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Neutrosophic Nano Semi-Frontier

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Abstract: Smarandache presented and built up the new idea of Neutrosophic set from the Intuitionistic fuzzy sets. A.A. Salama presented Neutrosophic topological spaces by utilizing the Neutrosophic sets. M.L.Thivagar et al., created Nano topological spaces and Neutrosophic nano topological spaces. Point of this paper is we present and study the properties of Neutrosophic Nano semi frontier in Neutrosophic nano topological spaces and its portrayal are talked about subtleties.

Keywords: Neutrosophic Nano semi open set, Neutrosophic Nano semi closed set, Neutrosophic Nano frontier, Neutrosophic Nano semi frontier, Neutrosophic nano topology.

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1. Introduction

Nano topology explored by M.L.Thivagar [15]et.al can be communicated as an assortment of nano approximations, Neutrosophic sets set up by F.Smarandache[14]. Neutrosophic set is illustrate by three functions: a membership, indeterminacy and nonmembership functions that are independently related. Neutrosophic set have wide scope of uses, all things considered. M.L.Thivagar et al., created Neutrosophic nano topological spaces .Neutrosophic nano semi closed, neutrosophic nano αclosed, neutrosophic nano pre closed, neutrosophic nano semi pre closed and neutrosophic nano regular closed are presented by M.Parimala[17] et al. Point of the current paper is we learned about properties of Neutrosophic Nano frontier, Neutrosophic Nano semi frontier in Neutrosophic nano topological spaces

2. PRELIMINARIES

In this section, we recall needed basic definition and operation of Neutrosophic sets

Definition 2.1 : [15]

Let U be a non-empty set and R be an equivalence relation on U. Let F be a neutrosophic set in U with the membership function μ_F , the indeterminacy function σ_F and the non-membership functionν_F . The neutrosophic nano lower,neutrosophic nano upper approximation and neutrosophic nano boundary of F in the approximation (U,R) denoted by $\underline{N(F)}$, $\overline{N(F)}$and $B_N(F)$are respectively defined as follows:

(i) $\underline{N(F)} = \{< u, \mu_{\underline{R}}(M_1^*)(u), \sigma_{\underline{R}}(M_1^*)(u), \nu_{\underline{R}}(M_1^*)(u) > / y \in [u]_R, u \in U\}$.

(ii) $\overline{N(F)} = \{< u, \mu_{\overline{R}}(M_1^*)(u), \sigma_{\overline{R}}(M_1^*)(u), \nu_{\overline{R}}(M_1^*)(u) > / y \in [u]_R, u \in U\}$.

(iii) $B_N(F)=\overline{N(F)} - \underline{N(F)}$

Definition 2.2 : [15]

Let U be an universe, R be an equivalence relation on U and F be a neutrosophic set in U and if the collection $N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(F)}, \overline{N(F)}, B_N(F)\}$

forms a topology then it is said to be a neutrosophic nano topology. We call (U, $N_N(\tau)$) as the neutrosophic nano topological space. The elements of $N_N(\tau)$ are called neutrosophic nano open sets.

Definition 2.3 : [15]

Let U be a nonempty set and the Neutrosophic sets M_1^* and M_2^* in the form M_1^*={< u:$\mu_{M_1^*}(u)$, $\sigma_{M_1^*}(u)$, $\nu_{M_1^*}(u)$>, u∈U},

M_2^*={< u:$\mu_{M_2^*}(u)$, $\sigma_{M_2^*}(u)$, $\nu_{M_2^*}(u)$>, u∈U}}.

Then the following statements hold:

(i) 0_{N_N} = {< u, 0, 0, 1 >: u∈U} and 1_{N_N} = {< u, 1, 1, 0 >: u∈U}.

(ii) $M_1^* \subseteq M_2^*$ iff { $\mu_{M_1^*}(u) \leq \mu_{M_2^*}(u)$, $\sigma_{M_1^*}(u) \leq \sigma_{M_2^*}(u)$, $\nu_{M_1^*}(u) \geq \nu_{M_2^*}(u)$, ∀u∈U}.

