



## Neutrosophic $g^*$ -Closed Sets and its maps

A.Atkinswestley<sup>1</sup> and S.Chandrasekar<sup>2\*</sup>

<sup>1</sup>Department of Mathematics, Roever College of Engineering and Technology, Elambalur,Perambalur(DT), Tamil Nadu, India

E-mail: ats.wesly@gmail.com

<sup>2</sup>Department of Mathematics, Arignar Anna Government Arts College, Namakkal(DT),Tamil Nadu, India. E-mail:

chandrumat@gmail.com.

\*Correspondence: chandrumat@gmail.com;

**Abstract:** Topology is one of the classical subjects in Mathematics. A lot of researchers have published their ideas.As a generalization of topological concepts many new kind of closed and open sets are published continuously. Salama presented Neutrosophic topological spaces by using Smarandache 's Neutrosophic sets. Many Researchers introduced so many closed sets in Neutrosophic topological spaces. Purpose of this research paper is we introduce Neutrosophic  $g^*$ -Closed sets and Neutrosophic $g^*$ -open sets in Neutrosophic topological spaces. Also we study about study about mappings of Neutrosophic  $g^*$ -Closed sets

**Keywords:** Neutrosophic  $g$ -Closed sets Neutrosophic  $g^*$ -Closed sets, Neutrosophic $g^*$ -open sets, Neutrosophic  $g^*$ -continuous.

### 1. Introduction

Smarandache [10,11] characterized the Neutrosophic set on three segment Neutrosophic sets(T Truth, I-Indeterminacy, F-Falsehood). Neutrosophic topological spaces(NS-T-S) presented by Salama [19,20]et al. Neutrosophic have wide scope of constant applications for the fields of Electrical & Electronic, Artificial Intelligence, Mechanics, Computer Science, Information Systems, Applied Mathematics , basic leadership. Prescription and Management Science and so on.

Neutrosophic semi closed,  $\alpha$ - closed, pre closed and regular closed sets are introduced by I. Arokiarani[6] et al.,R.Dhavaseelan[8] et al. introduced Neutrosophic  $g$  closed sets and  $g\alpha$  closed sets .Point of this paper is R .Dhavaseelan[9] and S.Jafari, are introduced Generalized Neutrosophic Closed sets . D.Jayanthi [13]presented  $\alpha G$  Closed Sets in Neutrosophic Topological Spaces, V.K.Shanthi [22] developed Neutrosophic  $gs$  and  $sg$  closed set. C.Mahesawri[14,15] et al introduced Neutrosophic  $gb$  closed sets.

Aim of this present paper is, we introduce and study the concepts of Neutrosophic  $g^*$ -Closed sets and Neutrosophic  $g^*$ -open sets in Neutrosophic topological spaces. Also we study about mappings of Neutrosophic $g^*$ -Closed sets

### 2. Preliminaries

In this section, we recall required and necessary definition and results of Neutrosophic sets

**Definition 2.1 [16,17]** Let  $Nu_X^*$  be a non-empty fixed set. A Neutrosophic set  $W_1^*$  is a object having the form  $W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$ ,

$\mu_{W_1^*}(w)$ - membership function

$\sigma_{W_1^*}(w)$ - Indeterminacy function

$\gamma_{W_1^*}(w)$ - Non-Membership function

**Definition 2.2 [16,17].** Neutrosophic set  $W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$ , on  $Nu_X^*$  and  $\forall w \in Nu_X^*$  then complement of  $W_1^*$  is

$$W_1^{*C} = \{ \langle w, \gamma_{W_1^*}(w), 1 - \sigma_{W_1^*}(w), \mu_{W_1^*}(w) \rangle : w \in Nu_X^* \}$$

**Definition 2.3 [16,17].** Let  $W_1^*$  and  $W_2^*$  are two Neutrosophic sets,  $\forall w \in Nu_X^*$

$$W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$$

$$W_2^* = \{ \langle w, \mu_{W_2^*}(w), \sigma_{W_2^*}(w), \gamma_{W_2^*}(w) \rangle : w \in Nu_X^* \}$$

Then  $W_1^* \subseteq W_2^* \Leftrightarrow \mu_{W_1^*}(w) \leq \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \leq \sigma_{W_2^*}(w) \ \& \ \gamma_{W_1^*}(w) \geq \gamma_{W_2^*}(w)$

**Definition 2.4[16,17].** Let  $Nu_X^*$  be a non-empty set, and Let  $W_1^*$  and  $W_2^*$  be two Neutrosophic sets are

$$W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}, W_2^* = \{ \langle w, \mu_{W_2^*}(w), \sigma_{W_2^*}(w), \gamma_{W_2^*}(w) \rangle : w \in Nu_X^* \}$$

Then  $W_1^* \cap W_2^* = \{ \langle w, \mu_{W_1^*}(w) \cap \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \cap \sigma_{W_2^*}(w), \gamma_{W_1^*}(w) \cup \gamma_{W_2^*}(w) \rangle : w \in Nu_X^* \}$

$$W_1^* \cup W_2^* = \{ \langle w, \mu_{W_1^*}(w) \cup \mu_{W_2^*}(w), \sigma_{W_1^*}(w) \cup \sigma_{W_2^*}(w), \gamma_{W_1^*}(w) \cap \gamma_{W_2^*}(w) \rangle : w \in Nu_X^* \}$$

**Definition 2.5 [19,20].** Let  $Nu_X^*$  be non-empty set and  $Nu_\tau$  be the collection of Neutrosophic subsets of  $Nu_X^*$  satisfying the accompanying properties:

1.  $0_{Nu}, 1_{Nu} \in Nu_\tau$
2.  $Nu_{T_1} \cap Nu_{T_2} \in Nu_\tau$  for any  $Nu_{T_1}, Nu_{T_2} \in Nu_\tau$
3.  $\cup Nu_{T_i} \in Nu_\tau$  for every  $\{Nu_{T_i} : i \in j\} \subseteq Nu_\tau$

Then the space  $(Nu_X^*, Nu_\tau)$ , is called a Neutrosophic topological space(NS-T-S) The component of  $Nu_\tau$  are called Nu-OS (Neutrosophic open set)and its complement is Nu-CS(Neutrosophic closed set)

**Example 2.6.** Let  $Nu_X^* = \{w\}$  and  $\forall w \in Nu_X^*, W_1^* = \langle w, \frac{6}{10}, \frac{6}{10}, \frac{5}{10} \rangle, W_2^* = \langle w, \frac{5}{10}, \frac{7}{10}, \frac{9}{10} \rangle$

$W_3^* = \langle w, \frac{6}{10}, \frac{7}{10}, \frac{5}{10} \rangle, W_4^* = \langle w, \frac{5}{10}, \frac{6}{10}, \frac{9}{10} \rangle$  Then the collection  $Nu_\tau = \{0_{Nu}, W_1^*, W_2^*, W_3^*, W_4^*, 1_{Nu}\}$  is called a NS-T-S on  $Nu_X^*$ .

**Definition 2.7.** Let  $(Nu_X^*, Nu_\tau)$ , be a NS-T-S

and  $W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$  be a Neutrosophic set in  $Nu_X^*$ . Then  $W_1^*$  is said to be

- [1] Neutrosophic  $\alpha$ -closed set [6] (Nu –  $\alpha$ CS in short)  $Nu-cl(Nu-in(Nu-cl(W_1^*))) \subseteq W_1^*$ ,
- [2] Neutrosophic pre-closed set [22] (Nu-PCS in short)  $Nu-cl(Nu-in(W_1^*)) \subseteq W_1^*$ ,
- [3] Neutrosophic regular closed set [6] (Nu -RCS in short)  $Nu-cl(Nu-in(W_1^*)) = W_1^*$ ,
- [4] Neutrosophic semi closed set [7] (Nu-SCS in short)  $Nu-in(Nu-cl(W_1^*)) \subseteq W_1^*$ ,
- [5] Neutrosophic generalized closed set [4] (Nu -GCS in short)  $Nu-cl(W_1^* \subseteq \mathcal{H}$  whenever  $W_1^* \subseteq \mathcal{H}$  and  $\mathcal{H}$  is a Nu -OS, in  $Nu_X^*$
- [6] Neutrosophic  $\alpha$  generalized closed set [13] (Nu - ( $\alpha$ G) CS in short)  $Nu\alpha cl(W_1^*) \subseteq \mathcal{H}$  whenever  $W_1^* \subseteq \mathcal{H}$  And  $\mathcal{H}$  is a Nu-OS, in  $Nu_X^*$
- [7] Neutrosophic generalized semi closed set [21]( Nu-GSCS in short)  $Nu-Scl(W_1^*) \subseteq \mathcal{H}$  whenever  $W_1^* \subseteq \mathcal{H}$  and  $\mathcal{H}$  is a Nu-OS in  $Nu_X^*$
- [8] Neutrosophic semi generalized closed set[21]( Nu-SGCS in short) if  $Nu scl(W_1^*) \subseteq \mathcal{H}$  whenever  $W_1^* \subseteq \mathcal{H}$  and  $\mathcal{H}$  is a Nu-SOS in  $Nu_X^*$  ,
- [9] Neutrosophic generalized alpha closed set[9]. (Nu-G $\alpha$ CS in short) if  $Nu-\alpha cl(W_1^*) \subseteq \mathcal{H}$  whenever  $W_1^* \subseteq \mathcal{H}$  and  $\mathcal{H}$  is a Nu- $\alpha$ OS in  $Nu_X^*$
- [10] Neutrosophic generalized b closed set[14](Nu-GbCS in short) if  $Nu-bcl(W_1^*) \subseteq \mathcal{H}$  whenever  $W_1^* \subseteq \mathcal{H}$  and  $\mathcal{H}$  is a Nu-OS in  $Nu_X^*$

