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Pythagorean Neutrosophic DEMATEL and AHP for Water Security Evaluation

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Abstract: This paper improves the current multi-criteria decision-making method for assessing water security by integrating the Pythagorean Neutrosophic Set (PNS) with the Analytic Hierarchy Process (AHP) and Decision-Making Trial and Evaluation Laboratory (DEMATEL). PNS effectively handles uncertainty and imprecision, offering a higher degree of truth, falsity, and indeterminacy compared to fuzzy sets. AHP excels at ranking objects by importance, while DEMATEL highlights cause-and-effect relationships. This study includes four key criteria: Water Availability (C1), Water Accessibility (C2), Water Quality (C3), and Climate Change (C4), along with twelve sub-criteria to generate a detailed understanding of how these criteria and sub-criteria influence water security. Using PNS-DEMATEL, the study develops a cause-and-effect diagram that maps causal linkages among the criteria, identifying key drivers to the water security problem. Concurrently, PNS-AHP ranks the criteria and sub-criteria based on expert assessments while highlighting their relative importance for sustainable decision-making. This integrated approach enhances the accuracy of water security evaluations and provides valuable insights for managing sustainable water resources. The findings offer policymakers actionable tools to prioritize interventions and craft legislation tailored to the region's unique challenges and needs.

Keywords: Pythagorean Neutrosophic Set; DEMATEL; AHP; decision making; water security assessment

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1. Introduction

Water security can be defined as reliable access to safe water, effective water management, and sustainable water quality, which are essential not only for human well-being but also for the survival of animals and ecosystems. It ensures that sufficient water is available for various needs without compromising the ability of future generations to meet their own demands [1]. However, rapid population growth and climate change have intensified global water challenges, increasing pressure on policymakers and researchers to develop integrated and sustainable solutions. In response, Integrated Water Reecosystemsgement (IWRM), which emphasizes the interconnectedness of water systems, communities, and ecosystems, has gained prominence, particularly through initiatives led by international organizations such as the United Nations [2]. Urban areas in developing countries are particularly vulnerable to water insecurity due to rapid urbanization and climate-induced changes in precipitation patterns [3]. These challenges are further exacerbated by human and environmental factors, including excessive groundwater extraction, industrial and agricultural pollution, and inefficient water usage [4,5]. While regions such as the Middle East and North Africa are widely recognized for severe water scarcity, similar concerns are increasingly emerging in non-arid regions [6].

In this context, Sarawak, Malaysia, despite its tropical climate and abundant rainfall, faces significant water security challenges. These arise from factors such as complex topography, uneven rainfall distribution, and anthropogenic pollution. The Sarawak River has been reported to contain elevated levels of heavy metals, including iron, cobalt, and copper, necessitating intensive treatment before the water becomes usable [7]. Additionally, Sarawak's hilly terrain, localized climatic conditions, and seasonal wind patterns further complicate water resource planning and management [8]. Addressing these issues requires comprehensive and integrated strategies, including infrastructure development, pollution mitigation, water reuse, and active public participation [9].

Similar challenges have been examined in different regions using various analytical approaches to assess water security. For example, Ding et al. [10] employed the Soil and Water Assessment Tool (SWAT) to evaluate water security by analysing the spatial and temporal distribution of blue and green water resources in the Weihe River Basin. On the other hand, Ogunbode et al. [11] evaluated water security in Iwo, Nigeria, by combining a 12-month household survey of 937 households on water sources and usage with a 31-year analysis of climatic data using descriptive and inferential statistical methods. Interestingly, many researchers have applied multi-criteria decision-making approaches to assess water security, including CRITIC-TOPSIS [12], VIKOR-TOPSIS [13], and BWM-VIKOR-TOPSIS [14].

Despite these developments, existing approaches often struggle to address uncertainty and the complex interdependence among environmental, social, and technical factors. This limitation reduces the ability of decision-makers to accurately identify critical issues, understand causal relationships, and implement effective interventions. Moreover, there remains a clear research gap, as no study has yet integrated Pythagorean Neutrosophic Sets (PNS) with both DEMATEL and the Analytic Hierarchy Process (AHP) within a single, unified framework. To overcome these limitations, this study proposes a hybrid decision-making framework that integrates Pythagorean Neutrosophic Sets (PNS) with DEMATEL and AHP.

PNS, an extension of neutrosophic and Pythagorean fuzzy sets, provides a powerful mechanism for modeling uncertainty through truth, indeterminacy, and falsity membership functions under a squared-sum constraint [15, 16]. This flexibility allows for more accurate representation of ambiguity in complex systems and has been successfully applied in areas such as risk assessment [17], decision support [18], and graph theory [19]. Building on this capability, the integration of PNS with DEMATEL enables the identification and analysis of causal relationships among water security factors, while the incorporation of AHP facilitates the prioritization of criteria within a structured hierarchical framework. Although alternative methods such as the Best-Worst Method (BWM), Full Consistency Method (FUCOM), Level-Based Weight Assessment (LBWA), and Double Intuitionistic Best–Worst Ratio (DIBR) have been developed to address some limitations of AHP, AHP remains a suitable choice in this study due to its strong compatibility with DEMATEL, widespread familiarity among practitioners, and extensive validation in the literature.

Furthermore, embedding AHP and DEMATEL within the PNS framework helps mitigate issues related to inconsistency and cognitive burden by explicitly modeling uncertainty in expert judgments. For example, Appasamy [20] and Paul and Mahapatra [21] used fuzzy DEMATEL, with the latter also combining fuzzy AHP to set priorities in sustainability issues. Çıkmak [22] and

Ayyildiz et al. [23] showed how Pythagorean Fuzzy AHP can be used to assess technology and transportation risks, proving its value for hierarchical decision-making under uncertainty.

In summary, this study contributes to the advancement of water security assessment by proposing a practical hybrid framework that enhances analytical real-world applicability. By integrating PNS, DEMATEL, and AHP, the proposed model provides a robust tool for evaluating water security under uncertainty and supports more informed and effective decision-making. The remainder of this paper is structured as follows. Section 2 reviews the relevant literature, while Section 3 presents the fundamental concepts underpinning the proposed methodology. Section 4 describes the research framework, followed by the implementation and results in Section 5. Finally, Section 6 concludes the paper.

2. Preliminaries

In this section, the definition of PNS introduced by Radha et al. [15] will be briefly explained since it is the fundamental part of this study. The PNS set operations facilitate more comprehensive data analysis, which acknowledges the complex and uncertain real-world issues.

Definition 1. [15]

Let X be a non-empty set (universe). A Pythagorean Neutrosophic Set with T and F as dependent neutrosophic components A on X is an object of the form

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\}, \tag{1}$$

where $T_A(x), I_A(x)$ and $F_A(x)$ are the truth, indeterminacy, and false membership respectively such that $T, I, F \in [0,1]$. Here when T and F are dependent components, then for all x in X:

$$T + F \le 1, \tag{2}$$

$$0 \le T^2 + F^2 \le 1, \tag{3}$$

$$0 \le T^2 + I^2 + F^2 \le 2, \tag{4}$$

Definition 2. [24] defined these basic operations on PNS which can be described as follows:

Let X be a non-empty set (universe). A Pythagorean Neutrosophic Set with ψ and κ as dependent neutrosophic components A and B of the form $A = \{\langle x, \psi_A(x), \varsigma_A(x), \kappa_A(x)\rangle | x \in X\}$ and $B = \{\langle x, \psi_B(x), \varsigma_B(x), \kappa_B(x)\rangle | x \in X\}$. The complement of A is

$$A^C = \{\langle x, \kappa_A(x), 1 - \varsigma_A(x), \psi_A(x)\rangle : x \in X\}., \tag{5}$$

The union and intersection of A and B are

$$\begin{aligned} A \cup B &= \{\max(\psi_A, \psi_B), \min(\varsigma_A, \varsigma_B), \min(\kappa_A, \kappa_B)\} \\ A \cap B &= \{\min(\psi_A, \psi_B), \max(\varsigma_A, \varsigma_B), \max(\kappa_A, \kappa_B)\}, \end{aligned} \tag{6}$$

