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On Operations of IndetermSoft Sets

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Abstract. In this study, we define the basic operations of IndetermSoft Operators, viz., $\partial-$subset, $\partial-$superset, $\partial-$equal, $\partial-$union, and $\partial-$intersection, which helps define the fundamental terms of an IndetermSoft Set, including the equality of two IndetermSoft Sets, subset, superset, and complement of an IndetermSoft Set. In addition, we define some operations, such as union, intersection, 'AND' and, 'OR' operations of two IndetermSoft Sets and also verify the validity of De Morgan's law in IndetermSoft Set theory.

Keywords: Soft Set; IndetermSoft Algebra; IndetermSoft Operators; IndetermSoft Set; $\partial-$union; $\partial-$intersection.

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1. Introduction

Soft Sets are useful tools for managing uncertainty and ambiguity in a number of fields, which was initially developed by Molodtsov [1]. Maji et al. [2] established numerous operations for Soft Sets. They developed the notion of subset, intersection, union, and complement of soft sets and looked into their properties, which were further improved by Ali et al. [3], Ge and Yang [4], and Sezgin and Atagun [5]. Maji et al. [6] demonstrated the effectiveness of the soft set theory as a tool for simulating uncertainty in decision-making situations.

Neutrosophy, a new branch of philosophy that deals with indeterminacy, was established by Smarandache. To address imprecise, uncertain, and inconsistent information, Smarandache [7] developed the concept of a neutrosophic set. Subsequently, Smarandache [8] introduced NeutroAlgebra, which is a partially well-defined, partially indeterminate, and partially outer-defined algebra (for more details, see [9, 10]). Subsequently, Smarandache [11] defined IndetermAlgebra as a subclass of NeutroAlgebra with indeterminacy (I $>$ 0) in its operators. A subclass of IndetermAlgebra, called IndetermSoft

Algebra (ISA), was created by Smarandache [11] using three indeterminate soft operators (disjoinOR, exclusiveOR, and NOT) and one determinate soft operator (joinAND). In 2022 [11], Smarandache introduced IndetermSoft Set, an extension of the Soft Set that handles indeterminate data. The term "IndetermSoft Operator Value Set" (ISOVS) refers to the collection of all possible outcomes after operating any one or more than one IndetermSoft Operators (joinAND, disjoinOR, exclusiveOR, and NOT) on specific elements. We used the terms "indeterminate soft operator value set" and "determinate soft operator value set" to refer to the collections of all possible outcomes after operating indeterminate soft operators and determinate soft operators on given elements, respectively.

In this work, we attempt to define IndetermSoft Set's(ISSs) operations. Considering this objective, we found that the first step was to define some basic ISOVS operations. The remainder of this paper is structured as follows: We begin by establishing several operations, such as $\partial-$subset and $\partial-$superset of an ISOVS, including $\partial-$union, $\partial-$intersection, and $\partial-$equality of two ISOVSs. We then examined their fundamental properties. Subsequently, definitions and corresponding examples of equality, union, intersection, 'AND' and 'OR' operations on the two ISSs are provided. Finally, we demonstrate how De Morgan's laws apply in the theory of IndetermSoft Sets concerning 'AND' and 'OR' operations. This study offers a theoretical analysis of the ISSs.

2. Materials and Methods

In this section, we revisit some fundamental concepts from the Soft Set theory and ISS theory. Consider $\mho$ as an initial universal set, ϕ as an empty element, and $P(\mho)$ as the power set of $\mho$.

Definition 2.1. [1] "Let $\pounds$ be a set of parameters. Subsequently, a pair $(F, \pounds)$ is called a Soft Set over $\mho$, where F is a mapping given by $F : \pounds \to \mathcal{P}(\mho)$."

Definition 2.2. [2] "Let $\pounds = \{\mathscr{C}_1, \mathscr{C}_2, ..., \mathscr{C}_n\}$ be the set of parameters. The *NOT* set of $\pounds$ is denoted by $\rceil\pounds$ and defined by $\rceil\pounds = \{\rceil\mathscr{C}_1, \rceil\mathscr{C}_2, ..., \rceil\mathscr{C}_n\}$ where $\rceil\mathscr{C}_i =$ not $\mathscr{C}_i, \forall i \in \{1, 2, ..., n\}$".

Proposition 2.3. *[2] "Let $\pounds$ be a set of parameters and $\mathfrak{X}, \mathfrak{P} \subseteq \pounds$. Then*

(i) $\rceil(\rceil\mathfrak{X}) = \mathfrak{X}$;

(ii) $\rceil(\mathfrak{X} \cup \mathfrak{P}) = \rceil\mathfrak{X} \cup \rceil\mathfrak{P}$;

(iii) $\rceil(\mathfrak{X} \cap \mathfrak{P}) = \rceil\mathfrak{X} \cap \rceil\mathfrak{P}$."

