Let $x \in X[I]$ be any point in $X[I]$ and $x \neq y \in X[I]$ be any ponit in $X[I]$. Then there are two type-1 neutrosophic open sets $G[I]$ and $U[I]$ in $(X[I], \tau[I])$ such that $x \in G[I]$, $y \in U[I]$ and $U[I] \cap G[I] = \emptyset[I]$. Take $H[I] = G[I]$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ containing x and so $y \notin H[I] \subseteq Cl_n(H[I])$.
Conversely, Let $x \neq y \in X[I]$ be any points in $(X[I], \tau[I])$. and By the hypothesis, there is a type-1 neutrosophic open set $H[I]$ in $(X[I], \tau[I])$ containing x such that $y \notin Cl_n(H[I])$. Then $X[I] - Cl_n(H[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ containing y such that $x \in H[I]$, $y \in X[I] - Cl_n(H[I])$ and $H[I] \cap X[I] - Cl_n(H[I]) = \emptyset[I]$. Then $(X[I], \tau[I])$ is $T_2[I]$ type-1 neutrosophic space. □

Theorem 4.8. A type-1 neutrosophic topology space $(X[I], \tau[I])$ is regular type-1 neutrosophic space if and only if for each $x \in X[I]$ and for each type-1 neutrosophic open set $M[I]$ in $(X[I], \tau[I])$ containing x, there is a type-1 neutrosophic open set $H[I]$ in $(X[I], \tau[I])$ containing x such that $Cl_n(H[I]) \subseteq M[I]$.

Proof. Suppose that $(X[I], \tau[I])$ is regular type-1 neutrosophic space. Let $x \in X[I]$ be any point in $X[I]$ and $M[I]$ be any type-1 neutrosophic open set in $(X[I], \tau[I])$ containing x. Then $X[I] - M[I]$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$ and $x \notin X[I] - M[I]$. Since $(X[I], \tau[I])$ is regular type-1 neutrosophic space then there are two type-1 neutrosophic open sets $G[I]$ and $U[I]$ in $(X[I], \tau[I])$ such that $(X[I] - M[I]) \subseteq G[I]$, $x \in U[I]$ and $U[I] \cap G[I] = \emptyset[I]$. Take $H[I] = U[I]$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ containing x. Then $H[I] = U[I] \subseteq (X[I] - G[I])$, this implies,

$$Cl_n(H[I]) \subseteq Cl_n(X[I] - G[I]) \subseteq (X[I] - G[I]) \subseteq M[I].$$

Conversely, Let $F[I]$ be any type-1 neutrosophic closed set in $(X[I], \tau[I])$ and $x \notin F[I]$. Then $x \in X[I] - F[I]$ and $X[I] - F[I]$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ containing x. By the hypothesis, there is a type-1 neutrosophic open set $H[I]$ in $(X[I], \tau[I])$ containing x such that $Cl_n(H[I]) \subseteq (X[I] - F[I])$. Then $F[I] \subseteq [X[I] - Cl_n(H[I])]$ and $X[I] - Cl_n(H[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$. Since $H[I]$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ containing x and $H[I] \cap [X[I] - Cl_n(H[I])] = \emptyset[I]$, then $(X[I], \tau[I])$ is regular type-1 neutrosophic space. □

Theorem 4.9. A type-1 neutrosophic topology space $(X[I], \tau[I])$ is normal type-1 neutrosophic space if and only if for each type-1 neutrosophic closed set $F[I]$ in $(X[I], \tau[I])$ and for each type-1 neutrosophic open set $G[I]$ in $(X[I], \tau[I])$ containing $F[I]$, there is a type-1 neutrosophic open set $V[I]$ in $(X[I], \tau[I])$ containing $F[I]$ such that $Cl_n(V[I]) \subseteq G[I]$.

Proof. Suppose that $(X[I], \tau[I])$ is normal type-1 neutrosophic space. Let $F[I]$ be any type-1 neutrosophic closed set in $(X[I], \tau[I])$ and $G[I]$ be any type-1 neutrosophic open set in $(X[I], \tau[I])$ containing $F[I]$. Then $X[I]-G[I]$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$ and $F[I] \cap (X[I]-G[I]) = \emptyset[I]$. Since $(X[I], \tau[I])$ is normal type-1 neutrosophic space then there are two type-1 neutrosophic open sets $H[I]$ and $U[I]$ in $(X[I], \tau[I])$ such that $(X[I] - G[I]) \subseteq U[I]$, $F[I] \subseteq H[I]$ and $U[I] \cap H[I] = \emptyset[I]$. Take $V[I] = H[I]$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ containing $F[I]$. Then $V[I] = H[I] \subseteq (X[I] - U[I])$, this implies,

$$Cl_n(V[I]) \subseteq Cl_n(X[I] - U[I]) \subseteq (X[I] - U[I]) \subseteq G[I].$$