Definition 3. [25] Given $x_1 = \left(\psi_{x_1}, \varsigma_{x_1}, \kappa_{x_1}\right)$, $x_2 = \left(\psi_{x_2}, \varsigma_{x_2}, \kappa_{x_2}\right)$ and $x = \left(\psi_x, \varsigma_x, \kappa_x\right)$ are any two PNSs, then the operational rules for PNSs such as addition, multiplication, scalar multiplication and power operations are defined as follows:

i. The PNS addition, denoted by $x_1 \oplus x_2$ are defined as:

$$\begin{aligned} x_1 \oplus x_2 &= \left(\psi_{x_1}, \varsigma_{x_1}, \kappa_{x_1}\right) + \left(\psi_{x_2}, \varsigma_{x_2}, \kappa_{x_2}\right) \\ &= \left(\sqrt{\psi_{x_1}{}^2 + \psi_{x_2}{}^2 - \psi_{x_1}{}^2 \psi_{x_2}{}^2}, \varsigma_{x_1}\varsigma_{x_2}, \kappa_{x_1}\kappa_{x_2}\right), \end{aligned} \tag{7}$$

ii. The PNS multiplication, denoted by $x_1 \otimes x_2$ are defined as:

$$\begin{aligned} x_1 \otimes x_2 &= \left(\psi_{x_1}, \varsigma_{x_1}, \kappa_{x_1}\right) \otimes \left(\psi_{x_2}, \varsigma_{x_2}, \kappa_{x_2}\right) \\ &= \left(\psi_{x_1}\psi_{x_2}, \varsigma_{x_1} + \varsigma_{x_2} - \varsigma_{x_1}\varsigma_{x_2}, \sqrt{\kappa_{x_1}{}^2 + \kappa_{x_2}{}^2 - \kappa_{x_1}{}^2 \kappa_{x_2}{}^2}\right), \end{aligned} \tag{8}$$

iii. The PNS scalar multiplication, denoted by μx where $\mu \in R$ and $\mu \geq 0$ are defined as:

$$A^C = \left\{\left\langle x, \kappa_A(x), 1 - \varsigma_A(x), \psi_A(x)\right\rangle : x \in X\right\}., \tag{9}$$

iv. The PNS power operation, denoted by x^μ where $\mu \in R$ and $\mu \geq 0$ are defined as:

$$\begin{aligned} x^\mu &= \left(\psi_x, \varsigma_x, \kappa_x\right)^\mu \\ &= \left(\psi_x{}^\mu, 1 - \left(1 - \varsigma_x\right)^\mu, \sqrt{1 - \left(1 - \kappa_x{}^2\right)^\mu}\right), \end{aligned} \tag{10}$$

3. Development of The Proposed PNS-DEMATEL and PNS-AHP Method

This section presents the development of a linguistic variable by PNS, the procedure method of PNS-DEMATEL and PNS-AHP. The development of the proposed method which is a unified Pythagorean Neutrosophic Set (PNS) DEMATEL with Pythagorean Neutrosophic Set (PNS) AHP method in Advancing Water Security Assessment in Sarawak. The primary features of this framework are the integration of PNS-DEMATEL and PNS-AHP methods, which provide a comprehensive approach to decision-making. Additionally, the framework incorporates the construction of linguistic variables specifically tailored for the PNS-AHP method, enhancing its precision and applicability in evaluating multiple criteria.

The framework begins with the data collection phase, where the main criteria for evaluation are identified, and relevant experts and decision-makers are selected to provide their insights. Following this, the aggregation phase is divided into two parts. The PNS-DEMATEL focuses on modelling interdependencies among criteria by constructing the initial direct influence matrix using PNS, computing average matrices, and performing deneutrosophication to derive crisp values. The process continues with normalizing the influence matrix, constructing the total-influence matrix, and calculating row and column sums. Meanwhile, the PNS-AHP emphasizes determining the hierarchical importance of criteria by generating a pairwise comparison matrix with PNS, performing deneutrosophication, normalizing the matrix, and ensuring consistency. In the result and discussion phase, the DEMATEL approach concludes by determining the threshold value and constructing a network relationship map (NRM), while the AHP approach provides a ranking of criteria based on normalized comparisons. This integrated framework offers the strengths of PNS, DEMATEL, and AHP to a better systematic and robust solution for multi-criteria decision-making problem. Figure 1 shows the framework of the proposed PNS-DEMATEL and PNS-AHP method.

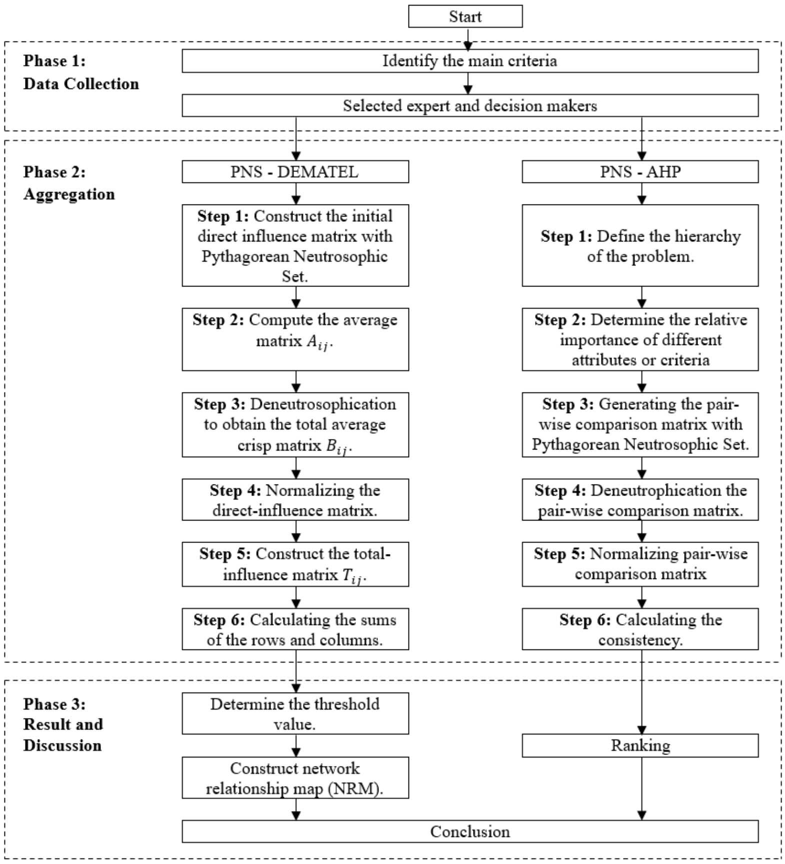

Figure 1. The proposed PNS-DEMATEL and PNS-AHP method.

3.1. Design the PNS linguistic variable

In the PNS-AHP framework, the linguistic variable leverages the Pythagorean Neutrosophic Set (PNS) to evaluate membership, non-membership, and indeterminacy, ensuring alignment with predefined criteria. Earlier applications, such as Saaty et al. [26], utilized 9-scale variables, as seen in Karamasa's [27] implementation of the extended single-valued neutrosophic set in Table 1.

Table 1. A 9-scale linguistic variable [27]

Saaty Scale	Linguistic Variable	Neutrosophic Triangular Scale
1	Equally influential	$\langle (1,1,1);0.50,0.50,0.50 \rangle$
3	Slightly influential	$\langle (2,3,4);0.30,0.75,0.70 \rangle$
5	Strongly influential	$\langle (4,5,6);0.80,0.15,0.20 \rangle$
7	Very strongly influential	$\langle (6,7,8);0.90,0.10,0.10 \rangle$
9	Absolutely influential	$\langle (9,9,9);1.00,1.00,1.00 \rangle$

2, 4, 6, 8	Sporadic values between two close scales	$\langle(1,2,3);0.40,0.65,0.60\rangle$ $\langle(3,4,5);0.60,0.35,0.40\rangle$ $\langle(5,6,7);0.70,0.25,0.30\rangle$ $\langle(7,8,9);0.85,0.10,0.15\rangle$

In the context of the proposed linguistic variable in PNS, pair-wise comparison scales are defined as the new PNS-AHP scale and its reciprocal, as illustrated in Table 2. These scales have now been integrated, replacing the conventional use of crisp numbers based on Table 1.

Table 2. A new PNS-AHP scale and its reciprocal

Saaty scale	**Linguistic Variable**	**PNS** (T I F)	**Reciprocal** (F I T)
1	Equal importance	$\langle 0.50,0.50,0.50\rangle$	$\langle 0.50,0.50,0.50\rangle$
3	Moderate importance	$\langle 0.30,0.75,0.70\rangle$	$\langle 0.70,0.75,0.30\rangle$
5	Strong importance	$\langle 0.80,0.15,0.20\rangle$	$\langle 0.20,0.15,0.80\rangle$
7	Very strong importance	$\langle 0.90,0.10,0.10\rangle$	$\langle 0.10,0.10,0.90\rangle$
9	Extreme importance	$\langle 1.00,0.00,0.00\rangle$	$\langle 0.00,0.00,1.00\rangle$
2, 4, 6, 8	Intermediate values	$\langle 0.40,0.65,0.60\rangle$ $\langle 0.60,0.35,0.40\rangle$ $\langle 0.70,0.25,0.30\rangle$ $\langle 0.85,0.10,0.15\rangle$	$\langle 0.60,0.65,0.40\rangle$ $\langle 0.40,0.35,0.60\rangle$ $\langle 0.30,0.24,0.70\rangle$ $\langle 0.15,0.10,0.85\rangle$

The scale of '2' in Table 2 is accompanied by a truth membership value of 0.4. The rationale behind attributing a truth membership of 0.4 in this context is to uphold equilibrium within the scale. The discrepancy between the higher extremity of the scale, denoted by score 6 with a truth membership of 0.7, and scale of 7 with a truth membership of 0.9, amounts to 0.1. To maintain a state of balance, it is imperative for the variance between the truth membership values of the lower end of the scale, particularly for scale 1, and 2 to also be 0.55. Through assigning a truth membership of 0.4 to the newly introduced scale of 2, the equilibrium between the lower and upper boundaries of the scale is effectively preserved. Therefore, while determining values for indeterminate and false membership, calculation according to the relevant truth value and ensure that meet the requirements for PNS as outlined in Eq. (2)-(4).

