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The domain of a Neutrosophic metric spaces of Tribonacci matrix defined by Triple sequence spaces

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Abstract. In this paper, we introduce Tribonacci triple sequence spaces by using Neutrosophic metric spaces. We introduce Neutrosophic metric spaces in Tribonacci triple sequence spaces and prove some topological and algebraic properties based on the results with respect to these spaces.

Keywords: Tribonacci matrix and sequence, t-norm , t-conorm, Neutrosophic metric space, Tribonacci analytic and Tribonacci entire, triple sequence spaces.

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1. Introduction

The theory of fuzzy sets was firstly given by Zadeh in 1965 [1]. Intuitionistic fuzzy sets for such cases were initiated by Atanassov [2], Atanassov et al. [3] used this concept in decision-making problems. More over, fuzzy metric space has used by practical researches as for example decision-making fixed point theory medical imaging.

Triangular norms space (t-norms) (TNS) were given by Menger [4] TNS are used to generalize with the probability distribution of triangle inequality in metric space terms. Triangular conorms space (t-conorms) (TCS) known as dual operations of TNS, TNS and TCS important for fuzzy operations.

The concept neutrosophy implies impartial knowledge of thought and then neutral describes the basic difference between neutral fuzzy, intuitive fuzzy set and logic. The neutrosophic set (NS) was investigated by F.Smarandache [5], who defined the degree of indeterminacy as an independent component. In [6], neutrosophic logic was firstly examined. It is a logic were each proposition is determined to have a degree truth (T) , falsity (F) and indeterminancy (I). A neutrosophic set (NS) is determined as a set where every component of the universe has a degree of T, F and I. Neutrosophic soft linear spaces (NSLS) where considered by Bera and Mahapatra [7]. Subsequently, in [8], the concept of neutrosophic soft normed linear space (NSNLS) was defined and the features of (NSNLS) were examined. Significant results on this topic can be found in [9]- [13]. Kirisci and Simsek [14] defined a new concept known as Neutrosophic Metric Space (NMS) with continuous t-norms and continuous t-conorms.

Neutrosophic Normed Space (NNS) and statistical convergence in NNS have been investigated by Kirisci and Simsek [15]. Neutrosophic set and Neutrosophic logic have used in applied science and theoretical science, such as decision making, robotics and summability theory.

A triple sequence (real or complex) can be defined as a function $X : \mathbb{N} \text{ x } \mathbb{N} \text{ x } \mathbb{N} \to \mathbb{R}(\mathbb{C})$where $\mathbb{N}, \mathbb{R}, \mathbb{C}$ denote the set of natural numbers , real numbers and complex numbers respectively. The different types of notions of triple sequences were introduced and investigated initially by Esi et al [16, 17], Subramanian et al [18–20]. Hazarika et al [21] and many others.

Let $x = (x_{mnk})$ be a triple sequence of real or complex numbers. Then the series $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} x_{mnk}$ is called a triple series. The triple series is said to be convergent if and only if the triple sequence (S_{mnk}) is convergent, where $(S_{mnk}) = \sum_{i,j,q=1}^{(m,n,k)} x_{ijq}$ ((m,n,k) = 1,2,3,...).

Consider a triple sequence $x = (x_{mnk})$ is said to be analytic
$Sup_{m,n,k}|x_{mnk}|^{\frac{1}{m+n+k}} < \infty$. The vector space of all triple analytic sequences are usually denoted by Λ^3. A triple sequence (x_{mnk}) is called triple entire sequence if $|x_{mnk}|^{\frac{1}{m+n+k}} \to 0$ as $m, n, k \to \infty$. The vector space of all triple entire sequence are usually denoted by Γ^3. Let the set of sequences with this property be denoted by Λ^3 and Γ^3 is a metric space with the metric $d(x, y) = Sup_{m,n,k}\{|x_{mnk} - y_{mnk}|^{\frac{1}{m+n+k}} : m, n, k = 1, 2, 3, ...\}$ for all $x = (x_{mnk})$ and $y = (y_{mnk})$ in Γ^3. Let ϕ={finite sequences}.

2. **Preliminaries**

Let λ and μ be two sequence spaces and $A = (a_{kl}^{mn})$ is an infinite four dimensional matrix of real or complex numbers a_{kl}^{mn}, where $k, l, m, n \in \mathbb{N}$. Then we say that A defines a matrix mapping from λ to μ and we denote it by writing $A : \lambda \to \mu$. If for every sequence $x = (x_{mnk}) \in \lambda$ the sequence $Ax = \{A(Ax)_{mnk}\} \in \mu$, then A transform for all $x \in \mu$ where $(Ax)_{mnk} = \sum_m \sum_n \sum_k a_{kl}^{mn} x_{mnk} (k, l \in \mathbb{N})$.

By $(\lambda : \mu)$, we denote the class of matrices A such that $A : \lambda \to \mu$. Thus $a \in (\lambda, \mu)$ if and only if $(Ax) \in \mu$ forever $x \in \lambda$. The approach of constructing the new sequence spaces by means of the matrix domain of a particular limitation method.

First, we give some background about Tribonacci numbers. The studies on Tribonacci numbers were first initiated by a 14-year old student Mark Feinberg in 1963. In 1963, Mark Feinberg [22] defined the sequence $(t_n)_{n\in\mathbb{N}}$ of Tribonacci numbers given by the third recurrence relation $t_n = t_{n-1} + t_{n-2} + t_{n-3}, n \geq 3$ with $t_0 = t_1 = 1, t_2 = 2$ and $t_3 = 4$.

Thus, the first few numbers of Tribonacci sequence are 1,1,2,4,7,13,24,44,81,... .
Some basic properties of Tribonacci sequence are $\lim_{n\to\infty} \frac{t_n}{t_{n-1}} = 0.54368901...$
$\sum_{k=0}^{n} t_k = \frac{t_{n+2}+t_{n-1}}{2}, n \geq 0 \lim_{n\to\infty} \frac{t_{n+1}}{t_n} = 1.83929$(approx.).

Throughout this paper, we use the lower triangular Tribonacci matrix $T = (t_{ik})$, defined as follows $t_{ik} = \begin{cases} \frac{2tk}{t_{n+2}+t_{n-1}} & \text{if} \quad 0 \leq k \leq n \\ 0 & k \geq n \end{cases}$
Equivalently,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots \\ 1/4 & 1/2 & 0 & 0 & 0 & \cdots \\ 1/4 & 1/4 & 1/2 & 0 & 0 & \cdots \\ 1/8 & 1/8 & 1/4 & 1/2 & 0 & \cdots \\ 1/15 & 1/15 & 2/15 & 4/15 & 7/15 & 0 \\ 1/28 & 1/28 & 1/14 & 1/7 & 1/14 & 13/28 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Now we define three dimensional infinite matrix $T = (t^{mnk}_{uvw})$ is given by
$t^{mnk}_{uvw} = \begin{cases} \frac{3t_{mnk}}{t_{u+2,v+2,w+2}+t_{uvw-1}} & \text{if} \quad 0 \leq m \leq u, 0 \leq n \leq v, 0 \leq k \leq w \\ 0 & (m > u, n > v, k > w) \end{cases}$

Equivalently, $T = (t^{mnk}_{uvw}) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \cdots & & \\ 0 & 1/2 & 0 & 0 & 0 & \cdots & & \\ 0 & 0 & 1/4 & 0 & 0 & \cdots & & \\ 0 & 0 & 0 & 1/2 & 0 & \cdots & & \\ 0 & 0 & 0 & 0 & 7/15 & 0 & \cdots & \\ 0 & 0 & 0 & 0 & 0 & 13/28 & 0 & \cdots \end{bmatrix}$

Lemma 2.1. *A matrix $A = (a^{mnk}_{uvw})$; $m.n.k$ and $u.v.w \in \mathbb{N}$ is said to be regular if and only if the following conditions hold:*

(1) *There exists $M > 0$ such that for every $u.v.w \in \mathbb{N}$, the equivalently $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sum_{k=1}^{\infty} |a^{mnk}_{uvw}| \leq M$ holds.*
(2) *$lim_{uvw\to\infty} a^{mnk}_{uvw} = 0$ for every $m, n, k \in \mathbb{N}$.*
(3) *$lim_{uvw\to\infty} \sum_{=1}^{\infty}\sum_{=1}^{\infty}\sum_{k=1}^{\infty} a^{mnk} = 1$.*

Clearly, Lemma 2.1 implies that the matrix T is regular. Define the sequence $y = (y_{uvw})$ which will be frequently used as the T by transform of the sequence $x = (x_{mnk})$

$$y_{uvw} = T_{uvw}x = \sum_{m=0}^{u}\sum_{n=0}^{v}\sum_{k=0}^{w}(\frac{3t_{mnk}}{t_{u+2,v+2,w+2} + t_{uvw-1}})x_{mnk}; u, v, w \in \mathbb{N} \qquad (2.1)$$

We also use the convention that any term with the negative subscripts example x_{-1-1-1} or t_{-1-1-1}, shall be considered as naught.