(iii) $M_1^* = M_2^*$ iff $M_1^* \subseteq M_2^*$ and $M_2^* \subseteq M_1^*$.

(iv) M_1^{*C} ={< u, $\nu_{M_1^*}(u)$, $1-\sigma_{M_1^*}(u)$, $\mu_{M_1^*}(u)$ >: u ∈ U}.

(v) $M_1^* \cap M_2^*$ ={u, $\mu_{M_1^*}(u) \wedge \mu_{M_2^*}(u)$, $\sigma_{M_1^*}(u) \wedge \sigma_{M_2^*}(u)$, $\nu_{M_1^*}(u) \vee \nu_{M_2^*}(u)$, ∀u∈U}.

(vi) $M_1^* \cup M_2^*$ ={u, $\mu_{M_1^*}(u) \vee \mu_{M_2^*}(u)$, $\sigma_{M_1^*}(u) \vee \sigma_{M_2^*}(u)$, $\nu_{M_1^*}(u) \wedge \nu_{M_2^*}(u)$, ∀u∈U}.

(vii) $\cup M_j^* = \langle u, \vee, \vee, \wedge \rangle$

(viii) $\cap M_j^* = \langle u, \wedge, \wedge, \vee \rangle$

(ix) $M_1^* - M_2^* = M_1^* \cap M_2^{*C}$

Proposition 2.4 [15]

For any Neutrosophic Nano set M_1^* in (U, $N_N(\tau)$)) we have

(1) N^NCl ((M_1^*)^C) = (N^NInt (M_1^*))^C,

(2) N^NInt ((M_1^*)^C) =(N^NCl (M_1^*))^C.

(3) $M_1^* \subseteq M_2^* \Rightarrow N^N$Int ($M_1^*$) ⊆ N^NInt (M_2^*),

(4) $M_1^* \subseteq M_2^* \Rightarrow N^N$Cl($M_1^*$) ⊆ N^NCl(M_2^*),

(5) N^NInt (N^NInt (M_1^*)) = N^NInt(M_1^*),

(6) N^NCl (N^NCl (M_1^*)) = N^NCl (M_1^*),

(7) N^NInt ($M_1^* \cap M_2^*$)) = N^NInt (M_1^*) ∩ N^NInt (M_2^*),

(8) N^NCl ($M_1^* \cup M_2^*$) = N^NCl(M_1^*) ∪ N^NCl(M_2^*),

(9) N^NInt (0_{N_N}) = 0_{N_N} ,

(10) N^NInt (1_{N_N}) = 1_{N_N} ,

(11) N^NCl (0_{N_N}) = 0_{N_N} ,

(12) N^NCl (1_{N_N}) = 1_{N_N} ,

(13) $M_1^* \subseteq M_2^* \Rightarrow$ ($M_2^{*C} \subseteq M_1^{*C}$,

(14) N^NCl ($M_1^* \cap M_2^*$)⊆N^NCl (M_1^*) ∩ N^NCl (M_2^*),

(15) N^NInt ($M_1^* \cup M_2^*$)⊇ N^NInt (M_1^*) ∪ N^NInt (M_2^*).

3. NEUTROSOPHIC NANO FRONTIER

In this section, the concepts of the Neutrosophic Nano frontier in Neutrosophic Nano topological space are introduced and also discussed their characterizations with some related examples.

Definition 3.1.

Let U be a *N-N-T-S* and let M_1^* ⊚⊚*NNS* (U). Neutrosophic Nano frontier of M_1^* and is denoted by *NFr* (M_1^*). i.e., $N^N Fr(M_1^*) = N^N Cl(M_1^*) \cap N^N Cl(M_1^*)^C$.

Proposition 3.2. For each M_1^* ⊚⊚*NNS*(U), $M_1^* \cup N^N Fr(M_1^*) \subseteq N^N Cl(M_1^*)$.