Step 1: Validate the linguistic variable of PNS.

As outlined in Table 2, is validated by assessing its compliance with the three requirements specified in Eq. (2)-(4). Applying Definition 1, consider $Y=\{y_1,y_2,\dots y_9)$ then

$$Y=\begin{Bmatrix} \langle y_1,0.50,0.50,0.50\rangle \\ \langle y_2,0.40,0.65,0.60\rangle \\ \langle y_3,0.30,0.75,0.70\rangle \\ \langle y_4,0.60,0.35,0.40\rangle \\ \langle y_5,0.80,0.15,0.20\rangle \\ \langle y_6,0.70,0.25,0.30\rangle \\ \langle y_7,0.90,0.10,0.15\rangle \\ \langle y_8,0.85,0.10,0.15\rangle \\ \langle y_9,1.00,0.00,0.00\rangle \end{Bmatrix},$$

A PN-subset of Y is a set-in which T is restricted to taking values from the set $\{0,1\}$. The values mentioned are inside the interval $[0,1]$, notably 0.10, 0.80, and 0.90. The accuracy of these values is confirmed by analyzing the conditions specified in Eq. (2)-(4). Let $Y_1=\langle 0.50,0.50,0.50\rangle$ and $Y_2=\langle 0.40,0.65,0.60\rangle$ are PNS numbers. Then, the verification of PNS numbers is presented in Table 3 which satisfies the three conditions mentioned in Eq. (2) to Eq. (4).

Table 3. The verification of PNS numbers

T	***I***	***F***	$T+F\le 1$	$T^2+F^2\le 1$	$T^2+I^2+F^2\le 2$
0.50	0.50	0.50	1	0.5	0.75
0.40	0.65	0.60	1	0.52	0.9425
0.30	0.75	0.70	1	0.58	1.1425
0.60	0.35	0.40	1	0.52	0.6425
0.80	0.15	0.20	1	0.68	0.7025
0.70	0.25	0.30	1	0.58	0.6425
0.90	0.10	0.10	1	0.82	0.83
0.85	0.10	0.15	1	0.745	0.755
1.00	0.00	0.00	1	1	1

3.2. Pythagorean Neutrosophic Sets – DEMATEL (PNS-DEMATEL) Method

The procedure of the PNS-DEMATEL method can be shown below:

Step 1: Construct the initial direct influence matrix.
A direct-relation matrix with Pythagorean neutrosophic numbers was constructed based on decision makers'(DMs) preferences. The *k*th DM gave the PNS score x_{ij}^k and the notation of x_{ij} shows the degree to which DM believes the criteria, i affects criteria, j and the diagonal components are set to zero for decision making, where the new PNS linguistic variable in Table 4.

Table 4. The new PNS linguistic variable [18]

Scale	Linguistic Variable	PNS Scale
1	No Influence	$\langle 0.10, 0.80, 0.90 \rangle$
2	Low Influence	$\langle 0.20, 0.70, 0.80 \rangle$
3	Medium Low Influence	$\langle 0.35, 0.60, 0.60 \rangle$
4	Medium Influence	$\langle 0.50, 0.40, 0.45 \rangle$
5	Medium High Influence	$\langle 0.65, 0.30, 0.25 \rangle$
6	High Influence	$\langle 0.80, 0.20, 0.15 \rangle$
7	Very High Influence	$\langle 0.90, 0.10, 0.10 \rangle$

The matrix contains PNS in the form $x_{ij}^k = \langle T_{ij}, I_{ij}, F_{ij} \rangle$, k is number of DMs as below,

$$X^k = \begin{bmatrix} 0 & x_{12}^k & \dots & x_{1n}^k \\ x_{21}^k & 0 & \dots & x_{2n}^k \\ \vdots & \vdots & 0 & \vdots \\ x_{n1}^k & x_{n2}^k & \dots & 0 \end{bmatrix}$$

Step 2: Compute the average matrix A_{ij}.

By combining the experts' opinions, the average direct-influence matrix A_{ij} can be obtained Eq. (11).

$$A_{ij}^k = \frac{1}{l} \sum_{k=1}^{l} x_{ij}^k, \quad i, j = 1, 2 \dots, n, \tag{11}$$

Step 3: Deneutrosophication to obtain the total average crisp matrix B_{ij}.

To deneutrosophicate the Pythagorean neutrosophic number $\left(T_A(x), I_A(x), F_A(x)\right)$ to a crisp value, the deneutrosophication formula is used as follows:

$$B_{ij} = \frac{T_A(x) + 0.5 I_A(x)}{1.5}, \tag{12}$$

Step 4: Normalizing the direct- influence matrix.

The normalized direct-influence matrix Z_{ij} be achieved by using Eq. (13).

$$Z_{ij} = \frac{B_{ij}}{s}, \tag{13}$$

where $s = \max\left(\max_{1 \le i \le n} \sum_{i=1}^{n} B_{ij}, \max_{1 \le j \le n} \sum_{j=1}^{n} B_{ij}, \right)$. All elements in the matrix Z_{ij} are complying with $0 < Z_{ij} < 1$.

Step 5: Create the total-influence matrix T_{ij} based on the normalized matrix Z as follows:

$$T_{ij} = Z_{ij}\left(I - Z_{ij}\right)^{-1}, \tag{14}$$

where I is the identity matrix.

Step 6: Calculating the sums of the rows and columns.
In this step, the vectors R and C representing the sum of the rows and the sum of columns from the total-influence matrix T_{ij} are defined by the following formulas:

$$R = \left[\sum_{j=1}^{n} T_{ij}\right]_{n\times 1}, \tag{15}$$

$$C = \left[\sum_{i=1}^{n} T_{ij}\right]_{1\times n}, \tag{16}$$

The $R+C$ and $R-C$ values are calculated in which these values reflect the importance and relation values, respectively.

3.3. Pythagorean Neutrosophic Sets – AHP (PNS-AHP) Method

The procedure of PNS-AHP method can be shown as below:
Step 1: Define the hierarchy of the problem.
The three-level hierarchy of the considered consists of Goal (Level 1), Criteria (Level 2) and Alternative (Level 3).
Step 2: Determine the relative importance of different attributes or criteria with respect to the goal.
AHP multiple pairwise comparisons are based on a nine-level standardized comparison scale in Table 3.
Step 3: Generating the pair-wise comparison matrix with Pythagorean Neutrosophic Set.
Definition 1 was used to propose PNS into AHP same goes with PNS-AHP. Let $C = \left\{C_j \mid j = 1,2,\ldots,n\right\}$ be the set of criteria. By using PNS-AHP scale in Table 3, the result of the pairwise comparison on n criteria can be summarized in an $(n\times n)$ evaluation matrix Y in which every element $y_{ij} = \left(i, j = 1,2,\ldots,n\right)$ is the quotient of weights of the criteria, as shown below:

$$Y = \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1n} \\ y_{21} & y_{22} & \cdots & y_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1} & y_{n2} & \cdots & y_{nn} \end{bmatrix},\ y_{ij} = 1, y_{ji} = \frac{1}{y_{ij}}, y_{ij} \neq 0., \tag{17}$$

Then, determine the aggregate of matrix E_{ij}. Combining the views of the I is the total of experts yields the average direct-influence matrix E_{ij}.

$$E_{ij}^{k} = \frac{1}{l}\sum_{k=1}^{l} y_{ij}^{k}, \quad i,j = 1,2,\ldots,n, \tag{18}$$

Step 4: Deneutrosophication the pair-wise comparison matrix.