Definition 2.2. [23] A binary operation is $\star : [0,1]^2 \times [0,1] \to [0,1]$. The operation $\star$ satisfying the following conditions, then it is called continuous TN, for each $s, t, u, v \in [0,1]$,

(1) s $\star$ 1 = s.
(2) If $s \leq u$ and $t \leq v$, then $s \star t \leq u \star v$.
(3) $\star$ is continuous.
(4) $\star$ is commutative and associative.

Definition 2.3. [23] A binary operation $\circ$ is satisfying the following conditions, then it is called continuous TC

(1) s $\circ$ 0 = s.
(2) If $s \leq u$ and $t \leq v$, then $s \circ t \leq u \circ v$.
(3) $\circ$ is continuous.
(4) $\circ$ is commutative and associative.

Remark 2.4. [23] If $0 < \nu_j < 1$, j= 1 to 7, $\star$ and $\circ$ are continuous t-norm, continuous t-conorm respectively. Then

(1) If we take $0 < \nu_1, \nu_2 < 1$ for $\nu_1 > \nu_2$ then there exists $0 \leq \nu_3, \nu_4 < 0, 1$ such that $\nu_1 \star \nu_3 \geq \nu_2$; $\nu_1 \geq \nu_4 \circ \nu_2$.
Further if we choose $\nu_5 \in (0,1)$ then there exist $\nu_6, \nu_7 \in (0,1)$ such that $\nu_6 \star \nu_6 \geq \nu_5$ and $\nu_7 \circ \nu_7 \circ \nu_5$.

Neutrosophic Metric Spaces:

Definition 2.5. [23] Take V to be an arbitrary set

$$N = \{< a, G(a), B(a), Y(a) >: a \in V\}$$

be a NS such that $N : V \times V \times \mathbb{R}^+ \to [0,1]$. Let $\star$ and $\circ$ show the continuous TN and continuous TC respectively. The four-tuple $(V, N, \star, \circ)$ is called a Neutrosophic metric space (NMS) when the following conditions are satisfied $\forall a, b, c \in V$

(1) $0 \leq G(a,b,\lambda) \leq 1, 0 \leq B(a,b,\lambda) \leq 1, 0 \leq Y(a,b,\lambda) \leq 1; \forall\lambda \in \mathbb{R}^+$.
(2) $G(a,b,\lambda) + B(a,b,\lambda) + Y(a,b,\lambda) \leq 3, (\forall\lambda \in \mathbb{R}^+)$

(3) $G(a,b,\lambda) = 1(for\lambda > 0) \Leftrightarrow a = b$

(4) $G(a,b,\lambda) = G(b,a,\lambda)(for\lambda > 0)$.

(5) $G(a,b,\lambda) \star G(b,c,\mu) \leq G(a,c,\lambda+\mu)(\forall\lambda,\mu > 0)$

(6) $G(a,b,\circ) : [0,\infty) \to [0,1]$ is continuous.

(7) $lim_{\lambda\to\infty} G(a,b,\lambda) = 1(\forall\lambda > 0)$

(8) $B(a,b,\lambda) = 0(for\lambda > 0) \Leftrightarrow a = b$.

(9) $B(a,b,\lambda) = B(b,a,\lambda)(for\lambda > 0)$

(10) $B(a,b,\lambda) \circ B(b,c,\mu) \geq B(a,c,\lambda+\mu)(\forall\lambda,\mu > 0)$

(11) $B(a,b,\circ) : [0,\infty) \to [0,1]$ is continuous.

(12) $lim_{\lambda\to\infty} B(a,b,\lambda) = 0(\forall\lambda > 0)$

(13) $Y(a,b,\lambda) = 0(\forall\lambda > 0) \Leftrightarrow a = b$

(14) $Y(a,b,\lambda) = Y(b,a,\lambda)(\forall\lambda > 0)$.

(15) $Y(a,b,\lambda).Y(b,c,\mu) \geq Y(a,c,\lambda+\mu)(\forall\lambda,\mu > 0)$.

(16) $Y(a,b,\circ) : [0,\infty) \to [0,1]$ is continuous.

(17) $lim_{\lambda\to\infty} Y(a,b,\lambda) = 0(for\lambda > 0)$

(18) If $\lambda \leq 0$ then $G(a,b,\lambda) = 0, B(a,b,\lambda) = 1andY(a,b,\lambda) = 1$

Then $N = (G \circ B \circ Y)$ is called Neutrosophic Metric (NM) on V. The functions $G(a,b,\lambda), B(a,b,\lambda)$ and $Y(a,b,\lambda)$ denote the degree of nearness, the degree of neutralness and the degree of non-neutralness between a and b with respect to λ respectively.

Example 2.6. [23] Let (V,d) be an NMS, the binary operations $\star$ and $\circ$ as default (min) TN $a \star b = min\{a,b\}$ and default (max) TC $a \circ b = max\{a,b\}$.
$G(a,b,\lambda) = \frac{\lambda}{\lambda+d(a,b)}$;

$B(a,b,\lambda) = \frac{d}{\lambda+d(a,b)}$;

$Y(a,b,\lambda) = \frac{d(a,b)}{\lambda}$; $\forall a,b \in V$ and $\lambda > 0$. Then $(V,N,\star,\circ)$ is NMS such that
$N : V \times V \times \mathbb{R}^+ \to [0,1]$.

Example 2.7. [23] Let (V,d) be an NMS, the binary operations $\star$ and $\circ$ as TN $a \star b = max\{0, a+b=1\}$ and TC $a \circ b = a+b-ab$ $\forall a,b \in V$ and $\lambda > 0$.

$$G(a,b,\lambda) = \begin{cases} \frac{a}{b} & (a \leq b) \\ \frac{b}{a} & (b \leq a) \end{cases};$$

$$B(a,b,\lambda) = \begin{cases} \frac{b-a}{Y} & (ax < b) \\ \frac{a-b}{x} & (b \leq a) \end{cases};$$

$$Y(a,b,\lambda) = \begin{cases} b-a & (a \leq b) \\ a-b & (b \leq a) \end{cases};$$

Then $(V,N,\star,\circ)$ is NMS such that $N : V \times V \times \mathbb{R}^+ \to [0,1]$.

Remark 2.8. [23] $N = \{< a, G(a), B(a), Y(a) >: a \in V\}$ defined.
case(i) : Suppose Example 2.6 is not an NM with TN $a \star b = max\{0, a + b - 1\}$ and TC $a \circ b = a + b - ab$. Case(ii): Suppose Example 2.7 is not an NM, then
$N = \{< a, G(a), B(a), Y(a) >: a \in V\}$ with TN $a \star b = min\{a, b\}$ and TC
$a \circ b = max\{a, b\}$.

Definition 2.9. Let V be an NMS the triple sequence $x = (x_{mnk}) \in V, 0 < \in < 1$ and $\lambda > 0$. Then the triple sequence $x = (x_{mnk})$ converges to L if and only if there exists $N \in \mathbb{N}^3$ such that $G(x_{mnk}, x.\lambda) > 1 - \epsilon$; $B(x_{mnk}, x.\lambda) < \epsilon$ and $Y(x_{mnk}, x.\lambda) < \epsilon$.
That is
$lim_{m,n,k\to\infty} G(x_{mnk}, x.\lambda) = 1$;
$lim_{m,n,k\to\infty} B(x_{mnk}, x.\lambda) = 0$;
$lim_{m,n,k\to\infty} Y(x_{mnk}, x.\lambda) = o$ as $\lambda > 0$;
In that case, the triple sequence $x = (x_{mnk})$ is called a convergent sequence in V. The convergent in NMS is denoted by $N - lim_{m,n,k\to\infty} x_{mnk} = L$.

Definition 2.10. Take a $(V, N, \star, \circ)$ to be NMS. A triple sequence $x = (x_{mnk})$ in V is called Cauchy if for each $\epsilon > 0$ and each $\lambda > 0$ there exists $N \in \mathbb{N}^3$ such that $G(x_{mnk}, x_{pqr}, \lambda) > 1 - \epsilon$;
$B(x_{mnk}, x_{pqr}, \lambda) < \epsilon$;
$Y(x_{mnk}, x_{pqr}, \lambda) < \epsilon; \forall m, n, k, p, q, r \geq N$.
Hence $(V, N, \star, \circ)$ is called complete if every Cauchy sequence is convergent with respect to τ_N.

Definition 2.11. Let $(V, N, \star, \circ)$ be an NMS. A subset A of V is called Neutrosophic bounded (NB) if there exist $\lambda > 0$ and $\epsilon \in (0, 1)$ such that $G(a, b, \lambda) > 1 - \epsilon$, $B(a, b, \lambda) < \epsilon$ and $Y(a, b, \lambda) < \epsilon \ \forall a, b \in A$.

