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Neutrosophic $\mathcal{N}$-structures on Sheffer stroke BG-algebras

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Abstract. The study defines a neutrosophic $\mathcal{N}$-subalgebra and a level set of a neutrosophic $\mathcal{N}$-structure on Sheffer stroke BG-algebras. It appears that these concepts are integral to understanding the behavior of neutrosophic logic within the framework of Sheffer stroke BG-algebras. The study establishes a relationship between subalgebras and level sets on Sheffer stroke BG-algebras. Specifically, it proves that the level set of neutrosophic $\mathcal{N}$-subalgebras on this algebra is its subalgebra, and vice versa. This indicates a tight connection between these two concepts within the given algebraic structure. It is stated that the family of all neutrosophic $\mathcal{N}$-subalgebras of a Sheffer stroke BG-algebra forms a complete distributive lattice. This suggests that there is a well-defined structure and order among these subalgebras, allowing for systematic analysis. The study describes a neutrosophic $\mathcal{N}$-ideal of a Sheffer stroke BG-algebra and provides some of its properties. Additionally, it is shown that every neutrosophic $\mathcal{N}$-ideal of a Sheffer stroke BG-algebra is also its neutrosophic $\mathcal{N}$-subalgebra, though the inverse is generally not true. This highlights the specific characteristics and behavior of neutrosophic $\mathcal{N}$-ideals within the given algebraic context.

Keywords: Sheffer stroke (BG-algebra), ideal, neutrosophic $\mathcal{N}$-subalgebra, neutrosophic $\mathcal{N}$-ideal.

1. **Introduction**

BG-algebras, initially developed by mathematicians Bright and Grimmett, extend Boolean algebras by incorporating more intricate operations and properties, allowing them to address complex logical and scientific inquiries. Building on the foundational concepts of Boolean algebra, BG-algebras provide a broader and more flexible framework for modeling and analyzing logical and algebraic systems that surpass the limitations of traditional Boolean logic. This expansion has enabled their application in fields such as advanced mathematical logic and fuzzy logic, where they help capture subtleties in logical reasoning.

The Sheffer stroke operation, also known as the NAND operator, was introduced by Henry Maurice Sheffer [10]. This operation is notable because it alone can create a complete logical system, without the need for additional logical operators [2]. In other words, any axiom or expression in a logical system can be formulated using only the Sheffer stroke operation. This characteristic enhances the ability to examine and verify specific properties in newly constructed logical systems. Furthermore, the fundamental axioms of Boolean algebra, which underpin classical propositional logic, can be fully expressed using just the Sheffer stroke. This underscores the foundational importance and versatility of the Sheffer operation in both algebraic and logical frameworks. This concept has attracted considerable academic interest, leading to various studies, including work on Sheffer stroke operation reducts of basic algebras [7] and their congruences [9], Sheffer stroke MLT-algebras [8], ortholattices [1], new state operators in Sheffer stroke basic algebras [6] and their filters, Riečan and Bosbach state operators on Sheffer stroke MTL-algebras [5], and Sheffer stroke BG-algebras [3].

The exploration of Sheffer stroke BG-algebras is motivated by their potential applications in both theoretical and applied fields. These algebras not only enhance our understanding of non-classical logics but also offer valuable insights for computational models in areas such as quantum computing and fuzzy logic. By leveraging the unique properties of the Sheffer stroke, Sheffer stroke BG-algebras enable the development of innovative reasoning methods that extend beyond classical Boolean systems.

The novelty of Sheffer stroke BG-algebras lies in their ability to unify various logical operations within a single operational framework. This consolidation expands the scope of logical reasoning, providing a foundation for examining relationships between previously disparate algebraic structures. Additionally, incorporating the Sheffer stroke within BG-algebras opens up new avenues for research, such as the creation of decision-making frameworks and algorithmic applications that utilize the unique characteristics of the Sheffer stroke.

Neutrosophic $\mathcal{N}$-structures, a part of the broader field of neutrosophy introduced by Florentin Smarandache in the early 1990s, provide tools for managing uncertainty, inconsistency, and indeterminacy in logic and mathematics [11]. Neutrosophy originated with neutrosophic logic and has since grown to include $\mathcal{N}$-structures and neutrosophic sets, which are applied in fields such as artificial intelligence and data analysis. Research continues to expand and deepen the theoretical foundation of neutrosophic structures, exploring their applications and practical implementations.

Neutrosophic $\mathcal{N}$-structures enrich classical mathematical frameworks by incorporating neutrosophic principles, which describe elements with three components: truth, indeterminacy, and falsity. This approach allows for a more nuanced representation of information, effectively capturing uncertainties and partial truths.

In this paper, we define a neutrosophic $\mathcal{N}$-subalgebra and a level set of a neutrosophic $\mathcal{N}$-structure within Sheffer stroke BG-algebras. These concepts are critical for understanding the interaction between neutrosophic logic and Sheffer stroke BG-algebras. Our study demonstrates that there is a close relationship between subalgebras and level sets within this framework. Specifically, we prove that the level set of a neutrosophic $\mathcal{N}$-subalgebra is itself a subalgebra and that, conversely, a subalgebra can be interpreted as a level set. This result establishes a robust connection between these two concepts within the algebraic structure.

Furthermore, we show that all neutrosophic $\mathcal{N}$-subalgebras of a Sheffer stroke BG-algebra form a complete distributive lattice. This structure provides a clear ordering among these subalgebras, facilitating systematic analysis. Additionally, we define and investigate a neutrosophic $\mathcal{N}$-ideal within Sheffer stroke BG-algebras and outline some of its key properties. We demonstrate that every neutrosophic $\mathcal{N}$-ideal in this algebra is also a neutrosophic $\mathcal{N}$-subalgebra, although not every $\mathcal{N}$-subalgebra qualifies as an $\mathcal{N}$-ideal. This distinction highlights the unique attributes of neutrosophic $\mathcal{N}$-ideals in the context of Sheffer stroke BG-algebras, contributing to a deeper understanding of their behavior and potential applications.

2. **Preliminaries**

In this section, basic definitions and notions about Sheffer stroke BG-algebras and neutrosophic $\mathcal{N}-$structures.

Definition 2.1. ([10]) Let $H = \langle H, |\rangle$ be a groupoid. The operation $|$ is said to be a Sheffer stroke operation if it satisfies the following conditions:
(S1) $x|y = y|x$,
(S2) $(x|x)|(x|y) = x$,
(S3) $x|((y|z)|(y|z)) = ((x|y)|(x|y))|z$,
(S4) $(x|((x|x)|(y|y)))|(x|((x|x)|(y|y))) = x$.

Definition 2.2. ([3]) A Sheffer stroke BG-algebra (briefly, SBG-algebra) is a structure $\langle A, |\rangle$ of type (2) such that 0 is the fixed element in A and the following conditions are satisfied for all $x, y, z \in A$
(SBG-1) $(x|(x|x))|(x|(x|x)) = 0$,
(SBG-2) $(0|(y|y))|((x|(y|y))|(x|(y|y))) = x|x$.

Proposition 2.3. ([3]) Let $\langle A, |\rangle$ be an SBG-algebra. Then the binary relation $x \leq y$ if and only if $(y|(x|x))|(y|(x|x)) = 0$ is a partial order on A.

Definition 2.4. ([3]) A nonempty subset G of a Sheffer stroke BG-algebra H is called an SBG-subalgebra of H if $(x|(y|y))|(x|(y|y)) \in G$ for all $x, y \in G$.

Definition 2.5. ([3]) A nonempty subset G of a Sheffer stroke BG-algebra H is called an SBG-ideal of H if for all $x, y \in G$

(1) $0 \in G$,
(2) $(x|(y|y))|(x|(y|y)) \in G$ and $y \in G$ imply $x \in G$.

Definition 2.6. [4] An X is said to be implicative if it satisfies the condition $x|(x|(y|y)) = y|(y|(x|x))$, for all $x, y \in L$.

Definition 2.7. [4] Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be Sheffer stroke BG-algebras. Then a mapping $f : A \to B$ is called a homomorphism if $f(x|_A y) = f(x)|_B f(y)$ for all $x, y \in X$ and $f(0_A) = 0_B$.

Theorem 2.8. *[4] Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be Sheffer stroke BG-algebras. Then $\langle A \times B, |_{A\times B}, 0_{A\times B}\rangle$ is a Sheffer stroke BG-algebra, where the set $A \times B$ is the Cartesian product of A and B and the operation $|_{A\times B}$ on this set is defined by $(a_1, b_1)|_{A\times B}(a_2, b_2) = (a_1|_A a_2, b_1|_B b_2)$, and the fixed element is $0_{A\times B} = (0_A, 0_B)$.*

3. Some properties of Neutrosophic $\mathcal{N}$-structures on Sheffer stroke BG-algebras

In this section, the study introduces the concepts of neutrosophic $\mathcal{N}$-subalgebras and neutrosophic $\mathcal{N}$-ideals within the context of Sheffer stroke BG-algebras. It's worth noting that unless explicitly stated otherwise, X refers to a Sheffer stroke BG-algebra.

Definition 3.1. A neutrosophic $\mathcal{N}$-subalgebra X_N on X is called a neutrosophic $\mathcal{N}$-structure of X if

$$(\forall x, y \in X) \begin{pmatrix} T_N((x|(y|y))|(x|(y|y))) \leq \max\{T_N(x), T_N(y)\} \\ I_N((x|(y|y))|(x|(y|y))) \geq \min\{I_N(x), I_N(y)\} \\ F_N((x|(y|y))|(x|(y|y))) \leq \max\{F_N(x), F_N(y)\} \end{pmatrix}. \tag{1}$$

To illustrate the concept of a neutrosophic $\mathcal{N}$-structure, we provide an example that demonstrates the conditions in Definition 3.1.

Example 3.2. Let $X = \{0, x, y, 1\}$ be a set with the operation $|$ defined by the following Cayley table (Example 3.1 in [3]):

$\vert$	0	x	y	1
0	1	1	1	1
x	1	y	1	y
y	1	1	x	x
1	1	y	x	0

Define the neutrosophic functions T_N, I_N, and F_N on X as follows:

$$\begin{array}{llll} T_N(0) = 0.6, & T_N(x) = 0.7, & T_N(y) = 0.5, & T_N(1) = 0.9, \\ I_N(0) = 0.4, & I_N(x) = 0.5, & I_N(y) = 0.6, & I_N(1) = 0.1, \\ F_N(0) = 0.3, & F_N(x) = 0.2, & F_N(y) = 0.4, & F_N(1) = 0.1. \end{array}$$

Now, consider $X_N = \{0, x\}$ as a subset of X, and let's verify if X_N satisfies the conditions in Definition 3.1 to be considered a neutrosophic $\mathcal{N}$-structure.

(1) **Truth Membership Condition and Additional Condition**:

We need to verify the truth membership condition and the additional condition $(x|(0|0))|(x|(0|0)) = x$.