To deneutrosophicate the PNS $\left(T_E(y), I_E(y), F_E(y)\right)$ to a crisp value at upper triangular matrix and the lower triangular matrix reciprocal as Eq. (19), the deneutrosophication formula is used as follows:

$$D_{ij} = \frac{T_E(y) + 0.5 I_E(y)}{1.5}, \tag{19}$$

Step 5: Normalizing pair-wise comparison matrix. The normalized and relative weight determination procedure for each matrix as follows:

$$F_{ij} = \frac{D_{ij}}{p}, \tag{20}$$

where $p = \sum_{i=1}^{n} D_{ij}$. All elements in the matrix F are complying with $0 < F_{ij} < 1$.Then, the largest eigenvalue, $\lambda_{\max}$ is represented by the right eigenvector, ω, which gives the criteria weights.

$$D_{\omega} = \lambda_{\max}\omega, \tag{21}$$

Another way is calculating the average of normalized pair-wise comparison matrix to find criteria weight and multiply with $p = \sum_{i=1}^{n} D_{ij}$ each. Then, $\lambda_{\max}$ value is the total of the results. The matrix E has rank 1 and $\lambda_{\max} = n$ (total of the criteria) if the pairwise comparisons are entirely consistent. In this instance, any of E's rows or columns can be normalized to obtain weight. Assuming perfect consistency in the pairwise comparisons, matrix E has rank 1 and $\lambda_{\max} = n$. In this case, normalizing any of E's rows or columns will provide weight.

Step 6: Calculating consistency.

The consistency of the pairwise comparison evaluations influence the accuracy with which the AHP produces its output. The relationship between matrix A that defines consistency: $E_{ij} \times E_{jk} = E_{ik}$. The Consistency Index (CI) formula as follows:

$$CI = \frac{\lambda_{\max} - n}{n-1}, \tag{22}$$

and the Consistency Ratio (CR) that defined as follows [27]:

$$CR = \frac{CI}{\text{Random Index}(RI)}, \tag{23}$$

The value of RI is related to the dimension of the matrix and will be extracted from Table 5. Note that consistency ratio lower than 0.10 verifies that the results of comparison are acceptable.

Table 5. The value of Random Index [28]

Dimension	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.5799	0.8921	1.1159	1.2358	1.3322	1.3952	1.4537	1.4882

4. Implementation of Water Security Evaluation

To address the specific problem of water security in Sarawak, Malaysia, this study applies the proposed PNS-AHP-DEMATEL framework to a real-world case study involving selected districts with varying water access and infrastructure conditions. Water security assessment here is defined as a systematic evaluation of the availability, accessibility, quality, and climate resilience of water resources. Sarawak's water challenges include inequitable distribution, inadequate supply in rural areas, seasonal extremes, and environmental degradation. Three experts were invited to provide their view on twelve sub-criteria under four main criteria. The proposed PNS-DEMATEL method will analyse the obtained data to identify the main causal factor affecting the others, while the proposed PNS-AHP will be employed to get the ranking of the criteria and sub-criteria. The findings offer a cause-effect diagram that can guide targeted interventions. This case study demonstrates the applicability and value of the proposed hybrid approach in supporting sustainable water management planning in Sarawak. The criteria, sub-criteria, and their descriptions are presented in Table 6.

Table 6. The description of criteria and sub-criteria

CRITERIA	**SUB-CRITERIA**	**DESCRIPTION**
Water Availability (C1)	Surface Water Availability (C11)	Refers to the amount of water that is accessible in rivers, lakes, and reservoirs for many purposes, including drinking, irrigation, and industrial use.
	Groundwater Availability (C12)	Refers to the volume of water contained underground in aquifers, which can be accessed by wells to provide drinking water and support agricultural irrigation.
	Seasonal Variability (C13)	Refers to variations in water availability over a year caused by changes in precipitation patterns, the melting of snow, and the flow of water, which impacts the dependability of water supply.

Water Accessibility (C2)	Physical Accessibility (C21)	Refers to the closeness and convenience of reaching water sources, such as wells, communal taps, and water distribution sites, to ensure that people may acquire water without facing excessive difficulty or having to travel long distances.
	Economic Accessibility (C22)	Refers to the accessibility of water services and infrastructure, considering household incomes, water prices, and procedures that ensure water remains available to all socio-economic groups.
	Infrastructure Accessibility (C23)	Encompasses the presence and effectiveness of water delivery infrastructure, including pipes, pumps, and storage tanks, to ensure that communities have sufficient access to secure and dependable water supply systems.
Water Quality (C3)	Chemical Contaminants (C31)	Encompasses contaminants such as toxic metals, insecticides, and industrial compounds that have the potential to pollute water supplies, hence endangering human well-being and the ecosystem when ingested or utilized for household, agricultural, or industrial activities.
	Microbiological Contaminants (C32)	Waterborne diseases, such as cholera, typhoid, and diarrhea, can be caused by pathogenic microorganisms such as bacteria, viruses, and parasites. These diseases occur when water becomes contaminated with sewage or fecal matter.
	Physical Characteristics (C33)	Encompasses the visual, gustatory, and olfactory aspects of water, which determine its aesthetic value and suitability for different purposes. Factors such as turbidity, color, and sedimentation influence how water quality is perceived and accepted.
Climate Change (C4)	Changes in Precipitation Patterns (C41)	Describes modifications in the timing, magnitude, and spatial pattern of precipitation and snowfall because of climate change, resulting in changes in water supply, periods of drought, and occurrences of floods in various places.
	Temperature Increases (C42)	Temperature change-induced warming can lead to increased evaporation rates, and changes in hydrological cycles, and worsen water scarcity and heat-related water stress in susceptible regions.
	Extreme Weather Events (C43)	Encompasses the presence of severe weather events such as hurricanes, cyclones, and storms, which can lead to flooding, infrastructure destruction, and water source pollution, thus impacting water security and

resilience.

Figure 2 illustrates the hierarchical structure of the criteria and sub-criteria used in the water security evaluation.

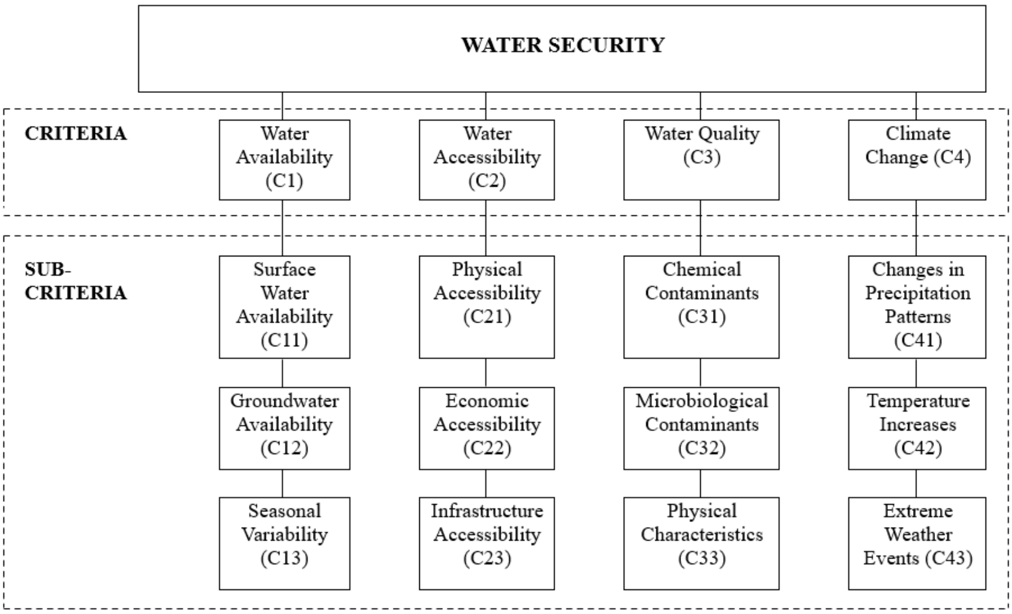

Figure 2. The Hierarchy of Advancing Water Security Assessment in Sarawak.

4.1 PNS-DEMATEL Method

During the initial phase, the proposed PNS-DEMATEL method is employed to identify the causal relationships among the separate groups in the research framework. Identifying the linkages at this stage is crucial as it can potentially unveil the hierarchical significance of the criteria and sub-criteria. Next, create a network relationship map (NRM) that illustrates the connections and interactions between the criterion and sub-criteria. The linguistic judgments provided by each expert are presented in Table 7 as the direct-relation matrix.