**Definition 2.8.[13]** An (NS)S  $W_1^*$  in an (NS)TS  $(Nu_X^*, Nu_\tau)$ , is said to be a Neutrosophic weakly generalized closed set ((Nu-WG)CS)  $Nu-cl(Nu-in(W_1^*)) \subseteq \mathcal{K}$  whenever  $W_1^* \subseteq \mathcal{K}$ ,  $\mathcal{K}$  is (Nu)OS in  $Nu_X^*$ .

**Definition 2.9.**  $(Nu_X^*, Nu_\tau)$ , be a NS-T-S and  $W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$   $Nu_X^*$ .

Then Neutrosophic closure of  $W_1^*$  is  $Nu-Cl(W_1^*) = \cap \{ \mathcal{H} : \mathcal{H} \text{ is a Nu-CS in } Nu_X^* \text{ and } W_1^* \subseteq \mathcal{H} \}$   
 Neutrosophic interior of  $W_1^*$  is  $Nu-Int(W_1^*) = \cup \{ M : M \text{ is a Nu-OS in } Nu_X^* \text{ and } M \subseteq W_1^* \}$ .

**Definition 2.10.[2]** Let  $(Nu_X^*, Nu_\tau)$ , be a NS-T-S and

- $W_1^* = \{ \langle w, \mu_{W_1^*}(w), \sigma_{W_1^*}(w), \gamma_{W_1^*}(w) \rangle : w \in Nu_X^* \}$
- $Nu-Sint(W_1^*) = \cup \{ \mathcal{H} / \mathcal{H} \text{ is a Nu-SOS in } Nu_X^* \text{ and } \mathcal{H} \subseteq W_1^* \}$ ,
- $Nu-Scl(W_1^*) = \cap \{ \mathcal{K} / \mathcal{K} \text{ is a Nu-SCS in } Nu_X^* \text{ and } W_1^* \subseteq \mathcal{K} \}$ .
- $Nu-\alpha int(W_1^*) = \cup \{ \mathcal{H} / \mathcal{H} \text{ is a Nu-}\alpha\text{OS in } Nu_X^* \text{ and } \mathcal{H} \subseteq W_1^* \}$ ,
- $Nu-\alpha cl(W_1^*) = \cap \{ \mathcal{K} / \mathcal{K} \text{ is a Nu-}\alpha\text{CS in } Nu_X^* \text{ and } W_1^* \subseteq \mathcal{K} \}$ .

### 3. NEUTROSOPHIC G\* CLOSED SETS

In this section we introduce Neutrosophic G\*-Closed sets and studied some of its basic properties.

**Definition 3.1:** An NS  $W_1^*$  in  $(Nu_X^*, Nu_\tau)$  is said to be a Neutrosophic G\*-Closed set (Nu-G\*CS in short) if  $Nu-cl(W_1^*) \subseteq \mathcal{K}$  whenever  $W_1^* \subseteq \mathcal{K}$  and  $\mathcal{K}$  is Nu-GOS in  $(Nu_X^*, Nu_\tau)$  .

The family of all Nu-G\*CS's of A NTS  $(Nu_X^*, Nu_\tau)$  is denoted by  $Nu-G^*C(Nu_X^*)$ .

**Example 3.2:** Let  $Nu_X^* = \{ w_1, w_2 \}$  and let  $Nu_\tau = \{ 0_{Nu}, \mathcal{K}, 1_{Nu} \}$  is NT on  $Nu_X^*$ , where  $\mathcal{K} = \langle w, (\frac{3}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{4}{10}, \frac{5}{10}, \frac{6}{10}) \rangle$ . Then the NS  $W_1^* = \langle w, (\frac{7}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{0}{10}) \rangle$  is Nu-G\*CS in  $(Nu_X^*, Nu_\tau)$

**Theorem 3.3:** Every Nu-CS is Nu-G\*CS .

**Proof:** Let  $W_1^*$  be a Nu-CS in  $(Nu_X^*, Nu_\tau)$  . Then  $Nu-cl(W_1^*) = W_1^*$ . Let  $W_1^* \subseteq \mathcal{K}$  and  $\mathcal{K}$  is Nu-GOS in  $(Nu_X^*, Nu_\tau)$  . Therefore  $Nu-cl(W_1^*) = W_1^* \subseteq \mathcal{K}$ . Thus  $W_1^*$  is Nu-G\*CS in  $Nu_X^*$ .

**Example 3.4:** Let  $Nu_X^* = \{ w_1, w_2 \}$  and let  $Nu_\tau = \{ 0_{Nu}, \mathcal{K}, 1_{Nu} \}$  is NT on  $Nu_X^*$ , where

$\mathcal{K} = \langle w, (\frac{4}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}) \rangle$  Then the NS  $W_1^* = \langle w, (\frac{6}{10}, \frac{5}{10}, \frac{1}{10}), (\frac{7}{10}, \frac{5}{10}, \frac{1}{10}) \rangle$  is Nu-G\*CS but not an

Nu-CS in  $Nu_X^*$ .

**Theorem 3.5:** Every Nu-G\*CS is Nu-GCS.

**Proof:** Let  $W_1^*$  be a Nu-G\*CS in  $(Nu_X^*, Nu_\tau)$ . Let  $W_1^* \subseteq \mathcal{K}$  and  $\mathcal{K}$  is Nu-OS in  $(Nu_X^*, Nu_\tau)$ . Since every Nu-OS is Nu-GOS and since  $W_1^*$  is Nu-G\*CS in  $Nu_X^*$ . Therefore  $Nu-cl(W_1^*) \subseteq \mathcal{K}$  whenever  $W_1^* \subseteq \mathcal{K}$ ,  $\mathcal{K}$  is Nu-OS in  $Nu_X^*$ . Thus  $W_1^*$  is Nu-GCS in  $Nu_X^*$ .

**Example 3.6:** Let  $Nu_X^* = \{w_1, w_2, w_3\}$  and

let  $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$  is NT on  $Nu_X^*$ , where  $\mathcal{K} = \langle w, (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{5}{10}) \rangle$ .

Then the NS  $W_1^* = \langle w, (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{7}{10}), (\frac{3}{10}, \frac{5}{10}, \frac{6}{10}) \rangle$  is Nu-GCS but not an Nu-G\*CS in  $Nu_X^*$ .

**Theorem 3.7:** Every Nu-G\*CS is Nu- $\alpha$ GCS.

**Proof:** Let  $W_1^*$  be a Nu-G\*CS in  $(Nu_X^*, Nu_\tau)$ . By Theorem 3.6  $W_1^*$  is Nu-GCS in  $Nu_X^*$ . Since  $Nu\alpha-cl(W_1^*) \subseteq Nu-cl(W_1^*)$  and  $W_1^*$  is a Nu-GCS in  $Nu_X^*$ . Therefore  $Nu\alpha-cl(W_1^*) \subseteq Nu-cl(W_1^*) \subseteq \mathcal{K}$  whenever  $W_1^* \subseteq \mathcal{K}$ ,  $\mathcal{K}$  is Nu-OS in  $Nu_X^*$ . Thus  $W_1^*$  is Nu- $\alpha$ GCS in  $Nu_X^*$ .

**Example 3.8:** Let  $Nu_X^* = \{w_1, w_2\}$  and let  $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$  is NT on  $Nu_X^*$ ,

where  $\mathcal{K} = \langle w, (\frac{1}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{4}{10}, \frac{5}{10}, \frac{5}{10}) \rangle$ . Then the NS  $W_1^* = \langle w, (\frac{3}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{5}{10}) \rangle$

is Nu- $\alpha$ GCS but not an Nu-G\*CS in  $Nu_X^*$ .