Definition 2.12. Give $(V, N, \star, \circ)$ be a NMS, $0 < \epsilon < 1$, $\lambda > 0$ and $a \in V$. The set $O(a, \epsilon, \lambda) = \{b \in V : G(a, b, \lambda) > 1 - \epsilon, B(a, b, \lambda) < \epsilon$ and $Y(a, b, \lambda) < \epsilon\}$ is said to be the open ball (OB) with center a and radius ϵ with respect to λ.
By using this T-transformation and the notion of Neutrosophic, we can define some sequence spaces, namely $L_{\Gamma^3(P,Q,R)}(T)$ and $L_{\Lambda^3(P,Q,R)}(T)$.

3. Main Results

In this section, we introduce the following sequence spaces.
$L_{\Gamma^3(P,Q,R)}(T) = lim_{m,n,k\to\infty}\{x^{\frac{1}{m+n+k}} \in w^3 : (T_{uvw}(x))\} = 0$
$= lim_{m,n,k\to\infty}\{x^{\frac{1}{m+n+k}} \in w^3 : \forall\epsilon > 0, \forall\lambda$, there exist $(p, q, r) \in \mathbb{N} : \forall u \geq p, v \geq q, w \geq r$,
$G(T(x_{mnk}), x \circ \lambda) > 1 - \epsilon$,
$B(T(x_{mnk}), x \circ \lambda) < \epsilon$ and

$Y(T(x_{mnk}), x \circ \lambda) < \epsilon\}$ (3.1)
$= lim_{m,n,k\to\infty}\{x = x_{mnk}^{\frac{1}{m+n+k}}\{(u,v,w \in \mathbb{N}\} \in w^3 : (u,v,w) \in \mathbb{N}, \forall\epsilon > 0$
$G(T(x_{mnk})x, \lambda) \leq 1 - \epsilon;$
$B(T(x_{mnk})x, \lambda) \geq \epsilon$ and
$Y(T(x_{mnk})x, \lambda) \geq \epsilon\} = 0\}$
$L_{\Lambda^3(P,Q,R)}(T) = sup_{(m,n,k)}\{x_{mnk}^{\frac{1}{m+n+k}} \in w^3 : (T_{uvw}(x)) \leq M\}$
$sup_{(m,n,k)}\{x_{mnk}^{\frac{1}{m+n+k}} \in w^3 : \forall\epsilon \in (0,1), \forall\lambda,$ there exist $(p,q,r) \in \mathbb{N} : \forall u \geq p, v \geq q, w \geq r,$
$G(T(x_{mnk}), x, \lambda) \geq 1 - \epsilon,$
$B(T(x_{mnk}), x, \lambda) \leq \epsilon$ and
$Y(T(x_{mnk}), x, \lambda) \leq \epsilon\} \leq M.\}$ (3.2)
$= sup_{m,n,k}\{x_{mnk}^{\frac{1}{m+n+k}} \in w^3 : \{(uvw) \in \mathbb{N} : \forall\epsilon > 0,$
$G(T(x_{mnk}), x, \lambda) \geq 1 - \epsilon,$
$B(T(x_{mnk}), x, \lambda) \geq \epsilon$ and
$Y(T(x_{mnk}), x, \lambda) \geq \epsilon\} < \infty\}$

Definition 3.1. Give $(V, N, \star, \circ)$ be an NMS, $0 < \epsilon < 1, \lambda > 0$ and $a \in V$.
The set $cl(a, \epsilon, \lambda) = \{b \in V : G(T(a), T(b), \lambda) < 1 - \epsilon; B(T(a), T(b), \lambda) > \epsilon$ and $Y(T(a), T(b), \lambda) > \epsilon$ is finite $\}$ is said to be a closed ball (CB) with center a radius $\in$ with respect to λ, then
If $(X_{mnk}) \in L_{(P,Q,R)}(T)$, then (x_{mnk}) converges to some $\beta \in Y$, denoted by $(x_{mnk})^{(PQR)} \to \beta$.

Theorem 3.2. *The inclusion relation* $L_{\Gamma^3(PQR)}(T) \subset L_{(PQR)}(T) \subset L_{\Lambda^3(P,Q,R)}(T)$ *holds.*

Proof. It can be easily seen that $L_{\Gamma^3(PQR)}(T) \subset L_{(PQR)}(T)$. Now to prove that $L_{(PQR)}(T) \subset L_{\Lambda^3(P,Q,R)}(T)$
Let $x = (x_{mnk})^{\frac{1}{m+n+k}} \in L_{(P.Q.R)}(T)$.
Then there exist β such that $N_{(P.Q.R)}(T) - lim_{mnk\to\infty}(x_{mnk}^{\frac{1}{m+n+k}}) = \beta$. Thus for every $0 < \epsilon < 1$ and $\lambda > 0$, the set A is finite.
$A = \{(u,v,w) \in \mathbb{N} :$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \frac{\beta}{2}, \frac{\lambda}{2}) > 1 - \epsilon; B(T_{unw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \frac{\beta}{2}, \frac{\lambda}{2}) < \epsilon,$
$Y(T_{uvw}(x_{mnk}), x, \frac{\beta}{2}, \frac{\lambda}{2}) < \epsilon\}$
Let $G(\beta, \frac{\lambda}{2}) = e; B = (\frac{\beta}{2}, \frac{\lambda}{2}) = f$, and $Y(\beta, \frac{\lambda}{2}) = g$ for all $\lambda > 0$.
Since $(e, f, g) \in (0,1)$ and $0 < \epsilon < 1$, there exist $s_1, s_2, s_3 \in (0,1)$ such that
$(1-\epsilon) \star e > 1 - s_1, \epsilon \circ f < s_2$ and $\epsilon \ \circ g < s_3$, we have
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \lambda) = G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \lambda) \geq G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, x, \frac{\beta}{2}, \frac{\lambda}{2}) \star G(\frac{\beta}{2}, \frac{\lambda}{2})$
$\geq (1-\epsilon) \star e$
$> 1 - s_1.$

$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \lambda) = B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \lambda)$
$\leq B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \frac{\beta}{2}, \frac{\beta}{2}, \frac{\lambda}{2}, \frac{\lambda}{2})$
$\leq B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \frac{\beta}{2}, \frac{\lambda}{2}) \star B(\frac{\beta}{2}, \frac{\lambda}{2}) < \epsilon \circ f < s_2$.
and $Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, x, \lambda) = Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \lambda)$
$\leq Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), \frac{\beta}{2}, \frac{\beta}{2}, \frac{\lambda}{2}, \frac{\lambda}{2})$
$\leq Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \frac{\beta}{2}, \frac{\beta}{2}, \frac{\lambda}{2}, \frac{\lambda}{2}) \star Y(\frac{\beta}{2}, \frac{\lambda}{2}) < \epsilon \star g < s_3$.
Taking $S = max\{s_1, s_2, s_3\}$, we have the set $\{(u, v, w) \in \mathbb{N} : \exists s \in (0, 1) :$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \lambda) > 1 - s$
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, x, \lambda) < s$ and
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, x, \lambda) < s$.
$\implies x = (x_{mnk}) \in L_{\Lambda^3(P,Q,R)}(T)$
$\implies L_{(P,Q,R)}(T) \subset L_{\Lambda^3(P,Q,R)}(T)$.
The converse is not valid. We present the following examples. □

Example 3.3. Let $(\mathbb{R}, |.,.|)$ be an NMS such that
$d(xy) = sup_{m,n,k}\{|x_{mnk} - y_{mnk}|^{\frac{1}{m+n+k}} : m, n, k = 1, 2, 3, \cdots\}$
Let $x \star y = min\{x, y\}$ and $x \circ y = max\{x, y\}, \forall x, y \in (0, 1)$.
Now define metric (P, Q, R) on $\eta^2 \times (0, \infty)$ as follows :
$G(x, y, \lambda) = \frac{\lambda}{\lambda + d(x.y)}$;
$B(x, y, \lambda) = \frac{d(x,y)}{\lambda + d(x,y)}$;
$Y(x, y, \lambda) = \frac{d(x,y)}{\lambda}$.
Then $(\mathbb{R}, N, \star, \circ)$ is an NMS.
Consider the sequence $(x_{mnk})^{\frac{1}{m+n+k}} = \{1\}$.
Hence $(x_{mnk}) \in L_{(P.Q.R)}(T)$ and $(x_{mnk}^{\frac{1}{m+n+k}}) \to \{1\}$ but $(x_{mnk}^{\frac{1}{m+n+k}}) \notin L_{\Lambda^3(P,Q,R)}(T)$.

Example 3.4. Let $(\mathbb{R}, |.,.|)$ be an NMS and (P, Q, R) be the NMS as defined in the above example. Consider the sequence $(x_{mnk}) = (-1)^{m+n+k}$. Then $(x_{mnk}) \in L_{\Lambda^3(P,Q,R)}(T)$,but $(x_{mnk}) \notin L_{(P,Q,R)}(T)$.