Table 7: The direct-relation matrix

Criteria	Expert	C1	C2	C3	C4
C1	E1	0	1	2	3
	E2	0	1	2	2
	E3	0	3	2	5
C2	E1	3	0	6	1
	E2	2	0	1	3
	E3	2	0	7	4
C3	E1	5	1	0	3
	E2	6	2	0	2
	E3	1	1	0	2
C4	E1	3	2	4	0
	E2	4	3	5	0
	E3	2	1	3	0

Table 8: The PNS direct-relation matrix, X_{ij}^{k}

Criteria	Expert	C1	C2	C3	C4
C1	E1	$\langle 0,0,0 \rangle$	$\langle 0.10,0.80,0.90 \rangle$	$\langle 0.20,0.70,0.80 \rangle$	$\langle 0.35,0.60,0.60 \rangle$
	E2	$\langle 0,0,0 \rangle$	$\langle 0.10,0.80,0.90 \rangle$	$\langle 0.20,0.70,0.80 \rangle$	$\langle 0.20,0.70,0.80 \rangle$
	E3	$\langle 0,0,0 \rangle$	$\langle 0.35,0.60,0.60 \rangle$	$\langle 0.20,0.70,0.80 \rangle$	$\langle 0.65,0.30,0.25 \rangle$
C2	E1	$\langle 0.35,0.60,0.60 \rangle$	$\langle 0,0,0 \rangle$	$\langle 0.80,0.20,0.15 \rangle$	$\langle 0.10,0.80,0.90 \rangle$
	E2	$\langle 0.20,0.70,0.80 \rangle$	$\langle 0,0,0 \rangle$	$\langle 0.10,0.80,0.90 \rangle$	$\langle 0.35,0.60,0.60 \rangle$
	E3	$\langle 0.20,0.70,0.80 \rangle$	$\langle 0,0,0 \rangle$	$\langle 0.90,0.10,0.10 \rangle$	$\langle 0.50,0.40,0.45 \rangle$
C3	E1	$\langle 0.65,0.30,0.25 \rangle$	$\langle 0.10,0.80,0.90 \rangle$	$\langle 0,0,0 \rangle$	$\langle 0.35,0.60,0.60 \rangle$
	E2	$\langle 0.80,0.20,0.15 \rangle$	$\langle 0.20,0.70,0.80 \rangle$	$\langle 0,0,0 \rangle$	$\langle 0.20,0.70,0.80 \rangle$
	E3	$\langle 0.10,0.80,0.90 \rangle$	$\langle 0.10,0.80,0.90 \rangle$	$\langle 0,0,0 \rangle$	$\langle 0.20,0.70,0.80 \rangle$
C4	E1	$\langle 0.35,0.60,0.60 \rangle$	$\langle 0.20,0.70,0.80 \rangle$	$\langle 0.50,0.40,0.45 \rangle$	$\langle 0,0,0 \rangle$
	E2	$\langle 0.50,0.40,0.45 \rangle$	$\langle 0.35,0.60,0.60 \rangle$	$\langle 0.65,0.30,0.25 \rangle$	$\langle 0,0,0 \rangle$
	E3	$\langle 0.20,0.70,0.80 \rangle$	$\langle 0.10,0.80,0.90 \rangle$	$\langle 0.35,0.60,0.60 \rangle$	$\langle 0,0,0 \rangle$

The matrix A_{ij} is computed by applying Eq. (11) to represent the evaluation made by three experts, as displayed in Table 9. The PNS value in the first row, second column, A_{12} can be obtained by taking the averaging of X_{12}^{1}, X_{12}^{2}, and X_{12}^{3}:

$$A_{12} = \left\langle \frac{0.1+0.1+0.35}{3}, \frac{0.8+0.8+0.6}{3}, \frac{0.9+0.9+0.6}{3} \right\rangle$$
$$= \langle 0.18, 0.73, 0.80 \rangle$$

Table 9. Average matrix, A_{ij}

Criteria	C1	C2	C3	C4
C1	$\langle 0,0,0 \rangle$	$\langle 0.18,0.73,0.80 \rangle$	$\langle 0.20,0.70,0.80 \rangle$	$\langle 0.40,0.53,0.55 \rangle$
C2	$\langle 0.25,0.67,0.73 \rangle$	$\langle 0,0,0 \rangle$	$\langle 0.60,0.37,0.38 \rangle$	$\langle 0.32,0.60,0.65 \rangle$
C3	$\langle 0.52,0.43,0.43 \rangle$	$\langle 0.13,0.77,0.87 \rangle$	$\langle 0,0,0 \rangle$	$\langle 0.25,0.67,0.73 \rangle$
C4	$\langle 0.35,0.57,0.62 \rangle$	$\langle 0.22,0.70,0.77 \rangle$	$\langle 0.50,0.43,0.43 \rangle$	$\langle 0,0,0 \rangle$

Table 10 displays the crisp averaging matrix B_{ij} which is derived using Eq. (12) for deneutrosophication. The crisp value in the first row, second column, B_{12} is demonstrated as follows:

$$B_{12} = \frac{T(x)+0.5I(x)}{1.5}$$

$$B_{12} = \frac{0.18+0.5(0.73)}{1.5} = 0.3667$$

Table 11 shows the crips averaging matrix, B_{ij} .

Table 10. The crips averaging matrix, B_{ij}

Criteria	C1	C2	C3	C4	Total Row
C1	0.0000	0.3667	0.3667	0.4444	1.1778
C2	0.3889	0.0000	0.5222	0.4111	1.3222
C3	0.4889	0.3444	0.0000	0.3889	1.2222
C4	0.4222	0.3778	0.4778	0.0000	1.2778
Total Column	1.3000	1.0889	**1.3667**	1.2444	

The matrix Z_{ij} is normalized by using Eq. (13) as shown in Table 11. The computation of Z_{12} is shown below:

$$Z_{12} = \frac{B_{12}}{s},$$

where $s = \max\left(\max_{1 \le i \le 4} \sum_{i=1}^{4} B_{ij}, \max_{1 \le j \le 4} \sum_{j=1}^{4} B_{ij} \right) = \max(1.3667, 1.3222) = 1.3667$

Thus, $Z_{12} = \frac{0.3667}{1.3667} = 0.2683$

Table 11. Normalized matrix, Z_{ij}

Criteria	C1	C2	C3	C4
C1	0.0000	0.2683	0.2683	0.3252
C2	0.2845	0.0000	0.3821	0.3008
C3	0.3577	0.2520	0.0000	0.2845
C4	0.3089	0.2764	0.3496	0.0000

Table 12 displays the total influence matrix T_{ij} which can be computed using Eq. (14).

Table 12. Total influence matrix, T_{ij}

Criteria	C1	C2	C3	C4	Total Row (*R*)
C1	2.4155	2.3069	2.6970	2.5719	9.9913
C2	2.8626	2.2886	2.9936	2.7718	10.9167
C3	2.7410	2.3489	2.5440	2.6062	10.2401
C4	2.8045	2.4427	2.8995	2.4717	10.6185
Total Column (*C*)	10.8237	9.3870	11.1342	10.4217	

The total of rows and columns is computed using Eq. (15) and Eq. (16) as displayed in Table 13. The criteria were divided into two groups which are cause and effect, based on the *R-C* values. The criteria in the cause group were water accessibility and climate change. Conversely, the criteria in the effect group were water availability and water quality. The significance of the criteria was subsequently established by assessing the $R+C$ value, with water security emerging as the most crucial criterion within the cause group.

Table 13. Rankings of criteria

Criteria	*R*	*C*	$R+C$	$R-C$	*Cause/Effect*
C1	9.9913	10.8237	20.8149	-0.8324	Effect
C2	10.9167	9.3870	20.3037	1.5297	Cause
C3	10.2401	11.1342	21.3742	-0.8941	Effect
C4	10.6185	10.4217	21.0402	0.1969	Cause

The network relationship map, shown in Figure 3 below, was created using $R+C$ and $R-C$ values. The criteria are ranked based on $R+C$ values from the highest to the lowest. Refer to the Table 14, the ranking of criteria as follows: $C3 \succ C4 \succ C1 \succ C2.$ "Water quality" is the most influential criteria, while "water accessibility" is the least influential.