Conversely, Let $F[I]$ and $H[I]$ be any two type-1 neutrosophic closed sets in $(X[I], \tau[I])$ such that $F[I] \cap H[I] = \emptyset[I]$. Then $H[I] \subseteq (X[I] - F[I])$ and $X[I] - F[I]$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ containing type-1 neutrosophic closed set $H[I]$. By the hypothesis, there is a type-1 neutrosophic open set $V[I]$ in $(X[I], \tau[I])$ containing $H[I]$ such that $Cl_n(V[I]) \subseteq (X[I] - F[I])$. Then $F[I] \subseteq X[I] - Cl_n(V[I])$ and $X[I] - Cl_n(V[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$. Since $V[I]$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ containing $H[I]$ and $V[I] \cap [X[I] - Cl_n(V[I])] = \emptyset[I]$, then $(X[I], \tau[I])$ is normal type-1 neutrosophic space. □

5. Continuous Type-1 Neutrosophic Functions

Definition 5.1. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is called continuous if $f_n^{-1}(U[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ for every a type-1 neutrosophic open set $U[I]$ in $Y[I]$.

Example 5.2. *Let $X = \{a, b\}$ and $Y = \{1, 2, 3\}$ be two classical sets with a classical type-1 neutrosophic function $f_c : X \longrightarrow Y$ such that $f_c(a) = 1$, and $f_c(b) = 2$, the type-1 neutrosophic set, which are generated by X, and Y are given by: $X[I] = \{a + aI, a + bI, b + aI, b + bI)\}$, and $Y[I] = \{1 + 1I, 1 + 2I, 1 + 3I, 2 + 1I, 2 + 2I, 2 + 3I, 3 + 1I, 3 + 2I, 3 + 3I\}$.*
The type-1 neutrosophic function fn of type-1 is given by:
$f_n(a + aI) = f_c(a) + f_n(aI) = f_c(a) + f_c(a)f_n(I) = 1 + 1I.$
$f_n(a + bI) = f_c(a) + f_n(bI) = f_c(a) + f_c(b)f_n(I) = 1 + 2I.$
$f_n(b + aI) = f_c(b) + f_n(aI) = f_c(b) + f_c(a)f_n(I) = 2 + 1I.$
$f_n(b + bI) = f_c(b) + f_n(bI) = f_c(b) + f_c(b)f_n(I) = 2 + 2I.$

and let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$, be a type-1 neutrosophic function between two type-1 neutrosophic topology spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$, where

$$\tau[I] = \{\emptyset[I], X[I], \{a + aI\}, \{a + bI\}, \{a + bI, a + aI\}\}.$$

And

$$\tau_1[I] = \{\emptyset[I], Y[I], \{1 + 1I\}, \{1 + 2I\}, \{1 + 1I, 1 + 2I\}\}$$

. The type-1 neutrosophic function is continuous, since for any type-1 neutrosophic open set in $(Y[I], \tau_1[I])$ we have the following:
$\{1 + 1I\} \in \tau_1[I]$, implies that, $f_n^{-1}(1 + 1I) = \{a + aI\}$, $\{1 + 2I\} \in \tau_1[I]$, implies that, $f_n^{-1}(\{1 + 2I\}) = \{a + bI\}$, $\{1 + 1I, 1 + 2I\} \in \tau_1[I]$, implies that, $f_n^{-1}(\{1 + 1I, 1 + 2I\}) = \{a + bI, a + aI\}$, $f_n^{-1}(Y[I]) = X[I]$ and $f_n^{-1}(\emptyset[I]) = \emptyset[I]$.

Theorem 5.3. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topology spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is continuous if and only if $f_n^{-1}(F[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$ for every a type-1 neutrosophic closed set $F[I]$ in $Y[I]$.

Proof. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a continuous and $F[I]$ be any a type-1 neutrosophic closed set in $Y[I]$. Then $f_n^{-1}(Y[I] - F[I]) = X[I] - f_n^{-1}(F[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$, that is, $f_n^{-1}(F[I])$ is type-1 neutrosophic closed set in $(X[I], \tau[I])$.
Conversely, suppose that $f_n^{-1}(F[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$ for every a type-1 neutrosophic closed set $F[I]$ in $Y[I]$. Let $U[I]$ be any a type-1 neutrosophic open set in $Y[I]$. Then by the hypothesis, $f_n^{-1}(Y[I] - U[I]) = X[I] - f_n^{-1}(U[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$, that is, $f_n^{-1}(U[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$. Hence f_n is a continuous □

Theorem 5.4. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topology spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is a continuous type-1 neutrosophic function if and only if for each $x \in X[I]$ and each a type-1 neutrosophic open set $U[I]$ in $Y[I]$ with $f_n(x) \in U[I]$, there exists a type-1 neutrosophic open set $V[I]$ in $(X[I], \tau[I])$ such that $x \in V[I]$ and $f_n(V[I]) \subseteq U[I]$.