Theorem 3.5. *The space $L_{\Gamma^3(P,Q,R)}(T)$ is linear space.*

Proof. Let $x = (x_{mnk}), y = (y_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$. Then $\exists \beta_1, \beta_2 \in Y$ such that (y_{mnk}) and (z_{mnk}) $\mathcal{N}$-convergence to β_1 and β_2 respectively. We shall prove that for any scalars ξ_1 and ξ_2 , the sequence $\xi_1 x_{mnk} + \xi_2 x_{mnk}$, N converges to $\eta_1\beta_1 + \eta_2\beta_2$. For $\lambda > 0$ and $0 < \epsilon < 1$. Consider the following finite sets ξ_1 and ξ_2 :
$\xi_1 = lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N} :$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2|\xi_1|}) \leq 1 - \epsilon$ or

$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2|\xi_1|}) \geq \epsilon$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2|\xi_1|}) \geq \epsilon\} = 0$

$\xi_1^c = lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2|\xi_1|}) > 1-\epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2|\xi_1|}) < \epsilon$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2|\xi_1|}) < \epsilon\} = 0$

$\xi_2 = lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta_2, \frac{\lambda}{2|\xi_2|}) \leq 1-\epsilon$ or
$B(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta_2, \frac{\lambda}{2|\xi_2|}) \geq \epsilon$
$Y(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta_2, \frac{\lambda}{2|\xi_2|}) \geq \epsilon\} = 0$

$\xi_2^c = lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta_2, \frac{\lambda}{2|\xi_2|}) > 1-\epsilon$ or
$B(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta_2, \frac{\lambda}{2|\xi_2|}) < \epsilon$
$Y(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta_2, \frac{\lambda}{2|\xi_2|}) < \epsilon\} = 0$

Define the set $\xi_3 = \xi_1 \cup \xi_2$ so that ξ_3 is finite. It follows that $\xi_3^c \neq \phi$. Now we prove that for each $x = (x_{mnk}), y = (y_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$,
$\xi_3^c \subset lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G((\xi_1 T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \xi_2 T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y), (\xi_1, \beta_1, \xi_2, \beta_2), \lambda) > 1-\epsilon$ or
$B((\xi_1 T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \xi_2 T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y), (\xi_1, \beta_1, \xi_2, \beta_2), \lambda) < \epsilon$
$Y((\xi_1 T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \xi_2 T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y), (\xi_1, \beta_1, \xi_2, \beta_2), \lambda) < \epsilon\} = 0$
Let $a, b, c \in \xi_3^c$. In this case, $lim_{m,n,k\to\infty}$
$\{G(T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2|\xi_1|}) > 1-\epsilon$ or
$B(T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2|\xi_1|}) < \epsilon$
$Y(T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2|\xi_1|}) < \epsilon = 0$ and

$= lim_{m,n,k\to\infty}\{G(T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), y, \beta_2, \frac{\lambda}{2|\xi_2|}) > 1-\epsilon$ or
$B(T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), y, \beta_2, \frac{\lambda}{2|\xi_2|}) < \epsilon$
$Y(T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), y, \beta_2, \frac{\lambda}{2|\xi_2|}) < \epsilon\} = 0$
$lim_{m,n,k\to\infty} G((\xi_1 T_{abc}(x), x, \xi_2 T_{abc}(y), y), (\xi_1 \circ \beta_1, \xi_1 \circ \beta_2), \lambda)$

$\geq lim_{m,n,k\to\infty}\{G(\xi_1 T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x, \xi_1, \beta_1, \frac{\lambda}{2}) \star G(\xi_2 T_{abc}(y_{mnk}^{\frac{1}{m+n+k}}), y, \xi_2, \beta_2, \frac{\lambda}{2})\}$
$= lim_{m,n,k\to\infty}\{G(T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2|\xi_1|} \star G(T_{abc}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta_2, \frac{\lambda}{2|\xi_2|})\}$
$> lim_{m,n,k\to\infty}\{(1-\epsilon) \star (1-\epsilon)\}$
$= lim_{m,n,k\to\infty}\{1-\epsilon\} = 0$
$\implies lim_{m,n,k\to\infty}\{G((\xi_1, T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x,\ \xi_2, T_{abc}(y_{mnk}^{\frac{1}{m+n+k}}), y), (\xi_1, \beta_1, \xi_2, \beta_2), \lambda)$
$> 1-\epsilon\} = 0$
In a similar way,
$lim_{m,n,k\to\infty}\{B((\xi_1, T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x,\ \xi_2, T_{abc}(y_{mnk}^{\frac{1}{m+n+k}}), y), (\xi_1 \circ \beta_1, \xi_2 \circ \beta_2), \lambda) < \epsilon\} = 0$ and
$lim_{m,n,k\to\infty}\{Y((\xi_1, T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x,\ \xi_2, T_{abc}(y_{mnk}^{\frac{1}{m+n+k}}), y), (\xi_1 \circ \beta_1, \xi_2 \circ \beta_2), \lambda) < \epsilon\} = 0$
We have, $\xi_3^c \subset lim_{m,n,k\to\infty(a,b,c)\in\mathbb{N}}$:
$G((\xi_1, T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x,\ \xi_2, T_{abc}(y_{mnk}^{\frac{1}{m+n+k}}), y), (\xi_1 \circ \beta_1, \xi_2 \circ \beta_2), \lambda)$
$> 1-\epsilon\}$,
$\{B((\xi_1, T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x,\ \xi_2, T_{abc}(y_{mnk}^{\frac{1}{m+n+k}}), y), (\xi_1 \circ \beta_1, \xi_2 \circ \beta_2), \lambda) < \epsilon\}$, or
$\{Y((\xi_1, T_{abc}(x_{mnk}^{\frac{1}{m+n+k}}), x,\ \xi_2, T_{abc}(y_{mnk}^{\frac{1}{m+n+k}}), y), (\xi_1 \circ \beta_1, \xi_2 \circ \beta_2), \lambda) < \epsilon\} = 0$
$\implies$ the sequence $(\xi_1 x_{mnk} + \xi_2 y_{mnk})$ $\mathcal{N}$-converges to $(\xi_1\beta_1 + \xi_2\beta_2)$.
Hence $(\xi_1 x_{mnk} + \xi_2 y_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$.
Hence $L_{\Gamma^3(P,Q,R)}(T)$ is a linear space. □

Theorem 3.6. *Every open ball with the center at v and the radius $r > 0$ with respect to the parameter if fuzziness $0 < \epsilon < 1$, i.e, $O_x(r.\epsilon)(T)$ is an open set in $L_{\Gamma^3(P,Q,R)}(T)$.*

Proof. Consider the open ball with center at x and radius $r > 0$ with the parameter of neutrosophic $0 < \epsilon < 1$,
$O_x(r,\epsilon)(T) = lim_{m,n,k\to\infty}\{x = (x_{uvw}) \in w : \{(u,v,w) \in \mathbb{N} :$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r) \leq 1-\epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r) \geq \epsilon$
$R(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r) \geq \epsilon\}$is finite $\}$ =0
Then $O_x^c(r,\epsilon)(T) = lim_{m,n,k\to\infty}\{x = (x_{uvw}) \in w : \{(u,v,w) \in \mathbb{N} :$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r) > 1-\epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r) < \epsilon$,
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r) < \epsilon\}$ =0$\}$
Let $y = (y_{uvw}) \in O_x(r,\epsilon)(T)$. Then the set $= lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N} :$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r) > 1-\epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r) < \epsilon$,
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r) < \epsilon\}$ =0$\exists r_0 \in (0,r)$ such that

$lim_{m,n,k\to\infty}\{G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r_0) > 1-\epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r_0) < \epsilon,$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r_0)) < \epsilon\} = 0$
Put $\epsilon_0 = G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r_0) > 1-\epsilon = \epsilon_0.$
then $\exists s \in (0,1)$ such that $\epsilon_0 > 1-s > 1-\epsilon$.
For $\epsilon_0 > 1-s$, we can have $\epsilon_1, \epsilon_2, \epsilon_3 \in (0,1)$ such that $\epsilon_0 \star \epsilon_1 > 1-s$;
$(1-\epsilon_0)\circ(1-\epsilon_2) < s$ and $(1-\epsilon_0)\circ(1-\epsilon_3) < s$.
Let $\epsilon_4 = max\{\epsilon_1, \epsilon_2, \epsilon_3\}$.
Now consider the open ball $O_x^c(r, r_0, 1-\epsilon_4)(T)$.
Now to show that $O_x^c(r, r_0, 1-\epsilon_4)(T) \subset O_x^c(r,\epsilon)(T)$.
Let $z = (z_{uvw}) \in O_z^c(r, r_0, 1-\epsilon_4)(T)$.
Then $= lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r, r_0) > \epsilon_4$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r, r_0) < 1-\epsilon_4;$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r, r_0) < 1-\epsilon_4\} = 0$
Therefore $lim_{m,n,k\to\infty}\{G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, r)$
$\geq \quad lim_{m,n,k\to\infty}G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r_0) \quad \star$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r, r_0)\}\}$
$\geq \epsilon_0 \star \epsilon_4 \geq \epsilon_0 \star \epsilon_1$
$> (1-s) > (1-\epsilon) = 0$
$\implies lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r) > 1-\epsilon\} = 0$
In similarly, $lim_{m,n,k\to\infty}B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r) < 0$
$= lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N} : B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r) < \epsilon\} = 0$
$= lim_{m,n,k\to\infty}Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r)$
$= lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N} : Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r) < \epsilon\} = 0$
Therefore the set $= lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r, r_0) > 1-\epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r, r_0) < \epsilon$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, (T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), z, r, r_0) < \epsilon\} = 0,$
$\implies x = (x_{mnk}) \in O_x^c(r,\epsilon)(T)$.
$\implies O_x^c(r, r_0, 1-\epsilon_4)(T) \subset O_y^c(r,\epsilon)(T)$. □

Remark 3.7. The space $L_{\Gamma^3(P,Q,R)}(T)$ is NMS with respect to the Neutrosophic metric (P,Q,R).