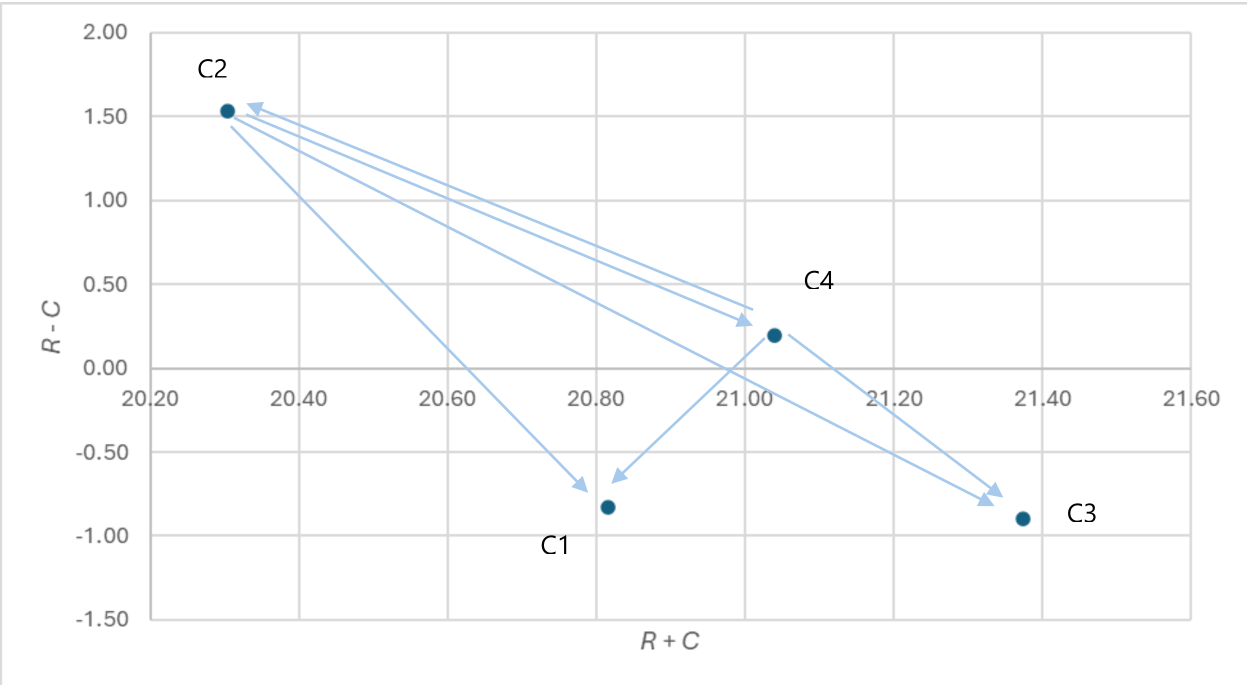

Figure 3. Network Relationship Map (NRM) of Criteria.

Figure 3 illustrates the network relationship map (NRM) of the criteria, depicting the interactions between causes and effects. The arrow from C2 to C4 indicates that C1 and C3 are the causes of the criteria. In an NRM, two arrows pointing at each other typically signify a bidirectional relationship, indicating mutual interaction between the nodes. For C2 and C4, the arrows pointing at each other demonstrate that these two criteria are effects. The significance of each sub-criterion can be determined by examining the sum of $R+C$ in Table 14. Additionally, their roles as causes or effects can be identified by analysing the difference $R-C$.

Table 14. Rankings of Sub-criteria

Criteria	*Sub-criteria*	*R*	*C*	$R+C$	$R-C$	*Cause / Effect*
C1	C11	61.7450	60.2662	122.0112	1.4788	Cause
	C12	60.2420	61.2373	121.4793	-0.9953	Effect
	C13	61.2858	61.7693	123.0551	-0.4835	Effect
C2	C21	14.0538	12.6290	26.6828	1.4248	Cause
	C22	12.3880	13.4102	25.7982	-1.0222	Effect
	C23	13.5660	13.9686	27.5347	-0.4026	Effect
C3	C31	17.1641	15.9497	33.1137	1.2144	Cause
	C32	17.5205	17.3810	34.9015	0.1395	Cause
	C33	16.4040	17.7579	34.1620	-1.3539	Effect
C4	C41	30.8692	30.3672	61.2364	0.5020	Cause
	C42	29.8652	30.3796	60.2448	-0.5144	Effect
	C43	29.8735	29.8611	59.7346	0.0124	Cause

The sum $R+C$ represents the aggregate significance of each sub-criterion within the network, with higher values indicating greater significance. For Water Availability (C1), Seasonal Variability (C13) holds the highest relevance, with a value of 123.0551, signifying its substantial role. In Water Accessibility (C2), Infrastructure Accessibility (C23) is the most significant factor, with a value of 27.5347. For Water Quality (C3), Microbiological Contaminants (C32) holds the highest significance, with a value of 34.9015. Climate Change (C4) is most significantly impacted by Change in Precipitation Pattern (C41), with a value of 61.2364.

The difference between $R-C$ (cause and effect) helps determine the role of each sub-criterion in terms of causality. Positive values indicate that the sub-criterion serves as a cause, while negative values suggest it acts as an effect. In Water Availability (C1), Surface Water Availability (C11) is a causal factor with an $R-C$ value of 1.4788, whereas C12 and C13 are effects with values of -0.9953 and -0.4835, respectively. In Water Accessibility (C2), Physical Accessibility (C21) is identified as a cause with a value of 1.4248, while C22 and C23 are effects. For Water Quality (C3), both C31 and C32 function as causes, with C32 having a modest value of 0.1395, and Physical Characteristics (C33) is identified as an effect. Climate Change (C4) identifies C41 and C43 as causal factors, with C43 having a minimal value of 0.0124, while Temperature Increase (C42) is an effect.

The sub-criteria with the highest $R+C$ values, demonstrating their relevance within each criterion, are C13, C23, C32, and C41. These sub-criteria play essential roles within their respective criteria. In cause-effect analysis, sub-criteria with positive $R-C$ values, such as C11, C21, C31, C32, C41, and C43, are considered causes, while those with negative $R-C$ values, such as C12, C13, C22, C23, C33, and C42, are considered effects.

The NRM provides crucial insights by visually depicting the significance of each sub-criterion, emphasizing nodes with high $R+C$ values, and displaying the causal relationships. Table 15 presents the conclusions from this analysis, sorting the importance order based on the $R+C$

value. Higher values indicate greater importance, while lower values indicate lesser importance. The $R-C$ value is used to determine whether the criteria are causes or effects, with positive values indicating causes and negative values indicating effects.

Table 15. Result of PNS-DEMATEL analysis

	Order of Important	Cause	Effect
Criteria	$C3 \succ C4 \succ C1 \succ C2$	C2, C4	C1, C3
Sub-Criteria	**C3:** $C32 \succ C33 \succ C31$	C31, C32	C33
	C4: $C41 \succ C42 \succ C43$	C41, C43	C42
	C1: $C13 \succ C11 \succ C12$	C11	C12, C13
	C2: $C23 \succ C21 \succ C22$	C21	C22, C23

4.2 PNS-AHP Method

The Pair-Wise Comparison Matrix, Y_{ij} in Table 16 displays the comparison of criteria (C1, C2, C3, and C4) with one another. Each element in the matrix denotes the relative significance between two criteria.

Table 16. The pair-wise comparison matrix, Y_{ij}

Criteria (E1)	C1	C2	C3	C4
C1	**1**	**7**	**1/5**	**3**
C2	**1/7**	**1**	**1/9**	**1/5**
C3	**5**	**9**	**1**	**5**
C4	**1/3**	**5**	**1/5**	**1**

Table 17 shows the average matrix, E_{ij}. By computing the average of the subjective opinions, the matrix helps one to get a fairer viewpoint on the relative importance of every criterion by using Eq. (18). It removes any anomalies and lays a better basis for further research. An example of a calculation matrix E_{12} as below:

Table 17. Average matrix, E_{ij}

Criteria	C1	C2	C3	C4
C1	1	$\langle 0.60, 0.12, 0.40 \rangle$	$\langle 0.40, 0.15, 0.60 \rangle$	$\langle 0.27, 0.55, 0.73 \rangle$
C2	$\langle 0.40, 0.12, 0.60 \rangle$	1	$\langle 0.37, 0.03, 0.63 \rangle$	$\langle 0.33, 0.47, 0.67 \rangle$
C3	$\langle 0.60, 0.15, 0.40 \rangle$	$\langle 0.63, 0.03, 0.37 \rangle$	1	$\langle 0.77, 0.35, 0.23 \rangle$
C4	$\langle 0.73, 0.55, 0.27 \rangle$	$\langle 0.67, 0.47, 0.33 \rangle$	$\langle 0.23, 0.35, 0.77 \rangle$	1

The aggregated pair-wise comparison matrix D_{ij} as illustrated in Table 18 results from the aggregated relative weights of every criterion. This matrix shows the whole relevance of every element by giving a general picture of the comparisons using Eq. (19). An example of a calculation matrix D_{12} as below:

Table 18. The aggregated pair-wise comparison matrix, D_{ij}

Criteria	C1	C2	C3	C4
C1	1.0000	0.4389	0.3167	0.3611
C2	2.2785	1.0000	0.2556	0.3778
C3	3.1579	3.9130	1.0000	0.6278
C4	2.7692	2.6471	1.5929	1.0000
Total Column	**9.2056**	**7.9990**	**3.1651**	**2.3667**

Table 19 normalizes F_{ij}, the aggregated pair-wise comparison matrix. The relative weight interpretation is simpler when the normalizing process assures that the total weight of all criteria is equal to one by Eq. (20). An example of a calculation matrix F_{ij} as below:

Table 19. Normalizing pair-wise comparison matrix, F_{ij}

Criteria	C1	C2	C3	C4	Weight	λ
C1	0.1086	0.0549	0.1000	0.1526	0.1040	0.9577
C2	0.2475	0.1250	0.0807	0.1596	0.1532	1.2256
C3	0.3430	0.4892	0.3159	0.2653	0.3534	1.1184
C4	0.3008	0.3309	0.5033	0.4225	0.3894	0.9215

The weights represent the computed values for each criterion (C1 through C4), highlighting their relative importance in the decision-making process. The eigenvector, λ helps verify the coherence of the comparisons. Typically, the consistency ratio should be below a certain threshold to ensure the reliability of the decisions. Table 20 performs a consistency test on the criteria to verify the reliability of the pairwise comparisons made in previous steps. This test assesses the coherence of judgments by calculating the Consistency Index (CI) and the Consistency Ratio (CR). In this case, the CR score is 0.0827, which is below 0.1, indicating that the judgments are consistent and acceptable.