Proof. Suppose type-1 neutrosophic function f_n is a continuous type-1 neutrosophic function. Let $x \in X[I]$ and $U[I]$ be any a type-1 neutrosophic open set in $Y[I]$ containing $f_n(x)$. Put $V[I] = f_n^{-1}(U[I])$. Since f_n is a continuous then $V[I]$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ such that $x \in V[I]$ and $f_n(V[I]) \subseteq U[I]$.
Conversely, Let $U[I]$ be any a type-1 neutrosophic open set in $Y[I]$ and $x \in f_n^{-1}(U[I])$. Then $f_n(x)[I] \in U[I]$ and hence by the hypothesis, there exists a type-1 neutrosophic open set $V[I]$ in $(X[I], \tau[I])$ such that $x \in V[I]$ and $f_n(V[I]) \subseteq U[I]$. Hence $x \in V[I] \subseteq f_n^{-1}(U[I])$ and $f_n^{-1}(U[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$. That is, f_n is a continuous. □

Theorem 5.5. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topology spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$. Then f_n is a continuous if and only if $f_n[Cl_n(A[I])] \subseteq Cl_n[f_n(A[I])]$ for all $A[I] \subseteq X[I]$.

Proof. Let f_n be a continuous and $A[I]$ be any subset of $X[I]$. Then $Cl_n[f(A[I])]$ is a type-1 neutrosophic closed set in $Y[I]$. Since f_n is a continuous then by Theorem 5.4, $f_n^{-1}Cl_n[f_n(A[I])]$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$. That is,

$$Cl_n[f_n^{-1}[Cl_n[f_n(A[I])]]] = f_n^{-1}[Cl_n[f_n(A[I])]].$$

Since $f_n(A[I]) \subseteq Cl_n[f_n(A[I])]$ then $A[I] \subseteq f_n^{-1}[Cl_n[f_n(A[I])]$. This implies,

$$Cl_n(A[I]) \subseteq Cl_n[f_n^{-1}[Cl_n f_n(A[I])]]] = f_n^{-1}[Cl_n[f_n(A[I])]].$$

Hence $f_n[Cl_n(A[I])] \subseteq Cl_n[f_n(A[I])]$.
Conversely, let $H[I]$ be any a type-1 neutrosophic closed set in $Y[I]$, that is, $Cl_n(H[I]) = H[I]$. Since $f_n^{-1}(H[I]) \subseteq X[I]$. Then by the hypothesis,

$$f_n[Cl_n[f_n^{-1}(H[I])]] \subseteq Cl_n[f_n(f_n^{-1}(H[I]))] \subseteq Cl_n(H[I]) = H[I].$$

This implies, $Cl_n[f_n^{-1}(H[I])] \subseteq f_n^{-1}(H[I])$. Hence $Cl_n[f_n^{-1}(H[I])] = f_n^{-1}(H[I])$, that is, $f_n^{-1}(H[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$. Therefore f_n is a continuous. □

Theorem 5.6. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topology space $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$. Then f_n is a continuous if and only if $Cl_n[f_n^{-1}(B[I])] \subseteq f_n^{-1}[Cl_n(B[I])]$ for all $B[I] \subseteq Y[I]$.

Proof. Let f_n be a continuous and $B[I]$ be any subset of $Y[I]$. Then $Cl_n(B[I])$ is a type-1 neutrosophic closed set in $Y[I]$. Since f_n is a continuous. Then by Theorem 5.4, $f_n^{-1}[Cl_n(B[I])]$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$. That is,

$$Cl_n[f_n^{-1}[Cl_n(B[I])]] = f_n^{-1}[Cl_n(B[I])].$$

Since $B[I] \subseteq Cl_n(B[I])$ then $f_n^{-1}(B[I]) \subseteq Cl_n[f_n^{-1}(B[I])]$ This implies,

$$f_n^{-1}Cl_n(B[I])] \subseteq Cl_n[f_n^{-1}[Cl_n(B[I])]] = f_n^{-1}[Cl_n(B[I])].$$

Hence $Cl_n[f_n^{-1}(B[I])] \subseteq f_n^{-1}[Cl_n(B[I])]$.
Conversely, Let $H[I]$ be any a type-1 neutrosophic closed set in $Y[I]$, that is, $Cl_n(H[I]) = H[I]$. Since $H[I] \subseteq Y[I]$. Then by the hypothesis,

$$Cl_n[f_n^{-1}(H[I])] \subseteq f_n^{-1}[(H[I])] = f_n^{-1}(H[I]).$$

This implies, $Cl_n[f_n^{-1}(H[I])] \subseteq f_n^{-1}(H[I])$. Hence $Cl_n[f_n^{-1}(H[I])] = f_n^{-1}(H[I])$, that is, $f_n^{-1}(H[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$. Hence f_n is a continuous. □

Theorem 5.7. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topology space $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$. Then f_n is a continuous if and only if $f_n^{-1}(Int_n(B[I])) \subseteq Int_n[f_n^{-1}(B[I])]$ for all $B[I] \subseteq Y[I]$.