Proof : Let M_1^* be the *NNS* in the N-N-T-S U. Using Definition 3.1.,

$M_1^* \cup N^N Fr(M_1^*) = M_1^* \cup (N^N Cl(M_1^*) \cap N^N Cl((M_1^{*C}))$

$= (M_1^* \cup N^N Cl(M_1^*)) \cap (M_1^* \cup N^N Cl((M_1^{*C}))$

$\subseteq N^N Cl(M_1^*) \cap N^N Cl(M_1^{*C})$

$\subseteq N^N Cl(M_1^*)$

Hence $M_1^* \cup N^N Fr(M_1^*) \subseteq N^N Cl(M_1^*)$.

Example 3.3.

Let U and $\mathcal{A}$ be two non-empty finite sets,

where U is the universe and $\mathcal{A}$ the set of attributes
The members of U= {P_1, P_2, P_3,P_4}are pressure patients
Let U/R ={{P_1, P_2, P_3}, {P_4}} be an equivalence relation
$\mathcal{A}$ = {Salt food , colostreal food } are two attributes
P_1= $\langle x, \left(\frac{5}{10},\frac{10}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{2}{10},\frac{5}{10}\right)\rangle$
P_2= $\langle x, \left(\frac{2}{10},\frac{5}{10},\frac{9}{10}\right),\left(\frac{0}{10},\frac{5}{10},\frac{10}{10}\right)\rangle$
P_3= $\langle x, \left(\frac{5}{10},\frac{10}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$
P_4= $\langle x, \left(\frac{2}{10},\frac{5}{10},\frac{9}{10}\right),\left(\frac{0}{10},\frac{2}{10},\frac{10}{10}\right)\rangle$
$N_N(\tau)$ = {0_{N_N}, 1_{N_N}, $\underline{N(F)}$, $\overline{N(F)}$,B_N(F)}

$\underline{N(F)} = \langle x, \left(\frac{2}{10},\frac{5}{10},\frac{9}{10}\right),\left(\frac{0}{10},\frac{5}{10},\frac{10}{10}\right)\rangle$
$\overline{N(F)} = \langle x, \left(\frac{5}{10},\frac{10}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$
BN(F) = $\langle \left(\frac{5}{10},\frac{5}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$

$N_N(\tau)$= {0_{N_N}, 1_{N_N}, $\langle x, \left(\frac{2}{10},\frac{5}{10},\frac{9}{10}\right),\left(\frac{0}{10},\frac{5}{10},\frac{10}{10}\right)\rangle$, $\langle \left(\frac{5}{10},\frac{10}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$,
$\langle \left(\frac{5}{10},\frac{5}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$, $\langle \left(\frac{2}{10},\frac{5}{10},\frac{9}{10}\right),\left(\frac{0}{10},\frac{2}{10},\frac{10}{10}\right)\rangle$}
Here N^NCl (P_3) = 1_{N_N} and N^NCl (P_3^c) = $\langle \left(\frac{2}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{9}{10}\right)\rangle$.
Using Definition 2.1, N^NFr (M_1^*) = $\langle \left(\frac{2}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{5}{10},\frac{5}{10},\frac{9}{10}\right)\rangle$.
Also M_1^* ∪◎N^NFr (M_1^*) = $\langle \left(\frac{5}{10},\frac{10}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{5}{10}\right)\rangle \subseteq 1_{N_N}$.
Therefore N^NCl (M_1^*) = $1_{N_N} \nsubseteq \langle \left(\frac{5}{10},\frac{10}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$.

Theorem 3.5.
For a NNS M_1^* in the N-N-T-S U, N^NFr (M_1^*) =N^NFr (M_1^{*C}).
Proof : Let M_1^* be the *NNS* in the N-N-T-S U. Using Definition 3.1.,
N^NFr (M_1^*) = N^NCl (M_1^*) ∩◎N^NCl (M_1^{*C})
= N^NCl (M_1^{*C}) ∩◎N^NCl (M_1^*)
= N^NCl (M_1^{*C}) ∩◎$N^NCl\left(M_1^{*C}\right)^C$
Again by Definition 3.1,= N^NFr (M_1^{*C})
Hence N^NFr (M_1^*) = N^NFr (M_1^{*C}).