Table 20. The consistency test for the criteria

Criteria	Weight	λ
C1	0.1040	0.9577
C2	0.1532	1.2256
C3	0.3534	1.1184
C4	0.3894	0.9215
n = 4	**1**	$\lambda_{max} = 4.1481$

The obtained weights are considered acceptable because the value of CR is less than 0.1.

Based on Eq. (22), the calculation to get Consistency Index is as follows:

$$CI = \frac{\lambda_{\max} - n}{n-1} = 0.0744$$

Hence, Eq. (23) is used to compute the Consistency Ratio,

$$CR = \frac{CI}{RI} = 0.0827$$

Table 21 extends the consistency test to include both criteria and sub-criteria. The table summarizes the criteria and sub-criteria, along with their respective weights and consistency values. Each criterion is divided into sub-criteria, each with its own weight. For example, the criterion "Water Availability" is further divided into sub-criteria such as "Surface Water Availability" and "Groundwater Availability," each assigned specific weights and consistency values. This step ensures that both the primary criteria and the specific sub-criteria are consistent, providing a comprehensive validation of the decision-making process. The consistency percentages for sub-criteria, such as 3.25% for "Surface Water Availability" and 5.93% for "Microbiological Contaminants," indicate the degree of consistency in the judgments made for each sub-criterion. The calculation of weights for sub-criteria follows the same procedure as the calculation of weights for criteria in step 5 of the PNS-AHP method.

Table 21. The consistency test for the criteria and sub-criteria

Criteria	Weight	Consistency	Sub-criteria	Weight	Consistency
Water Availability	0.1040	0.0827 (8.27%)	Surface Water Availability	0.1789	0.0325 (3.25%)
			Ground Availability	0.2399	
			Seasonal Variability	0.5812	
Water Accessibility	0.1532		Physical Accessibility	0.2369	0.0300 (3.00%)
			Economic Accessibility	0.2943	
			Infrastructure Accessibility	0.4687	
Water Quality	0.3534		Chemical Contaminants	0.2313	0.0593 (5.93%)
			Microbiological Contaminants	0.3311	
			Physical Characteristics	0.4376	
Climate Change	0.3894		Change in Precipitation Patterns	0.2220	0.0326 (3.26%)

	Temperature Increases	0.3013
	Extreme Weather Events	0.4767

The consistency test is crucial for verifying the logical validity and reliability of the pairwise comparisons conducted during the AHP process. The criteria have an overall consistency ratio of 0.0827, equivalent to 8.27%. Given that, a consistency ratio below 10% is deemed acceptable [42], the judgments made in this analysis are considered logically consistent. The weights assigned to the criteria are as follows: Water Availability (0.1040), Water Accessibility (0.1532), Water Quality (0.3534), and Climate Change (0.3894), indicating their relative significance in the overall evaluation. The weights and consistency ratios within the sub-criteria confirm the analysis's reliability. For Water Availability, both Surface Water Availability and Groundwater Availability exhibit a consistency ratio of 3.25%, which falls within the permissible range. These factors are assigned weights of 0.1789 and 0.2399, respectively. The sub-criterion of Seasonal Variability, under Water Availability, carries a weight of 0.5812, suggesting its utmost significance despite the absence of an exact consistency ratio. Regarding Water Accessibility, Economic Accessibility has a consistency ratio of 3.00% and a weight of 0.2943. Physical Accessibility and Infrastructure Accessibility have weights of 0.2369 and 0.4687, respectively, though their exact consistency ratios are not provided. These weights highlight the relative importance of economic and infrastructural elements in determining water accessibility.

The sub-criteria for Water Quality include Chemical Contaminants with a consistency ratio of 5.93% and a weight of 0.2313, Microbiological Contaminants with a weight of 0.3311, and Physical Characteristics with a weight of 0.4376. Although the consistency ratio for Chemical Contaminants is higher, it remains within the acceptable range, confirming the reliability of these judgments. For Climate Change, the consistency ratio for the change in precipitation patterns is 3.26%, with a weight of 0.2220. The weights assigned to Temperature Increases and Extreme Weather Events are 0.3013 and 0.4767, respectively, indicating their significant impact despite the absence of specific consistency ratios. Overall, the table demonstrates that the criteria and sub-criteria in the AHP analysis are logically consistent, with all consistency ratios within the permissible range. The overall consistency ratio of 8.27% signifies coherent and dependable judgments. The analytical results are reinforced by the acceptable consistency ratios observed in each sub-criterion of Water Availability, Water Accessibility, and Climate Change. The highest consistency ratio is 5.93% for Chemical Contaminants under Water Quality, which falls within acceptable standards, confirming the reliability and strength of the pairwise comparisons conducted in this research.

Table 22 summarises the overall weight and ranking of different criteria and sub-criteria about Water Security in Sarawak. The study reveals that Climate Change (C4) is the most prominent factor, carrying a weight of 0.3894. This emphasizes its utmost significance among the parameters considered. As a result of Climate Change, the weight of Water Quality (C3) is at 0.3534, making it the second most significant element. The criteria of water accessibility and availability have lower influence, with weights of 0.1532 and 0.1040, respectively. Overall weight is calculated as follows:

Table 22. Overall weight for criteria and sub-criteria

Criteria	Weight	Sub-criteria	Weight	Overall Weight	Rank
Water Availability (C1)	0.1040	Surface Water Availability (C11)	0.1789	0.0186	12
		Ground Availability (C12)	0.2399	0.0249	11
		Seasonal Variability (C13)	0.5812	0.0604	8
Water Accessibility (C2)	0.1532	Physical Accessibility (C21)	0.2369	0.0363	10
		Economic Accessibility (C22)	0.2943	0.0451	9
		Infrastructure Accessibility (C23)	0.4687	0.0718	7
Water Quality (C3)	0.3534	Chemical Contaminants (C31)	0.2313	0.0817	6
		Microbiological Contaminants (C32)	0.3311	0.1170	4
		Physical Characteristics (C33)	0.4376	0.1546	2
Climate Change (C4)	0.3894	Change in Precipitation Patterns (C41)	0.2220	0.0864	5
		Temperature Increases (C42)	0.3013	0.1173	3
		Extreme Weather Events (C43)	0.4767	0.1856	1

A detailed examination of the sub-criteria reveals subtle and significant insights. The top three sub-criteria, ranked by importance, are Extreme Weather Events (rank 1), Physical Characteristics (rank 2), and Temperature Increases (rank 3). Conversely, the sub-criteria with the least significance are Surface Water Availability (rank 23), Groundwater Availability (rank 11), and Physical Accessibility (rank 10). This thorough analysis underscores the prominent impact of Climate Change, particularly Extreme Weather Events, on water resource management. Additionally, it highlights the crucial elements of Water Quality and Water Accessibility, while acknowledging that Water Availability, though important, has a comparatively less influence.

The PNS-DEMATEL and PNS-AHP evaluations assess various criteria and sub-criteria to determine their relative relevance and interconnections. Together, these evaluations provide a comprehensive view of the fundamental factors affecting the system and their interdependencies, guiding strategic development and resource allocation. The PNS-DEMATEL analysis highlights the complex interactions among sub-criteria within the same criterion. For instance, in Water Availability (C1), C11 and C21 act as causative elements, significantly impacting C12 and C13. Similarly, in Water Accessibility (C2), C22 and C23 are influenced by C31 and C32, respectively. These interdependencies suggest that addressing the fundamental causes (like C11 and C21 for C1) would improve the associated sub-criteria.