**Theorem 3.9:** Every Nu-RCS is Nu-G\*CS.

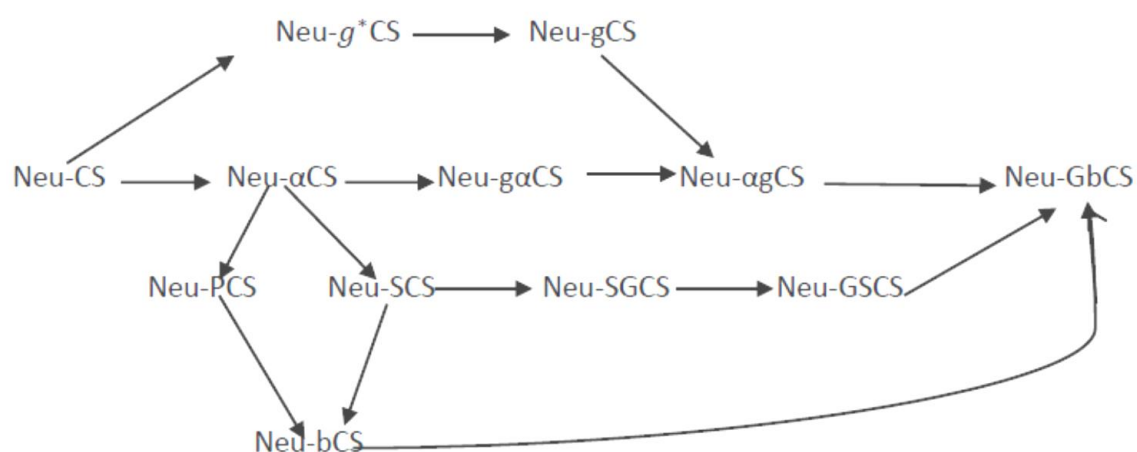
**Proof:** Let  $W_1^*$  be a Nu-RCS in  $(Nu_X^*, Nu_\tau)$ . Then  $W_1^* = Nu-cl(Nu-int(W_1^*))$ . Let  $W_1^* \subseteq \mathcal{K}$  and  $\mathcal{K}$  is Nu-GOS in  $(Nu_X^*, Nu_\tau)$ . Therefore  $Nu-cl(W_1^*) \subseteq Nu-cl(Nu-int(W_1^*))$ . This implies  $Nu-cl(W_1^*) \subseteq \mathcal{K}$ . Thus  $W_1^*$  is Nu-G\*CS in  $Nu_X^*$ .

**Example 3.10:** Let  $Nu_X^* = \{w_1, w_2\}$  and let  $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$  is NT on  $Nu_X^*$ , Where

$\mathcal{K} = \langle w, (\frac{3}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{7}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$  Then NS  $W_1^* = \langle w, (\frac{7}{10}, \frac{5}{10}, \frac{3}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}) \rangle$  is Nu-G\*CS but not an

Nu-RCS in  $Nu_X^*$ .

**Diagram:I**



**Remark 3.11:**

Nu-G\*CS is independent from Nu- $\alpha$ CS, Nu-SCS, Nu-PCS, and Nu-bCS as seen from the following example.

**Example 3.12:** Let  $Nu_X^* = \{w_1, w_2\}$  and let  $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$  is NT on  $Nu_X^*$ , where

$$\mathcal{K} = \langle w \left( \frac{2}{10}, \frac{5}{10}, \frac{6}{10} \right), \left( \frac{4}{10}, \frac{5}{10}, \frac{5}{10} \right) \rangle. \text{ Then } NS W_1^* = \langle w, \left( \frac{3}{10}, \frac{5}{10}, \frac{5}{10} \right), \left( \frac{4}{10}, \frac{5}{10}, \frac{5}{10} \right) \rangle$$

is NuSCS, Nu-bCS, but not an Nu-G\*CS in  $Nu_X^*$ .

**Example 3.13:** Let  $Nu_X^* = \{w_1, w_2\}$  and let  $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$  is NT on  $Nu_X^*$ , where

$$\mathcal{K} = \langle w \left( \frac{6}{10}, \frac{5}{10}, \frac{2}{10} \right), \left( \frac{5}{10}, \frac{5}{10}, \frac{4}{10} \right) \rangle. \text{ Then } NS W_1^* = \langle w, \left( \frac{1}{10}, \frac{5}{10}, \frac{7}{10} \right), \left( \frac{4}{10}, \frac{5}{10}, \frac{5}{10} \right) \rangle$$

is Nu-PCS, Nu- $\alpha$ CS, but not an Nu-G\*CS in  $Nu_X^*$ .

**Example 3.14:** Let  $Nu_X^* = \{w_1, w_2\}$  and let  $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$  is NT on  $Nu_X^*$ , where

$$\mathcal{K} = \langle w, \left( \frac{6}{10}, \frac{5}{10}, \frac{2}{10} \right), \left( \frac{5}{10}, \frac{5}{10}, \frac{2}{10} \right) \rangle \text{ Then the } NS W_1^* = \langle w, \left( \frac{1}{10}, \frac{5}{10}, \frac{3}{10} \right), \left( \frac{4}{10}, \frac{5}{10}, \frac{2}{10} \right) \rangle$$

is Nu-G\*CS but not NuSCS, Nu-bCS  $Nu_X^*$ .

**Example 3.15:** Let  $Nu_X^* = \{w_1, w_2\}$  and let  $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$  is NT on  $Nu_X^*$ , where

$$\mathcal{K} = \langle w, \left( \frac{2}{10}, \frac{5}{10}, \frac{6}{10} \right), \left( \frac{3}{10}, \frac{5}{10}, \frac{5}{10} \right) \rangle \text{ Then the } NS W_1^* = \langle w, \left( \frac{3}{10}, \frac{5}{10}, \frac{1}{10} \right), \left( \frac{3}{10}, \frac{5}{10}, \frac{3}{10} \right) \rangle$$

is Nu-G\*CS but not Nu $\alpha$ CS, Nu-PCS  $Nu_X^*$ .

**Theorem 3.16:** The union of two Nu-G\*CS's is Nu-G\*CS

**Proof:** Let  $W_1^*$  and  $W_2^*$  be the two Nu-G\*CS's in  $Nu_X^*$  and let  $W_1^* \cup W_2^* \subseteq \mathcal{K}$ , where  $\mathcal{K}$  is a Nu-GOS in  $Nu_X^*$ . Therefore  $W_1^* \subseteq \mathcal{K}$  or  $W_2^* \subseteq \mathcal{K}$  or both contained  $\mathcal{K}$ . Since  $W_1^*$  and  $W_2^*$  are Nu-G\*CS, Nu-cl( $W_1^*$ ) $\subseteq \mathcal{K}$  and Nu-cl( $W_2^*$ ) $\subseteq \mathcal{K}$ . Therefore Nu-cl( $W_1^* \cup W_2^*$ ) $\subseteq \mathcal{K}$ . Thus  $W_1^* \cup W_2^*$  is Nu-G\*CS.

**Remark 3.17:** The intersection of any two Nu-G\*CSs is not an Nu-G\*CS in general as seen in the following example.

**Example 3.18:** Let  $Nu_X^* = \{w_1, w_2\}$  and let  $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$  is NT on  $Nu_X^*$ ,

$$\text{where } \mathcal{K} = \langle w, \left( \frac{5}{10}, \frac{5}{10}, \frac{1}{10} \right), \left( \frac{1}{10}, \frac{5}{10}, \frac{8}{10} \right) \rangle.$$

$$\text{Then NS's } W_1^* = \langle w, \left( \frac{2}{10}, \frac{5}{10}, \frac{5}{10} \right), \left( \frac{7}{10}, \frac{5}{10}, \frac{0}{10} \right) \rangle \quad W_2^* = \langle w, \left( \frac{6}{10}, \frac{5}{10}, \frac{0}{10} \right), \left( \frac{3}{10}, \frac{5}{10}, \frac{7}{10} \right) \rangle$$

are Nu-G\*CS's in  $Nu_X^*$  but  $W_1^* \cap W_2^*$  is not a Nu-G\*CS in  $Nu_X^*$ .

**Theorem 3.19:** If  $W_1^*$  is Nu-G\*CS in  $(Nu_X^*, Nu_\tau)$ , such that  $W_1^* \subseteq W_2^* \subseteq Nu-cl(W_1^*)$ . Then  $W_2^*$  is also a Nu-G\*CS of  $(Nu_X^*, Nu_\tau)$

**Proof:** Let  $\mathcal{K}$  be a Nu-GOS in  $(Nu_X^*, Nu_\tau)$  such that  $W_2^* \subseteq \mathcal{K}$ . Since  $W_1^* \subseteq W_2^*$ ,  $W_1^* \subseteq \mathcal{K}$  and  $\mathcal{K}$  be a Nu-GOS. Also since  $W_1^*$  is Nu-G\*CS, Nu-cl( $W_1^*$ ) $\subseteq \mathcal{K}$ . By hypothesis  $W_2^* \subseteq Nu-cl(W_1^*)$ . This implies Nu-cl( $W_2^*$ ) $\subseteq Nu-cl(Nu-cl(W_1^*)) \subseteq \mathcal{K}$ . Therefore Nu-cl( $W_2^*$ ) $\subseteq \mathcal{K}$ . Hence  $W_2^*$  is Nu-G\*CS of  $Nu_X^*$ .