Smarandache [11] defined the determinate and indeterminate soft operators as follows:

Definition 2.4. [11] "*joinAND*, or put together, denoted by $\Cap$, defined as
$\mathcal{X} \Cap \mathcal{Y} = \mathcal{X}$ and $\mathcal{Y}$, or put together $\mathcal{X}$ and $\mathcal{Y}$; Herein, the conjunction "and" has the same meaning as in the natural language.
$\mathcal{X} \Cap \mathcal{Y} = \{\mathcal{X}, \mathcal{Y}\}$ denotes a set of two objects. *joinAND*is a determinate soft operator, since one gets one clear output."

Definition 2.5. [11] "*disjoinOR*, denoted by $\Cup$, defined as
$\mathcal{X}$ disjoinOR $\mathcal{Y} = \mathcal{X} \Cup \mathcal{Y} = \{\mathcal{X}\},$ or $\{\mathcal{Y}\},$ or both $\{\mathcal{X}, \mathcal{Y}\} = \mathcal{X},$ or $\mathcal{Y},$ or both $\mathcal{X}$ and $\mathcal{Y}$; But there is some indeterminacy (uncertainty) to choose among three alternatives. *disjoinOR* is an indeterminate soft operator, since it does not have a clear unique output, but three possible alternative outputs to choose from."

Definition 2.6. [11] "exclusiveOR, means either one or the other; it is an indeterminate soft operator. $\mathcal{X} \Cup_E \mathcal{Y} =$ either $\mathcal{X}, or\ \mathcal{Y}$, and not both $\{\mathcal{X}, \mathcal{Y}\}$."

Definition 2.7. [11] "*NOT*, or no, or sub-complement, denoted by $\rightleftharpoons$, where $NOT(\mathcal{X})$ = no $\mathcal{X}$, in other words, all elements from $\mathcal{H}$, except $\mathcal{X}$, either single elements, or two elements ,..., or $n-1$ elements from $\mathcal{H} - \{X\}$, or the empty element ϕ. If we let $\mathcal{H} = \{a, b, c, d\}$ then

$$\rightleftharpoons b = (\emptyset, \{a\},\ or\ \{c\},\ or\ \{d\},\ or\ \{a, c\},\ or\ \{a, d\},\ or\ \{c, d\},\ or\ \{a, c, d\})."$$

Definition 2.8. [11] "The set $\mathcal{H}$ ($\Cap, \Cup, \Cup_E, \rightleftharpoons$) closed under the four IndetermSoft Operators, which includes one determinate soft operator and three indeterminate soft operators such as
joinAND (denoted by $\Cap$), disjoinOR (denoted by $\Cup$), exclusiveOR (denoted by $\Cup_E$), and subnegation / subcomplement NOT (denoted by $\rightleftharpoons$), is called an ISA".

Definition 2.9. [11] "Let $\mathcal{H}$ be a non-empty subset of $\mho$ and $\mathcal{H}(\Cap, \Cup, \Cup_E, \rightleftharpoons)$ the ISA generated by closing the set $\mathcal{H}$ under the operators $\Cap, \Cup, \Cup_E$, and $\rightleftharpoons$. Let e be an attribute with a set of attribute values, denoted by $\pounds$. Then, the pair $(F, \pounds)$, where $F : \pounds \to \mathcal{H}(\Cap, \Cup, \Cup_E, \rightleftharpoons)$, is called an ISS over $\mathcal{H}$".

Example 2.10. Consider the set of four vehicles, $\mathcal{H} = \{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\}$ and the ISA that results from closing the set $\mathcal{H}$ under the soft operators $\Cap, \Cup, \Cup_E$ and $\rightleftharpoons$. Let $\pounds$ be the set of parameters that give rise to the expression $\pounds = \{\mathscr{C}_1 =$ cheap, $\mathscr{C}_2 =$ compact size, $\mathscr{C}_3 =$ big size, $\mathscr{C}_4 =$ costly$\}$. We assume that $F(\mathscr{C}_1) = \mathcal{V}_1 \Cap \mathcal{V}_3$. $F(\mathscr{C}_2) = \mathcal{V}_1 \Cup \mathcal{V}_4$, $F(\mathscr{C}_3) = \mathcal{V}_2 \Cup \mathcal{V}_4$, $F(\mathscr{C}_4) = \rightleftharpoons \mathcal{V}_3$.