Theorem 3.8. *The topology $\tau_{(P,Q,R)}(T)$ on the space $L_{\Gamma^3(P,Q,R)}(T)$ is first countable.*

Proof. For each $x = (x_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$.
Consider the set $B = \{O_x(\frac{1}{efg}, \frac{1}{efg}) : e, f, g = 1, 2, 3, \cdots\}$ which is a countable local base at x. Therefore, the topology $\tau_{(P,Q,R)}(T)$ on the space $L_{\Gamma^3(P,Q,R)}(T)$ is first countable. □

4. On the Tribonacci triple sequence T_{uvw}

Definition 4.1. Give $(V, N, \star, \circ)$ be a NMS, A tripe sequence $x = (x_{mnk}) \in V$ is said to be Tribonacci convergent to $\beta \in \eta$ if for every $\epsilon > 0$ the set B_1 is finite, where

$$B_1 = \{(u, v, w) \in \mathbb{N} : |T_{uvw}(x_{mnk}), x, \beta| \geq \epsilon\}$$

Definition 4.2. Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{mnk}) \in V$ is said to be Neutrosophic Tribonacci convergent to $\beta \in \eta$ with respect to Neutrosophic metric (P, Q, R) denoted by $(x_{mnk}) \to \beta$ is for every $\epsilon \in (0, 1)$ and $\lambda > 0$, the set T_1 is finite, where $T_1 =$ $\{(u, v, w) \in \mathbb{N} :$
$G(T_{uvw}(x_{mnk}), x, \beta, \lambda) \leq 1 - \epsilon$ or
$B(T_{uvw}(x_{mnk}), x, \beta, \lambda) \geq \epsilon,$
$YS(T_{uvw}(x_{mnk}), x, \beta, \lambda) \geq \epsilon\}$
We write $N_{(P,Q,R)}(T) - lim_{m,n,k\to\infty}(x_{mnk}) = \beta$.

Definition 4.3. Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{m,n,k}) \in V$ is said to be Tribonacci Cauchy if for every $\epsilon > 0$, then $\exists(m, n, k) = (mnk(\epsilon)) \in \mathbb{N}$ such that the set B_2 is finite where

$$B_2 = \{(u, v, w) \in \mathbb{N} : |T_{uvw}(x_{mnk}), T_{uvw}(x_{pqr)|} \geq \epsilon.$$

Definition 4.4. Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{m,n,k}) \in V$ is said to be Neutrosophic Tribonacci Cauchy with respect to Neutrosophic metric (P,Q,R) is for every $\epsilon \in (0, 1)$ and $\lambda > 0$ there exist $(P, Q, R) \in \mathbb{N}$ such that the set T_2 is finite, where $T_2 = \{(u, v, w) \in \mathbb{N} : G(T_{uvw}(x_{mnk}), T_{uvw}(x_{pqr})\lambda) \leq 1 - \epsilon.$ or
$B(T_{uvw}(x_{mnk}), T_{uvw}(x_{pqr})\lambda) \geq \epsilon.$
$Y(T_{uvw}(x_{mnk}), T_{uvw}(x_{pqr})\lambda) \geq \epsilon.$

Definition 4.5. Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{m,n,k}) \in V$ is said to be Tribonacci bounded if there exists $M > 0$ such that the set

$$B_3 = \{(u, v, w) \in \mathbb{N} : |T_{uvw}(x_{mnk})| > M\}$$

Definition 4.6. Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{m,n,k}) \in V$ and a subset D of V is said to be Neutrosophic Tribonacci bounded with respect to Neutrosophic metric

(P,Q,R) if $\forall x \in D\ \exists\ 0 < \epsilon < 1$ and $\lambda > 0$ such that the set $\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}), \lambda) \leq 1 - \epsilon$ or
$B(T_{uvw}(x_{mnk}), \lambda) \geq \epsilon,$
$Y(T_{uvw}(x_{mnk}), \lambda) \geq \epsilon\}$ (4.6.1)

Theorem 4.7. *Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{mnk}) \in V$ is neutrosophic Tribonacci entire, then the $N_{(P,Q,R)}(T)$-limit is unique.*

Proof. Let $x = (x_{mnk}^{\frac{1}{m+n+k}}) \in V$ such that $(x_{mnk}^{\frac{1}{m+n+k}})$ is Neutrosophic Tribonacci entire. Let $N_{(P,Q,R)}(T) - lim_{m,n,k\to\infty}(x_{mnk}^{\frac{1}{m+n+k}}) - \beta_1$ and $N_{(P,Q,R)}(T) - lim_{m,n,k\to\infty}(x_{mnk}^{\frac{1}{m+n+k}}) - \beta_2$. To prove that $\beta_1 = \beta_2$. Now, for given $\epsilon \in (0,1), \exists \epsilon \in (0,1)$ such that
$(1-\epsilon_1) \star (1-\epsilon_1) > 1 - \epsilon$ and $\epsilon_1 \circ \epsilon_2 < \epsilon$. Therefore the sets ξ_1 and ξ_2 are finite, where
$\xi_1 = lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2}) \leq 1 - \epsilon_1$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2}) \geq \epsilon_1$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2}) \geq \epsilon_1\} = 0$
$\xi_2 = lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_2, \frac{\lambda}{2}) \leq 1 - \epsilon_1$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_2, \frac{\lambda}{2}) \geq \epsilon_1$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_2, \frac{\lambda}{2}) \geq \epsilon_1\} = 0$

$\xi_1^c = lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2}) > 1 - \epsilon_1$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2}) < \epsilon_1$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2}) < \epsilon_1\} = 0$
$\xi_2^c = lim_{m,n,k\to\infty}\{(u,v,w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_2, \frac{\lambda}{2}) > 1 - \epsilon_1$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_2, \frac{\lambda}{2}) < \epsilon_1$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_2, \frac{\lambda}{2}) < \epsilon_1\} = 0$
Then $\xi_1^c \cap \xi_2^c \neq \phi$. Taking $(u,v,w) \in (\xi_1^c \cap \xi_2^c)$, we have
$lim_{m,n,k\to\infty}(G(\beta_1, \beta_2, \lambda))$ $\geq$
$lim_{m,n,k\to\infty}\{G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2}) \star G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_2, \frac{\lambda}{2})$
$> \{(1-\epsilon_1) \star (1-\epsilon_1)$
$> (1-\epsilon_1) = 1 - \epsilon$
$= lim_{m,n,k\to\infty} B(\beta_1, \beta_2, \lambda)$
$\leq lim_{m,n,k} \quad \{B(T \quad (x^{\frac{1}{m+n+k}}), x, \beta_1 \ \frac{\lambda}{}) \circ B(T \quad (x^{\frac{1}{m+n+k}}), x, \beta_2 \ \frac{\lambda}{}\}$

$\leq \{\epsilon_1 \circ \epsilon_2\} \leq \epsilon_1 = \epsilon_1$ and
$lim_{m,n,k\to\infty} B(\beta_1, \beta_2, \lambda)$
$\leq lim_{m,n,k\to\infty}\{Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_1, \frac{\lambda}{2})\circ Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta_2, \frac{\lambda}{2}\}$
$< lim_{m,n,k\to\infty}(\epsilon_1 \circ \epsilon_2) < \epsilon_1 = \epsilon$
Since $\epsilon \in (0, 1)$ is arbitrary,therefore $G(\beta_1, \beta_2, \lambda) = 1, B(\beta_1, \beta_2, \lambda) = 0$ and
$Y(\beta_1, \beta_2, \lambda) = 0$ forall $\lambda > 0$.
Hence $\beta_1 = \beta_2$.
Thus $N_{(P,Q,R)}(T)-$ limit is unique. □

Theorem 4.8. *Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{mnk}) \in V$ is Neutrosophic Tribonacci entire of every subset D of $L_{\Gamma^3(P,Q,R)}(T)$ is Neutrosophic Tribonacci bounded.*

Proof. The proof of this theorem follows from Theorem 3.2 and can be verified on similar. □