**Theorem 3.20:** If  $W_1^*$  is both Nu-GOS and Nu-G\*CS of  $(Nu_X^*, Nu_\tau)$ , then  $W_1^*$  is Nu-CS in  $Nu_X^*$ .

**Proof:** Let  $W_1^*$  is Nu-GOS in  $Nu_X^*$ . Since  $W_1^* \subseteq W_1^*$ , by hypothesis Nu-cl( $W_1^*$ ) $\subseteq W_1^*$ . But from the Definition,  $W_1^* \subseteq Nu-cl(W_1^*)$ . Therefore Nu-cl( $W_1^*$ ) $= W_1^*$ . Hence  $W_1^*$  is Nu-CS of  $Nu_X^*$ .

**Theorem 3.21:** Let  $(Nu_X^*, Nu_\tau)$  be a NTS. Then Nu-GO( $Nu_X^*$ ) $= Nu-GC(Nu_X^*)$  iff every NS in  $(Nu_X^*, Nu_\tau)$  is Nu-G\*CS in  $Nu_X^*$ .

**Proof:**

**Necessity:** Suppose that Nu-GO( $Nu_X^*$ ) $= Nu-GC(Nu_X^*)$ . Let  $W_1^* \subseteq \mathcal{K}$  and  $\mathcal{K}$  is Nu-GOS in  $Nu_X^*$ . This implies Nu-cl( $W_1^*$ ) $\subseteq Nu-cl(\mathcal{K})$ . Since  $\mathcal{K}$  is Nu-GOS in  $Nu_X^*$ . Since by hypothesis  $\mathcal{K}$  is Nu-GCS in  $Nu_X^*$ , Nu-cl( $\mathcal{K}$ ) $\subseteq \mathcal{K}$ . This implies Nu-cl( $W_1^*$ ) $\subseteq \mathcal{K}$ . Therefore  $W_1^*$  is Nu-G\*CS in  $Nu_X^*$ .

**Sufficiency:** Suppose that every NS in  $(Nu_X^*, Nu_\tau)$  is Nu-G\*CS in  $Nu_X^*$ . Let  $\mathcal{K} \subseteq Nu-O(Nu_X^*)$ , then

$\mathcal{K} \subseteq \text{Nu-GO}(\text{Nu}_X^*)$ . Since  $\mathcal{K} \subseteq \mathcal{K}$  and  $\mathcal{K}$  is Nu-OS in  $\text{Nu}_X^*$ , by hypothesis  $\text{Nu-cl}(\mathcal{K}) \subseteq \mathcal{K}$ . I.e.,  $\mathcal{K} \subseteq \text{Nu-GC}(\text{Nu}_X^*)$ . Hence  $\text{Nu-GO}(\text{Nu}_X^*) \subseteq \text{Nu-GC}(\text{Nu}_X^*)$ . Let  $W_1^* \in \text{Nu-GC}(\text{Nu}_X^*)$  then  $W_1^{*c}$  is an Nu-GOS in  $\text{Nu}_X^*$ . But  $\text{Nu-GO}(\text{Nu}_X^*) \subseteq \text{Nu-GC}(\text{Nu}_X^*)$ . Therefore  $W_1^{*c} \subseteq \text{Nu-GC}(\text{Nu}_X^*)$ . I.e.,  $W_1^* \in \text{Nu-GO}(\text{Nu}_X^*)$ . Hence  $\text{Nu-GC}(\text{Nu}_X^*) \subseteq \text{Nu-GO}(\text{Nu}_X^*)$ . Thus  $\text{Nu-GO}(\text{Nu}_X^*) \subseteq \text{Nu-GC}(\text{Nu}_X^*)$ .

**Theorem 3.22:** If  $W_1^*$  is Nu-OS and an Nu-G\*CS in  $(\text{Nu}_X^*, \text{Nu}_\tau)$ , then

$W_1^*$  is Nu-ROS in  $\text{Nu}_X^*$

$W_1^*$  is Nu-RCS in  $\text{Nu}_X^*$

**Proof:** (i) Let  $W_1^*$  be a Nu-OS and a Nu-G\*CS in  $\text{Nu}_X^*$ . Then  $\text{Nu-cl}(W_1^*) \subseteq W_1^*$ . I.e.,  $\text{Nu-int}(\text{Nu-cl}(W_1^*)) \subseteq W_1^*$ . Since  $W_1^*$  is a Nu-OS,  $W_1^*$  is Nu-POS in  $\text{Nu}_X^*$ . Hence  $W_1^* \subseteq \text{Nu-int}(\text{Nu-cl}(W_1^*))$ . Therefore  $W_1^* = \text{Nu-int}(\text{Nu-cl}(W_1^*))$ . Hence  $W_1^*$  is Nu-ROS in  $\text{Nu}_X^*$ .

(ii): Let  $W_1^*$  be a Nu-OS and an Nu-G\*CS in  $\text{Nu}_X^*$ . Then  $\text{Nu-cl}(W_1^*) \subseteq W_1^*$ . I.e.,  $\text{Nu-cl}(\text{Nu-int}(W_1^*)) \subseteq W_1^*$ . Since  $W_1^*$  is a Nu-OS,  $W_1^*$  is Nu-OS in  $\text{Nu}_X^*$ . Hence  $W_1^* \subseteq \text{Nu-cl}(\text{Nu-int}(W_1^*))$ . Therefore  $W_1^* = \text{Nu-int}(\text{Nu-cl}(W_1^*))$ . Hence  $W_1^*$  is Nu-RCS in  $\text{Nu}_X^*$ .

#### 4. NEUTROSOPHIC g\*-OPEN SETS

In this section we introduce Neutrosophic g\*-open sets and studied some of its properties.

**Definition 4.1:** An NS  $W_1^*$  is said to be a Neutrosophic g\*-open set (Nu-G\*OS in short) in  $(\text{Nu}_X^*, \text{Nu}_\tau)$  if the complement  $W_1^{*c}$  is Nu-G\*CS in  $\text{Nu}_X^*$ . The family of all Nu-G\*OS's of A NTS  $(\text{Nu}_X^*, \text{Nu}_\tau)$  is denoted by  $\text{Nu-G}^*\text{O}(\text{Nu}_X^*)$ .

**Theorem 4.2:** A subset  $W_1^*$  of  $(\text{Nu}_X^*, \text{Nu}_\tau)$  is Nu-G\*OS iff  $W_2^* \subseteq \text{Nu-int}(W_1^*)$  whenever  $W_2^*$  is Nu-GCS in  $\text{Nu}_X^*$  and  $W_2^* \subseteq W_1^*$ .

**Proof:** Necessity: Let  $W_1^*$  is Nu-G\*OS in  $\text{Nu}_X^*$ . Let  $W_2^*$  be a Nu-GCS in  $\text{Nu}_X^*$  and  $W_2^* \subseteq W_1^*$ . Then  $W_2^{*c}$  is Nu-GOS in  $\text{Nu}_X^*$  such that  $W_1^{*c} \subseteq W_2^{*c}$ . Since  $W_1^{*c}$  is Nu-G\*CS, we have  $\text{Nu-cl}(W_1^{*c}) \subseteq W_2^{*c}$ . Hence  $\text{Nu-int}(W_1^*)^c \subseteq W_2^{*c}$ . Therefore  $W_2^* \subseteq \text{Nu-int}(W_1^*)$ .

Sufficiency: Let  $W_2^* \subseteq \text{Nu-int}(W_1^*)$  whenever  $W_2^*$  is Nu-GCS in  $\text{Nu}_X^*$  and  $W_2^* \subseteq W_1^*$ . Then  $W_1^{*c} \subseteq W_2^{*c}$  and  $W_2^{*c}$  is Nu-GOS. By hypothesis,  $(\text{Nu-int}(W_1^*))^c \subseteq W_2^{*c}$ , which implies  $\text{Nu-cl}(W_1^{*c}) \subseteq W_2^{*c}$ . Therefore  $W_1^{*c}$  is Nu-G\*CS of  $\text{Nu}_X^*$ . Hence  $W_1^*$  is Nu-G\*OS in  $\text{Nu}_X^*$ .

**Theorem 4.3:** Every Nu-OS is Nu-G\*OS .

**Proof:** Let  $W_1^*$  be a Nu-OS. Then  $W_1^{*c}$  is Nu-CS. By Theorem 3.3, every Nu-CS is Nu-G\*CS. Therefore  $W_1^{*c}$  is Nu-G\*CS. Hence  $W_1^*$  is Nu-G\*OS.