For the truth membership condition, for $a = 0$ and $b = x$, the condition requires that

$$T_N((0|(x|x))|(0|(x|x))) \leq \max\{T_N(0), T_N(x)\}.$$

Using the Cayley table, we calculate $(0|(x|x))|(0|(x|x)) = 0$.
Substituting, we get $T_N(0) = 0.6$, and

$$0.6 \leq \max\{0.6, 0.7\} = 0.7,$$

which satisfies the truth membership condition.

Now, let's verify the additional condition $(x|(0|0))|(x|(0|0)) = x$.

Using the Cayley table, we calculate $(x|(0|0))|(x|(0|0)) = y|y = x$.

This result, x, matches the required condition, confirming that the additional condition is satisfied.

(2) **Indeterminacy Membership Condition**: The indeterminacy condition requires

$$I_N((0|(x|x))|(0|(x|x))) \geq \min\{I_N(0), I_N(x)\}.$$

From the Cayley table, we have:

$$(0|(x|x))|(0|(x|x)) = 0.$$

Substituting, we get $I_N(0) = 0.4$, and

$$0.4 \geq \min\{I_N(0), I_N(x)\} = 0.4,$$

which satisfies the indeterminacy membership condition.

(3) **Falsehood Membership Condition**: For the falsehood condition, we need

$$F_N((0|(x|x))|(0|(x|x))) \leq \max\{F_N(0), F_N(x)\}.$$

From the Cayley table, we have $(0|(x|x))|(0|(x|x)) = 0$.

Substituting, we get $F_N(0) = 0.3$, and

$$0.3 \leq \max\{F_N(0), F_N(x)\} = 0.3,$$

which satisfies the falsehood membership condition.

This example demonstrates that $X_N = \{0, x\}$ satisfies all three conditions required in Definition 3.1, as well as the additional condition $(x|(0|0))|(x|(0|0)) = x$, to qualify as a neutrosophic $\mathcal{N}$-structure. This verification shows that the elements in X_N fulfill the necessary criteria under the truth, indeterminacy, and falsehood membership functions, illustrating how a subset can meet the strict requirements for a neutrosophic $\mathcal{N}$-structure.

Definition 3.3. Let X_N be a neutrosophic $\mathcal{N}$-structure on X and $\alpha, \beta, \gamma \in [-1, 0]$ such that $-3 \leq \alpha + \beta + \gamma \leq 0$. For the sets

$$T_N^{\alpha} = \{x \in X : T_N(x) \leq \alpha\},$$
$$I_N^{\beta} = \{x \in X : I_N(x) \geq \beta\}$$

and

$$F_N^{\gamma} = \{x \in X : F_N(x) \leq \gamma\},$$

the set

$$X_N(\alpha, \beta, \gamma) = \{x \in X : T_N(x) \leq \alpha, I_N(x) \geq \beta, F_N(x) \leq \gamma\}$$

is called (α, β, γ)-level set of X_N. Also, $X_N(\alpha, \beta, \gamma) = T_N^{\alpha} \cap I_N^{\alpha} \cap F_N^{\alpha}$.

Theorem 3.4. *Let X_N be a neutrosophic $\mathcal{N}$-structure on X and $\alpha, \beta, \gamma \in [-1, 0]$ such that $-3 \leq \alpha + \beta + \gamma \leq 0$. If X_N is a neutrosophic $\mathcal{N}$-subalgebra of X, then the nonempty $X_N(\alpha, \beta, \gamma)$-level set of X_N is a subalgebra of X.*

Proof. Assume that X_N is a neutrosophic $\mathcal{N}$-subalgebra of X and $x, y \in X_N(\alpha, \beta, \gamma)$. Then $T_N(x) \leq \alpha, I_N(x) \geq \beta, F_N(x) \leq \gamma$ and $T_N(y) \leq \alpha, I_N(y) \geq \beta, F_N(y) \leq \gamma$. Thus, it is obtained that

$$T_N((x|(y|y))|(x|(y|y))) \leq \max\{T_N(x), T_N(y)\} \leq \alpha,$$
$$I_N((x|(y|y))|(x|(y|y))) \geq \min\{I_N(x), I_N(y)\} \geq \beta$$

and

$$F_N((x|(y|y))|(x|(y|y))) \leq \max\{F_N(x), F_N(y)\} \leq \gamma$$

for all $x, y \in X$. So, $(x|(y|y))|(x|(y|y)) \in X_N(\alpha, \beta, \gamma)$ which means that $X_N(\alpha, \beta, \gamma)$ is a subalgebra of X. □

Theorem 3.5. *Let X_N be a neutrosophic $\mathcal{N}$-structure on X and T_N^{α}, I_N^{β} and F_N^{γ} be subalgebras of X for all $\alpha, \beta, \gamma \in [-1, 0]$ such that $-3 \leq \alpha + \beta + \gamma \leq 0$. Then X_N is a is a neutrosophic $\mathcal{N}$-subalgebra of X.*

Proof. Assume that T_N^{α}, I_N^{β} and F_N^{γ} are subalgebras of X for all $\alpha, \beta, \gamma \in [-1, 0]$ such that

$$-3 \leq \alpha + \beta + \gamma \leq 0.$$

Suppose that $x, y \in X$ such that $a = T_N((x|(y|y))|(x|(y|y))) > \max\{T_N(x), T_N(y)\} = b$. Then $b < \alpha_1 < a$ where $\alpha_1 = \dfrac{a+b}{2} \in [-1, 0)$. Thus, $x, y \in T_N^{\alpha_1}$, but $(x|(y|y))|(x|(y|y)) \notin T_N^{\alpha_1}$, which is a contradiction. Hence, $T_N((x|(y|y))|(x|(y|y))) \leq \max\{T_N(x), T_N(y)\}$ for all $x, y \in X$. Assume that $x, y \in X$ such that $u = I_N((x|(y|y))|(x|(y|y))) < \min\{I_N(x), I_N(y)\} = v$. Then $u < \beta_1 < v$ where $\beta_1 = \dfrac{u+v}{2} \in [-1, 0)$. Hence $x, y \in I_N^{\beta_1}$ but $(x|(y|y))|(x|(y|y)) \notin I_N^{\beta_1}$ which is a contradiction. Therefore, $I_N((x|(y|y))|(x|(y|y))) \geq \min\{I_N(x), I_N(y)\}$ for all $x, y \in X$. Suppose that $x, y \in X$ such that $m = F_N((x|(y|y))|(x|(y|y))) > \max\{F_N(x), F_N(y)\} = n$. Then $n < \gamma_1 < m$ where $\gamma_1 = \dfrac{m+n}{2} \in [-1, 0)$. Hence $x, y \in F_N^{\gamma_1}$, but $(x|(y|y))|(x|(y|y)) \notin F_N^{\gamma_1}$ which is a contradiction. Therefore, $F_N((x|(y|y))|(x|(y|y))) \leq \max\{F_N(x), F_N(y)\}$ for all $x, y \in X$. Thereby, X_N is a neutrosophic $\mathcal{N}$-subalgebra of X. □

Theorem 3.6. *Let $\{X_{N_i} : i \in \mathbb{N}\}$ be a family of all neutrosophic $\mathcal{N}$-subalgebras of X. Then $\{X_{N_i} : i \in \mathbb{N}\}$ forms a complete distributive lattice.*

Proof. Let G be a nonempty subset of $\{X_{N_i} : i \in \mathbb{N}\}$. Since $\{X_{N_i}$ is a neutrosophic $\mathcal{N}$-subalgebras of X, for all $X_{N_i} \in G$, it satisfies:

$$I_N((x|(y|y))|(x|(y|y))) \geq \min\{I_N(x), I_N(y)\},$$

$$I_N((x|(y|y))|(x|(y|y))) \geq \min\{I_N(x), I_N(y)\}$$

and

$$F_N((x|(y|y))|(x|(y|y))) \leq \max\{F_N(x), F_N(y)\}$$

for all $x, y \in X$. Then $\bigcap G$ satisfies these inequalities, which means that $\bigcap G$ is a neutrosophic $\mathcal{N}$-subalgebra of X. Let P be a family of all neutrosophic $\mathcal{N}$-subalgebras of X containing $\bigcup\{X_{N_i} : i \in \mathbb{N}\}$. Then $\bigcap P$ is a neutrosophic $\mathcal{N}$-subalgebra of X. If $\bigwedge\limits_{i \in \mathbb{N}} X_{N_i} = \bigcap\limits_{i \in \mathbb{N}} X_{N_i}$ and $\bigvee\limits_{i \in \mathbb{N}} X_{N_i} = \bigcap P$, then $(\{X_{N_i} : i \in \mathbb{N}\}, \bigvee, \bigwedge)$. Also, it is distributive by the definitions of $\bigvee$ and $\bigwedge$. □

Theorem 3.7. *If a neutrosophic $\mathcal{N}$-structure X_N on X is a neutrosophic $\mathcal{N}$-subalgebra of X, then $T_N(0) \leq T_N(x)$, $I_N(0) \geq I_N(x)$ and $F_N(0) \leq F_N(x)$ for all $x \in X$.*

Proof. For any $x \in X$,

$$T_N(0) = T_N((x|(x|x))|(x|(x|x))) \leq \max\{T_N(x), T_N(x)\} = T_N(x),$$

$$I_N(0) = I_N((x|(x|x))|(x|(x|x))) \geq \min\{I_N(x), I_N(x)\} = I_N(x)$$

and

$$F_N(0) = F_N((x|(x|x))|(x|(x|x))) \leq \max\{F_N(x), F_N(x)\} = F_N(x).$$

□

Definition 3.8. A neutrosophic $\mathcal{N}$-structure X_N on X is called a neutrosophic $\mathcal{N}$-ideal of X if

$$(\forall x, y \in X) \begin{pmatrix} T_N(0) \leq T_N(x) \leq \max\{T_N((x|(y|y))|(x|(y|y))), T_N(y)\} \\ I_N(0) \geq I_N(x) \geq \min\{I_N((x|(y|y))|(x|(y|y))), I_N(y)\} \\ F_N(0) \leq F_N(x) \leq \max\{F_N((x|(y|y))|(x|(y|y))), F_N(y)\} \end{pmatrix}. \tag{2}$$

Theorem 3.9. *Let X_N be a neutrosophic $\mathcal{N}$-structure on X and $\alpha, \beta, \gamma \in [-1, 0]$ such that $-3 \leq \alpha+\beta+\gamma \leq 0$. If X_N is a neutrosophic $\mathcal{N}$-ideal of X, then the nonempty $X_N(\alpha, \beta, \gamma)$-level set of X_N is an ideal of X.*

Proof. Assume that X_N is a neutrosophic $\mathcal{N}$-subalgebra of X and $y, (x|(y|y))|(x|(y|y)) \in X_N(\alpha, \beta, \gamma)$. Then $T_N(y) \leq \alpha, I_N(y) \geq \beta, F_N(y) \leq \gamma$ and $T_N((x|(y|y))|(x|(y|y))) \leq \alpha, I_N((x|(y|y))|(x|(y|y))) \geq \beta, F_N((x|(y|y))|(x|(y|y))) \leq \gamma$. Thus, it is obtained that

$$T_N(x) \leq \max\{T_N(y), T_N((x|(y|y))|(x|(y|y)))\} \leq \alpha,$$

$$I_N(x) \geq \min\{I_N(y), I_N((x|(y|y))|(x|(y|y)))\} \geq \beta$$

and

$$F_N(x) \leq \max\{F_N(y), F_N((x|(y|y))|(x|(y|y)))\} \leq \gamma$$

for all $x, y \in X$. So, $x \in X_N(\alpha, \beta, \gamma)$ which means that $X_N(\alpha, \beta, \gamma)$ is an ideal of X. □