Theorem 4.9. *Let $(V, N, \star, \circ)$ be an NMS, $L_{\Gamma^3(P,Q,R)}(T)$ and $\tau_{(P,Q,R)}(T)$ be the topology on $L_{\Gamma^3(P,Q,R)}(T)$. Let $x = (x_{mnk}^{\frac{1}{m+n+k}})^{efg}$ be a triple sequence of points in $L_{\Gamma^3(P,Q,R)}(T)$. Then $(x_{mnk} - x) \to 0$ as $m, n, k \to \infty$ if and only if*
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), \lambda) \to 1,$
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), \lambda) \to 0$ *and*
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), \lambda) \to 0$ *as* $m, n, k \to \infty$

Proof. Let $(x_{mnk} - x) \to 0$ as $m, n, k \to \infty$. Fix $r > 0$. Then, for $0 < \epsilon < 1$, there exists $(e, f, g) \in \mathbb{N}$ such that $(x_{mnk}) \in O_x(r, \epsilon)(T)$ for all $m \geq e, n \geq f, k \geq g$. Then the set A is finite, where
$A = lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N} :$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) \leq 1 - \epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) \geq \epsilon,$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) \geq \epsilon\} = 0.$

ie. $A^c = lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N} :$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) > 1 - \epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) < \epsilon,$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) < \epsilon\} = 0$

For $\{(u, v, w) \in \mathbb{N} :\} \subseteq A^c$,
ie. $lim_{m,n,k\to\infty}\{G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r > 1 - \epsilon$

$\implies 1 - G(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) > \epsilon$
$B(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) < \epsilon$ and
$Y(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) < \epsilon\} = 0$
Therefore,
$\implies 1 - G(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 0$
$B(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 0$
$Y(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 0$ as $m, n, k \to \infty$
$\implies G(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 1$
$B(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 0$
$Y(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 0$ as $m, n, k \to \infty$

Conversely, suppose that for each $(m, n, k) > 0$,
$G(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 1$
$B(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 0$ and
$Y(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 0$ as $m, n, k \to \infty$.
Then for each $\epsilon \in (0, 1)$ there exists $(e, f, g) \in \mathbb{N}$: such that
$\implies 1 - G(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 0$
$B(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 0$ and
$Y(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) \to 0$ as $m, n, k \to \infty$
for all $m \geq e, n \geq f, k \geq g$
$G(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) > 1 - \epsilon$
ie. $1 - G(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) > \epsilon$
$B(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) < \epsilon$
$Y(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) < \epsilon$
Consider that the Neutrosophic $\mathcal{N}$ generated by the set $\{a < e, b < f, c \leq g\}$ implies the collection of sets generated by the set
$\{(m, n, k) \in \mathbb{N} : u \geq e, v \geq f, w \geq g$. Thus $= lim_{m,n,k \to \infty}\{(u, v, w) \in \mathbb{N}$:
$G(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) > 1 - \epsilon$ or
$B(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) < \epsilon$
$Y(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) < \epsilon\} = 0$
$\implies (x_{mnk} \in O_x(r, \epsilon)(T)$ for all $m \geq e, n \geq f, k \geq g$
Hence $(x_{mnk} - x) \to 0$ as $m, n, k \to \infty$.
ie. $(x_{mnk}) \to x$ as $m, n, k \to \infty$ □

Theorem 4.10. *Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{mnk}) \in V$ is Neutrosophic Tribonacci of every closed ball with the center at x and the radius $r > 0$ with respect to the parameter of fuzziness $0 <\in< 1$, ie. $O_x[r, \epsilon](T)$ is a closed set in $L_{\Gamma^3(P,Q,R)}(T)$.*

Proof. Let $x = (x_{mnk}) \in V$ be such that $x \in \overline{O_x[r, \in](T)}$. Then there exists a triple sequence $(x_{mnk}) = (x_{mnk})^{efg} \in O_x[r, \in](T)$ such that $(x_{mnk} - x) \to 0$ as $m, n, k \to \infty$. This implies the set

$X = lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) \geq 1 - \epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) \leq \epsilon$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) \leq \epsilon\} = 0$
Since $(x_{mnk} - x) \to 0$ as $m, n, k \to \infty$
ie.$(x_{mnk}) \to x$ as $m, n, k \to \infty$ by Theorem 4.9,
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) \to 1$
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) \to 0$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), r) \to 0$ as $m, n, k \to \infty$ for all$(a, b, c) > 0$ as $\lambda \to \infty$.
Hence for $(m, n, k) \in x$.
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), \lambda + r)$
$\geq lim_{m,n,k\to\infty,u,v,w\to\infty}\{G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), \lambda) \star G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r)\}$
$\geq 1 \star (1 - \epsilon) = 1 - \epsilon$ as $m, n, k \to \infty$.
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), \lambda + r)$
$\leq lim_{m,n,k\to\infty,u,v,w\to\infty}\{B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), \lambda) \circ B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r)\}$
$\leq 0 \circ \epsilon = \epsilon$ and
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), \lambda + r)$
$\leq lim_{m,n,k\to\infty,u,v,w\to\infty}\{Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), \lambda) \circ Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r)$
$\leq 0 \circ \epsilon = \epsilon$
In particular, for $(e, f, g) \in \mathbb{N}$, take $\lambda = \frac{1}{efg}$. Then
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r) =$
$lim_{mnk\to\infty,e,f,g\to\infty}\{G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r + \frac{1}{efg}) \geq 1 - \epsilon\}$
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r) =$
$lim_{mnk\to\infty,e,f,g\to\infty}\{B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r + \frac{1}{efg}) \leq \epsilon\}$ and
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r) =$
$lim_{mnk\to\infty,e,f,g\to\infty}\{Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r + \frac{1}{efg}) \leq \epsilon\}$ and
$\implies lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N} : G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r) \geq 1 - \epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r) \leq \epsilon$

$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y), r) \leq \epsilon\} = 0.$
$y \in O_x[r, \in](T).$
Therefore $O_x[r, \epsilon](T)$ is a closed set. □

Theorem 4.11. *Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$. Then for some $\beta \in \eta, (x_{mnk}) \to \beta$ if and only if for every $\epsilon \in (0,1)$ and $\lambda > 0$ there exists positive integers $N = N(x, \epsilon, \lambda)$ such that*
$lim_{m,n,k\to\infty}\{N \in \mathbb{N}:$
$G(T_N(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) > 1 - \epsilon$
$B(T_N(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) < \epsilon$
$Y(T_N(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) < \epsilon\} = 0$

Proof. Suppose $(x_{mnk}) \to \beta$ for some $\beta \in Y$. For given $\epsilon \in (0,1)$ such that
$(1-\epsilon) \star (1-\epsilon) > 1 - r$ and $\epsilon \circ \epsilon < r$. Since $(x_{mnk}) \to \beta$, for all $\lambda > 0$,
$S = lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N}:$
$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) \leq 1 - \epsilon$ or
$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) < \epsilon$
$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) < \epsilon\} = 0$
Conversely, let us choose $N \in S^c$. Then,
$G(T_N(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) > 1 - \epsilon$
$B(T_N(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) < \epsilon$
$Y(T_N(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) < \epsilon$ as $m, n, k \to \infty$.
To prove that there exists a positive integer $N = N(x, \epsilon, \lambda)$ such that
$lim_{m,n,k\to\infty}\{(e, f, g) \in \mathbb{N}:$
$G(T_{efg}(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda) \leq 1 - r$ or
$B(T_{efg}(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda) \geq r$
$Y(T_{efg}(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda) \geq r\} = 0.$
We have $x = (x_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$ define,
$\xi = lim_{m,n,k\to\infty}\{(e, f, g) \in \mathbb{N}:$
$G(T_{efg}(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda) \leq 1 - r$ or
$B(T_{efg}(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda) \geq r$
$Y(T_{efg}(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda) \geq r\} = 0.$
We shall prove that $\xi \subset \eta$. Let on the contrary$\xi \nsubseteq \eta$. ie, $\exists m \in \xi$ such that $m \notin \eta$. Then
$G(T_m(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda) \leq 1 - r$ or
$G(T_m(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \frac{\lambda}{2}) > 1 - \epsilon$ as $m, n, k \to \infty$

In particular ,$G(T_N(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) > 1-\epsilon$ as $m,n,k \to \infty$.

Therefore, we have $1-r \geq G(T_m(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda)$

$\geq G(T_m(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) \star G(T_m(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2})$

$\geq (1-\epsilon) \star (1-\epsilon)$

$> 1-\epsilon = 1-r.$

which is a contradiction. Similarly,

$B(T_m(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda) \geq r$ or

$B(T_m(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) < \epsilon$ as $m,n,k \to \infty$.

In particular $B(T_N(x), \beta, \frac{\lambda}{2}) < \epsilon$.