**Example 4.4:** Let  $\text{Nu}_X^* = \{w_1, w_2\}$  and let  $\text{Nu}_\tau = \{0_{\text{Nu}}, \mathcal{K}, 1_{\text{Nu}}\}$  is NT on  $\text{Nu}_X^*$ , where

$$\mathcal{K} = \langle w, \left(\frac{2}{10}, \frac{5}{10}, \frac{2}{10}\right), \left(\frac{5}{10}, \frac{5}{10}, \frac{8}{10}\right) \rangle. \text{ Then NS } W_1^* = \langle w, \left(\frac{2}{10}, \frac{5}{10}, \frac{3}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{8}{10}\right) \rangle$$

is Nu-G\*OS but not an Nu-OS in  $\text{Nu}_X^*$ .

**Theorem 4.5:** Every Nu-ROS is Nu- G\*OS .

**Proof:** Let  $W_1^*$  be a Nu-WS. Then  $W_1^{*c}$  is Nu-RCS. By Theorem 3.15, every Nu-RCS is Nu-G\*CS. Therefore  $W_1^{*c}$  is Nu-G\*CS. Hence  $W_1^*$  is Nu-G\*OS.

**Example 4.6:** Let  $\text{Nu}_X^* = \{w_1, w_2\}$  and let  $\text{Nu}_\tau = \{0_{\text{Nu}}, \mathcal{K}, 1_{\text{Nu}}\}$  is NT on  $\text{Nu}_X^*$ , where

$$\mathcal{K} = \langle w, \left(\frac{3}{10}, \frac{5}{10}, \frac{6}{10}\right), \left(\frac{7}{10}, \frac{5}{10}, \frac{3}{10}\right) \rangle \text{ Then NS } W_1^* = \langle w, \left(\frac{3}{10}, \frac{5}{10}, \frac{7}{10}\right), \left(\frac{4}{10}, \frac{5}{10}, \frac{6}{10}\right) \rangle \text{ is Nu-G*OS but not an}$$

Nu-ROS in  $\text{Nu}_X^*$ .

**Theorem 4.7:** Every Nu-G\*OS is Nu-GOS .

**Proof:** Let  $W_1^*$  be a Nu-G\*OS in  $(\text{Nu}_X^*, \text{Nu}_\tau)$  . Then  $W_1^{*c}$  is Nu-G\*CS. By Theorem 3.6, every Nu-G\*CS is Nu-GCS. Therefore  $W_1^{*c}$  is Nu-GCS. Hence  $W_1^*$  is Nu-GOS.

**Example 4.8:** Let  $Nu_X^* = \{w_1, w_2\}$  and let  $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$  is NT on  $Nu_X^*$ , where

$\mathcal{K} = \langle w, (\frac{5}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{2}{10}) \rangle$ . Then NS  $W_1^* = \langle w, (\frac{4}{10}, \frac{5}{10}, \frac{6}{10}), (\frac{2}{10}, \frac{5}{10}, \frac{6}{10}) \rangle$  is Nu-GOS but not an Nu-G\*OS in  $Nu_X^*$ .

**Theorem 4.9:** Every Nu-G\*OS is Nu- $\alpha$ GOS .

**Proof:** Let  $W_1^*$  be a Nu-G\*OS in  $(Nu_X^*, Nu_\tau)$ . Then  $W_1^{*C}$  is Nu-G\*CS. By Theorem 3.9, every Nu-G\*CS is Nu- $\alpha$ GCS. Therefore  $W_1^{*C}$  is Nu- $\alpha$ GCS. Hence  $W_1^*$  is Nu- $\alpha$ GOS.

**Example 4.10:** Let  $Nu_X^* = \{w_1, w_2\}$  and let  $Nu_\tau = \{0_{Nu}, \mathcal{K}, 1_{Nu}\}$  is NT on  $Nu_X^*$ , where  $\mathcal{K} = \langle w, (\frac{4}{10}, \frac{5}{10}, \frac{2}{10}), (\frac{6}{10}, \frac{5}{10}, \frac{7}{10}) \rangle$ . Then the NS  $W_1^* = \langle w, (\frac{6}{10}, \frac{5}{10}, \frac{4}{10}), (\frac{5}{10}, \frac{5}{10}, \frac{3}{10}) \rangle$  is Nu- $\alpha$ GOS but not an Nu-G\*OS in  $Nu_X^*$ .

**Theorem 4.11:** The intersection of two Nu-G\*OS's is Nu-G\*OS.

**Proof:** Let  $W_1^*$  and  $W_2^*$  be the two Nu-G\*OS's in  $Nu_X^*$ ,  $W_1^{*C}$  and  $W_2^{*C}$  are Nu-G\*CS. By Theorem 3.28  $W_1^{*C} \cup W_2^{*C}$  is Nu-G\*CS in  $Nu_X^*$ . Therefore  $(W_1^* \cap W_2^*)$  is Nu-G\*CS. Thus  $W_1^* \cap W_2^*$  is Nu-G\*OS in  $Nu_X^*$ .

**Theorem 4.12:** Let  $(Nu_X^*, Nu_\tau)$  be a NTS. If  $W_1^*$  is NS of  $Nu_X^*$ . Then for every  $W_1^* \in Nu-G^*O(Nu_X^*)$  and every  $W_2^* \in (Nu_X^*), Nu-int(W_1^*) \subseteq W_2^* \subseteq W_1^*$  implies  $W_2^* \in Nu-G^*O(Nu_X^*)$ .

**Proof:** By hypothesis  $Nu-int(W_1^*) \subseteq W_2^* \subseteq W_1^*$ . Taking complement on both sides, we get  $W_1^{*C} \subseteq W_2^{*C} \subseteq Nu-cl(W_1^{*C})$ . Let  $W_2^{*C} \in \mathcal{K}$  and  $\mathcal{K}$  is Nu-GOS in  $Nu_X^*$ . Since  $W_1^{*C} \subseteq W_2^{*C}$ ,  $W_1^{*C} \subseteq \mathcal{K}$ . Since  $W_1^{*C}$  is Nu-G\*CS,  $Nu-cl(W_1^{*C}) \subseteq \mathcal{K}$ . Therefore  $Nu-cl(W_2^{*C}) \subseteq Nu-cl(W_1^{*C}) \subseteq \mathcal{K}$ . Hence  $W_2^{*C}$  is Nu-G\*CS in  $Nu_X^*$ . Therefore  $W_2^*$  is Nu-G\*OS in  $Nu_X^*$ . I.e.,  $W_2^* \in Nu-G^*O(Nu_X^*)$

**Definition: 4.13:** For any Nu. set  $W_1^*$  in any NSTS,

$$Nu-g^*cl(W_1^*) = \cap \{ \mathcal{U} : \mathcal{U} \text{ is Nu-g*CS Nu. set and } W_1^* \subseteq \mathcal{U} \}$$

$$Nu-g^*int(W_1^*) = \cup \{ \mathcal{V} : \mathcal{V} \text{ is Nu-g* OS and } W_1^* \supseteq \mathcal{V} \}$$

**Theorem: 4.14:** In a Its  $(Nu_X^*, Nu_\tau)$  a Nu. set  $W_1^*$  is Nu-g\*- CS iff  $W_1^* = Nu-g^*cl(W_1^*)$ .

**Proof:** Let  $W_1^*$  be a Nu-g\*CS Nu. set in NSTS  $(Nu_X^*, Nu_\tau)$ . Since  $W_1^* \subseteq W_1^*$  and  $W_1^*$  is Nu-g\*CS ,  $W_1^* \in \{ \mathcal{K} : \mathcal{K} \text{ is a Nu-g*CS Nu. set and } W_1^* \subseteq \mathcal{K} \}$  and  $W_1^* \subseteq \mathcal{K} \Rightarrow W_1^* = \cap \{ \mathcal{K} : \mathcal{K} \text{ is Nu-g*CS and } W_1^* \subseteq \mathcal{K} \}$  that is  $W_1^* = Nu-g^*cl(W_1^*)$ .

Conversely, suppose that  $W_1^* = Nu-g^*cl(W_1^*)$ , that is  $W_1^* = \cap \{ \mathcal{K} : \mathcal{K} \text{ is a Nu-g*- CS Nu. set and } W_1^* \subseteq \mathcal{K} \}$ . This denotes that  $W_1^* \in \{ \mathcal{K} : \mathcal{K} \text{ is a Nu-g*CS Nu. set and } W_1^* \subseteq \mathcal{K} \}$ . From now  $W_1^*$  is Nu-g\*CS Nu. set.