Visually, we may represent it as $F(\mathscr{C}_1) = \Big\{ \{\mathcal{V}_1, \mathcal{V}_3\}$; $F(\mathscr{C}_2) = \begin{cases} \{\mathcal{V}_1\} \\ \{\mathcal{V}_4\} \\ \{\mathcal{V}_1, \mathcal{V}_4\} \end{cases}$;

$F(\mathscr{C}_3) = \begin{cases} \{\mathcal{V}_2\} \\ \{\mathcal{V}_4\} \\ \{\mathcal{V}_2, \mathcal{V}_4\} \end{cases}$; $F(\mathscr{C}_4) = \begin{cases} \{\mathcal{V}_1\} \\ \{\mathcal{V}_2\} \\ \{\mathcal{V}_4\} \\ \{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_4\} \\ \{\mathcal{V}_1, \mathcal{V}_2\} \\ \{\mathcal{V}_1, \mathcal{V}_4\} \\ \{\mathcal{V}_2, \mathcal{V}_4\} \\ \phi \end{cases}$. The set $(F, \pounds)$ defines ISS.

3. Results

Let us consider $\mho$ as initial universal set; ϕ represents an empty element; $P(\mho)$ will be the power set of $\mho$; $\aleph$ a non-empty subset of $\mho$ where $\aleph(\Cap, \Cup, \Cup_E, \neg)$ represent the ISA produced by closing the set $\aleph$ under the operators $\Cap, \Cup, \Cup_E$ and $\neg$.

3.1. *Operations of IndetermSoft Operator Value Sets*

Definition 3.1. Let $\mathfrak{X}$ and $\mathfrak{P}$ be two ISOVSs over the common ISA $\aleph$ then, $\mathfrak{X}$ is said to be $\partial-$super set of $\mathfrak{P}$, if every element of $\mathfrak{P}$ is contained in every element of $\mathfrak{X}$. It is written as $\mathfrak{X} \Supset_\partial \mathfrak{P}$.

Definition 3.2. If $\mathfrak{X}$ and $\mathfrak{P}$ are two ISOVSs over a common ISA $\aleph$, $\mathfrak{X}$ is identified as a $\partial-$subset of $\mathfrak{P}$, if $\mathfrak{P}$ is $\partial-$ superset of $\mathfrak{X}$. This is expressed as $\mathfrak{X} \Subset_\partial \mathfrak{P}$.

Example 3.3. Consider the set of four houses, $\aleph = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4\}$. Let $\mathfrak{X}$ and $\mathfrak{P}$ be two ISOVSs, corresponding to $\hbar_1 \Cap \hbar_2$ and $\hbar_1 \Cup \hbar_2$ respectively. Here, $\mathfrak{X} = \{\hbar_1, \hbar_2\}$ and $\mathfrak{P} = (\{\hbar_1\}$, or $\{\hbar_2\}$, or $\{\hbar_1, \hbar_2\})$ and $\mathfrak{P} \Subset_\partial \mathfrak{X}$. Since, every element of $\mathfrak{P}$ is contained in every element of $\mathfrak{X}$.

Definition 3.4. Let $\mathfrak{X}$ and $\mathfrak{P}$ be two ISOVSs over a common ISA $\aleph$ then, $\mathfrak{X}$ and $\mathfrak{P}$ are said to be $\partial-$equal if they contain the same values.

Definition 3.5. ∂-union of any two ISOVSs over ISA $\aleph$ is the collection of the union of every possible combination of the components of both ISOVSs. This is denoted as $\Cup_\partial$.

Example 3.6. Consider the set of four cars, $\varsigma = \{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\}$. Let $\mathfrak{X}$ and $\mathfrak{P}$ be two ISOVSs, representing $\mathcal{V}_1 \Cup \mathcal{V}_2$ and $\mathcal{V}_2 \Cap \mathcal{V}_3$ respectively. Then, their $\partial-$union is defined as
$\mathfrak{X} \Cup_\partial \mathfrak{P} = (\{\mathcal{V}_1\}$, or $\{\mathcal{V}_2\}$, or $\{\mathcal{V}_1, \mathcal{V}_2\}) \Cup_\partial (\{\mathcal{V}_2, \mathcal{V}_3\}) = (\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3\}$, or $\{\mathcal{V}_2, \mathcal{V}_3\})$, which is indeterminate, because there are two possible alternative outputs rather than a single outcome.

Definition 3.7. ∂-intersection of any two ISOVSs over $\aleph$ is the collection of the intersections of every possible combination of the components of both ISOVSs. This is denoted as $\Cap_\partial$.

Example 3.8. Let $\varsigma = \{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\}$ be the set of cars, $\mathfrak{X}$ and $\mathfrak{P}$ be two ISOVSs, representing values of $\neg \mathcal{V}_3$ and $\mathcal{V}_2 \Cup \mathcal{V}_3$ respectively, then their $\partial-$intersection is defined as
$\mathfrak{X} \Cap_\partial \mathfrak{P} = (\{\mathcal{V}_1\}$, or $\{\mathcal{V}_2\}$, or $\{\mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_2\}$, or $\{\mathcal{V}_1, \mathcal{V}_4\}$, or $\{\mathcal{V}_2, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_4\}$, or $\phi)$ $\Cap_\partial (\{\mathcal{V}_2\}$, or $\{\mathcal{V}_3\}$, or $\{\mathcal{V}_2, \mathcal{V}_3\}) = (\{\mathcal{V}_2\}$, or $\phi)$.