Proof. Let f_n be a continuous and $B[I]$ be any subset of $Y[I]$. Then $Int_n(B[I])$ is a type-1 neutrosophic open set in $Y[I]$. Since f_n is a continuous we get $f_n^{-1}(Int_n(B))$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$. That is,

$$Int_n[f_n^{-1}[Int_n(B[I])]] = f_n^{-1}[Int_n(B[I])].$$

Since $Int_n(B[I]) \subseteq B[I]$ then $f_n^{-1}[Int_n(B[I])] \subseteq f_n^{-1}(B[I])$. This implies,

$$f_n^{-1}[Int_n(B[I])] = Int_n[f_n^{-1}[Int_n(B[I])]] \subseteq Int_n[f_n^{-1}[(B[I])].$$

Hence $f_n^{-1}[Int_n(B[I])] \subseteq Int_n[f_n^{-1}(B[I])]$. Conversely, let $U[I]$ be any a type-1 neutrosophic open set in $Y[I]$, that is, $Int_n(U[I]) = U[I]$. Since $U[I] \subseteq Y[I]$. Then by the hypothesis,

$$f_n^{-1}(U[I]) = f_n^{-1}[Int_n(U[I])] \subseteq Int_n(f_n^{-1}U[I])].$$

This implies, $f_n^{-1}(U[I]) \subseteq Int_n[f_n^{-1}(U[I])]$. Hence $f_n^{-1}(U[I]) = Int_n[f_n^{-1}(U[I])]$, that is, $f_n^{-1}(U[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$. Hence f_n is a continuous. □

Definition 5.8. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is called a type-1 neutrosophic closed function if $f_n(G[I])$ is a type-1 neutrosophic closed set in $(Y[I], \tau[I])$ for every type-1 neutrosophic closed set $G[I]$ in $(X[I], \tau[I])$.

Theorem 5.9. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is a type-1 neutrosophic closed function if and only if $Cl_n[f_n(A[I])] \subseteq f_n[Cl_n(A[I])]$ for all $A[I] \subseteq X[I]$.

Proof. Suppose that f_n is a type-1 neutrosophic closed function and $A[I]$ be any type-1 neutrosophic subset of $X[I]$. Since $Cl_n(A[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$ and f_n is a type-1 neutrosophic closed then $f_n[Cl_n(A[I])]$ is a a type-1 neutrosophic closed set in $Y[I]$. That is,

$$Cl_n[f_n[Cl_n(A[I])]] = f_n[Cl_n(A[I])].$$

Since $A[I] \subseteq Cl_n(A[I])$ then $f_n(A[I]) \subseteq f_n[(A[I])]$. This implies,

$$Cl_n[f_n(A[I])] \subseteq Cl_n[f_n[Cl_n(A[I])]] = f_n[(Cl_nA[I])].$$

Hence $Cl_n[f_n(A[I])] \subseteq f_n[Cl_n(A[I])]$.
Conversely, let $F[I]$ be any type-1 neutrosophic closed set in $(X[I], \tau[I])$, that is, $Cl_n(F[I]) = F[I]$. Since $F[I] \subseteq X[I]$. Then by the hypothesis,

$$Cl_n[f_n(F[I])] \subseteq f_n[Cl_n(F[I])] = f_n(F[I]).$$

This implies, $Cl_n[f_n(F[I])] \subseteq f_n(F[I])$. Hence $Cl_n[f_n(F[I])] = f_n(F[I])$, that is, $f_n(F[I])$ is a a type-1 neutrosophic closed set in $Y[I]$. Hence $F[I]$ is a type-1 neutrosophic closed. □

Definition 5.10. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is called a type-1 neutrosophic open function if $f(G[I])$ is a type-1 neutrosophic open set in $(Y[I], \tau_1[I])$ for every type-1 neutrosophic open set $G[I]$ in $(X[I], \tau[I])$.

Theorem 5.11. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is a type-1 neutrosophic open function if and only if $f[Int_n(A[I])] \subseteq Int_n[f(A[I])]$ for all $A[I] \subseteq X[I]$.

Proof. Suppose that f_n is a type-1 neutrosophic open function and $A[I]$ be any type-1 neutrosophic subset of $X[I]$. Since $Int_n(A[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ and f_n is a type-1 neutrosophic open then $f_n[Int_n(A[I])]$ is a type-1 neutrosophic open set in $Y[I]$. That is,

$$Int_n[f_n[Int_n(A[I])]] = f_n[Int_n(A[I])].$$

Since $Int_n(A[I]) \subseteq A[I]$ then $f_n[Int_n(A[I])] \subseteq f_n(A[I])$. This implies,

$$Int_n[f_n[Int_n(A[I])]] \subseteq Int_n[f_n(A[I])] = f_n[Int_n(A[I])].$$

Hence $f_n[Int_n(A[I])] \subseteq Int_n[f_n(A[I])]$.
Conversely, let $F[I]$ be any type-1 neutrosophic open set in $(X[I], \tau[I])$, that is, $Int_n(F[I]) = F[I]$. Since $F[I] \subseteq X[I]$. Then by the hypothesis,

$$f_n[Int_n(F[I])] \subseteq Int_n[f_n(F[I])] = f_n(F[I]).$$

This implies, $f_n(F[I]) \subseteq Int_n[f_n(F[I])]$. Hence $Int_n[f_n(F[I])] = f_n(F[I])$, that is, $f_n(F[I])$ is a type-1 neutrosophic open set in $Y[I]$. Hence f_n is a type-1 neutrosophic open. □

6. Contra Type-1 Neutrosophic Functions

Definition 6.1. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is called contra continuous type-1 Neutrosophic function if $f_n^{-1}(V[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$ for every type-1 neutrosophic open set $V[I]$ in $Y[I]$.