Theorem 3.6.
If a N^NS M_1^* is a *NCS*, then N^NFr (M_1^*) ⊆◎M_1^*.
Proof :
Let M_1^* be the N^NS in the Neutrosophic Nano topological space U. Using Definition 3.1.,
N^NFr (M_1^*) = N^NCl (M_1^*) ∩ N^NCl (M_1^{*C}) ⊆ N^NCl (M_1^*)
By Propositon (2.4) ,= M_1^*
Hence N^NFr (M_1^*) ⊆◎M_1^*, if M_1^* is N^NCS in U.
The converse of the above theorem needs not be true as shown by the following example.

Theorem 3.7.
If a *NNS* M_1^* is N^NOS, then N^NFr (M_1^*) ⊆◎◎M_1^{*C}.
Proof :
Let M_1^* be the *NNS* in the N-N-T-S U. Using Definition 3.1 ,
M_1^* is N^NOS implies $M_1^{*C}$is N^NCS in U. By Theorem 3.6, N^NFr (M_1^{*C}) ⊆ $M_1^{*C}$and by Theorem 3.5, we get N^NFr (M_1^*) ⊆ M_1^{*C}

Theorem 3.8.
For a *NNS* M_1^* in the N^NTS U, (N^NFr (M_1^*))^C= N^NInt (M_1^*) ∪◎N^NInt (M_1^{*C}).
Proof :
Let M_1^* be the N^NS in the N-N-T-S U. Using Definition 3.1.,
(N^NFr (M_1^*))^C = (N^NCl (M_1^*))^C(∩◎N^NCl (M_1^{*C})))

By Propositon (2.4) ,= $(N^N Cl\ (M_1^*)\ )^C \cup\circledcirc (N^N Cl\ (M_1^{*C})))^C$

By Propositon (2.4),= $N^N Int\ (M_1^{*C}) \cup\circledcirc N^N Int\ (M_1^*)$

Hence $(N^N Fr\ (M_1^*))^C = N^N Int\ (M_1^*) \cup\circledcirc N^N Int\ (M_1^{*C})$.

Theorem 3.9

Let $M_1^* \subseteq\circledcirc M_2^*$ and $M_2^* \circledcirc\circledcirc N^N C$ (U) (resp.,$M_2^* \circledcirc\circledcirc N^N O$ (U)). Then $N^N Fr\ (M_1^*) \subseteq\circledcirc M_2^*$ (resp., $N^N Fr\ (M_1^*) \subseteq (N^N Cl\ (M_2^*)\ )^C$), where $N^N C$ (U) (resp., $N^N O$ (U)) denotes the class of Neutrosophic Nano closed (resp., Neutrosophic Nano open) sets in U.

Proof : Use Prop.,2.4 , $M_1^* \subseteq\circledcirc M_2^*$,

$N^N Cl\ (M_1^*) \subseteq\circledcirc N^N Cl\ (M_2^*)$ -------------------- (1).

By Definition 3.1.,

$N^N Fr\ (M_1^*) = N^N Cl\ (M_1^*) \cap\circledcirc N^N Cl\ (M_1^{*C})$

$\subseteq\circledcirc N^N Cl\ (M_2^*) \cap\circledcirc N^N Cl\ (M_1^{*C})$ by (1)

$\subseteq\circledcirc N^N Cl\ (M_2^*) = M_2^*$

Hence $N^N Fr\ (M_1^*) \subseteq\circledcirc M_2^*$.

Theorem 3.10

Let M_1^* be the *NNS* in the *N-N-T-S* U. Then

$N^N Fr\ (M_1^*) = N^N Cl\ (M_1^*) - N^N Int\ (M_1^*)$.

Proof : Let M_1^* be the *NNS* in the N-N-T-S U. By Propositon (2.4),

$((N^N Cl\ (M_1^{*C})))^C = N^N Int\ (M_1^*)$ and by Definition 3.1.,

$N^N Fr\ (M_1^*) = N^N Cl\ (M_1^*) \cap\circledcirc N^N Cl\ (M_1^{*C})$

$= N^N Cl\ (M_1^*) - (NCl\ (M_1^{*C}))^C$

by using $M_1^* - M_2^* = M_1^* \cap\circledcirc M_2^{*C}$

By Propositon (2.4),

$= N^N Cl\ (M_1^*) - N^N Int\ (M_1^*)$

Hence $N^N Fr\ (M_1^*) = N^N Cl\ (M_1^*) - N^N Int\ (M_1^*)$.