Remark 3.9. ∂-union and ∂-intersection of two determinate soft operator value sets over the common ISA $\aleph$ function as the union and intersection of classical sets.

3.1.1. *Properties of IndetermSoft Operator Value Sets*

In classical set theory, $\mathfrak{X} \cup \mathfrak{X} = \mathfrak{X}$ and $\mathfrak{X} \cap \mathfrak{X} = \mathfrak{X}$. However, this need not be true in the case of ISOVSs.

Remark 3.10. Let $\mathfrak{X}$ be any ISOVS over common ISA $\aleph$, then

(i) $\mathfrak{X} \Cup_\partial \mathfrak{X}$ need not be equal to $\mathfrak{X}$.

(ii) $\mathfrak{X} \Cap_\partial \mathfrak{X}$ need not be equal to $\mathfrak{X}$.

Example 3.11. Let $\mathfrak{X}$ be an ISOVS representing values of $\hbar_1 \mathbb{V}_E \hbar_2$. So, $\mathfrak{X} = (\{\hbar_1\}$, or $\{\hbar_2\})$ and $\mathfrak{X} \Cup_\partial \mathfrak{X} = (\{\hbar_1\}$, or $\{\hbar_2\}$, or $\{\hbar_1, \hbar_2\})$, which is not equal to $\mathfrak{X}$. Again, $\mathfrak{X} \Cap_\partial \mathfrak{X} = (\{\hbar_1\}$, or $\{\hbar_2\}$, or $\phi)$, which is not equal to $\mathfrak{X}$.

According to the definitions above, the following results are evident.

Proposition 3.12. *Let $\mathfrak{X}$ and $\mathfrak{P}$ be two ISOVSs then,*

(i) $\mathfrak{X} \Cup_\partial \phi = \mathfrak{X}$;

(ii) $\mathfrak{X} \Cap_\partial \phi = \phi$;

(iii) If $\mathfrak{X} \Subset_\partial \mathfrak{P}$ then

(a) $\mathfrak{X} \Cup_\partial \mathfrak{P} = \mathfrak{P}$;

(b) $\mathfrak{X} \Cap_\partial \mathfrak{P} = \mathfrak{X}$.

Remark 3.13. $\partial-$union of the indeterminate and determinate soft operator value sets over $\varkappa$ can be either determinate or indeterminate.

In example 3.3 $\partial-$union of $\mathfrak{X}$(determinate soft operator value set) and $\mathfrak{P}$(indeterminate soft operator value set) is equal to $\{\hbar_1, \hbar_2\}$, which is a determinate set. In Example 3.6, $\partial-$union of the determinate and indeterminate soft operator value sets are indeterminate.

Theorem 3.14. *$\partial-$union of the two indeterminate soft operator value sets is always indeterminate.*

Proof. Let $\mathfrak{X}$ and $\mathfrak{P}$ be two indeterminate soft operator value sets. Then three cases arise:

(1) $\mathfrak{X} \Cap_\partial \mathfrak{P} = \phi$;

(2) One of them is $\partial-$subset of other i.e. either $\mathfrak{X} \Supset_\partial \mathfrak{P}$ or $\mathfrak{X} \Subset_\partial \mathfrak{P}$;

(3) $\mathfrak{X} \Cap_\partial \mathfrak{P} \neq \phi$ and neither $\mathfrak{X} \Supset_\partial \mathfrak{P}$ nor $\mathfrak{X} \Subset_\partial \mathfrak{P}$.

By definition, $\partial-$union is the collection of the unions of every possible combination between the components of $\mathfrak{X}$ and $\mathfrak{P}$. Since, $\mathfrak{X}$ and $\mathfrak{P}$, are both indeterminate soft operator value sets, they contain more than one value. Thus, $\mathfrak{X} \Cup_\partial \mathfrak{P}$ has more than one possible output in all the three cases mentioned above.

Thus, $\mathfrak{X} \Cup_\partial \mathfrak{P}$ is always indeterminate. □

Theorem 3.15. *$\partial-$intersection of two indeterminate soft operator value sets is indeterminate only if their $\partial-$intersection is non-empty.*

Proof. Let $\mathfrak{X}$ and $\mathfrak{P}$ be two indeterminate soft operator value sets.

If $\mathfrak{X} \Cap_\partial \mathfrak{P} = \phi$, then clearly it is determinate.

If $\mathfrak{X} \Cap_\partial \mathfrak{P} \neq \phi$, then two cases arises:

Case I- Either $\mathfrak{X} \Supset_\partial \mathfrak{P}$ or $\mathfrak{X} \Subset_\partial \mathfrak{P}$.