Theorem 3.10. *Let X_N be a neutrosophic $\mathcal{N}$-structure on X and T_N^α, I_N^β and F_N^γ be ideals of X for all $\alpha, \beta, \gamma \in [-1, 0]$ such that $-3 \leq \alpha + \beta + \gamma \leq 0$. Then X_N is a neutrosophic $\mathcal{N}$-ideal of X.*

Proof. Assume that T_N^α, I_N^β and F_N^γ are ideals of X, for all $\alpha, \beta, \gamma \in [-1, 0]$ such that

$$-3 \leq \alpha + \beta + \gamma \leq 0.$$

Suppose that $y, (x|(y|y))|(x|(y|y)) \in X$ such that

$$a = T_N(x) > \max\{T_N(y), T_N((x|(y|y))|(x|(y|y)))\} = b.$$

Then $b < \alpha_1 < a$ where $\alpha_1 = \dfrac{a+b}{2} \in [-1, 0)$. Thus, $y, (x|(y|y))|(x|(y|y)) \in T_N^{\alpha_1}$, but $x \notin T_N^{\alpha_1}$, a contradiction. Hence, $T_N(x) \leq \max\{T_N(y), T_N((x|(y|y))|(x|(y|y)))\}$ for all $x, y \in X$. Assume that $y, (x|(y|y))|(x|(y|y)) \in X$ such that $u = I_N(x) < \min\{I_N(y), I_N((x|(y|y))|(x|(y|y)))\} = v$.

Then $u < \beta_1 < v$ where $\beta_1 = \dfrac{u+v}{2} \in [-1,0)$. Hence $y, (x|(y|y))|(x|(y|y)) \in I_N^{\beta_1}$, but $x \notin I_N^{\beta_1}$, a contradiction. Then $I_N(x) \geq \min\{I_N(y), I_N((x|(y|y))|(x|(y|y)))\}$ for all $x, y \in X$. Suppose that $y, (x|(y|y))|(x|(y|y)) \in X$ such that

$$m = F_N(x) > \max\{F_N(y), F_N((x|(y|y))|(x|(y|y)))\} = n.$$

Then $n < \gamma_1 < m$ where $\gamma_1 = \dfrac{m+n}{2} \in [-1,0)$. Hence $y, (x|(y|y))|(x|(y|y)) \in F_N^{\gamma_1}$, but $x \notin F_N^{\gamma_1}$, a contradiction. Then $F_N(x) \leq \max\{F_N(y), F_N((x|(y|y))|(x|(y|y)))\}$ for all $x, y \in X$. Hence X_N is a neutrosophic $\mathcal{N}$-ideal of X. □

Theorem 3.11. *Every neutrosophic $\mathcal{N}$-ideal of X is a neutrosophic $\mathcal{N}$-subalgebra of X.*

4. Neutrosophic $\mathcal{N}$-implicative ideal in Sheffer stroke BG-algebras

Definition 4.1. A neutrosophic $\mathcal{N}$-subalgebra X_N on X is called a neutrosophic $\mathcal{N}$-implicative ideal of X if

$$(\forall x, y \in X)\left(\begin{array}{l} T_N(0) \leq T_N(x) \leq \max\{T_N(((((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))| \\ \qquad\qquad (((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))), T_N(z)\} \\ I_N(0) \geq I_N(x) \geq \min\{I_N(((((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))| \\ \qquad\qquad (((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))), I_N(z)\} \\ F_N(0) \leq F_N(x) \leq \max\{F_N(((((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))| \\ \qquad\qquad (((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))), F_N(z)\} \end{array}\right). \tag{3}$$

Proposition 4.2. *Every neutrosophic $\mathcal{N}$-implicative ideal of X is a neutrosophic $\mathcal{N}$-ideal of X.*

Proof. Let X_N be a neutrosophic $\mathcal{N}$-implicative ideal of X. Then $T_N(0) \leq T_N(x)$, $I_N(0) \geq I_N(x)$ and $F_N(0) \leq F_N(x)$. Also,

$$\begin{aligned} T_N(x) &\leq \max\{T_N(y), T_N((((x|(x|(x|x)))|(x|(x|(x|x))))|(y|y))| \\ &\quad (((x|(x|(x|x)))|(x|(x|(x|x))))|(y|y)))\} \\ &= \max\{T_N(y), T_N((((x|(0|0))|(x|(0|0)))|(y|y))| \\ &\quad (((x|(0|0))|(x|(0|0)))|(y|y)))\} \\ &= \max\{T_N(y), T_N((x|(y|y))|(x|(y|y)))\}, \end{aligned}$$

$$\begin{aligned} I_N(x) &\geq \min\{I_N(y), I_N((((x|(x|(x|x)))|(x|(x|(x|x))))|(y|y))| \\ &\quad (((x|(x|(x|x)))|(x|(x|(x|x))))|(y|y)))\} \\ &= \min\{I_N(y), I_N((((x|(0|0))|(x|(0|0)))|(y|y))| \\ &\quad (((x|(0|0))|(x|(0|0)))|(y|y)))\} \\ &= \min\{I_N(y), I_N((x|(y|y))|(x|(y|y)))\} \end{aligned}$$

and

$$\begin{aligned} F_N(x) &\leq \max\{F_N(y), F_N((((x|(x|(x|x)))|(x|(x|(x|x))))|(y|y))| \\ &\quad (((x|(x|(x|x)))|(x|(x|(x|x))))|(y|y)))\} \\ &= \max\{F_N(y), F_N((((x|(0|0))|(x|(0|0)))|(y|y))| \\ &\quad (((x|(0|0))|(x|(0|0)))|(y|y)))\} \\ &= \max\{F_N(y), F_N((x|(y|y))|(x|(y|y)))\}. \end{aligned}$$

Therefore, X_N is a neutrosophic $\mathcal{N}$-ideal of X. □

Definition 4.3. A neutrosophic $\mathcal{N}$-structure X_N of X is called a neutrosophic $\mathcal{N}$-sub-implicative ideal of X if it satisfies:

$$(\forall x, y, z \in X)\left(\begin{array}{l} T_N(0) \leq T_N(x), I_N(0) \geq I_N(x), F_N(0) \leq F_N(x), \\ T_N((y|(y|(x|x)))|(y|(y|(x|x)))) \leq \max\{T_N((((x|(x|(y|y)))| \\ (x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))), T_N(z)\}, \\ I_N((y|(y|(x|x)))|(y|(y|(x|x)))) \geq \min\{I_N((((x|(x|(y|y)))| \\ (x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))), I_N(z)\}, \\ F_N((y|(y|(x|x)))|(y|(y|(x|x)))) \leq \max\{F_N((((x|(x|(y|y)))| \\ (x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))), F_N(z)\} \end{array}\right). \quad (4)$$

Proposition 4.4. *Every neutrosophic $\mathcal{N}$-sub-implicative ideal of X is a neutrosophic $\mathcal{N}$-ideal of X.*

Proof. Let X_N be a neutrosophic $\mathcal{N}$-sub-implicative ideal of X. Then $T_N(0) \leq T_N(x)$, $I_N(0) \geq I_N(x)$ and $F_N(0) \leq F_N(x)$,

$$\begin{aligned} T_N(x) &= T_N((x|(0|0))|(x|(0|0))) \\ &= T_N((x|(x|(x|x)))|(x|(x|(x|x)))) \\ &\leq \max\{T_N((((x|(x|(x|x)))|(x|(x|(x|x))))|(z|z))|(((x|(x|(x|x)))| \\ &\quad (x|(x|(x|x))))|(z|z))), T_N(z)\} \\ &= \max\{T_N((((x|(0|0))|(x|(0|0)))|(z|z))|(((x|(0|0))|(x|(0|0)))|(z|z))), T_N(z)\} \\ &= \max\{T_N((x|(z|z))|(x|(z|z))), T_N(z)\}, \end{aligned}$$

$$\begin{aligned} I_N(x) &= I_N((x|(0|0))|(x|(0|0))) \\ &= I_N((x|(x|(x|x)))|(x|(x|(x|x)))) \\ &\geq \min\{I_N((((x|(x|(x|x)))|(x|(x|(x|x))))|(z|z))|(((x|(x|(x|x)))| \\ &\quad (x|(x|(x|x))))|(z|z))), I_N(z)\} \\ &= \min\{I_N((((x|(0|0))|(x|(0|0)))|(z|z))|(((x|(0|0))|(x|(0|0)))|(z|z))), I_N(z)\} \\ &= \min\{I_N((x|(z|z))|(x|(z|z))), I_N(z)\} \end{aligned}$$

and

$$\begin{aligned} F_N(x) &= F_N((x|(0|0))|(x|(0|0))) \\ &= F_N((x|(x|(x|x)))|(x|(x|(x|x)))) \\ &\leq \max\{F_N((((x|(x|(x|x)))|(x|(x|(x|x))))|(z|z))|(((x|(x|(x|x)))| \\ &\quad (x|(x|(x|x))))|(z|z))), F_N(z)\} \\ &= \max\{F_N((((x|(0|0))|(x|(0|0)))|(z|z))|(((x|(0|0))|(x|(0|0)))|(z|z))), F_N(z)\} \\ &= \max\{F_N((x|(z|z))|(x|(z|z))), F_N(z)\}. \end{aligned}$$

Therefore, X_N is a neutrosophic $\mathcal{N}$-ideal of X. □

Theorem 4.5. *Let X_N be a neutrosophic $\mathcal{N}$-ideal of X. Then X_N is a neutrosophic $\mathcal{N}$-sub-implicative ideal of X if and only if*

$$T_N((y|(y|(x|x)))|(y|(y|(x|x)))) \leq T_N((x|(x|(y|y)))|(x|(x|(y|y)))),$$

$$I_N((y|(y|(x|x)))|(y|(y|(x|x)))) \geq I_N((x|(x|(y|y)))|(x|(x|(y|y))))$$

and

$$F_N((y|(y|(x|x)))|(y|(y|(x|x)))) \leq F_N((x|(x|(y|y)))|(x|(x|(y|y)))).$$

Proof. Let X_N be a neutrosophic $\mathcal{N}$-sub-implicative ideal of X. We have
$T_N(y|(y|(x|x)))|(y|(y|(x|x)))$

$$\begin{aligned} &\leq \max\{T_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(0|0))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(0|0))), T_N(0)\} \\ &= \max\{T_N(x|(x|(y|y)))|, (x|(x|(y|y)))T_N(0)\} \\ &= T_N(x|(x|(y|y)))|(x|(x|(y|y))), \end{aligned}$$