**Theorem: 4.15** In a NSTS  $Nu_X^*$  the subsequent results hold for Nu-g\* - closure.

- 1)  $Nu-g^*cl(0_{Nu}) = 0_{Nu}$ .
- 2)  $Nu-g^*cl(W_1^*)$  is Nu-g\*CS Nu. set in  $Nu_X^*$ .
- 3)  $Nu-g^*cl(W_1^*) \subseteq Nu-g^*cl(W_2^*)$  if  $W_1^* \subseteq W_2^*$ .
- 4)  $Nu-g^*cl(Nu-g^*cl(W_1^*)) = Nu-g^*cl(W_1^*)$ .
- 5)  $Nu-g^*cl(W_1^* \cup W_2^*) \supseteq Nu-g^*cl(W_1^*) \cup Nu-g^*cl(W_2^*)$ .
- 6)  $Nu-g^*cl(W_1^* \cap W_2^*) \subseteq Nu-g^*cl(W_1^*) \cap Nu-g^*cl(W_2^*)$ .

Proof: easy

**Theorem: 4.16** In a NSTS  $Nu_X^*$ , a Nu. set  $W_1^*$  is Nu-g\* OS iff  $W_1^* = Nu-g^*int(W_1^*)$ .

**Proof:** Let  $W_1^*$  be Nu-g\*OS Nu. set in  $Nu_X^*$ . Since  $W_1^* \subseteq W_1^*$  and  $W_1^*$  is Nu-g\* OS and  $W_1^* \in \{ \mathcal{K} : \mathcal{K} \text{ is a Nu-g* OS Nu. set and } W_1^* \supseteq \mathcal{K} \}$  and  $W_1^* \supseteq \mathcal{K} \Rightarrow W_1^* = \cup \{ \mathcal{K} : \mathcal{K} \text{ is Nu-g*OS and } W_1^* \supseteq \mathcal{K} \}$ . That is  $W_1^* = Nu-g^*int(W_1^*)$ .

Conversely, suppose that  $W_1^* = \text{Nu-g}^*\text{-int}(W_1^*)$ , that is  $W_1^* = \cup \{ \mathcal{K} : \mathcal{K} \text{ is Nu-g}^*\text{-OS and } W_1^* \supseteq \mathcal{K} \}$ . This implies that  $W_1^* \in \{ \mathcal{K} : \mathcal{K} \text{ is Nu-g}^*\text{-OS and } W_1^* \supseteq \mathcal{K} \}$ . Hence  $W_1^*$  is Nu-g<sup>\*</sup>-OS Nu. set.

**Theorem: 4.17** In a NSTS  $\text{Nu}_X^*$ , the following hold for Nu-g<sup>\*</sup>-interior.

- 1)  $\text{Nu-g}^*\text{-int}(0_{\text{Nu}}) = 0_{\text{Nu}}$
- 2)  $\text{Nu-g}^*\text{-int}(W_1^*) \subseteq \text{Nu-g}^*\text{-int}(W_2^*)$  if  $W_1^* \subseteq W_2^*$ .
- 3)  $\text{Nu-g}^*\text{-int}(W_1^*)$  is Nu-g<sup>\*</sup>-OS in  $\text{Nu}_X^*$ .
- 4)  $\text{Nu-g}^*\text{-int}(\text{Nu-g}^*\text{-int}(W_1^*)) = \text{Nu-g}^*\text{-int}(W_1^*)$ .
- 5)  $\text{Nu-g}^*\text{-int}(W_1^* \cup W_2^*) \supseteq \text{Nu-g}^*\text{-int}(W_1^*) \cup \text{Nu-g}^*\text{-int}(W_2^*)$ .
- 6)  $\text{Nu-g}^*\text{-int}(W_1^* \cap W_2^*) \subseteq \text{Nu-g}^*\text{-int}(W_1^*) \cap \text{Nu-g}^*\text{-int}(W_2^*)$ .

Proof: proof is as usual.

### 5. NEUTROSOPHIC g<sup>\*</sup>- CONTINUOUS

In this section we introduce Neutrosophic g<sup>\*</sup>-continuous and studied some properties of neutrosophic g<sup>\*</sup>- open map and closed map.

**Definition:5.1** Let  $\text{Nu}_X^*$  and  $\text{Nu}_Y^*$  be two NTS. A function  $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$  is said to be neutrosophic g<sup>\*</sup>- continuous (Nu-g<sup>\*</sup>- continuous) if the inverse image of every neutrosophic open set in  $\text{Nu}_Y^*$  is g<sup>\*</sup>- open in  $\text{Nu}_X^*$ .

**Theorem:5.2** A function  $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$  is Nu-g<sup>\*</sup>- continuous iff the inverse image of every Nu-closed set in  $\text{Nu}_Y^*$  is g<sup>\*</sup>- closed set in  $\text{Nu}_X^*$ .

**Proof:** Suppose the function  $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$  is Nu-g<sup>\*</sup>- continuous. Let  $\mathcal{F}$  be Nu-closed set in  $\text{Nu}_Y^*$ . Then  $\mathcal{F}^c$  is Nu- open set in  $\text{Nu}_Y^*$ . Since  $f$  is Nu-g<sup>\*</sup>- continuous,  $f^{-1}(\mathcal{F}^c)$  is Nu- g<sup>\*</sup>- open in  $\text{Nu}_X^*$ . But  $f^{-1}(\mathcal{F}^c) = (f^{-1}(\mathcal{F}))^c$  and so  $f^{-1}(\mathcal{F})$  is Nu-g<sup>\*</sup>- closed in  $\text{Nu}_X^*$ .

Conversely, assume that the inverse image of every Nu-closed set in  $\text{Nu}_Y^*$  is Nu-g<sup>\*</sup>- closed in  $\text{Nu}_X^*$ . Let  $\mathcal{V}$  be neutrosophic open set in  $\text{Nu}_Y^*$ . Then  $\mathcal{V}^c$  is Nu-closed in  $\text{Nu}_Y^*$ . By hypothesis,  $f^{-1}(\mathcal{V}^c)$  is Nu-g<sup>\*</sup>-closed set in  $\text{Nu}_X^*$ . But  $f^{-1}(\mathcal{V}^c) = (f^{-1}(\mathcal{V}))^c$  and so  $f^{-1}(\mathcal{V})$  is Nu-g<sup>\*</sup>- open set in  $\text{Nu}_X^*$ . Hence  $f$  is Nu-g<sup>\*</sup>-continuous.

**Theorem:5.3** Every Nu- continuous function is Nu-g<sup>\*</sup>- continuous.

**Proof:** Let  $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$  be Nu-continuous. Let  $\mathcal{F}$  be Nu-closed set in  $\text{Nu}_Y^*$ . Then  $f^{-1}(\mathcal{F})$  is Nu-closed set in  $\text{Nu}_X^*$  since  $f$  is neutrosophic continuous. And therefore  $f^{-1}(\mathcal{F})$  is Nu-g<sup>\*</sup>- closed in  $\text{Nu}_X^*$ . Hence  $f$  is Nu-g<sup>\*</sup>- continuous.

**Theorem:5.4** Every Nu-g<sup>\*</sup>-continuous function is Nu-g-continuous.

**Proof:** Let  $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$  be Nu-g<sup>\*</sup>- continuous. Let  $\mathcal{F}$  be a Nu-closed set in  $\text{Nu}_Y^*$ . Since  $f$  is Nu-g<sup>\*</sup>- continuous,  $f^{-1}(\mathcal{F})$  is Nu-g<sup>\*</sup>- closed in  $\text{Nu}_X^*$ . And therefore  $f^{-1}(\mathcal{F})$  is Nu-g- closed in  $\text{Nu}_X^*$  as every Nu-g<sup>\*</sup>-closed set is Nu-g- closed. Hence  $f$  is Nu-g- continuous.

The converse of the above theorem need not be true as seen from the following example.

**Theorem:5.5** If  $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$  is Nu-g<sup>\*</sup>- continuous and  $\text{Nu}_X^*$  is neutrosophic -T<sub>1/2</sub> NTS. Then  $f$  is neutrosophic -continuous.

**Proof:** Let  $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$  be Nu-g<sup>\*</sup>- continuous . Let  $\mathcal{F}$  be Nu-closed set in  $\text{Nu}_Y^*$ . Then  $f^{-1}(\mathcal{F})$  is  $f^{-1}(\mathcal{F})$  Nu- g<sup>\*</sup>- closed in  $\text{Nu}_X^*$  since  $f$  is Nu-g<sup>\*</sup>- continuous. Also since  $\text{Nu}_X^*$  is neutrosophic - T<sub>1/2</sub>,  $f^{-1}(\mathcal{F})$  is closcl in  $\text{Nu}_X^*$ . Hence  $f$  is Nu-continuous.

**Theorem:5.6** If  $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$  is Nu-g- continuous and  $\text{Nu}_X^*$  is neutrosophic - T<sub>1/2</sub> NTS. Then  $f$  is Nu-g<sup>\*</sup>- continuous.