Theorem 3.11.

For a *NNS* M_1^* in the $N^N TS$ U, $N^N Fr\ (N^N Int\ (M_1^*)) \subseteq\circledcirc N^N Fr\ (M_1^*)$.

Proof :

Let M_1^* be the NNS in the N-N-T-S U. Using Definition 3.1.,

$N^N Fr\ (N^N Int\ (M_1^*)) = N^N Cl\ (N^N Int\ (M_1^*)) \cap N^N Cl\ (N^N Int\ (M_1^*))\ )^C$

By Propositon (2.4),

$= N^N Cl\ (N^N Int\ (M_1^*)) \cap N^N Cl\ (N^N Cl\ (M_1^{*C})))$

By Propositon (2.4),

$= N^N Cl\ (N^N Int\ (M_1^*)) \cap N^N Cl\ (M_1^{*C})\ )$

. By Propositon (2.4), ,

$\subseteq\circledcirc N^N Cl\ (M_1^*) \cap N^N Cl\ (M_1^{*C}))$

Again by Definition 3.1.,

$= N^N Fr\ (M_1^*)$

Hence $N^N Fr\ (N^N Int\ (M_1^*)) \subseteq\circledcirc N^N Fr\ (M_1^*)$.

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Example 3.12.

Let U and $\mathcal{A}$ be two non-empty finite sets,

where U is the universe and $\mathcal{A}$ the set of attributes

The members of U= {P_1, P_2, P_3,P_4}are pressure patients

Let U/R ={{P_1, P_2, P_3}, {P_4}} be an equivalence relation

$\mathcal{A} = \{\text{Head ache, Temperature}\}$ are two attributes

$P_1 = \langle x, \left(\frac{5}{10}, \frac{6}{10}, \frac{7}{10}\right), \left(\frac{10}{10}, \frac{9}{10}, \frac{4}{10}\right)\rangle$

$P_2 = \langle x, \left(\frac{3}{10}, \frac{9}{10}, \frac{2}{10}\right), \left(\frac{4}{10}, \frac{1}{10}, \frac{6}{10}\right)\rangle$

P3= ⟨x, $\left(\frac{5}{10},\frac{9}{10},\frac{2}{10}\right),\left(\frac{4}{10},\frac{9}{10},\frac{4}{10}\right)$⟩
P4= ⟨x, $\left(\frac{3}{10},\frac{6}{10},\frac{7}{10}\right),\left(\frac{1}{10},\frac{1}{10},\frac{6}{10}\right)$⟩
$N_N(\tau)$ = {0_{N_N}, 1_{N_N}, $\underline{\text{N(F)}}$, $\overline{N(\text{F})}$, BN(F)}

$\underline{\text{N(F)}}$ = ⟨x, $\left(\frac{3}{10},\frac{6}{10},\frac{7}{10}\right),\left(\frac{4}{10},\frac{1}{10},\frac{6}{10}\right)$⟩
$\overline{N(\text{F})}$ = ⟨x, $\left(\frac{5}{10},\frac{9}{10},\frac{2}{10}\right),\left(\frac{10}{10},\frac{9}{10},\frac{4}{10}\right)$⟩
BN(F) = ⟨x, $\left(\frac{5}{10},\frac{4}{10},\frac{3}{10}\right),\left(\frac{6}{10},\frac{9}{10},\frac{4}{10}\right)$⟩

$N_N(\tau)$= {0_{N_N}, 1_{N_N}, ⟨x, $\left(\frac{3}{10},\frac{6}{10},\frac{7}{10}\right),\left(\frac{4}{10},\frac{1}{10},\frac{6}{10}\right)$⟩, ⟨ $\left(\frac{5}{10},\frac{9}{10},\frac{2}{10}\right),\left(\frac{10}{10},\frac{9}{10},\frac{4}{10}\right)$⟩,
⟨$\left(\frac{5}{10},\frac{4}{10},\frac{3}{10}\right),\left(\frac{6}{10},\frac{9}{10},\frac{4}{10}\right)$⟩, $\left(\frac{3}{10},\frac{6}{10},\frac{7}{10}\right),\left(\frac{1}{10},\frac{1}{10},\frac{6}{10}\right)$}

$M_3^* = \langle\left(\frac{5}{10},\frac{4}{10},\frac{3}{10}\right),\left(\frac{6}{10},\frac{9}{10},\frac{4}{10}\right)\rangle$

Therefore by Definition 3.1., $N^N Fr\ (M_3^*)$ = ⊈ $N^N Fr\ (N^N Int\ (M_3^*))$.