If $\mathfrak{X} \Supset_\partial \mathfrak{P}$ then $\mathfrak{X} \Cap_\partial \mathfrak{P} = \mathfrak{P}$ and if $\mathfrak{X} \Subset_\partial \mathfrak{P}$, $\mathfrak{X} \Cap_\partial \mathfrak{P} = \mathfrak{X}$. As both $\mathfrak{X}$ and $\mathfrak{P}$ are indeterminate, this implies $\mathfrak{X} \Cap_\partial \mathfrak{P}$ is also indeterminate.

Case II- Neither $\mathfrak{X} \Supset_\partial \mathfrak{P}$ nor $\mathfrak{X} \Subset_\partial \mathfrak{P}$.

Since $\mathfrak{X} \Cap_\partial \mathfrak{P} \neq \phi$, So, $\mathfrak{X} \Cap_\partial \mathfrak{P}$ must contain an element different from ϕ.

Also, neither $\mathfrak{X} \Supset_\partial \mathfrak{P}$ nor $\mathfrak{X} \Subset_\partial \mathfrak{P}$. Therefore, there exists an element in $\mathfrak{X}$ that is not in $\mathfrak{P}$ and an element in $\mathfrak{P}$ that is not in $\mathfrak{X}$. The intersection of these two elements is ϕ, which is an element of $\mathfrak{X} \Cap_\partial \mathfrak{P}$.

Therefore, $\mathfrak{X} \Cap_\partial \mathfrak{P}$ contains at least ϕ and an element. Therefore, there is a possibility of more than one output. Therefore, $\mathfrak{X} \Cap_\partial \mathfrak{P}$ is indeterminate. □

Remark 3.16. $\partial-$intersection of the indeterminate soft operator value sets and determinate soft operator value sets can be either determinate or indeterminate.

Example 3.17. Let $\mathcal{H} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4\}$ be the set of houses and $\mathfrak{X}$ and $\mathfrak{P}$ be two ISOVSs, corresponding to $\hbar_1 \mathbin{\mathbb{A}} \hbar_3$ and $\hbar_2 \mathbb{V}_E \hbar_4$ respectively. Here, $\mathfrak{X} = (\{\hbar_1, \hbar_3\})$, $\mathfrak{P} = (\{\hbar_2\}$, or $\{\hbar_4\})$, and $\mathfrak{X} \Cap_\partial \mathfrak{P} = \phi$ is a determinate set. Whereas, in example 3.6 $\mathfrak{X} \Cap_\partial \mathfrak{P} = (\{\mathcal{V}_2, \}$, or $\phi)$, which is indeterminate.

3.2. *Operations on IndetermSoft Sets*

Definition 3.18. For two ISSs $(F, \mathfrak{X})$ and $(\eth, \mathfrak{P})$ over a common ISA $\mathcal{H}$, $(F, \mathfrak{X})$ is an IndetermSoft subset of $(\eth, \mathfrak{P})$, if

(1) $\mathfrak{X} \subset \mathfrak{P}$
(2) $\forall \epsilon \in \mathfrak{X}, F(\epsilon)$ and $\eth(\epsilon)$ have identical outcome.

We write $(F, \mathfrak{X}) \tilde{\Subset}_\partial (\eth, \mathfrak{P})$:

Remark 3.19. $(F, \mathfrak{X})$ is IndetermSoft super set of $(\eth, \mathfrak{P})$ if $(\eth, \mathfrak{P})$ is an IndetermSoft subset of $(F, \mathfrak{X})$ and written as $(F, \mathfrak{X}) \tilde{\Supset}_\partial (\eth, \mathfrak{P})$.

Example 3.20. Let $\varsigma = \{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4, \mathcal{V}_5\}$, $\mathfrak{X} = \{t_1, t_2\}$ and $\mathfrak{P} = \{t_1, t_2, t_3\}$.

Consider two ISSs $(F, \mathfrak{X})$= $\{(t_1,\ \mathcal{V}_1 \mathbb{V}\ \mathcal{V}_3), (t_2,\ \mathcal{V}_3 \mathbb{V}_E\ \mathcal{V}_5)\}$

and $(\eth, \mathfrak{P})$= $\{(t_1,\ \mathcal{V}_1 \mathbb{V}\ \mathcal{V}_3), (t_2,\ \mathcal{V}_3 \mathbb{V}_E\ \mathcal{V}_5), (t_3, \fallingdotseq \mathcal{V}_5)\}$. Here, $(F, \mathfrak{X})$ is an IndetermSoft subset of $(\eth, \mathfrak{P})$.