Theorem 6.2. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is contra continuous if and only if $f_n^{-1}(F[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$ for every a type-1 neutrosophic closed set $F[I]$ in $Y[I]$.

Proof. Suppose that f_n is contra continuous. Let $G[I]$ be any a type-1 neutrosophic closed set in $Y[I]$. Then $Y[I] - G[I]$ is a type-1 neutrosophic open set in $Y[I]$. Since f_n is contra continuous then $X[I] - f_n^{-1}(G[I]) = f_n^{-1}(Y[I] - G[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$. That is, $f_n^{-1}(G[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$. Conversely, Let $G[I]$ be any a type-1 neutrosophic open set in $Y[I]$. Then $Y[I] - G[I]$ is a type-1 neutrosophic closed set in $Y[I]$. Then by the hypothesis, $f_n^{-1}(Y[I] - G[I]) = X[I] - f_n^{-1}(G[I])$

is a type-1 neutrosophic open set in $(X[I], \tau[I])$. That is, $f_n^{-1}(G[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$. Hence f_n is contra continuous. □

Theorem 6.3. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is contra continuous if and only if for each $x \in X[I]$ and each type-1 neutrosophic closed set $G[I]$ in $Y[I]$ containing $f_n(x)$, there is a type-1 neutrosophic open set $U[I]$ in $(X[I], \tau[I])$ containing x such that $f_n(U[I]) \subseteq G[I]$.

Proof. Suppose that f_n is contra continuous. Let $x \in X[I]$ and G[I] be a type-1 neutrosophic closed set in $Y[I]$ containing $f_n(x)$. Then by the Theorem 6.2, $U[I] = f_n^{-1}(G[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$. Since $f(x) \in G[I]$ then $x \in f_n^{-1}(G[I]) = U[I]$ and $f_n(U[I]) = f(f_n^{-1}(G[I])) \subseteq G[I]$.
Conversely, Let $G[I]$ be a a type-1 neutrosophic closed set in $Y[I]$. For each $x \in f_n^{-1}(G[I])$, $f_n(x) \in G[I]$. Then by the hypothesis, there is a type-1 neutrosophic open set $U[I]_x$ in $(X[I], \tau[I])$ containing x such that $f_n(U[I]_x) \subseteq G[I]$. Therefore, we obtain

$$f_n^{-1}(G[I]) = \cup\{U[I]_x : x \in f_n^{-1}(G[I])\}.$$

Then $f_n^{-1}(G[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$. Hence by the theorem 6.2, f_n is a contra continuous. □

Theorem 6.4. The set of all points x in $X[I]$ at which $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is not a contra continuous is identical with the union of the frontier of the inverse images of type-1 neutrosophic closed sets of $Y[I]$ containing $f_n(x)$.

Proof. Suppose that f_n is not contra continuous at $x \in X[I]$. Then by Theorem 6.3 , there is a type-1 neutrosophic closed set $G[I]$ in $Y[I]$ containing $f_n(x)$ such that $f_n(U[I]) \subseteq G[I]$ for all a type-1 neutrosophic open set $U[I]$ in $(X[I], \tau[I])$. containing x. That is, for all a type-1 neutrosophic open set $U[I]$ in $(X[I], \tau[I])$. containing x, $f_n(U[I]) \cap (Y[I] - G[I]) \neq \emptyset[I]$ and this implies

$$U[I] \cap f_n^{-1}(Y[I] - G[I]) \neq \emptyset[I].$$

Therefore we have

$$x \in Cl_n[f_n^{-1}(Y[I] - G[I])] = Cl_n[X[I] - f_n^{-1}(G[I])].$$

However, since $f_n(x) \in G[I]$, then

$$x \in f_n^{-1}(G[I]) \subseteq Cl_n(f_n^{-1}G[I])].$$

Then

$$x \in Cl_n[X[I] - f_n^{-1}(G[I])] \cap Cl_n[f_n^{-1}(G[I])] = \Gamma_n[f_n^{-1}(G[I])].$$

Conversely, Suppose that $x \in X[I]$ and $x \in \Gamma_n(f_n^{-1}G[I])]$ for some a type-1 neutrosophic closed sets $G[I]$ in $Y[I]$ containing $f_n(x)$. If f_n is a contra continuous at x then there is a type-1 neutrosophic set $U[I]$ in $(X[I], \tau[I])$. containing x such that $f_n(U[I]) \subseteq G[I]$. Therefore we have $x \in U[I] \subseteq f_n^{-1}(G[I])$. That is,

$$x \in Int_n[f_n^{-1}(G[I])] \subseteq \Gamma_n[f_n^{-1}(G[I])].$$

This is a contra diction. Hence by Theorem 6.3, f_n is not contra continuous at x. □

Definition 6.5. The kernel of a subset $H[I]$ of a type-1 neutrosophic topology space $(X[I], \tau[I])$ is denoted by $Ker_n(H[I])$ and defined by $Ker_n(H[I]) = \cap\{U[I] : H[I] \subseteq U[I] \text{and } U[I] \text{is a type-1 neutrosophic open set} in X[I]\}$.