**Proof:** Let  $f: \text{Nu}_X^* \rightarrow \text{Nu}_Y^*$  be Nu-g- continuous. Let  $\mathcal{F}$  be Nu-closed set in  $\text{Nu}_Y^*$ , then  $f^{-1}(\mathcal{F})$  is g-closed in  $\text{Nu}_X^*$ . Since  $X$  is neutrosophic - T<sub>1/2</sub>,  $f^{-1}(\mathcal{F})$  is Nu- g<sup>\*</sup>- closed in  $\text{Nu}_X^*$ . Hence  $f$  is Nu-g<sup>\*</sup>-



continuous.

**Theorem:5.7** If  $f: Nu_X^* \rightarrow Nu_Y^*$  is Nu-g\* - continuous and  $g: Nu_Y^* \rightarrow Nu_Z^*$  is Nu-continuous then  $g \circ f: Nu_X^* \rightarrow Nu_Z^*$  is Nu-g\* - continuous.

**Proof:** Let  $\mathcal{F}$  be Nu-closed set in  $Nu_Z^*$ . Then  $g^{-1}(\mathcal{F})$  is closed in  $Nu_Y^*$  since  $g$  is Nu-continuous. And then  $f^{-1}(g^{-1}(\mathcal{F}))$  is Nu-g\* - closed in  $Nu_X^*$  since  $f$  is Nu-g\* - continuous.

Now  $(g \circ f)^{-1}(\mathcal{F}) = f^{-1}(g^{-1}(\mathcal{F}))$  is Nu-g\* - closed in  $Nu_X^*$ . Hence  $g \circ f: Nu_X^* \rightarrow Nu_Z^*$  is Nu-g\* - continuous.

**Theorem:5.8** If  $f: Nu_X^* \rightarrow Nu_Y^*$  is Nu-g\* - continuous and  $g: Nu_Y^* \rightarrow Nu_Z^*$  is Nu-g\* - continuous and  $Nu_Y^*$  is neutrosophic  $-T_{1/2}$  space. Then  $g \circ f: Nu_X^* \rightarrow Nu_Z^*$  is Nu-g\* - continuous.

**Proof:** Let  $\mathcal{F}$  be Nu-closed set in  $Nu_Z^*$ . Then  $g^{-1}(\mathcal{F})$  is Nu-g\*CS in  $Nu_Y^*$  since  $g$  is Nu-g\* - continuous. Since  $Nu_Y^*$  is neutrosophic  $-T_{1/2}$ ,  $g^{-1}(\mathcal{F})$  is Nu-closed in  $Nu_Y^*$ . And then  $f^{-1}(g^{-1}(\mathcal{F}))$  is Nu-g\*CS in  $Nu_X^*$  as  $f$  is Nu-g\* - continuous. Now  $(g \circ f)^{-1}(\mathcal{F}) = f^{-1}(g^{-1}(\mathcal{F}))$  is Nu-g\*CS in  $Nu_X^*$ . Hence  $g \circ f$  is Nu-g\* - continuous.

**Definition:5.9** A map  $f: Nu_X^* \rightarrow Nu_Y^*$  is said to be neutrosophic g\* - open if the image of every neutrosophic open set in  $Nu_X^*$  is Nu-g\*-open set in  $Nu_Y^*$ .

**Definition:5.10** A map  $f: Nu_X^* \rightarrow Nu_Y^*$  is said to be neutrosophic g\* - closed if the image of every Nu-closed set in  $Nu_X^*$  is Nu-g\*-closed set in  $Nu_Y^*$ .

**Theorem: 5.11** Every neutrosophic open map is neutrosophic g\* - open.

**Proof:** Let  $f: Nu_X^* \rightarrow Nu_Y^*$  be a neutrosophic open map let  $\mathcal{V}$  be an neutrosophic open set in  $Nu_X^*$  then  $f(\mathcal{V})$  is Nu-open in  $Nu_Y^*$  since  $f$  is neutrosophic open map. And therefore  $f(\mathcal{V})$  is Nu-g\* - open in  $Nu_Y^*$ . Hence  $f$  is neutrosophic g\* open map.

**Theorem :5.12** If  $f: Nu_X^* \rightarrow Nu_Y^*$  is Nu-g\*-open map and  $Nu_Y^*$  is neutrosophic  $-T_{1/2}$ , then  $f$  is a Nu-open map.

**Proof :** Let  $f: Nu_X^* \rightarrow Nu_Y^*$  is neutrosophic g\* - open map. Let  $\mathcal{V}$  be neutrosophic open set in  $Nu_X^*$ . Then  $f(\mathcal{V})$  is Nu-g\* - open in  $Nu_Y^*$ . Since  $Nu_Y^*$  is neutrosophic  $-T_{1/2}$ ,  $f(\mathcal{V})$  is neutrosophic open set in  $Nu_Y^*$ . Hence  $f$  is Nu- open map.

**Theorem:5.13** Every Nu-g\* - open map is neutrosophic g - open.

**Proof:** Let  $f: Nu_X^* \rightarrow Nu_Y^*$  be a Nu-g\* - open map. Let  $\mathcal{V}$  be neutrosophic open set in  $Nu_X^*$ . Then  $f(\mathcal{V})$  is Nu-g\* - open in  $Nu_Y^*$  since  $f$  is Nu-g\* - open map. And therefore  $f(\mathcal{V})$  is Nu-g - open set in  $Nu_Y^*$ . Hence  $f$  is neutrosophic g - open map.

**Theorem : 5.14** If  $f: Nu_X^* \rightarrow Nu_Y^*$  is neutrosophic g - open and  $Nu_Y^*$  is neutrosophic  $-T_{1/2}$  space, then  $f$  is in Nu-g\* - open map.

**Proof:** Let  $\mathcal{V}$  be neutrosophic open set in  $Nu_X^*$ . Then  $f(\mathcal{V})$  is Nu-g - open in  $Nu_Y^*$ . Since  $Nu_Y^*$  is neutrosophic  $-T_{1/2}$ ,  $f(\mathcal{V})$  is Nu-g\* - open in  $Nu_Y^*$ . And hence  $f$  is Nu-g\* - open map.

**Theorem : 5.15** Every Nu-closed map is Nu-g\* - closed map.

**Proof:** Let  $f: Nu_X^* \rightarrow Nu_Y^*$  be Nu-closed map. Let  $\mathcal{F}$  be Nu-closed set in  $Nu_X^*$ . Then  $f(\mathcal{F})$  is closed in  $Nu_Y^*$ . And therefore  $f(\mathcal{F})$  is Nu-g\* - closed in  $Nu_Y^*$ . And hence  $f$  is Nu-g\* - closed map.

**Theorem :5.16** If  $f: Nu_X^* \rightarrow Nu_Y^*$  is Nu-g\* - closed and  $Nu_Y^*$  is neutrosophic  $-T_{1/2}$ . Then  $f$  is Nu-closed map.

**Proof:** Let  $f: Nu_X^* \rightarrow Nu_Y^*$  be Nu-g\* - closed map. Let  $\mathcal{F}$  be Nu-closed set in  $Nu_X^*$ . Then  $f^{-1}(\mathcal{F})$  is Nu-g\* - closed in  $Nu_X^*$ . Since  $Nu_Y^*$  is neutrosophic  $-T_{1/2}$ ,  $f(\mathcal{F})$  is Nu-closed in  $Nu_Y^*$ . Hence  $f$  is neutrosophic closed map.

**Theorem: 5.17** A map  $f: Nu_X^* \rightarrow Nu_Y^*$  is Nu-g\* - closed iff for each neutrosophic set  $\mathcal{S}$  of  $Nu_Y^*$  and for

each neutrosophic open set  $\mathcal{U}$  such that  $f^{-1}(\mathcal{S}) \subseteq \mathcal{U}$  there is a Nu-g\*-open set  $\mathcal{V}$  of  $\text{Nu}_Y^*$  such that  $\mathcal{S} \subseteq \mathcal{V}$  and  $f^{-1}(\mathcal{V}) \subseteq \mathcal{U}$ .

**Proof:** Suppose  $f$  is Nu-g\* - closed map. Let  $\mathcal{S}$  be a neutrosophic set of  $\text{Nu}_X^*$  and  $\mathcal{U}$  be a neutrosophic open set of  $\text{Nu}_X^*$  such that  $f^{-1}(\mathcal{U}) \subseteq \mathcal{U}$ . Then  $\mathcal{V} = \text{Nu}_Y^* - f(\mathcal{U}^c)$  is a Nu-g\*-open set in  $\text{Nu}_Y^*$  such that  $\mathcal{S} \subseteq \mathcal{V}$  and  $f^{-1}(\mathcal{V}) \subseteq \mathcal{U}$ .