Definition 3.21. Two ISSs $(F, \mathfrak{X})$ and $(\eth, \mathfrak{P})$ over a common ISA are considered to be IndetermSoft equal if $(F, \mathfrak{X}) \tilde{\Subset}_\partial (\eth, \mathfrak{P})$ and $(F, \mathfrak{X}) \tilde{\Supset}_\partial (\eth, \mathfrak{P})$.

Definition 3.22. Let $(F, \mathfrak{X})$ be an ISS. Then, the complement of $(F, \mathfrak{X})$ is represented by $(F, \mathfrak{X})^c$ and defined by $(F, \mathfrak{X})^c = (F^c, \rceil\mathfrak{X})$, where $F^c : \rceil\mathfrak{X} \to \mathcal{H}(\mathbb{A}, \mathbb{V}, \mathbb{V}_E, \fallingdotseq)$ is a mapping given by $F^c(\mu) = \mathcal{H} - F(\rceil\mu),\ \forall \mu \in \rceil\mathfrak{X}$.

Example 3.23. Consider the example (2.10).

We have $(F, \mathcal{L})^c = \{$not cheap cars= $\{\mathcal{V}_2, \mathcal{V}_4\}$, not compact size cars = $(\{\mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3\}$, or $\{\mathcal{V}_2, \mathcal{V}_3\})$, not big size cars = $(\{\mathcal{V}_1, \mathcal{V}_3, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3\}$, or $\{\mathcal{V}_1, \mathcal{V}_3\})$, not costly cars $= (\{\mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_3, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3\}$, or $\{\mathcal{V}_3\}$, or $\{\mathcal{V}_3, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_3\}$, or $\{\mathcal{V}_2, \mathcal{V}_3\}$, or $\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\})\}$.

Definition 3.24. Union of two ISSs $(\digamma, \mathfrak{X})$ and $(\eth, \mathfrak{P})$ over a common ISA Ӿ is the ISS (K, ς), where $\varsigma = \mathfrak{X} \cup \mathfrak{P}$ and $\forall \mathscr{C} \in \varsigma$,

$$K(\mathscr{C}) = \begin{cases} \digamma(\mathscr{C}), & \text{if } \mathscr{C} \in \mathfrak{X} - \mathfrak{P} \\ \eth(\mathscr{C}), & \text{if } \mathscr{C} \in \mathfrak{P} - \mathfrak{X} \\ \digamma(\mathscr{C}) \Cup_\partial \eth(\mathscr{C}), & \text{if } \mathscr{C} \in \mathfrak{X} \cap B \end{cases}$$

we write $(\digamma, \mathfrak{X}) \tilde{\Cup}_\partial (\eth, \mathfrak{P}) = (K, \varsigma)$.

Example 3.25. Let us consider the ISS $(\digamma, \mathfrak{X})$ that represents the price of motor vehicles and the ISS $(\eth, \mathfrak{P})$ that represents aspects of motor vehicles. Suppose $\mho$ be a common universe of discourse and ISA Ӿ$(\Cap, \Cup, \Cup_E, \eqcirc) = \{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\}$ be a set of four motor vehicles.
Let $\mathfrak{X}$= {expensive, cheap}
and $\mathfrak{P}$ = {cheap, latest model, luxury} be the set of parameters.
Let $\digamma(expensive) = \mathcal{V}_2 \Cap \mathcal{V}_4$, $\digamma(cheap) = \mathcal{V}_1 \Cup \mathcal{V}_3$ and
$\eth(cheap) = \mathcal{V}_1 \Cup_E \mathcal{V}_3$, $\eth(latest\ model) = \mathcal{V}_1 \Cup \mathcal{V}_4$, $\eth(luxury) = \mathcal{V}_2 \Cup \mathcal{V}_4$.
Then, union of two ISSs $(\digamma, \mathfrak{X})$ and $(\eth, \mathfrak{P})$ is the ISS (K, ς), where ς = {expensive, cheap, latest model, luxury} and $(K, \varsigma) = (\digamma, \mathfrak{X}) \tilde{\Cup}_\partial (\eth, \mathfrak{P})$ and $K(expensive) = (\{\mathcal{V}_2, \mathcal{V}_4\})$, $K(latest\ model) = (\{\mathcal{V}_1\}$, or $\{\mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_4\})$, $K(luxury) = (\{\mathcal{V}_2\}$, or $\{\mathcal{V}_4\}$,...,or $\{\mathcal{V}_2, \mathcal{V}_4\})$, and $K(cheap) = (\{\mathcal{V}_1, \mathcal{V}_3\}$, or $\{\mathcal{V}_1\}$, or $\{\mathcal{V}_3\})$.