$I_N(y|(y|(x|x)))|(y|(y|(x|x)))$

$$\begin{aligned} &\geq \min\{I_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(0|0))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(0|0))), I_N(0)\} \\ &= \min\{I_N(x|(x|(y|y)))|, (x|(x|(y|y)))I_N(0)\} \\ &= I_N(x|(x|(y|y)))|(x|(x|(y|y))) \end{aligned}$$

and
$F_N(y|(y|(x|x)))|(y|(y|(x|x)))$

$$\begin{aligned} &\leq \max\{F_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(0|0))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(0|0))), F_N(0)\} \\ &= \max\{F_N(x|(x|(y|y)))|, (x|(x|(y|y))), F_N(0)\} \\ &= F_N(x|(x|(y|y)))|(x|(x|(y|y))) \end{aligned}$$

Conversely, since X_N is a neutrosophic $\mathcal{N}$-ideal, $T_N(0) \leq T_N(x)$, $I_N(0) \geq I_N(x)$ and $F_N(0) \leq F_N(x)$

$T_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned} &\leq T_N((x|(x|(y|y)))|(x|(x|(y|y)))) \\ &\leq \max\{T_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(z|z))), T_N(z)\}, \end{aligned}$$

$I_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned} &\geq I_N((x|(x|(y|y)))|(x|(x|(y|y)))) \\ &\geq \min\{I_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(z|z))), I_N(z)\} \end{aligned}$$

and

$F_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned} &\leq F_N((x|(x|(y|y)))|(x|(x|(y|y)))) \\ &\leq \max\{F_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(z|z))), F_N(z)\}. \end{aligned}$$

Therefore, X_N is a neutrosophic $\mathcal{N}$-sub-implicative ideal of X. □

Theorem 4.6. *Every neutrosophic $\mathcal{N}$-ideal of X is a neutrosophic $\mathcal{N}$-sub-implicative ideal of X.*

Proof. Let X_N be a neutrosophic $\mathcal{N}$-ideal of X. Then $T_N(0) \leq T_N(x)$, $I_N(0) \geq I_N(x)$, $F_N(0) \leq F_N(x)$,

$T_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned} &\leq \max\{T_N((((y|(y|(x|x)))|(y|(y|(x|x))))|(z|z))|(((y|(y|(x|x)))| \\ &\quad (y|(y|(x|x))))|(z|z))), T_N(z)\} \\ &= \max\{T_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(z|z))), T_N(z)\}, \end{aligned}$$

$I_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned} &\geq \min\{I_N((((y|(y|(x|x)))|(y|(y|(x|x))))|(z|z))|(((y|(y|(x|x)))| \\ &\quad (y|(y|(x|x))))|(z|z))), I_N(z)\} \\ &= \min\{I_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(z|z))), I_N(z)\}, \end{aligned}$$

and

$F_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned}
&\leq \max\{F_N((((y|(y|(x|x)))|(y|(y|(x|x))))|(z|z))|(((y|(y|(x|x)))| \\
&\quad (y|(y|(x|x))))|(z|z))), F_N(z)\} \\
&= \max\{F_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\
&\quad (x|(x|(y|y))))|(z|z))), F_N(z)\}.
\end{aligned}$$

Thereby, X_N is a neutrosophic $\mathcal{N}$-sub-implicative ideal of X. □

Lemma 4.7. *In X, the following property holds:*

$$((x|(x|(y|y)))|(x|(x|(y|y))))|(y|(x|x)) = x|(x|(y|y)),$$

for all $x, y \in X$.

Theorem 4.8. *Let for any $x, y \in X$ satisfies the condition $x|(x|(y|y)) = y|y$. Then every neutrosophic $\mathcal{N}$-ideal of X is a neutrosophic $\mathcal{N}$-sub-implicative ideal of X.*

Proof. Assume that X_N is a neutrosophic $\mathcal{N}$-ideal of X. It is obtained that $T_N(0) \leq T_N(x)$, $I_N(0) \geq I_N(x)$, $F_N(0) \leq F_N(x)$,

$T_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned}
&= T_N(x) \\
&\leq \max\{T_N((x|(z|z))|(x|(z|z))), T_N(z)\} \\
&= \max\{T_N((((y|(y|(x|x)))|(y|(y|(x|x))))|(z|z))|(((y|(y|(x|x)))| \\
&\quad (y|(y|(x|x))))|(z|z))), T_N(z)\} \\
&= \max\{T_N((((((x|(x|(y|y)))|(x|(x|(y|y))))|(y|(x|x)))|(((x|(x|(y|y)))| \\
&\quad (x|(x|(y|y))))|(y|(x|x)))|(z|z))|((((x|(x|(y|y)))|(x|(x|(y|y))))| \\
&\quad (y|(x|x)))|(((x|(x|(y|y)))|(x|(x|(y|y))))|(y|(x|x)))|(z|z))), T_N(z)\} \\
&= \max\{T_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\
&\quad (x|(x|(y|y))))|(z|z))), T_N(z)\},
\end{aligned}$$

$I_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned}
&= I_N(x) \\
&\geq \min\{I_N((x|(z|z))|(x|(z|z))), I_N(z)\} \\
&= \min\{I_N((((y|(y|(x|x)))|(y|(y|(x|x))))|(z|z))|(((y|(y|(x|x)))| \\
&\quad (y|(y|(x|x))))|(z|z))), I_N(z)\} \\
&= \min\{I_N((((((x|(x|(y|y)))|(x|(x|(y|y))))|(y|(x|x)))|(((x|(x|(y|y)))| \\
&\quad (x|(x|(y|y))))|(y|(x|x)))|(z|z))|((((x|(x|(y|y)))|(x|(x|(y|y))))| \\
&\quad (y|(x|x)))|(((x|(x|(y|y)))|(x|(x|(y|y))))|(y|(x|x)))|(z|z))), I_N(z)\} \\
&= \min\{I_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\
&\quad (x|(x|(y|y))))|(z|z))), I_N(z)\}
\end{aligned}$$

and

$$
\begin{aligned}
&F_N((y|(y|(x|x)))|(y|(y|(x|x))))\\
&\quad= F_N(x)\\
&\quad\leq \max\{F_N((x|(z|z))|(x|(z|z))), F_N(z)\}\\
&\quad= \max\{F_N((((y|(y|(x|x)))|(y|(y|(x|x))))|(z|z))|(((y|(y|(x|x)))|\\
&\qquad (y|(y|(x|x))))|(z|z))), F_N(z)\}\\
&\quad= \max\{F_N(((((x|(x|(y|y)))|(x|(x|(y|y))))|(y|(x|x)))|(((x|(x|(y|y)))|\\
&\qquad (x|(x|(y|y))))|(y|(x|x)))|(z|z))|((((x|(x|(y|y)))|(x|(x|(y|y))))|\\
&\qquad (y|(x|x)))|(((x|(x|(y|y)))|(x|(x|(y|y))))|(y|(x|x)))|(z|z))), F_N(z)\}\\
&\quad= \max\{F_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))|\\
&\qquad (x|(x|(y|y))))|(z|z))), F_N(z)\}.
\end{aligned}
$$

Hence, X_N is a neutrosophic $\mathcal{N}$-sub-implicative ideal of X. □

Theorem 4.9. *If X satisfying*

$$
(\forall x,y,z\in X)\left(\begin{array}{l}
T_N((y|(z|z))|(y|(z|z))) \leq T_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|\\
\qquad\qquad (((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z)))\\
I_N((y|(z|z))|(y|(z|z))) \geq I_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|\\
\qquad\qquad (((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z)))\\
F_N((y|(z|z))|(y|(z|z))) \leq F_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|\\
\qquad\qquad (((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z)))
\end{array}\right), \quad (5)
$$

then every neutrosophic $\mathcal{N}$-ideal of X is a neutrosophic $\mathcal{N}$-sub-implicative ideal of X.

Proof. It is obtained that

$$
\begin{aligned}
&T_N((y|(y|(x|x)))|(y|(y|(x|x))))\\
&\quad\leq T_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(y|(x|x)))|(((x|(x|(y|y)))|\\
&\qquad (x|(x|(y|y))))|(y|(x|x))))\\
&\quad= T_N((x|(x|(y|y)))|(x|(x|(y|y)))),
\end{aligned}
$$

$$
\begin{aligned}
&I_N((y|(y|(x|x)))|(y|(y|(x|x))))\\
&\quad\geq I_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(y|(x|x)))|(((x|(x|(y|y)))|\\
&\qquad (x|(x|(y|y))))|(y|(x|x))))\\
&\quad= I_N((x|(x|(y|y)))|(x|(x|(y|y))))
\end{aligned}
$$

and

$$
\begin{aligned}
&F_N((y|(y|(x|x)))|(y|(y|(x|x))))\\
&\quad\leq F_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(y|(x|x)))|(((x|(x|(y|y)))|\\
&\qquad (x|(x|(y|y))))|(y|(x|x))))\\
&\quad F\ \ ((x|(x|(y|y)))|(x|(x|(y|y))))
\end{aligned}
$$

Thus, X_N is a neutrosophic $\mathcal{N}$-sub-implicative ideal of X. $\square$

Theorem 4.10. *Let X_N be a neutrosophic $\mathcal{N}$-ideal of X. Then X_N is a neutrosophic $\mathcal{N}$-implicative ideal of X if and only if X_N satisfies the following condition:*

$$(\forall x, y \in X)\left(\begin{array}{l} T_N(x) \leq T_N((x|(y|(x|x)))|(x|(y|(x|x)))) \\ I_N(x) \geq I_N((x|(y|(x|x)))|(x|(y|(x|x)))) \\ F_N(x) \leq F_N((x|(y|(x|x)))|(x|(y|(x|x)))) \end{array}\right). \tag{6}$$