Conversely, suppose that  $\mathcal{F}$  is a Nu-closed set of  $\text{Nu}_X^*$ . Then  $f^{-1}(f(\mathcal{F}^c)) \subseteq \mathcal{F}^c$  and  $\mathcal{F}^c$  is Nu-open. By hypothesis, there is a Nu-g\*-open set  $\mathcal{V}$  of  $\text{Nu}_Y^*$  such that  $f(\mathcal{F}^c) \subseteq \mathcal{V}$  and  $f^{-1}(\mathcal{V}) \subseteq \mathcal{F}^c$ . Therefore  $\mathcal{F} \subseteq f^{-1}(\mathcal{V})^c$ . Hence  $\mathcal{V}^c \subseteq f(\mathcal{F}) \subseteq f(f^{-1}(\mathcal{V})^c) \subseteq \mathcal{V}^c$  which implies  $f(\mathcal{F}) = \mathcal{V}^c$ . Since  $\mathcal{V}^c$  is Nu-g\* - closed,  $f(\mathcal{F})$  is Nu-g\*CS and thus  $f$  is a Nu-g\*- closed map.

### Conclusion

In this paper, we have defined the neutrosophic g\* closed sets and open sets. Then we discussed about neutrosophic g\* continuity. Then, we have presented some properties of these operations. We have also investigated neutrosophic topological structures of neutrosophic sets. Hence, we hope that the findings in this paper will help researchers enhance and promote the further study on neutrosophic topology.

**Funding:** This research received no external funding.

**Acknowledgments:** The authors are highly grateful to the Referees for their constructive suggestions.

**Conflicts of Interest:** The authors declare no conflict of interest

### References

- [1] Abdel-Basset, M., Mohamed, R., Elhoseny, M., & Chang, V. (2020). Evaluation framework for smart disaster response systems in uncertainty environment. *Mechanical Systems and Signal Processing*, 145, 106941.
- [2] Abdel-Basset, M., Ali, M., & Atef, A. (2020). Uncertainty assessments of linear time-cost tradeoffs using neutrosophic set. *Computers & Industrial Engineering*, 141, 106286.
- [3] Abdel-Basset, M., Ali, M., & Atef, A. (2020). Resource levelling problem in construction projects under neutrosophic environment. *The Journal of Supercomputing*, 76(2), 964-988.
- [4] Abdel-Basset, M., Gamal, A., Son, L. H., & Smarandache, F. (2020). A Bipolar Neutrosophic Multi Criteria Decision Making Framework for Professional Selection. *Applied Sciences*, 10(4), 1202.
- [5] K. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems* 20(1986), 87-94.
- [6] I. Arokiarani, R. Dhavaseelan, S. Jafari, M. Parimala, On Some New Notions and Functions In Neutrosophic Topological Spaces, *Neutrosophic Sets and Systems*, Vol. 16, 2017, (16-19)
- [7] V. Banu priya, S. Chandrasekar, Neutrosophic  $\alpha$  generalized semi closed set, *Neutrosophic Sets and Systems*, Vol. 28, 2019, 162-170.
- [8] R. Dhavaseelan and S. Jafari, Generalized Neutrosophic closed sets, *New trends in Neutrosophic theory and applications Volume II*- 261-273, (2018).
- [9] R. Dhavaseelan, S. Jafari and md. Hanif page, Neutrosophic generalized  $\alpha$ -contra-continuity, *creat. math. inform.* 27 (2018), no. 2, 133 – 139
- [10] Florentin Smarandache, Neutrosophic and Neutrosophic Logic, *First International Confer On*

- Neutrosophic Logic, Set, Probability, and Statistics University of New Mexico, Gallup, NM 87301, USA (2002), smarand@unm.edu
- [11] Floretin Smaradache, Neutrosophic Set: - A Generalization of Intuitionistic Fuzzy set, Journal of Defense resources Management. 1(2010), 107-114.
- [12] Ishwarya, P and Bageerathi, K., On Neutrosophic semi open sets in Neutrosophic topological spaces, International Jour. of Math. Trends and Tech. 2016, 214-223.
- [13] D.Jayanthi,  $\alpha$  Generalized Closed Sets in Neutrosophic Topological Spaces, International Journal of Mathematics Trends and Technology (IJMTT)- Special Issue ICRMIT March 2018.
- [14] Maheswari, C., Sathyabama, M., Chandrasekar, S., Neutrosophic generalized b-closed Sets Neutrosophic Topological Spaces, Journal of physics Conf. Series, 1139 (2018) 012065.
- [15] C.Maheswari, S. Chandrasekar, Neutrosophic gb-closed Sets and Neutrosophic gb-Continuity, Neutrosophic Sets and Systems, Vol. 29, 2019, 89-99
- [16] T Rajesh kannan, S.Chandrasekar, Neutrosophic  $\omega\alpha$ -closed sets in Neutrosophic topological space, Journal Of Computer And Mathematical Sciences, vol.9(10), 1400-1408 Octobe 2018.
- [17] T Rajesh kannan, S.Chandrasekar, Neutrosophic  $\alpha$ -continuous multifunction in Neutrosophic topological space, The International Journal of Analytical and Experimental Modal Analysis, Volume XI, Issue IX, September 2019, 1360-9367
- [18] T.RajeshKannan, and S.Chandrasekar, Neutrosophic  $\alpha$ -Irresolute Multifunction In Neutrosophic Topological Spaces, " Neutrosophic Sets and Systems 32, 1 (2020), 390-400.  
[https://digitalrepository.unm.edu/nss\\_journal/vol32/iss1/25](https://digitalrepository.unm.edu/nss_journal/vol32/iss1/25)
- [19] A.A. Salama and S.A. Alblowi, Generalized Neutrosophic Set and Generalized Neutrosophic Topological Spaces, Journal computer Sci. Engineering, Vol.(ii) No.(7)(2012).
- [20] A.A.Salama and S.A.Alblowi, Neutrosophic set and Neutrosophic topological space, ISOR J.mathematics, Vol.(iii) ,Issue(4),(2012).pp-31-35.
- [21] V.K.Shanthi, S.Chandrasekar, K.SafinaBegam, Neutrosophic Generalized Semi Closed Sets In Neutrosophic Topological Spaces, International Journal of Research in Advent Technology, Vol.6, No.7, July 2018, 1739-1743
- [22] V. Venkateswara Rao, Y. Srinivasa Rao, Neutrosophic Pre-open Sets and Pre-closed Sets in Neutrosophic Topology, International Journal of ChemTech Research, Vol.10, pp 449- 458, 2017
- [23] Fatimah M. Mohammed and Sarah W. Raheem, Generalized b Closed Sets and Generalized b Open Sets in Fuzzy Neutrosophic bi-Topological Spaces, Neutrosophic Sets and Systems, vol. 35, 2020, pp. 188-197. DOI: 10.5281/zenodo.3951661
- [24] R. Suresh and S. Palaniammal, Neutrosophic Weakly Generalized open and Closed Sets Neutrosophic Sets and Systems, vol. 33, 2020, pp. 67-77. DOI: 10.5281/zenodo.3782855
- [25] G.Jayaparthasarathy, M.Arockia Dasan, V.F.Little Flower and R.Ribin Christal, New Open Sets in N-Neutrosophic Supra Topological Spaces, Neutrosophic Sets and Systems, vol. 31, 2020, pp. 44-62. DOI: 10.5281/zenodo.3638252
- [26] R. Vijayalakshmi, A. Savitha Mary and S. Anjalmoose: Neutrosophic Semi-Baire Spaces, Neutrosophic Sets and Systems, vol. 30, 2019, pp. 132-142. DOI: 10.5281/zenodo.3569673
- [27] I. Mohammed Ali Jaffer and K. Ramesh: Neutrosophic Generalized Pre Regular Closed Sets, Neutrosophic Sets and Systems, vol. 30, 2019, pp. 171-181. DOI: 10.5281/zenodo.3569681

- [28] T. Nandhini, M. Vigneshwaran:  $\alpha g^{\#}\psi$ -closed map and  $N\alpha g^{\#}\psi$ -homomorphism in neutrosophic topological spaces, *Neutrosophic Sets and Systems*, vol. 29, 2019, pp. 186-196, DOI: 10.5281/zenodo.3514429
- [29] Anitha S, Mohana K, F. Smarandache: On NGSR Closed Sets in Neutrosophic Topological Spaces, *Neutrosophic Sets and Systems*, vol. 28, 2019, pp. 171-178. DOI: 10.5281/zenodo.3382534
- [30] R. Narmada Devi, R. Dhavaseelan and S. Jafari: A Novel on NSR Contra Strong Precontinuity, *Neutrosophic Sets and Systems*, vol. 27, 2019, pp. 70-79. DOI: 10.5281/zenodo.3275708

Received: May 4, 2020. Accepted: September 23, 2020