Lemma 6.6. Let $H[I]$ and $G[I]$ be a subset of a type-1 neutrosophic topological space $(X[I], \tau[I])$. The following holds:

(1) $x \in Ker_n(H[I])$ if and only if $H[I] \cap U[I] \neq \emptyset[I]$ for any type-1 neutrosophic closed set $U[I]$ containing x.

(2) $H[I] \subseteq Ker_n(H[I])$ and $H[I] = Ker_n(H[I])$ if $H[I]$ is a type-1 neutrosophic open set in $X[I]$.

(3) If $H[I] \subseteq G[I]$ then $Ker_n(H[I]) \subseteq Ker_n(G[I])$.

Proof. (1) Let $V[I]$ be any type-1 neutrosophic closed subset of $X[I]$, containing x. Suppose that $H[I] \cap V[I] = \emptyset[I]$. Then $H[I] \subseteq X[I] - V[I]$. Since $X[I] - V[I]$ is type-1 neutrosophic open set, then $Ker_n(H[I]) \subseteq X[I] - V[I]$. Hence $x \in X[I] - V[I]$ but this is contradiction. Therefore $H[I] \cap V[I] \neq \emptyset[I]$. Conversely, Suppose that $x \notin Ker_n(H[I])$. Then there is at least type-1 neutrosophic open set containing H[I] and $x \notin V[I]$. Then $X[I] - V[I]$ is type-1 neutrosophic closed set containing x and $H[I] \cap (X[I] - V[I]) = \emptyset[I]$. This is contradiction with the hypothesis. Hence $x \in Ker_n(H[I])$.

(2) By the definition of $Ker_n(H[I])$, we get that $H[I] \subseteq Ker_n(H[I])$. If $H[I]$ is type-1 neutrosophic open set, then $H[I]$ is the smallest type-1 neutrosophic open set containing $H[I]$. That is, $Ker_n(H[I]) = H[I]$.

(3) Let $x \in Ker_n(H[I])$. Then by the part (1), $H[I] \cap V[I] \neq \emptyset[I]$ for any type-1 neutrosophic closed set $V[I]$ containing x. Since $H[I] \subseteq G[I]$, then $G[I] \cap V[I] \neq \emptyset[I]$. Hence $x \in Ker_n(G[I])$. That is, $Ker_n(H[I]) \subseteq Ker_n(G[I])$.

□

Theorem 6.7. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is a contra continuous if and only if $f_n[Cl_n[(H[I])]] \subseteq Ker[f(H[I])]$ for every subset $H[I]$ of $X[I]$.

Proof. Suppose that f_n is a contra continuous. Let $H[I]$ be any subset of $X[I]$. Let $y \notin Ker[f_n(H[I])]$. Then by Lemma 6.6 , there is a a type-1 neutrosophic closed set $F[I]$ in $Y[I]$ containing $Y[I]$ such that $f_n(H[I]) \cap F[I] = \emptyset[I]$. Then $H[I] \cap f_n^{-1}(F[I]) = \emptyset[I]$. Since $F[I]$ is a a type-1 neutrosophic closed set in $Y[I]$ and f_n is a contra continuous then by Theorem 5.3, $f_n^{-1}(F[I])$ is a type-1 neutrosophic open set in $(X[I], \tau[I])$. Then $X[I] - f_n^{-1}(F[I])$ is a type-1 neutrosophic closed set in $(X[I], \tau[I])$, that is,

$$Cl_n[X[I] - f_n^{-1}(F[I])] = X[I] - f_n^{-1}(F[I]).$$

Since $H[I] \cap f_n^{-1}(F[I]) = \emptyset[I]$. then $H[I] \subseteq X[I] - f_n^{-1}(F[I])$, this implies,

$$Cl_n(H[I]) \subseteq Cl_n[X[I] - f_n^{-1}(F[I])] = X[I] - f_n^{-1}(F[I]).$$

Hence $Cl_n(H[I]) \cap f_n^{-1}(F[I]) = \emptyset[I]$. Then $f_n[Cl_n(H[I])] \cap F[I] = \emptyset[I]$. Hence $y \notin f_n[Cl_n(H[I])]$. Therefore $f_n[Cl_n(H[I])] \subseteq Ker[f_n(H[I])]$. Conversely, Suppose that $f_n[Cl_n(H[I])] \subseteq Ker_n[f_n(H[I])]$ for every subset $H[I]$ of $X[I]$. Let $V[I]$ be any type-1 neutrosophic open subset of $Y[I]$. Then $f_n^{-1}(V[I]) \subseteq X[I]$. Then by the hypothesis, $f_n[(f_n^{-1}(V[I]))] \subseteq Ker_n[f_n(f_n^{-1}(V[I]))]$. Since $V[I]$ is a type-1 neutrosophic open in $Y[I]$ then by Lemma 6.6,

$$f_n[Cl_n(f_n^{-1}(V[I]))] \subseteq Ker_n[f_n(f_n^{-1}(V[I]))] \subseteq Ker_n(V[I]) = V[I].$$

This implies, $Cl_n(f_n^{-1}(V[I])) \subseteq f_n^{-1}(V[I])$, that is, $f_n^{-1}(V[I])$ is a type-1 neutrosophic closed in $(X[I], \tau[I])$. Hence f_n is contra continuous. □

Theorem 6.8. Let $f_n : (X[I], \tau[I]) \longrightarrow (Y[I], \tau_1[I])$ be a type-1 neutrosophic function between two type-1 neutrosophic topological spaces $(X[I], \tau[I])$ and $(Y[I], \tau_1[I])$ is a contra continuous if and only if $Cl_n[f_n^{-1}(G[I])] \subseteq f_n^{-1}[Ker_n(G[I])]$ for every subset $G[I]$ of $Y[I]$.