Proof. Let X_N be a neutrosophic $\mathcal{N}$-implicative ideal of X. Then

$$\begin{aligned} T_N(x) &\leq \max\{T_N(0), T_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(0|0))|(((x|(y|(x|x)))| \\ &\quad (x|(y|(x|x))))|(0|0)))\} \\ &= \max\{T_N(0), T_N((x|(y|(x|x)))|(x|(y|(x|x))))\} \\ &= T_N((x|(y|(x|x)))|(x|(y|(x|x)))), \end{aligned}$$

$$\begin{aligned} I_N(x) &\geq \min\{I_N(0), I_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(0|0))|(((x|(y|(x|x)))| \\ &\quad (x|(y|(x|x))))|(0|0)))\} \\ &= \min\{I_N(0), I_N((x|(y|(x|x)))|(x|(y|(x|x))))\} \\ &= I_N((x|(y|(x|x)))|(x|(y|(x|x)))) \end{aligned}$$

and

$$\begin{aligned} F_N(x) &\leq \max\{F_N(0), F_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(0|0))|(((x|(y|(x|x)))| \\ &\quad (x|(y|(x|x))))|(0|0)))\} \\ &= \max\{F_N(0), F_N((x|(y|(x|x)))|(x|(y|(x|x))))\} \\ &= F_N((x|(y|(x|x)))|(x|(y|(x|x)))), \end{aligned}$$

for all $x, y \in X$. Conversely, let X_N be a neutrosophic $\mathcal{N}$-ideal of X satisfying the inequality (6). Then it is clear that $T_N(0) \leq T_N(x)$, $I_N(0) \geq I_N(x)$ and $F_N(0) \leq F_N(x)$ for all $x \in X$. Since

$$\begin{aligned} T_N(x) &\leq T_N((x|(y|(x|x)))|(x|(y|(x|x)))) \\ &\leq \max\{T_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))|(((x|(y|(x|x)))| \\ &\quad (x|(y|(x|x))))|(z|z))), T_N(z)\}, \end{aligned}$$

$$\begin{aligned} I_N(x) &\geq I_N((x|(y|(x|x)))|(x|(y|(x|x)))) \\ &\geq \min\{I_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))|(((x|(y|(x|x)))| \\ &\quad (x|(y|(x|x))))|(z|z))), I_N(z)\} \end{aligned}$$

and

$$\begin{aligned} F_N(x) &\leq F_N((x|(y|(x|x)))|(x|(y|(x|x)))) \\ &\leq \max\{F_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))|(((x|(y|(x|x)))| \\ &\quad (x|(y|(x|x))))|(z|z))), F_N(z)\}, \end{aligned}$$

for all $x, y, z \in A$, we have X_N is a neutrosophic $\mathcal{N}$-implicative ideal of X. $\square$

Theorem 4.11. *If X satisfying*

$$(\forall x, y \in X)\left(\begin{array}{l} T_N((x|(x|(y|y)))|(x|(x|(y|y)))) \leq T_N((x|(y|(x|x)))|(x|(y|(x|x)))) \\ I_N((x|(x|(y|y)))|(x|(x|(y|y)))) \geq I_N((x|(y|(x|x)))|(x|(y|(x|x)))) \\ F_N((x|(x|(y|y)))|(x|(x|(y|y)))) \leq F_N((x|(y|(x|x)))|(x|(y|(x|x)))) \end{array}\right), \qquad (7)$$

then every neutrosophic $\mathcal{N}$-sub-implicative ideal of X is a neutrosophic $\mathcal{N}$-implicative ideal of X.

Proof. Let X_N be a neutrosophic $\mathcal{N}$-sub-implicative ideal of X satisfying the inequality (7). Then we obtain that

$$\begin{aligned} T_N(x) &= T_N((y|(y|(x|x)))|(y|(y|(x|x)))) \\ &\leq \max\{T_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(0|0))| \\ &\quad (((x|(x|(y|y)))|(x|(x|(y|y))))|(0|0))), T_N(0)\} \\ &= \max\{T_N((x|(x|(y|y)))|(x|(x|(y|y)))), T_N(0)\} \\ &= T_N((x|(x|(y|y)))|(x|(x|(y|y)))) \\ &\leq T_N((x|(y|(x|x)))|(x|(y|(x|x)))), \end{aligned}$$

$$\begin{aligned} I_N(x) &= I_N((y|(y|(x|x)))|(y|(y|(x|x)))) \\ &\geq \min\{I_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(0|0))| \\ &\quad (((x|(x|(y|y)))|(x|(x|(y|y))))|(0|0))), I_N(0)\} \\ &= \min\{I_N((x|(x|(y|y)))|(x|(x|(y|y)))), I_N(0)\} \\ &= I_N((x|(x|(y|y)))|(x|(x|(y|y)))) \\ &\geq I_N((x|(y|(x|x)))|(x|(y|(x|x)))) \end{aligned}$$

and

$$\begin{aligned} F_N(x) &= F_N((y|(y|(x|x)))|(y|(y|(x|x)))) \\ &\leq \max\{F_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(0|0))| \\ &\quad (((x|(x|(y|y)))|(x|(x|(y|y))))|(0|0))), F_N(0)\} \\ &= \max\{F_N((x|(x|(y|y)))|(x|(x|(y|y)))), F_N(0)\} \\ &= F_N((x|(x|(y|y)))|(x|(x|(y|y)))) \\ &\leq F_N((x|(y|(x|x)))|(x|(y|(x|x)))). \end{aligned}$$

Thus, X_N is a neutrosophic $\mathcal{N}$-implicative ideal of X. □

Theorem 4.12. *Let X be an implicative algebra. Then every neutrosophic $\mathcal{N}$-implicative ideal of X is a neutrosophic $\mathcal{N}$-sub-implicative ideal of X.*

Proof. Let X_N be a neutrosophic $\mathcal{N}$-implicative ideal of an implicative algebra X. Then X_N is a neutrosophic $\mathcal{N}$-ideal of X. So, it is obvious that $T_N(0) \leq T_N(x)$, $I_N(0) \geq I_N(x)$ and

$F_N(0) \leq F_N(x)$ for all $x \in X$. Thus

$T_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned} &= T_N((x|(x|(y|y)))|(x|(x|(y|y)))) \\ &\leq \max\{T_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(z|z))), T_N(z)\}, \end{aligned}$$

$I_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned} &= I_N((x|(x|(y|y)))|(x|(x|(y|y)))) \\ &\geq \min\{I_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(z|z))), I_N(z)\} \end{aligned}$$

and

$F_N((y|(y|(x|x)))|(y|(y|(x|x))))$

$$\begin{aligned} &= F_N((x|(x|(y|y)))|(x|(x|(y|y)))) \\ &\leq \max\{F_N((((x|(x|(y|y)))|(x|(x|(y|y))))|(z|z))|(((x|(x|(y|y)))| \\ &\quad (x|(x|(y|y))))|(z|z))), F_N(z)\}, \end{aligned}$$

for all $x, y, z \in X$. Hence X_N is a neutrosophic $\mathcal{N}$-sub-implicative ideal of X. □

Definition 4.13. A neutrosophic $\mathcal{N}$-ideal X_N of X is said to be neutrosophic $\mathcal{N}$-closed if

$$(\forall x \in X) \begin{pmatrix} T_N((0|(x|x))|(0|(x|x))) \leq T_N(x) \\ I_N((0|(x|x))|(0|(x|x))) \geq I_N(x) \\ F_N((0|(x|x))|(0|(x|x))) \leq F_N(x) \end{pmatrix}. \tag{8}$$

Definition 4.14. Let X_N be a neutrosophic $\mathcal{N}$-ideal of X. Then X_N is called a neutrosophic $\mathcal{N}$-completely closed ideal of X if

$$(\forall x, y \in X) \begin{pmatrix} T_N((x|(y|y))|(x|(y|y))) \leq \max\{T_N(x), T_N(y)\}, \\ I_N((x|(y|y))|(x|(y|y))) \geq \min\{I_N(x), I_N(y)\}, \\ F_N((x|(y|y))|(x|(y|y))) \leq \max\{F_N(x), F_N(y)\} \end{pmatrix}. \tag{9}$$

Theorem 4.15. *If X satisfying the following condition*

$$(\forall x, y \in X) \begin{pmatrix} ((((x|(y|y))|(x|(y|y)))|(x|(z|z)))|(((x|(y|y))|(x|(y|y)))| \\ (x|(z|z))))|(z|(y|y)) = 0|0. \end{pmatrix}, \tag{10}$$

then X is implicative if and only if every neutrosophic $\mathcal{N}$-closed ideal of X is a neutrosophic $\mathcal{N}$-implicative ideal of X.

Proof. Let X be satisfying (10). Assume that X is implicative and X_N is a neutrosophic $\mathcal{N}$-closed ideal of X. Then X_N is a neutrosophic $\mathcal{N}$-ideal of X. Thus, $T_N(0) \leq T_N(x)$,

$I_N(0) \geq I_N(x)$ and $F_N(0) \leq F_N(x)$. Also

$$\begin{aligned} T_N(x) &\leq \max\{T_N(z), T_N((x|(z|z))|(x|(z|z)))\} \\ &= \max\{T_N(z), T_N(((((x|x)|(x|x))|((x|x)|y))|(z|z))| \\ &\quad ((((x|x)|(x|x))|((x|x)|y))|(z|z)))\} \\ &= \max\{T_N(z), T_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))| \\ &\quad (((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z)))\}, \end{aligned}$$

$$\begin{aligned} I_N(x) &\geq \min\{I_N(z), I_N((x|(z|z))|(x|(z|z)))\} \\ &= \min\{I_N(z), I_N(((((x|x)|(x|x))|((x|x)|y))|(z|z))| \\ &\quad ((((x|x)|(x|x))|((x|x)|y))|(z|z)))\} \\ &= \min\{I_N(z), I_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))| \\ &\quad (((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z)))\} \end{aligned}$$

and

$$\begin{aligned} F_N(x) &\leq \max\{F_N(z), F_N((x|(z|z))|(x|(z|z)))\} \\ &= \max\{F_N(z), F_N(((((x|x)|(x|x))|((x|x)|y))|(z|z))| \\ &\quad ((((x|x)|(x|x))|((x|x)|y))|(z|z)))\} \\ &= \max\{F_N(z), F_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z))| \\ &\quad (((x|(y|(x|x)))|(x|(y|(x|x))))|(z|z)))\}, \end{aligned}$$

which means that X_N is a neutrosophic $\mathcal{N}$-implicative ideal of X. Conversely, suppose that every neutrosophic $\mathcal{N}$-closed ideal of X is a neutrosophic $\mathcal{N}$-implicative ideal of X. So, it follows from the equation (10) that $z|(y|y) = (x|(z|z))|((x|(y|y))|(x|(y|y)))$. Since $z|(y|y) = (x|(z|z))|((x|(y|y))|(x|(y|y))) = ((x|(x|(z|z)))|(x|(x|(z|z))))|(y|y)$, $z = (x|(x|(z|z)))|(x|(x|(z|z)))$. Thus,

$$\begin{aligned} x|(x|(y|y)) &= ((y|(y|(x|x)))|(y|(y|(x|x))))|(((y|(y|(x|x)))|(y|(y|(x|x))))|(y|y)) \\ &= ((y|(y|(x|x)))|(y|(y|(x|x))))|(y|(((y|y)|(y|(x|x)))|((y|y)|(y|(x|x))))) \\ &= ((y|(y|(x|x)))|(y|(y|(x|x))))|(y|(y|y)) \\ &= (((y|(y|y))|((y|y)|(y|y)))|((y|(y|y))|((y|y)|(y|y))))|(y|(x|x)) \\ &= y|(y|(x|x)), \end{aligned}$$

for all $x, y \in X$, which means that X is implicative. □

Proposition 4.16. *Let A be an implicative algebra satisfying the equation* (10). *Then every neutrosophic $\mathcal{N}$-completely closed ideal of A is a neutrosophic $\mathcal{N}$-implicative ideal of A.*