Definition 3.26. Intersection of two ISSs $(\digamma, \mathfrak{X})$ and $(\eth, \mathfrak{P})$ over a common ISA Ӿ is the ISS (Q, ς), where $\varsigma = \mathfrak{X} \cup \mathfrak{P}$ and $\forall \mathscr{C} \in \varsigma$,

$$Q(\mathscr{C}) = \begin{cases} \digamma(\mathscr{C}), & \text{if } \mathscr{C} \in \mathfrak{X} - \mathfrak{P} \\ \eth(\mathscr{C}), & \text{if } \mathscr{C} \in \mathfrak{P} - \mathfrak{X} \\ \digamma(\mathscr{C}) \Cap_\partial \eth(\mathscr{C}), & \text{if } \mathscr{C} \in \mathfrak{X} \cap \mathfrak{P} \end{cases}$$

we write $(\digamma, \mathfrak{X}) \tilde{\Cap}_\partial (\eth, \mathfrak{P}) = (Q, \varsigma)$.

Example 3.27. In example 3.25, intersection of two ISSs $(\digamma, \mathfrak{X})$ and $(\eth, \mathfrak{P})$ is the ISS (Q, ς), where ς = {expensive, cheap, latest model, luxury} and $Q(expensive) = (\{\mathcal{V}_2, \mathcal{V}_4\})$, $Q(latest\ model) = (\{\mathcal{V}_1\}$, or $\{\mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_4\})$, $Q(luxury) = (\{\mathcal{V}_2\}$, or$\{\mathcal{V}_4\}$, or $\{\mathcal{V}_2, \mathcal{V}_4\})$, and $Q(cheap) = (\{\mathcal{V}_1\}$, or $\{\mathcal{V}_3\}$ or $\phi)$.

Definition 3.28. If $(\digamma, \mathfrak{X})$ and $(\eth, \mathfrak{P})$ are two ISSs then $(\digamma, \mathfrak{X})$ AND $(\eth, \mathfrak{P})$ denoted by $(\digamma, \mathfrak{X}) \wedge (\eth, \mathfrak{P})$ is defined by $(\digamma, \mathfrak{X}) \wedge (\eth, \mathfrak{P}) = (M, \mathfrak{X} \times \mathfrak{P})$,
where $M(\mu, \nu) = \digamma(\mu) \Cap_\partial \eth(\nu), \quad \forall (\mu, \nu) \in \mathfrak{X} \times \mathfrak{P}$.

Example 3.29. In example 3.25, $(\digamma, \mathfrak{X}) \wedge (\eth, \mathfrak{P}) = (M, \mathfrak{X} \times \mathfrak{P})$,
where, M(expensive, cheap) = ϕ, M(expensive, latest Model) = (ϕ, or $\{\mathcal{V}_4\}$), M(expensive, luxury) = ($\{\mathcal{V}_2\}$, or $\{\mathcal{V}_4\}$, or $\{\mathcal{V}_2, \mathcal{V}_4\}$), M(cheap, cheap) = ($\{\mathcal{V}_1\}$, or $\{\mathcal{V}_3\}$, or ϕ), M(cheap, latest model) = ($\{\mathcal{V}_1\}$, or ϕ), M(cheap, luxury) = ϕ.

Definition 3.30. If $(\digamma, \mathfrak{X})$ and $(\eth, \mathfrak{P})$ are two ISSs, then $(\digamma, \mathfrak{X})$ OR $(\eth, \mathfrak{P})$ denoted by $(\digamma, \mathfrak{X}) \vee (\eth, \mathfrak{P})$ is defined by $(\digamma, \mathfrak{X}) \vee (\eth, \mathfrak{P}) = (T, \mathfrak{X} \times \mathfrak{P})$, where $T(\mu, \nu) = \digamma(\mu) \Cup_{\partial} \eth(\nu)$, $\forall(\mu, \nu) \in \mathfrak{X} \times \mathfrak{P}$.

Example 3.31. Consider the Example 3.25.
$(\digamma, \mathfrak{X}) \vee (\eth, \mathfrak{P}) = (T, \mathfrak{X} \times \mathfrak{P})$, where T(expensive, cheap) = $(\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_4\}$, or $\{\mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\})$,
T(expensive, latest model) = $(\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_4\}$, or $\{\mathcal{V}_2, \mathcal{V}_4\})$,
M(expensive, luxury) = $(\{\mathcal{V}_2, \mathcal{V}_4\})$, M(cheap, cheap) = $(\{\mathcal{V}_1, \mathcal{V}_3\})$,
M(cheap, latest model) = $(\{\mathcal{V}_1\}$, or $\{\mathcal{V}_1, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_3\}$, or $\{\mathcal{V}_3, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_3, \mathcal{V}_4\})$,
M(cheap, luxury) = $(\{\mathcal{V}_1, \mathcal{V}_2\}$, or $\{\mathcal{V}_1, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_4\}$, or $\{\mathcal{V}_3, \mathcal{V}_2\}$, or $\{\mathcal{V}_3, \mathcal{V}_4\}$, or $\{\mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3\}$, or $\{\mathcal{V}_1, \mathcal{V}_3, \mathcal{V}_4\}$, or $\{\mathcal{V}_1, \mathcal{V}_2, \mathcal{V}_3, \mathcal{V}_4\})$.