Therefore we have

$r \leq B(T_m(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda)$

$\leq B(T_m(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) \circ B(T_N(x), x, \beta, \frac{\lambda}{2})$

$\leq \epsilon \circ \epsilon < \epsilon = r$

Similarly, in the other way,

$Y(T_m(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda) \geq r$

$Y(T_m(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) < \epsilon$ as $m,n,k \to \infty$.

In particular $Y(T_N(x), x, \beta, \frac{\lambda}{2}) < \epsilon$.

$r \leq Y(T_m(x_{mnk}^{\frac{1}{m+n+k}}), T_N(x), \lambda)$

$Y(T_m(x_{mnk}^{\frac{1}{m+n+k}}), x, \beta, \frac{\lambda}{2}) \circ Y(T_N(x), x, \beta, \frac{\lambda}{2})$

$\leq \epsilon \circ \epsilon < \epsilon = r$

which is a contradiction.

Hence $\xi \nsubseteq \eta$ since $\eta \in \mathbb{N} \implies \xi \in N$. □

Definition 4.12. Consider $D \subseteq L_{\Gamma^3(P,Q,R)}(T)$. Then D is compact if and only if every open cover of D by the open set of $\tau_{(P,Q,R)}(T)$ has a finite subcover.

Theorem 4.13. *Every finite subset D of $L_{\Gamma^3(P,Q,R)}(T)$ is compact.*

Proof. Let

$$D = \begin{pmatrix} x_{111}^{\frac{1}{3}} & x_{112}^{\frac{1}{4}} & \cdots & x_{11n}^{\frac{1}{2+n}} \\ x_{221}^{\frac{1}{4}} & x_{222}^{\frac{1}{6}} & \cdots & x_{22n}^{\frac{1}{4+n}} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m11}^{\frac{1}{2+m}} & x_{m22}^{\frac{1}{4+m}} & \cdots & x_{mnk}^{\frac{1}{m+n+k}} \end{pmatrix}$$

be the finite subsets of $L_{\Gamma^3(P,Q,R)}(T)$. For $r > 0$ and $0 < \epsilon < 1$.

Let us assume that $\{O_x(r,\epsilon)(T) : x \in D\}$ is an open cover of D. Then $D \subseteq U_{x \in D} O_x(r,\epsilon)(T)$.

Now for all $(x_{mnk}^{\frac{1}{m+n+k}}) \in D, m, n, k = 1, 2, 3, \cdots (a, b, c)$, we have $(x_{mnk}^{\frac{1}{m+n+k}}) \in U_{x_{mnk} \in D} O_{mnk}(r, \epsilon)(T)$.

$\implies x = (x_{mnk}^{\frac{1}{m+n+k}}) \in O_{x_{mnk}}(r, \epsilon)(T)$

for some $(m, n, k) \in (1, 2, 3, \cdots (a, b, c))$

Then $\{O_{x_{mnk}(r,\epsilon)(T): m,n,k=1,2,3,\cdots(abc)\}}$

is a finite subcover of D.

Hence D is compact. □

Theorem 4.14. *Let $D \subseteq L_{\Gamma^3(P,Q,R)}(T)$. Then D is compact iff every triple sequence in D has an entire subsequence.*

Proof. Suppose that D is compact subset of $L_{\Gamma^3(P,Q,R)}(T)$.

Let $x = (x_{mnk}^{\frac{1}{m+n+k}})$ be a triple sequence in D. For given $0 < t < 1$ and $r > 0$, let $\{O_x(\frac{r}{3}, \epsilon)(T) : x = (x_{mnk}) \in D\}$. be an open cover of S.

$\implies x = (x_{mnk}^{\frac{1}{m+n+k}}) \in \cup_{x \in D} O_x(\frac{r}{3}, \epsilon)(T)$.

Then there exists some $x = (x_{mnk}) \in D$ such that $x = (x_{mnk}^{\frac{1}{m+n+k}}) \in O_x(\frac{r}{3}, \epsilon)(T)$. We have

$\eta_1 = lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N} :$

$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, T_{uvw}(x), \frac{r}{3}) > 1 - \epsilon$ or

$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, T_{uvw}(x), \frac{r}{3}) < \epsilon$

$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, T_{uvw}(x), \frac{r}{3}) < \epsilon\} = 0$

Since D is compact, there exist a finite subcover $\{O_{x_{mnk}}(\frac{r}{3}, \epsilon)(T) : (x_{mnk}) \in D$ and m,n,k=1,2,... (a,b,c)} .

of D such that $D \subseteq \cup_{m,n,k=1}^{a,b,c} O_{x_{mnk}}(\frac{r}{3}, \epsilon)(T)$.

Let $x = (x_{m_p n_p k_p})$ be a subsequence of (x_{mnk}). Then

$x = (x_{m_p n_p k_p}) \in \cup_{m,n,k=1}^{a,b,c} O_{x_{mnk}}(\frac{r}{3}, \epsilon)(T)$

$x = (x_{m_p n_p k_p}) \in O_{x_{mnk}}(\frac{r}{3}, \epsilon)(T)$

for some $x_{mnk} \in D$. We have

$\eta_2 = lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N} :$

$G(T_{uvw}(x_{m_p n_p k_p}^{\frac{1}{m_p+n_p+k_p}}), T_{uvw}(x_{mnk}), \frac{r}{3}) > 1 - \epsilon$ or

$B(T_{uvw}(x_{m_p n_p k_p}^{\frac{1}{m_p+n_p+k_p}}), T_{uvw}(x_{mnk}), \frac{r}{3}) < \epsilon$

$Y(T_{uvw}(x_{m_p n_p k_p}^{\frac{1}{m_p+n_p+k_p}}), T_{uvw}(x_{mnk}), \frac{r}{3}) < \epsilon\} = 0$

Now for $(e, f, g) \in \eta_1 \cap \eta_2$,

$lim(G(T_{uvw}(x_{m_p n_p k_p}^{\frac{1}{m_p+n_p+k_p}}), T_{uvw}(x), r)$

$\geq lim\{G(T_{uvw}(x_{m_p n_p k_p}^{\frac{1}{m_p+n_p+k_p}}), T_{uvw}(x), \frac{r}{3}) \star G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), \frac{r}{3}) \star G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(x), \frac{r}{3})\} = 0$

$> (1-\epsilon) \star (1-\epsilon) \star (1-\epsilon) = (1-\epsilon) = 0$

$\implies \{G(T_{uvw}(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}), T_{uvw}(x), r) > 1-\epsilon\} = 0$ as $m_p, n_p, k_p \to \infty$

Similarly $lim B(T_{uvw}(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}), T_{uvw}(x), r)$

$\leq \quad lim\{B(T_{uvw}(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}), T_{uvw}(x), \frac{r}{3}) \quad \circ \quad B(T_{uvw}(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}), T_{uvw}(x), \frac{r}{3}) \quad \circ$

$B(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), \frac{r}{3})\} = 0$ as $m, n, k \to \infty$.

$< \epsilon \circ \epsilon \circ \epsilon = \epsilon$.

and $lim Y(T_{uvw}(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}), T_{uvw}(x), r)$

$\leq \quad lim\{Y(T_{uvw}(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}), T_{uvw}(x), \frac{r}{3}) \quad \circ \quad Y(T_{uvw}(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}), T_{uvw}(x_{m,n,k}), \frac{r}{3}) \quad \circ$

$Y(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), \frac{r}{3})\} = 0$ as $m, n, k \to \infty$.

$< \epsilon \circ \epsilon . \epsilon = \epsilon$.

Take $\epsilon = \frac{1}{p}$. Then

$lim_{p\to\infty} G(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) = lim_{p\to\infty} 1 - \frac{1}{p} = 1.$

$lim_{p\to\infty} B(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) = lim_{p\to\infty} \frac{1}{p} = 0.$ and

$lim_{p\to\infty} Y(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) = lim_{p\to\infty} \frac{1}{p} = 0.$

Hence, by Theorem 4.9, $(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}} \to 0)$ as $p \to \infty$.

$(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}) \to x$.

Conversely, suppose that $(x_{m_p n_p k_p})$ is a subsequence of the triple sequence (x_{mnk}) in D such that $(x_{m_p n_p k_p} - x) \to 0$ in D.

ie. $(x_{m_p n_p k_p}) \to x$ in D. Let on the contrary D be not a compact subset of $L_{\Gamma^3(P,Q,R)}(T)$.

Suppose that $\{O_x(r,\epsilon)(T) : x \in D\}$ is an open cover of D.

$\implies D \subseteq \cup_{x\in D} O_x(r,\epsilon)(T)$.We have

$lim_{m,n,k\to\infty}\{(u,v,w) \in V :$

$G(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) > 1-\epsilon$ or

$B(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) < \epsilon$

$Y(T_{uvw}(x_{mnk}^{\overline{\frac{1}{m+n+k}}}), T_{uvw}(x), r) < \epsilon\} = 0$

Since D is not compact, there exist a finite subcover $O_{x_{mnk}}(r,\epsilon)(T) : (x_{mnk} \in D\} \exists : D \nsubseteq \cup_{x\in D} O_{x_{mnk}}(r,\epsilon)(T)$

$lim_{m,n,k\to\infty}\{(m,n,k) \in \mathbb{N} :$

$G(T_{uvw}(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}), T_{uvw}(x), r) > 1-\epsilon$ or

$B(T_{uvw}(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}), T_{uvw}(x), r) < \epsilon$

$Y(T_{uvw}(x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}), T_{uvw}(x), r) < \epsilon\} = 0$

For any $0 <\in< 1$ and $r > 0, (x_{m_p n_p k_p}^{\overline{\frac{1}{m_p+n_p+k_p}}}) \notin O_x(r,\epsilon)$.