Proof. Let X_N be a neutrosophic $\mathcal{N}$-completely closed ideal of an implicative algebra X. Then X_N is a neutrosophic $\mathcal{N}$-ideal of X. Since

$$T_N((0|(y|y))|(0|(y|y))) \leq \max\{T_N(0), T_N(y)\} = T_N(y),$$

$$I_N((0|(y|y))|(0|(y|y))) \geq \min\{I_N(0), I_N(y)\} = I_N(y)$$

and

$$F_N((0|(y|y))|(0|(y|y))) \leq \max\{F_N(0), F_N(y)\} = F_N(y),$$

it is obtained that X_N is a neutrosophic $\mathcal{N}$-closed ideal of X. Therefore, X_N is a neutrosophic $\mathcal{N}$-implicative ideal of X. $\square$

Definition 4.17. A neutrosophic $\mathcal{N}$-structure X_N of X is called a neutrosophic $\mathcal{N}$-p-ideal of X if

$$(\forall x, y, z \in X) \left(\begin{array}{l} T_N(0) \leq T_N(x), I_N(0) \geq I_N(x), F_N(0) \leq F_N(x) \\ T_N(x) \leq \max\{T_N((((x|(z|z))|(x|(z|z)))|(y|(z|z)))| \\ \qquad\qquad (((x|(z|z))|(x|(z|z)))|(y|(z|z)))), T_N(y)\} \\ I_N(x) \geq \min\{I_N((((x|(z|z))|(x|(z|z)))|(y|(z|z)))| \\ \qquad\qquad (((x|(z|z))|(x|(z|z)))|(y|(z|z)))), I_N(y)\} \\ F_N(x) \leq \max\{F_N((((x|(z|z))|(x|(z|z)))|(y|(z|z)))| \\ \qquad\qquad (((x|(z|z))|(x|(z|z)))|(y|(z|z)))), F_N(y)\} \end{array} \right). \tag{11}$$

Definition 4.18. The subset $X^+ = \{x \in X : (0|(x|x))|(0|(x|x)) = 0\}$ of X is called the BCA-part of X.

Theorem 4.19. *If $X = X^+$, then every neutrosophic $\mathcal{N}$-p-ideal of X is a neutrosophic $\mathcal{N}$-implicative ideal of X.*

Proof. Let X_N be a neutrosophic $\mathcal{N}$-p-ideal of X. Then

$$\begin{array}{rcl} T_N(x) & \leq & \max\{T_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(0|(y|(x|x))))| \\ & & (((x|(y|(x|x)))|(x|(y|(x|x))))|(0|(y|(x|x))))), T_N(0)\} \\ & = & \max\{T_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(0|0))| \\ & & (((x|(y|(x|x)))|(x|(y|(x|x))))|(0|0))), T_N(0)\} \\ & = & \max\{T_N((x|(y|(x|x)))|(x|(y|(x|x)))), T_N(0)\}, \\ & = & T_N((x|(y|(x|x)))|(x|(y|(x|x)))), \end{array}$$

$$\begin{array}{rcl} I_N(x) & \geq & \min\{I_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(0|(y|(x|x))))| \\ & & (((x|(y|(x|x)))|(x|(y|(x|x))))|(0|(y|(x|x))))), I_N(0)\} \\ & = & \min\{I_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(0|0))| \\ & & (((x|(y|(x|x)))|(x|(y|(x|x))))|(0|0))), I_N(0)\} \\ & = & \min\{I_N((x|(y|(x|x)))|(x|(y|(x|x)))), I_N(0)\} \\ & = & I_N((x|(y|(x|x)))|(x|(y|(x|x)))), \end{array}$$

and

$$\begin{array}{rcl} F_N(x) & \leq & \max\{F_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(0|(y|(x|x))))| \\ & & (((x|(y|(x|x)))|(x|(y|(x|x))))|(0|(y|(x|x))))), F_N(0)\} \\ & = & \max\{F_N((((x|(y|(x|x)))|(x|(y|(x|x))))|(0|0))| \\ & & (((x|(y|(x|x)))|(x|(y|(x|x))))|(0|0))), F_N(0)\} \\ & = & \max\{F_N((x|(y|(x|x)))|(x|(y|(x|x)))), F_N(0)\} \\ & = & F_N((x|(y|(x|x)))|(x|(y|(x|x)))). \end{array}$$

Hence X_N is a neutrosophic $\mathcal{N}$-implicative ideal of X. □

5. **Homomorphism on Sheffer stroke BG-algebras**

Theorem 5.1. *Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be Sheffer stroke BG-algebras, $f : A \to B$ be a surjective homomorphism and $B_N = (T_N, I_N, F_N)$ be a neutrosophic $\mathcal{N}$-structure on B. Then B_N is a neutrosophic $\mathcal{N}$-ideal of B if and only if $B_N^f = (T_N^f, I_N^f, F_N^f)$ is a neutrosophic $\mathcal{N}$-ideal of A, where the $\mathcal{N}$-function $T_N^f, I_N^f, F_N^f \to [-1, 0]$ on A are defined by $T_N^f = T_N(f(x)), I_N^f = I_N(f(x)), F_N^f = F_N(f(x))$ for all $x \in A$, respectively.*

Proof. Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be Sheffer stroke BG-algebras, $f : A \to B$ be a surjective homomorphism and X_N be a neutrosophic $\mathcal{N}$-ideal of B. Let $x_1, x_2, x_3 \in A$. Then

$$T_N^\alpha(0_A) = T_N(\alpha(0_A)) \leq T_N(\alpha(x_1)) = T_N^\alpha(x_1),$$

$$I_N^\alpha(0_A) = I_N(\alpha(0_A)) \geq I_N(\alpha(x_1)) = I_N^\alpha(x_1)$$

and

$$F_N^\alpha(0_A) = F_N(\alpha(0_A)) \leq F_N(\alpha(x_1)) = F_N^\alpha(x_1),$$

$$\begin{array}{rcl} T_N^\alpha(x_1) & = & T_N(\alpha(x_1)) \\ & \leq & \max\{T_N(\alpha(x_2)), T_N((\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2))))\} \\ & = & \max\{T_N(\alpha(x_2)), I_N(\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))))\} \\ & = & \max\{T_N^\alpha(x_2), T_N^\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))\} \end{array}$$

and

$$\begin{array}{rcl} I_N^\alpha(x_1) & = & I_N(\alpha(x_1)) \\ & \geq & \min\{I_N(\alpha(x_2)), I_N((\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2))))\} \\ & = & \min\{I_N(\alpha(x_2)), I_N(\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))))\} \\ & = & \min\{I_N^\alpha(x_2), I_N^\alpha((x_1|_A(x_2|_Ax_1))|_A(x_1|_A(x_2|_Ax_2)))\}, \end{array}$$

$$\begin{array}{rcl} F_N^\alpha(x_1) & = & F_N(\alpha(x_1)) \\ & \leq & \max\{F_N(\alpha(x_2)), F_N((\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2))))\} \\ & = & \max\{F_N(\alpha(x_2)), F_N(\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))))\} \\ & = & \max\{F_N^\alpha(x_2), F_N^\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))\}. \end{array}$$

Hence f^{α} is a neutrosophic $\mathcal{N}$-ideal of A.

Conversely, let f^{α} be a neutrosophic $\mathcal{N}$-ideal of A. Let $y_1, y_2 \in B$ such that $\alpha(x_1) = y_1$ and $\alpha(x_2) = y_2$ for $x_1, x_2 \in A$. Then

$$T_N(0_B) = T_N(\alpha(0_A)) \leq T_N^{\alpha}(0_A) = T_N(\alpha(x_1)) = T_N(y_1),$$

$$I_N(0_B) = I_N(\alpha(0_A)) \geq I_N^{\alpha}(x_1) = I_N(\alpha(x_1)) = I_N(y_1)$$

and

$$F_N(0_B) = F_N(\alpha(0_A)) \leq F_N^{\alpha}(0_A) = F_N(\alpha(x_1)) = F_N(y_1),$$

Furthermore, we have

$$\begin{aligned}
T_N(y_1) &= T_N(\alpha(x_1)) \\
&= T_N^{\alpha}(x_1) \\
&\leq \max\{T_N^{\alpha}(x_2), T_N^{\alpha}((x_1|_A(x_2|_A x_2))|_A(x_1|_A(x_2|_A x_2)))\} \\
&= \max\{T_N(\alpha(x_2)), T_N(\alpha((x_1|_A(x_2|_A x_2))|_A(x_1|_A(x_2|_A x_2))))\} \\
&= \max\{T_N(\alpha(x_2)), T_N((\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2))))\} \\
&= \max\{T_N(y_2), T_N((y_1|_B(y_2|_B y_2))|_B(y_1|_B(y_2|_B y_2)))\},
\end{aligned}$$

$$\begin{aligned}
I_N(y_1) &= I_N(\alpha(x_1)) \\
&= I_N^{\alpha}(x_1) \\
&\geq \min\{I_N^{\alpha}(x_2), I_N^{\alpha}((x_1|_A(x_2|_A x_2))|_A(x_1|_A(x_2|_A x_2)))\} \\
&= \min\{I_N(\alpha(x_2)), I_N(\alpha((x_1|_A(x_2|_A x_2))|_A(x_1|_A(x_2|_A x_2))))\} \\
&= \min\{I_N(\alpha(x_2)), I_N((\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2))))\} \\
&= \min\{I_N(y_2), I_N((y_1|_B(y_2|_B y_2))|_B(y_1|_B(y_2|_B y_2)))\}
\end{aligned}$$

and

$$\begin{aligned}
F_N(y_1) &= F_N(\alpha(x_1)) \\
&= F_N^{\alpha}(x_1) \\
&\leq \max\{F_N^{\alpha}(x_2), F_N^{\alpha}((x_1|_A(x_2|_A x_2))|_A(x_1|_A(x_2|_A x_2)))\} \\
&= \max\{F_N(\alpha(x_2)), F_N(\alpha((x_1|_A(x_2|_A x_2))|_A(x_1|_A(x_2|_A x_2))))\} \\
&= \max\{F_N(\alpha(x_2)), F_N((\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2))))\} \\
&= \max\{F_N(y_2), F_N((y_1|_B(y_2|_B y_2))|_B(y_1|_B(y_2|_B y_2)))\}.
\end{aligned}$$

Hence X_N is a neutrosophic $\mathcal{N}$-ideal of B. □

Theorem 5.2. *Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be Sheffer stroke BG-algebras, $f : A \to B$ be a surjective homomorphism and X_N be a neutrosophic $\mathcal{N}$-structure on B. Then X_N is a neutrosophic $\mathcal{N}$-subalgebra of B if and only if f^{α} is a neutrosophic $\mathcal{N}$-subalgebra of A, where $f^{\alpha} \to [-1, 0]$ on A are defined by $f^{\alpha} = f(\alpha(x))$ for all $x \in A$, respectively.*

Proof. Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be Sheffer stroke BG-algebras, $f : A \to B$ be a surjective homomorphism and X_N be a neutrosophic $\mathcal{N}$-subalgebra of B. Let $x_1, x_2 \in A$. Then

$$\begin{aligned}
&T_N^{\alpha}((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))) \\
&\quad= T_N(\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))) \\
&\quad= T_N(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))) \\
&\quad\leq \max\{T_N(\alpha(x_1)), T_N(\alpha(x_2))\} \\
&\quad= \max\{T_N^{\alpha}(x_1), T_N^{\alpha}(x_2)\},
\end{aligned}$$

$$\begin{aligned}
&I_N^{\alpha}((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))) \\
&\quad= I_N(\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))) \\
&\quad= I_N(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))) \\
&\quad\geq \min\{I_N(\alpha(x_1)), I_N(\alpha(x_2))\} \\
&\quad= \min\{I_N^{\alpha}(x_1), I_N^{\alpha}(x_2)\}
\end{aligned}$$

and

$$\begin{aligned}
&F_N^{\alpha}((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))) \\
&\quad= F_N(\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))) \\
&\quad= F_N(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))) \\
&\quad\leq \max\{F_N(\alpha(x_1)), F_N(\alpha(x_2))\} \\
&\quad= \max\{F_N^{\alpha}(x_1), F_N^{\alpha}(x_2)\}.
\end{aligned}$$

Hence f^{α} is a neutrosophic $\mathcal{N}$-subalgebra of A.