3.3. *De Morgan's laws in IndetermSoft Set theory*

Proposition 3.32. *Let $(\digamma, \mathfrak{X})$ and $(\eth, \mathfrak{P})$ be two ISSs over a common ISA $\varkappa$, then*

(i) $((\digamma, \mathfrak{X}) \vee (\eth, \mathfrak{P}))^c = (\digamma, \mathfrak{X})^c \wedge (\eth, \mathfrak{P})^c$;
(ii) $((\digamma, \mathfrak{X}) \wedge (\eth, \mathfrak{P}))^c = (\digamma, \mathfrak{X})^c \vee (\eth, \mathfrak{P})^c$.

Proof. (i) Suppose that $(\digamma, \mathfrak{X}) \vee (\eth, \mathfrak{P}) = (T, \mathfrak{X} \times \mathfrak{P})$.
Therefore, $((\digamma, \mathfrak{X}) \vee (\eth, \mathfrak{P}))^c = (T, \mathfrak{X} \times \mathfrak{P})^c = (T^c, \rceil(\mathfrak{X} \times \mathfrak{P}))$.
$Now,$

$$\begin{aligned}(\digamma, \mathfrak{X})^c \wedge (\eth, \mathfrak{P})^c &= (\digamma^c, \rceil\mathfrak{X}) \wedge (\eth^c, \rceil\mathfrak{P}),\\ &= (\jmath, \rceil\mathfrak{X}\times\rceil\mathfrak{P}),\\ &= (\jmath, \rceil(\mathfrak{X} \times \mathfrak{P})),\end{aligned}$$

where, $\jmath(\mu, \nu) = \digamma^c(\mu) \Cap_{\partial} \eth^c(\nu)$.
Now, take $(\rceil\mu, \rceil\nu) \in \rceil(\mathfrak{X} \times \mathfrak{P})$.
Therefore,

$$\begin{aligned}T^c(\rceil\mu, \rceil\nu) &= \varkappa - T(\mu, \nu),\\ &= \varkappa - [\digamma(\mu) \Cup_{\partial} \eth(\nu)],\\ &= [\varkappa - \digamma(\mu)] \Cap_{\partial} [\varkappa - \eth(\nu)],\\ &= \digamma^c(\mu) \Cap_{\partial} \eth^c(\nu),\\ &= \jmath(\rceil\mu, \rceil\nu),\end{aligned}$$

$\implies T^c$ and $\jmath$ are the same.
(ii) Suppose that $(\digamma, \mathfrak{X}) \wedge (\eth, \mathfrak{P}) = (M, \mathfrak{X} \times \mathfrak{P})$.
Therefore, $((\digamma, \mathfrak{X}) \wedge (\eth, \mathfrak{P}))^c = (M, \mathfrak{X} \times \mathfrak{P})^c = (M^c, \rceil(\mathfrak{X} \times \mathfrak{P}))$.

$Now,$

$$
\begin{aligned}
(\digamma, \mathfrak{X})^c \vee (\eth, \mathfrak{P})^c &= (\digamma^c, \rceil\mathfrak{X}) \vee (\eth^c, \rceil\mathfrak{P}),\\
&= (P, \rceil\mathfrak{X} \times \rceil\mathfrak{P}),\\
&= (P, \rceil(\mathfrak{X} \times \mathfrak{P})),
\end{aligned}
$$

where, $P(\mu, \nu) = \digamma^c(\mu) \Cup_\partial \eth^c(\nu)$.

Now, take $(\rceil\mu, \rceil\nu) \in \rceil(\mathfrak{X} \times \mathfrak{P})$.

Therefore,

$$
\begin{aligned}
M^c(\rceil\mu, \rceil\nu) &= \text{Ӿ} - M(\mu, \nu),\\
&= \text{Ӿ} - [\digamma(\mu) \Cap_\partial \eth(\nu)],\\
&= [\text{Ӿ} - \digamma(\mu)] \Cup_\partial [\text{Ӿ} - \eth(\nu)],\\
&= \digamma^c(\mu) \Cup_\partial \eth^c(\nu),\\
&= P(\rceil\mu, \rceil\nu),
\end{aligned}
$$

$\implies M^c$ and P are the same. □

4. Conclusions

Smarandache is known to invent the notion of ISS, which is based on ISA. In this study, we defined the operations of union, intersection, complement, OR, and AND on the ISSs using examples. Several ISOVS's operations like $\partial-$subset, $\partial-$superset, $\partial-$equal, $\partial-$union, and $\partial-$intersection have been given and their fundamental properties are discussed. This research seeks to improve the practical applicability of these notions in real-world circumstances.

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