The PNS-AHP study ranks sub-criteria based on their overall weight, emphasizing their relative significance. The top-ranked sub-criteria are related to Climate Change, specifically "Extreme Weather Events" (0.1856) and "Temperature Increases" (0.1173), highlighting the urgency of addressing climate change issues. High-ranking sub-criteria for Water Quality include "Physical Characteristics" (0.1546) and "Microbiological Contaminants" (0.1170), underscoring the importance of maintaining water quality. The study highlights that climate change (C4) is the most significant factor influencing water security in Sarawak, with a weight of 0.3894, followed by water quality (C3) at 0.3534. Among the sub-criteria, extreme weather events stand out as the most critical, highlighting the pressing need to address climate-related challenges. By integrating PNS-DEMATEL method, the interconnected impacts of causative elements like C11 (surface water availability) and C21 (groundwater availability) on other sub-criteria become evident.

4.3 Comparative Analysis

To demonstrate the applicability and feasibility of the proposed procedures, this section presents a twofold comparative analysis. First, a comparison of the performance of the proposed Pythagorean Neutrosophic DEMATEL–AHP approach with conventional DEMATEL and AHP methods using crisp numbers. The results, summarized in Table 23, highlight the impact of incorporating PNS in handling uncertainty and modelling the interrelationships among criteria. By allowing independent degrees of truth, indeterminacy, and falsity, the proposed approach provides a unique representation of expert judgments compared to traditional crisp methods. Second, a conceptual comparison with recent MCDM models, particularly by Radovanović et al. [40] and Fujita [41]. The study by Radovanović et al. [40] integrates Spherical Fuzzy AHP with Grey MARCOS for evaluating university professors, employing spherical fuzzy sets to capture uncertainty and the MARCOS method for ranking alternatives. Fujita [41] introduces Hyperfuzzy VIKOR and Hyperfuzzy DEMATEL, extending classical fuzzy models to handle hierarchical and multi-layered uncertainty within a hyperfuzzy framework. In contrast, the proposed Pythagorean Neutrosophic DEMATEL–AHP approach differs in both methodology and application focus. The key advantages of the proposed method include enhanced uncertainty modelling, causal relationship analysis, domain-specific applicability, and a balance between accuracy and interpretability. Overall, this dual comparison demonstrates not only the quantitative performance improvements of the proposed approach over traditional crisp methods but also its conceptual advantages over existing fuzzy and hyperfuzzy MCDM models, thereby reinforcing its suitability for water security evaluation. Table 23 summarises the findings of the proposed method with the conventional DEMATEL and AHP methods.

Table 23. Comparative analysis with existing methods

Criteria	Sub-criteria	AHP		PNS-AHP		DEMATEL			PNS-DEMATEL		
		Overall Weight	Rank	Overall Weight	Rank	R+C	R-C	Causal group	R+C	R-C	Causal group
Water Availability (C1)	Surface Water Availability (C11)	0.151	4	0.019	10	41.6	1.27	Cause	122.0	1.47	Cause
	Groundwater Availability (C12)	0.017	9	0.025	9	40.8	-0.66	Effect	121.5	-0.99	Effect
	Seasonal Variability (C13)	0.066	5	0.060	6	42.4	-0.62	Effect	123.0	-0.48	Effect
Water Accessibility (C2)	Physical Accessibility (C21)	0.055	6	0.036	8	10.4	0.48	Cause	26.7	1.42	Cause
	Economic Accessibility	0.005	12	0.045	7	11.7	0.04	Cause	25.8	-1.02	Effect

	(C22)										
	Infrastructure Accessibility (C23)	0.013	10	0.072	5	11.8	-0.52	Effect	27.5	-0.40	Effect
Water Quality (C3)	Chemical Contaminants (C31)	0.222	1	0.082	4	14.7	1.42	Cause	33.1	1.21	Cause
	Microbiological Contaminants (C32)	0.181	3	0.117	2	15.4	-1.01	Effect	34.9	0.13	Cause
	Physical Characteristics (C33)	0.201	2	0.155	1	17.0	-0.41	Effect	34.2	-1.35	Effect
Climate Change (C4)	Change in Precipitation Patterns (C41)	0.055	7	0.086	3	22.47	-0.26	Effect	61.23	0.50	Cause
	Temperature Increases (C42)	0.011	11	0.117	10	21.76	-0.25	Effect	60.24	-0.51	Effect
	Extreme Weather Events (C43)	0.023	8	0.186	9	21.14	0.51	Cause	59.73	0.01	Cause

Table 23 presents a comparative analysis of the evaluation results obtained through AHP, the proposed PNS-AHP, DEMATEL, and the proposed PNS-DEMATEL. The results highlight several important insights. From the AHP analysis, Chemical Contaminants (C31) emerged as the most influential sub-criterion, followed by Physical Characteristics (C33) and Microbiological Contaminants (C32), indicating that water quality factor is prioritized in traditional weight-based decision-making. However, when uncertainty and indeterminacy elements are incorporated in the proposed PNS-AHP method, a slight shift is observed. Although Physical Characteristics (C33) and Microbiological Contaminants (C32) remain highly ranked, Change in Precipitation Patterns (C41) rises to the top position, highlighting the growing importance of climate-related risks when uncertainty is explicitly considered.

The DEMATEL results further refined these insights by identifying the structural relationships among sub-criteria. Surface Water Availability (C11), Chemical Contaminants (C31), and Physical Accessibility (C21) are classified as causal factors with positive (R–C) values, suggesting that these sub-criteria play a driving role and exert influence over other components of the system. In contrast, Groundwater Availability (C12), Seasonal Variability (C13), and Infrastructure Accessibility (C23) were categorized as effect factors, indicating that they are more responsive to upstream changes rather than being primary drivers. The integration of PNS with DEMATEL strengthened the interpretation of these interdependencies under uncertain conditions. For instance, Surface Water Availability (C11) (R–C = 1.47) and Chemical Contaminants (C31) (R–C = 1.21) remained dominant causal factors, reinforcing their role as primary levers in managing water-related challenges. At the same time, Physical Characteristics (C33) shifted strongly into the effect group (R–C = –1.35), suggesting that while it holds high weight in terms of importance, it is not a root cause but rather a consequence of other factors.

This comparative analysis provides two layers of understanding. The weight-based methods (AHP, PNS-AHP) highlight which sub-criteria are perceived as the most important, while the causal–effect methods (DEMATEL, PNS-DEMATEL) uncover the structural drivers of the system. Practically, this implies that effective water management strategies should prioritize controlling causal sub-criteria such as surface water availability and chemical contamination, as interventions in these areas are likely to reduce downstream impacts, including those from extreme weather events and physical characteristics. The methodological contribution is also noteworthy, as integrating PNS with both AHP and DEMATEL demonstrates the value of incorporating uncertainty, leading to more robust and realistic decision-making outcomes.

5. Conclusions

This study advances the theoretical landscape of decision-making under uncertainty by integrating the Pythagorean Neutrosophic Set (PNS) with AHP and DEMATEL. The combination contributes a novel hybrid framework capable of simultaneously handling imprecise information (via PNS), prioritizing decision elements (via AHP), and identifying causal relationships (via DEMATEL). The proposed framework can be generalized and adapted to other complex, multidimensional problems beyond water security, especially where ambiguity and interdependencies exist. Practically, this study provides a decent policymaker implication in evaluating and improving water security strategies. By identifying key drivers such as water availability and climate change, and mapping their interrelations, policymakers can make informed decisions that target the root causes. This structured evaluation enables governments, water management authorities, and NGOs to allocate resources more effectively, prioritize actions, and design targeted interventions in line with the Sustainable Development Goals (SDGs). This study provides a detailed understanding of water security in Sarawak, revealing Climate Change as the most critical factor, with a weight of 0.3894, followed closely by Water Quality at 0.3534. The analysis highlights the profound impact of Extreme Weather Events and Temperature Increases, which rank as the top sub-criteria, underscoring the urgent need to address climate-related challenges. While Water Accessibility and Water Availability are also important, their relatively lower weights suggest that Sarawak's current water resources are less immediately threatened, though they remain vulnerable to future climate-driven changes. The integration of PNS-DEMATEL and PNS-AHP offers a practical implication for evaluating the complex interdependencies among criteria and sub-criteria, providing valuable insights for strategic resource allocation and policy development. While the integrated PNS-AHP-DEMATEL approach offers several strengths, the study has some limitations. First, the reliance on expert judgments may introduce bias or subjectivity despite the robustness of the methods. Second, the scope is limited to predefined criteria and sub-criteria; additional factors relevant in other geographical or socio-political contexts may not be captured. Finally, the framework's effectiveness depends on the quality and availability of local data, which may pose a challenge in data-scarce regions.

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