Conversely, let f^f be a neutrosophic $\mathcal{N}$-subalgebra of A. Let $y_1, y_2 \in B$ such that $\alpha(x_1) = y_1$ and $\alpha(x_2) = y_2$ for $x_1, x_2 \in A$. Then

$$\begin{aligned}
&T_N((y_1|_B(y_2|_By_2))|_B(y_1|_B(y_2|_By_2))) \\
&\quad= T_N((\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))) \\
&\quad= T_N(\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))) \\
&\quad= T_N^{\alpha}((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))) \\
&\quad\leq \max\{T_N^{\alpha}(x_1), T_N^{\alpha}(x_2)\} \\
&\quad= \max\{T_N(\alpha(x_1)), T_N(\alpha(x_2))\} \\
&\quad= \max\{T_N(y_1), T_N(y_2)\},
\end{aligned}$$

$$\begin{aligned}
&I_N((y_1|_B(y_2|_By_2))|_B(y_1|_B(y_2|_By_2))) \\
&\quad= I_N((\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))) \\
&\quad= I_N(\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))) \\
&\quad= I_N^{\alpha}((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))) \\
&\quad\geq \min\{I_N^{\alpha}(x_1), I_N^{\alpha}(x_2)\} \\
&\quad= \min\{I_N(\alpha(x_1)), I_N(\alpha(x_2))\} \\
&\quad= \min\{I_N(y_1), I_N(y_2)\}
\end{aligned}$$

and

$$
\begin{aligned}
&F_N((y_1|_B(y_2|_By_2))|_B(y_1|_B(y_2|_By_2))) \\
&\quad = F_N((\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))|_B(\alpha(x_1)|_B(\alpha(x_2)|_B\alpha(x_2)))) \\
&\quad = F_N(\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2)))) \\
&\quad = F_N^\alpha((x_1|_A(x_2|_Ax_2))|_A(x_1|_A(x_2|_Ax_2))) \\
&\quad \leq \max\{F_N^\alpha(x_1), F_N^\alpha(x_2)\} \\
&\quad = \max\{F_N(\alpha(x_1)), F_N(\alpha(x_2))\} \\
&\quad = \max\{F_N(y_1), F_N(y_2)\}.
\end{aligned}
$$

Hence X_N is a neutrosophic $\mathcal{N}$-subalgebra of B. □

Definition 5.3. Let X be a Sheffer stroke BG-algebra. We define

$$X_N^{x_t} = \{x \in X : T_N(x) \leq T_N(x_t)\},$$

$$X_N^{x_i} = \{x \in X : I_N(x) \geq I_N(x_t)\},$$

$$X_N^{x_f} = \{x \in X : F_N(x) \leq F_N(x_t)\},$$

for all $x_t, x_i, x_f \in X$. Obviously, $x_t \in X_N^{x_t}$, $x_i \in X_N^{x_i}$ and $x_f \in X_N^{x_f}$.

Theorem 5.4. *If X_N is a neutrosophic $\mathcal{N}$-ideal of X, then $X_N^{x_t}$, $X_N^{x_i}$ and $X_N^{x_f}$ are ideals of X, where $x_t, x_i, x_f \in X$.*

Proof. Assume that X_N is a neutrosophic $\mathcal{N}$-ideal of X. Let $y_1 \in X_N^{x_t}$, $y_2 \in X_N^{x_i}$ and $y_3 \in X_N^{x_f}$. Hence $T_N(y_1) \leq T_N(x_t)$, $I_N(y_2) \geq I_N(x_t)$ and $F_N(y_3) \leq F_N(x_t)$. This shows that $T_N((y_1|(x_1|x_1))|(y_1|(x_1|x_1))) \leq T_N(y_1) \leq T_N(x_t)$, $I_N((y_2|(x_2|x_2))|(y_2|(x_2|x_2))) \geq I_N(y_2) \geq I_N(x_t)$ and $F_N((y_3|(x_3|x_3))|(y_3|(x_3|x_3))) \leq F_N(y_3) \leq F_N(x_t)$. Thus, $(y_1|(x_1|x_1))|(y_1|(x_1|x_1)) \in T_N(x_t)$, $(y_2|(x_2|x_2))|(y_2|(x_2|x_2)) \in I_N(x_i)$ and $(y_3|(x_3|x_3))|(y_3|(x_3|x_3)) \in F_N(x_f)$. Let $x_1, (y_1|(x_1|x_1))|(y_1|(x_1|x_1)) \in X_N^{x_t}$, $x_2, (y_2|(x_2|x_2))|(y_2|(x_2|x_2)) \in X_N^{x_i}$ and $x_3, (y_3|(x_3|x_3))|(y_3|(x_3|x_3)) \in X_N^{x_f}$. Then $T_N(x_1) \leq T_N(x_t)$, $T_N((y_1|(x_1|x_1))|(y_1|(x_1|x_1))) \leq T_N(x_t)$, $I_N(x_2) \geq I_N(x_i)$, $I_N((y_2|(x_2|x_2))|(y_2|(x_2|x_2))) \geq I_N(x_t)$, $F_N(x_3) \leq F_N(x_f)$ and $F_N((y_3|(x_3|x_3))|(y_3|(x_3|x_3))) \leq F_N(x_t)$. Then

$$T_N(y_1) \leq \max\{T_N(x_1), T_N((y_1|(x_1|x_1))|(y_1|(x_1|x_1)))\} \leq T_N(x_t),$$

$$I_N(y_2) \geq \min\{I_N(x_2), I_N((y_2|(x_2|x_2))|(y_2|(x_2|x_2)))\} \geq I_N(x_i)$$

and

$$F_N(y_3) \leq \max\{F_N(x_3), F_N((y_3|(x_3|x_3))|(y_3|(x_3|x_3)))\} \leq F_N(x_t).$$

Hence $y_1 \in T_N(x_t)$, $y_2 \in I_N(x_i)$ and $y_3 \in F_N(x_f)$. Therefore, $X_N^{x_t}$, $X_N^{x_i}$ and $X_N^{x_f}$ are ideals of X. □

Theorem 5.5. *If X_N is a neutrosophic $\mathcal{N}$-subalgebra of X, then $X_N^{x_t}$, $X_N^{x_i}$ and $X_N^{x_f}$ are subalgebras of X, where $x_t, x_i, x_f \in X$.*

Proof. Let X_N be a neutrosophic $\mathcal{N}$-subalgebra of X. Let $x_1, y_1 \in X_N^{x_t}$, $x_2, y_2 \in X_N^{x_i}$ and $x_3, y_3 \in X_N^{x_f}$. Then $T_N(x_1) \leq T_N(x_t)$, $T_N(y_1) \leq T_N(x_t)$, $I_N(y_1) \geq I_N(x_t)$, $I_N(y_2) \geq I_N(x_t)$, $F_N(x_3) \leq F_N(x_t)$ and $F_N(y_3) \leq F_N(x_t)$. Then

$$T_N(x_1|(y_1|y_1))|(x_1|(y_1|y_1))) \leq \max\{T_N(x_1), T_N(y_1)\} \leq T_N(x_t),$$

$$I_N(x_2|(y_2|y_2))|(x_2|(y_2|y_2))) \geq \min\{I_N(x_2), I_N(y_2)\} \geq I_N(x_t)$$

and

$$F_N(x_3|(y_3|y_3))|(x_3|(y_3|y_3))) \leq \max\{F_N(x_3), F_N(y_3)\} \leq F_N(x_t).$$

It follows that $(x_1|(y_1|y_1))|(x_1|(y_1|y_1)) \in T_N(x_t)$, $(x_2|(y_2|y_2))|(x_2|(y_2|y_2)) \in I_N(x_i)$ and $(x_3|(y_3|y_3))|(x_3|(y_3|y_3)) \in F_N(x_f)$. Therefore, $X_N^{x_t}$, $X_N^{x_i}$ and $X_N^{x_f}$ are subalgebras of X. □

Theorem 5.6. *Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be Sheffer stroke BG-algebras. If X_{A_N} and X_{B_N} are neutrosophic $\mathcal{N}$-subalgebra of $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$, respectively, then $X_{A\times B_N}$ is a neutrosophic $\mathcal{N}$-subalgebra of $\langle A\times B, |_{A\times B}, 0_{A\times B}\rangle$.*

Proof. Let $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$ be Sheffer stroke BG-algebras and X_{A_N} and X_{B_N} be neutrosophic $\mathcal{N}$-subalgebra of $\langle A, |_A, 0_A\rangle$ and $\langle B, |_B, 0_B\rangle$, respectively. Let $(a_1, b_1), (a_2, b_2) \in A\times B$. Then

$$\begin{aligned}
&T_{A\times B_N}(((a_1,b_1)|_{A\times B}((a_2,b_2)|_{A\times B}(a_2,b_2)))|_{A\times B}((a_1,b_1)|_{A\times B}((a_2,b_2)|_{A\times B}(a_2,b_2))))\\
&= T_{A\times B_N}((a_1|_A(a_2|_Aa_2))|_A(a_1|_A(a_2|_Aa_2)), (b_1|_B(b_2|_Bb_2))|_B(b_1|_B(b_2|_Bb_2)))\\
&= \max\{T_{A_N}((a_1|_A(a_2|_Aa_2))|_A(a_1|_A(a_2|_Aa_2)), T_{B_N}(b_1|_B(b_2|_Bb_2))|_B(b_1|_B(b_2|_Bb_2)))\}\\
&\leq \max\{\max\{T_{A_N}(a_1), T_{A_N}(a_2)\}, \max\{T_{B_N}(b_1), T_{B_N}(b_2)\}\}\\
&= \max\{\max\{T_{A_N}(a_1), T_{B_N}(b_1)\}, \max\{T_{A_N}(a_2), T_{B_N}(b_2)\}\}\\
&= \max\{T_{A\times B_N}(a_1,b_1), T_{A\times B_N}(a_2,b_2)\},
\end{aligned}$$