$\implies (x_{m_pn_pk_p}^{\frac{1}{m_p+n_p+k_p}} - x) \nrightarrow 0$

ie. $(x_{m_pn_pk_p}^{\frac{1}{m_p+n_p+k_p}}) \nrightarrow x$ which is a contradiction.

Hence D is compact. □

Theorem 4.15. *Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$. Let $r > 0$ and $\epsilon_1\epsilon_2 \in (0, 1)$ such that $(1 - \epsilon_1) \star (1 - \epsilon_1) \geq (1 - \epsilon_1)$ and $\epsilon_1 \circ \epsilon = \epsilon_1$ Then for any $x = (x_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$, $O_x(\frac{r}{2}, \epsilon_1)(T) \subseteq O_x(\frac{r}{2}, \epsilon_2)(T)$.*

Proof. Let $s = (s_{mnk}) \in \overline{O_x(\frac{r}{2}, \epsilon_1)(T)}$ and $O_s(\frac{r}{2}, \epsilon_1)(T)$ be an open ball with center at s and the radius ϵ_1.

Hence $O_s(\frac{r}{2}, \epsilon_1)(T) \cap O_x(\frac{r}{2}, \epsilon_1)(T) \neq \phi$.

Suppose $z = (z_{mnnk}) \in O_s(\frac{r}{2}, \epsilon_1)(T) \cap O_x(\frac{r}{2}, \epsilon_1)(T)$.

Then the sets $\eta_1 = lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N} :$

$G(T_{uvw}(s_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), s, z, \frac{r}{2}) \geq 1 - \epsilon_1$ or

$B(T_{uvw}(s_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), s, z, \frac{r}{2}) < \epsilon_1$

$Y(T_{uvw}(s_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), s, z, \frac{r}{2}) < \epsilon_1\} = 0.$

$\eta_2 = lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N} :$

$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), x, z, \frac{r}{2}) \geq 1 - \epsilon_1$ or

$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), x, z, \frac{r}{2}) < \epsilon_1$

$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(z_{mnk}^{\frac{1}{m+n+k}}), x, z, \frac{r}{2}) < \epsilon_1\} = 0.$

Consider $(e, f, g) \in \eta_1 \cap \eta_2$.

$lim_{m,n,k\to\infty}\{G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{efg}(s_{mnk}^{\frac{1}{m+n+k}}), x, s, r) \geq$

$lim_{m,n,k\to\infty}\{G(T_{efg}(x_{mnk}^{\frac{1}{m+n+k}}), T_{efg}(z_{mnk}^{\frac{1}{m+n+k}}), x, z, \frac{r}{2}) \star$

$G(T_{efg}(s_{mnk}^{\frac{1}{m+n+k}}), T_{efg}(z_{mnk}^{\frac{1}{m+n+k}}), s, z, \frac{r}{2})\} = 0.$

$> (1 - \epsilon_1) \star (1 - \epsilon_1) \geq (1 - \epsilon_2)$

$lim_{mnk\to\infty}\{B(T_{efg}(x_{mnk}^{\frac{1}{m+n+k}}), T_{efg}(s_{mnk}^{\frac{1}{m+n+k}}), x, s, r)$

$\leq B(T_{efg}(x_{mnk}^{\frac{1}{m+n+k}}), T_{efg}(z_{mnk}^{\frac{1}{m+n+k}}), x, z, \frac{r}{2}) \circ B(T_{efg}(s_{mnk}^{\frac{1}{m+n+k}}), T_{efg}(z_{mnk}^{\frac{1}{m+n+k}}), s, z, \frac{r}{2})$ and

$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{efg}(s_{mnk}^{\frac{1}{m+n+k}}), x, z, r)$

$\leq Y(T_{efg}(x_{mnk}^{\frac{1}{m+n+k}}), T_{efg}(z_{mnk}^{\frac{1}{m+n+k}}), x, z, \frac{r}{2}) \circ Y(T_{efg}(s_{mnk}^{\frac{1}{m+n+k}}), T_{efg}(z_{mnk}^{\frac{1}{m+n+k}}), s, z, \frac{r}{2})$

$< \epsilon_1 \circ \epsilon_1 \leq \epsilon_2.$

Therefore we have $lim_{u,v,w} \to \infty\{(u, v, w) \in \mathbb{N}$

$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(s_{mnk}^{\frac{1}{m+n+k}}), x, s, r) > 1 - \epsilon_1$ or

$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(s_{mnk}^{\frac{1}{m+n+k}}), x, s, r) < \epsilon_1$

$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(s_{mnk}^{\frac{1}{m+n+k}}), x, s, r) < \epsilon_1\} = 0$

$\implies s = (s_{mnk}) \in O_x(r, \epsilon_2)(T)$.

Hence $O_x(\frac{r}{2}, \epsilon_1)(T) \subseteq O_x(\frac{r}{2}, \epsilon_2)(T)$. □

Theorem 4.16. *Let $(V, N, \star, \circ)$ be an NMS. A triple sequence $x = (x_{mnk}) \in V$. If there exists a triple sequence $y = (y_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$ such that $T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}) = T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}})$ for almost all (u, v, w) relative to Neutrosophic $\mathcal{N}$, then $x \in L_{\Gamma^3(P,Q,R)}(T)$.*

Proof. Suppose $T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}) = T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}})$ for almost all (u, v, w) relative to $\mathcal{N}$.

Then $lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N}:$

$T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}) \neq T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}) = 0\} \in N$

$\implies lim_{m,n,k\to\infty}\{(u, v, w) \in \mathbb{N} : T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}) = T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}})\} = 0$

Therefore, for all $\lambda > 0$

$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), x, y, \frac{\lambda}{2})\} = 1.$ and

$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), x, y, \frac{\lambda}{2})\} = 0.$

$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), x, y, \frac{\lambda}{2})\} = 0.$ as $m, n, k \to \infty$.

Since $y = (y_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$, let $lim_{m,n,k\to\infty}(z_{mnk}^{\frac{1}{m+n+k}}) = \beta$.

Then for every $\epsilon \in (0, 1)$ and $\lambda > 0$,

Since $y = (y_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$, let

$\eta_1 = lim_{mnk\to\infty} \in \{(m, n, k) \in \mathbb{N}:$

$G(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta, \frac{\lambda}{2}) > 1 - \epsilon$ or

$B(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta, \frac{\lambda}{2}) < \epsilon,$

$Y(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta, \frac{\lambda}{2}) < \epsilon\} = 0$

Consider the set

$\eta_2 = lim_{mnk\to\infty} \in \mathbb{N}:$

$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, x, \beta, \lambda) > 1 - \epsilon$ or

$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, x, \beta, \lambda) < \epsilon,$

$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, x, \beta, \lambda) < \epsilon\} = 0$

To prove that $\eta_1 \subset \eta_2$. So far $\lambda \in \eta_1$, we have

$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, x, \beta, \lambda) \geq$

$G(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), x, y, \frac{\lambda}{2}) \star G(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta, \frac{\lambda}{2})$

$> 1 \star (1 - \epsilon) = 1 - \epsilon.$

$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, x, \beta, \lambda) \leq$

$B(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), x, y, \frac{\lambda}{2}) \circ B(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta, \frac{\lambda}{2})$

$< 0 \circ \epsilon = \epsilon.$

$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}, x, \beta, \lambda) \leq$

$Y(T_{uvw}(x_{mnk}^{\frac{1}{m+n+k}}), T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), x, y, \frac{\lambda}{2}) \circ Y(T_{uvw}(y_{mnk}^{\frac{1}{m+n+k}}), y, \beta, \frac{\lambda}{2})$

$< 0 \circ \epsilon$ as $m, n, k \to \infty$.

$\Longrightarrow (u, v, w) \in \eta_2$.

Hence $\eta_1 \subset \eta_2$.

Hence $x = (x_{mnk}) \in L_{\Gamma^3(P,Q,R)}(T)$ □

5. Conclusions

In this paper, we explore various directions by introducing a Tribonacci sequence space with aid of a Neutrosophic triple sequence space. We expect that our results will serve as a reference for further studies and also motivate young researchers in this field. We have defined the tribonacci matrix from neutrosophic triple entire sequence spaces and examined some topological and algebraic properties.

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Availability of data and materials

Not applicable.

Declarations

Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors contributed equally to the writing of this paper and also read and approved the final manuscript.

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