Proof. Suppose that f_n is a contra continuous. Let $G[I]$ be any type-1 neutrosophic subset of $Y[I]$. Since $f_n^{-1}(G[I])$ is a type-1 neutrosophic subset of $X[I]$ and f_n is a contra continuous then by theorem **??**, $f_n[Cl_n[f_n^{-1}(G[I])]] \subseteq Ker_n[f_n(f_n^{-1}(G[I]))]$. This implies that,

$$f_n[Cl_n[f_n^{-1}(G[I])]] \subseteq Ker[f_n(f_n^{-1}(G[I]))] \subseteq Ker(G[I]).$$

Hence $Cl_n[f_n^{-1}(G[I])] \subseteq f_n^{-1}[Ker_n(G[I])]$.
Conversely, Suppose that $Cl_n[f_n^{-1}(G[I])] \subseteq f_n^{-1}[Ker_n(G[I])]$ for every type-1 neutrosophic

subset $G[I]$ of $Y[I]$. Let $V[I]$ be any type-1 neutrosophic open subset of $Y[I]$. By the hypothesis and Lemma 6.6,

$$Cl_n f_n^{-1}(V[I])] \subseteq f_n^{-1}[Ker_n(V[I])] \subseteq f_n^{-1}(V[I]).$$

That is, $Cl_n[f_n^{-1}(V[I])] = f_n^{-1}(V[I])$ and so $f_n^{-1}(V[I])$ is a type-1 neutrosophic closed in $(X[I], \tau[I])$. Therefore f_n is a contra continuous. □

References

1. Agboola, A.A.A. and S. A. Akinleye. Neutrosophic Vector Spaces. Neutrosophic Sets and Systems (2014),4, 1,pp.9-18. $DOI : https : //digitalrepository.unm.edu/nss_journal/vol4/iss1/3$
2. Al-Odhari, A. M. An Introduction to the Algebraic Structure of Type-1 Neutrosophic-Set Theory. International Journal of Neutrosophic Science(IJNS)(2026),27,2. pp.511-541. $DOI : https : //doi.org10.54216/IJNS.270242$
3. Al-Odhari, A. M. Neutrosophic Power-Set and Neutrosophic Hyper-Structure of Neutrosophic Set of Three Types. Annals of Pure and Applied Mathematics (2025), 31, 2, pp.125-146. $DOI : http : //dx.doi.org/10.22457/apam.v31n2a05964$
4. Al-Odhari, A. Invertible Neutrosophic Functions on Neutrosophic Set Theory of Three Types. Neutrosophic Knowledge (2025), pp. 21-29. $DOI : https : //doi.org/10.5281/zenodo.14740796$
5. Al-Odhari, A. Basic Introduction of Neutrosophic Set Theory. Plithogenic Logic and Computation (2024), 2 pp. 20-28. $DOI : https : //doi.org/10.61356/j.plc.2024.2327$
6. Al-Odhari, A. Some Aspects of Neutrosophic Set Theory. Neutrosophic Optimization and intelligent Systems (2024), v 4, pp. 31-38. $DOI : https : //doi.org/10.61356/j.nois.2024.4359$
7. Al-Odhari, A. On the Generalization of Neutrosophic Set Operations: Testing Proofs by Examples. HyperSoft Set Methods in Engineering (2024), 2, pp.72-82, doi:10.61356/j.2321.2024.
8. Al-Odhari, A. Some Results of Neutrosophic Relations. Neutrosophic Knowledge (2024),5, pp. 77-86. https://doi.org/10.5281/zenodo.14181753
9. Al-Odhari , A. M. Axiomatic of Neutrosophic Groups. Sana'a University Journal of Applied Sciences and Technology (2024), 2(2), 205–214. https://doi.org/10.59628/jast.v2i2.793
10. Al-Odhari, A. M. (2025). The Neutrosophic Cosets and Neutrosophic Normal Subgroups of Neutrosophic Groups Associated With Gt1 [I]. Sana'a University Journal of Applied Sciences and Technology, 3(3), 848–860. https://doi.org/10.59628/jast.v3i3.1666
11. Al-Omeri, W. Neutrosophic crisp Sets via Neutrosophic crisp Topological Spaces. Neutrosophic Sets and Systems (2016), 13, 1 . $https : //digitalrepository.unm.edu/nss_journal/vol13/iss1/13$
12. Atanassov, K. Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems (1986),20 ,87–96.
13. Atanassov, K. Intuitionistic Fuzzy Sets: Theory and Applications. Springer-Verlag Berlin Heidelberg GmbH,1999.
14. Brown, R. Topology: A Geometric Account of General Topology, Homotopy Types and the Fundamental Groupoid, Halsted Press: a division of John Wiley and Sons, 1988.
15. Bera, T. N. ;Mahapatra, K. On neutrosophic soft linear spaces. Fuzzy Information and Engineering (2017), 9, 3,pp. 299–324.
16. Biswas S.; Moi, S. and Sarkar, S. P. Neutrosophic Riemann Integration and its properties. Soft Computing (2021), 25, 22, pp. 13987–13999, .
17. Chang, C. Fuzzy Topological Spaces. J. Math. Anal. Appl (1968), 24, pp. 182-190. .