$$\begin{aligned}
&I_{A\times B_N}(((a_1,b_1)|_{A\times B}((a_2,b_2)|_{A\times B}(a_2,b_2)))|_{A\times B}((a_1,b_1)|_{A\times B}((a_2,b_2)|_{A\times B}(a_2,b_2))))\\
&= I_{A\times B_N}((a_1|_A(a_2|_Aa_2))|_A(a_1|_A(a_2|_Aa_2)), (b_1|_B(b_2|_Bb_2))|_B(b_1|_B(b_2|_Bb_2)))\\
&= \max\{I_{A_N}((a_1|_A(a_2|_Aa_2))|_A(a_1|_A(a_2|_Aa_2)), T_{B_N}(b_1|_B(b_2|_Bb_2))|_B(b_1|_B(b_2|_Bb_2)))\}\\
&\geq \max\{\min\{I_{A_N}(a_1), I_{A_N}(a_2)\}, \min\{I_{B_N}(b_1), I_{B_N}(b_2)\}\}\\
&= \max\{\min\{I_{A_N}(a_1), I_{B_N}(b_1)\}, \min\{I_{A_N}(a_2), i_{B_N}(b_2)\}\}\\
&= \max\{I_{A\times B_N}(a_1,b_1), i_{A\times B_N}(a_2,b_2)\}
\end{aligned}$$

and

$$
\begin{aligned}
&F_{A\times B_N}(((a_1,b_1)|_{A\times B}((a_2,b_2)|_{A\times B}(a_2,b_2)))|_{A\times B}((a_1,b_1)|_{A\times B}((a_2,b_2)|_{A\times B}(a_2,b_2))))\\
&\quad = F_{A\times B_N}((a_1|_A(a_2|_Aa_2))|_A(a_1|_A(a_2|_Aa_2)),(b_1|_B(b_2|_Bb_2))|_B(b_1|_B(b_2|_Bb_2)))\\
&\quad = \max\{F_{A_N}((a_1|_A(a_2|_Aa_2))|_A(a_1|_A(a_2|_Aa_2))),F_{B_N}(b_1|_B(b_2|_Bb_2))|_B(b_1|_B(b_2|_Bb_2)))\}\\
&\quad \leq \max\{\max\{F_{A_N}(a_1),F_{A_N}(a_2)\},\max\{F_{B_N}(b_1),F_{B_N}(b_2)\}\}\\
&\quad = \max\{\max\{F_{A_N}(a_1),F_{B_N}(b_1)\},\max\{F_{A_N}(a_2),F_{B_N}(b_2)\}\}\\
&\quad = \max\{F_{A\times B_N}(a_1,b_1),F_{A\times B_N}(a_2,b_2)\}.
\end{aligned}
$$

Hence $X_{A\times B_N}$ is a neutrosophic $\mathcal{N}$-subalgebra of $\langle A\times B,|_{A\times B},0_{A\times B}\rangle$. □

Theorem 5.7. *Let $\langle A,|_A,0_A\rangle$ and $\langle B,|_B,0_B\rangle$ be Sheffer stroke BG-algebras. If X_{A_N} and X_{B_N} are neutrosophic $\mathcal{N}$-ideal of $\langle A,|_A,0_A\rangle$ and $\langle B,|_B,0_B\rangle$, respectively, then $X_{A\times B_N}$ is a neutrosophic $\mathcal{N}$-ideal of $\langle A\times B,|_{A\times B},0_{A\times B}\rangle$.*

Proof. Let $\langle A,|_A,0_A\rangle$ and $\langle B,|_B,0_B\rangle$ be Sheffer stroke BG-algebras and X_{A_N} and X_{B_N} be neutrosophic $\mathcal{N}$-ideal of $\langle A,|_A,0_A\rangle$ and $\langle B,|_B,0_B\rangle$, respectively. Let $(a_1,b_1),(a_2,b_2)\in A\times B$. Then

$$
\begin{aligned}
&T_{A\times B_N}(((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1)))|_{A\times B}((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1))))\\
&\quad = \max\{T_{A_N}((a_2|_A(a_1|_Aa_1))|_A(a_2|_A(a_1|_Aa_1))),T_{B_N}((b_2|_B(b_1|_Bb_1))|_B(b_2|_B(b_1|_Bb_1)))\}\\
&\quad \leq \max\{T_{A_N}(a_2),T_{B_N}(b_2)\}\\
&\quad = T_{A\times B_N}(a_2,b_2),
\end{aligned}
$$

$$
\begin{aligned}
&I_{A\times B_N}(((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1)))|_{A\times B}((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1))))\\
&\quad = \max\{I_{A_N}((a_2|_A(a_1|_Aa_1))|_A(a_2|_A(a_1|_Aa_1))),I_{B_N}((b_2|_B(b_1|_Bb_1))|_B(b_2|_B(b_1|_Bb_1)))\}\\
&\quad \geq \max\{I_{A_N}(a_2),I_{B_N}(b_2)\}\\
&\quad = I_{A\times B_N}(a_2,b_2),
\end{aligned}
$$

$$
\begin{aligned}
&F_{A\times B_N}(((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1)))|_{A\times B}((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1))))\\
&\quad = \max\{F_{A_N}((a_2|_A(a_1|_Aa_1))|_A(a_2|_A(a_1|_Aa_1))),F_{B_N}((b_2|_B(b_1|_Bb_1))|_B(b_2|_B(b_1|_Bb_1)))\}\\
&\quad \leq \max\{F_{A_N}(a_2),F_{B_N}(b_2)\}\\
&\quad = F_{A\times B_N}(a_2,b_2),
\end{aligned}
$$

$$
\begin{aligned}
&T_{A\times B_N}(a_2,b_2)\\
&\quad = \max\{T_{A_N}(a_2),T_{B_N}(b_2)\}\\
&\quad \leq \max\{\max\{T_{A_N}(a_1),T_{A_N}((a_2|_A(a_1|_Aa_1))|_A(a_2|_A(a_1|_Aa_1)))\},\\
&\qquad \max\{T_{B_N}(b_1),T_{B_N}((b_2|_B(b_1|_Bb_1))|_B(b_2|_B(b_1|_Bb_1)))\}\}\\
&\quad = \max\{\max\{T_{A_N}(a_1),T_{B_N}(b_1)\},\max\{T_{A_N}((a_2|_A(a_1|_Aa_1))|_A(a_2|_A(a_1|_Aa_1))),\\
&\qquad T_{B_N}((b_2|_B(b_1|_Bb_1))|_B(b_2|_B(b_1|_Bb_1)))\}\}\\
&\quad = \max\{T_{A\times B_N}(a_1,b_1),\\
&\qquad T_{A\times B_N}(((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1)))|_{A\times B}((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1))))\},
\end{aligned}
$$

$$
\begin{aligned}
&I_{A\times B_N}(a_2,b_2)\\
&\quad= \max\{I_{A_N}(a_2), I_{B_N}(b_2)\}\\
&\quad\geq \max\{\min\{I_{A_N}(a_1), I_{A_N}((a_2|_A(a_1|_Aa_1))|_A(a_2|_A(a_1|_Aa_1)))\},\\
&\qquad \min\{I_{B_N}(b_1), I_{B_N}((b_2|_B(b_1|_Bb_1))|_B(b_2|_B(b_1|_Bb_1)))\}\}\\
&\quad= \max\{\min\{I_{A_N}(a_1), I_{B_N}(b_1)\}, \min\{I_{A_N}((a_2|_A(a_1|_Aa_1))|_A(a_2|_A(a_1|_Aa_1))),\\
&\qquad I_{B_N}((b_2|_B(b_1|_Bb_1))|_B(b_2|_B(b_1|_Bb_1)))\}\}\\
&\quad= \max\{I_{A\times B_N}(a_1,b_1),\\
&\qquad I_{A\times B_N}(((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1)))|_{A\times B}((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1))))\}
\end{aligned}
$$

and

$$
\begin{aligned}
&F_{A\times B_N}(a_2,b_2)\\
&\quad= \max\{F_{A_N}(a_2), F_{B_N}(b_2)\}\\
&\quad\leq \max\{\max\{F_{A_N}(a_1), F_{A_N}((a_2|_A(a_1|_Aa_1))|_A(a_2|_A(a_1|_Aa_1)))\},\\
&\qquad \max\{F_{B_N}(b_1), F_{B_N}((b_2|_B(b_1|_Bb_1))|_B(b_2|_B(b_1|_Bb_1)))\}\}\\
&\quad= \max\{\max\{F_{A_N}(a_1), F_{B_N}(b_1)\}, \max\{F_{A_N}((a_2|_A(a_1|_Aa_1))|_A(a_2|_A(a_1|_Aa_1))),\\
&\qquad F_{B_N}((b_2|_B(b_1|_Bb_1))|_B(b_2|_B(b_1|_Bb_1)))\}\}\\
&\quad= \max\{F_{A\times B_N}(a_1,b_1),\\
&\qquad F_{A\times B_N}(((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1)))|_{A\times B}((a_2,b_2)|_{A\times B}((a_1,b_1)|_{A\times B}(a_1,b_1))))\}.
\end{aligned}
$$

Hence $X_{A\times B_N}$ is a neutrosophic $\mathcal{N}$-ideal of $\langle A\times B, |_{A\times B}, 0_{A\times B}\rangle$. □

6. Conclusion

In this study, we introduced and examined the concepts of neutrosophic $\mathcal{N}$-subalgebras and level sets within the framework of Sheffer stroke BG-algebras. We demonstrated that there exists a close relationship between subalgebras and level sets in Sheffer stroke BG-algebras, proving that a level set of a neutrosophic $\mathcal{N}$-subalgebra is itself a subalgebra, and conversely, any subalgebra can be viewed as a level set. This connection emphasizes the structural consistency within this algebraic setting and provides a foundation for further exploration of neutrosophic logic in algebraic systems.

Additionally, our results established that all neutrosophic $\mathcal{N}$-subalgebras of a Sheffer stroke BG-algebra form a complete distributive lattice, which supports systematic analysis and applications in logical and computational models. We also defined neutrosophic $\mathcal{N}$-ideals within this framework and highlighted their unique properties, showing that while every neutrosophic $\mathcal{N}$-ideal is a neutrosophic $\mathcal{N}$-subalgebra, the reverse does not necessarily hold. This insight reveals the specific behaviors and structural distinctions of $\mathcal{N}$-ideals, contributing to a more nuanced understanding of Sheffer stroke BG-algebras.

Building on this foundation, future studies could explore the practical applications of Sheffer stroke BG-algebras in fields such as quantum computing and artificial intelligence, where logical operations are crucial. Additionally, the incorporation of advanced neutrosophic structures, such as neutrosophic $\mathcal{N}$-implicative ideals and neutrosophic $\mathcal{N}$-closed ideals, could provide richer frameworks for modeling uncertainty and partial truth in computational systems. Another promising direction is the development of algorithms and decision-making frameworks that utilize the unique properties of Sheffer stroke operations, potentially opening new avenues in computational logic and information processing.

This study not only advances the theoretical understanding of neutrosophic structures in Sheffer stroke BG-algebras but also sets the stage for diverse applications and further theoretical development in algebraic logic and computational fields.

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