18. Coker, D. An Introduction to Intuitionistic Fuzzy Topological Spaces. Fuzzy Sets and Systems (1997), 88, 1, pp. 81-89.
19. Cetkin, V.; Aygun, H. An approach to neutrosophic subgroup and its fundamental properties. Journal of Intelligent and Fuzzy Systems 2015, 29, 5, pp. 1941–1947.
20. Dugundji, J. Topology. Universal Book Stall, 5 Ansari Road, New Delhi-110002, 1990.
21. Dummitm D. S.; Foote, R. M. Abstract Algebra. Wiley,Hoboken, NJ, USA, 2004.
22. Elrawy, A; Abdalla, M. and Saleem,M. A. Some Results on Neutrosophic Group. Journal of Mathematics 2023. , 1-6.
23. Herstein, I. N. Topics in Algebra, John Wiley, Sons, Hoboken, NJ, USA, 2006.
24. Fujita, T.; Smarandache, F. Advancing uncertain combinatorics through graphization, hyperization, and uncertainization: Fuzzy, neutrosophic, soft, rough, and beyond. 2nd ed. Infinite Study, (2025).
25. Fujita, T.; Smarandache, F. Representing Higher-Order Networks: A Survey of Graph-Based Frameworks. Neutrosophic Science International Association (NSIA) Publishing House,(2026)
26. Kandasamy, W. V.;Smarandache, F. Basic neutrosophic algebraic structures and their application to fuzzy and neutrosophic models, Infnite Study, FlagstaO, AZ, USA, (2004).
27. Kandasamy, W. V.; Smarandache, F. Some neutrosophic algebraic structures and neutrosophic N-algebraic structures. Infinite Study.(2006)
28. Karatas, S.; Kuru. C. Neutrosophic Topology. Neutrosophic Sets and Systems(2016), 13, 1 . $https://digitalrepository.unm.edu/nss_journal/vol13/iss1/12$
29. Lupia˜nez, F. G. On neutrosophic topology. The International Journal of Systems and Cybernetics (2008), 37,6 , 797–800.
30. Lupia˜nez, F. G. On neutrosophic paraconsistent topology. The International Journal of Systems and Cybernetics (2010), 39(4), 598–601.
31. Lowen, R. Fuzzy Topological Spaces and Fuzzy Compactness. JOURNAL OF MATHEMATICAL ANALYSIS AND APPLICATIONS (1976),56, pp. 621-633.
32. Munkres, J. R. Topology a first Course, Prentice-Hall, Inc. Englewood cliff's,New Jersey, (1975).
33. Oyebo, Y. T. Neutrosophic groups and subgroups (2012), https: www.researchgate.net/publication.
34. Omran, S; Elrawy, A. Continuous and bounded operators on neutrosophic normed spaces. Neutrosophic Sets and Systems (2021), 46, pp. 276–289.
35. Salama, A.A.; Alblowi, S.A. Neutrosophic Set and Neutrosophic Topological Spaces. ISORJ. Mathematics (2012),3,3, pp-31-35.
36. Salama, A.A.; Alblowi, S.A. Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces. Journal Computer Sci. Engineering (2012),27, pp 129-132 .
37. Salama, A.; Florentin Smarandache; and Valeri Kromov,(2014)."Neutrosophic Closed Set and Neutrosophic Continuous Functions." Neutrosophic Sets and Systems 4, 1.$https://digitalrepository.unm.edu/nss_journal/vol4/iss1/2$
38. Saleem, M. A.; Abdalla, M. and Elrawy, A. On a matrix over NC and multiset NC semigroups. Journal of Mathematics 2022,1,pp.1-6.
39. Smarandache, F. Neutrosophy, Neutrosophic Probability, Set and Logic, American Rescue Press, Rehoboth, DE, USA, (1998).
40. Smarandache, F. Neutrosophic set - a generalization of the Intuitionistic fuzzy set, International Journal of Pure and Applied Mathematics (2005), 24,3, 287–297.
41. Smarandache, F. and Ali, M. Neutrosophic triplet group. Neural Computing and Applications(2018), 29, 7, pp. 595-601.
42. Siddique, S.; Ahmad U. and Akram, M. A study on generalized graphs representations of complex neutrosophic information, Journal of Applied Mathematics and Computing (2021), 65, 1-2, pp. 481–514.

43. Zadeh, L. A. Fuzzy Sets. Information and Control(1965),18,pp. 338-353.

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