Theorem 3.13.

For a *NNS* M_1^* in the *N-N-T-S* U, $N^N Fr\ (N^N Cl\ (M_1^*)) \subseteq N^N Fr\ (M_1^*)$.

Proof : Let M_1^* be the *NNS* in the N-N-T-S U. Using Definition 3.1.,
$N^N Fr\ (N^N Cl\ (M_1^*)) = N^N Cl\ (N^N Cl\ (M_1^*)) \cap N^N Cl\ ((N^N Cl\ (M_1^*)))\ )^C$ By Propositon (2.4),
$= N^N Cl\ (M_1^*) \cap N^N Cl\ (N^N Int\ ({M_1^*}^C))$By Propositon (2.4),
$\subseteq N^N Cl\ (M_1^*) \cap N^N Cl\ ({M_1^*}^C)$Again by Definition 3.1.,
$= N^N Fr\ (M_1^*)$
Hence $N^N Fr\ (N^N Cl\ (M_1^*)) \subseteq N^N Fr\ (M_1^*)$.

Theorem 3.14.

Let M_1^* be the *NNS* in the *N-N-T-S* U. Then $N^N Int\ (M_1^*) \subseteq_{\circledcirc} M_1^* - N^N Fr\ (M_1^*)$.

Proof : Let M_1^* be the *NNS* in the N-N-T-S U. Now by Definition 3.1.,
$M_1^* - N^N Fr\ (M_1^*) = M_1^* - (N^N Cl\ (M_1^*) \cap_{\circledcirc} N^N Cl\ ({M_1^*}^C))$
$= (M_1^* - N^N Cl\ (M_1^*)) \cup (M_1^* - N^N Cl\ ({M_1^*}^C))$
$= M_1^* - N^N Cl\ ({M_1^*}^C)$
$N^N Int\ (M_1^*)$.
Hence $N^N Int\ (M_1^*) \subseteq M_1^* - N^N Fr\ (M_1^*)$.

Remark 3.15.

In general topology, the following conditions are hold :
$N^N Fr\ (M_1^*) \cap_{\circledcirc} N^N Int\ (M_1^*)$ = 0N,
$N^N Int\ (M_1^*) \cup_{\circledcirc} N^N Fr\ (M_1^*) = N^N Cl\ (M_1^*)$,
$N^N Int\ (M_1^*) \cup_{\circledcirc} N^N Int\ ({M_1^*}^C) \cup_{\circledcirc} N^N Fr\ (M_1^*) = 1_{N_N}$.

Theorem 3.16.

Let M_1^* and M_2^* be the two *NNSs* in the *N-N-T-S*.
Then $N^N Fr\ (M_1^* \cup_{\circledcirc} M_2^*) \subseteq_{\circledcirc} N^N Fr\ (M_1^*) \cup_{\circledcirc} N^N Fr\ (M_2^*)$.

Proof : Let M_1^* and M_2^* be the two *NNSs* in the *N-N-T-S* U.
Using Definition 3.1.,
$N^N Fr\ (M_1^* \cup_{\circledcirc} M_2^*) = N^N Cl\ (M_1^* \cup_{\circledcirc} M_2^*) \cap_{\circledcirc} N^N Cl\ ((M_1^* \cup M_2^*)^C$
. By Propositon (2.4),
$= N^N Cl\ (M_1^* \cup_{\circledcirc} M_2^*) \cap_{\circledcirc} N^N Cl\ (\ {M_1^*}^C \cap_{\circledcirc} {M_2^*}^C)$
$\subseteq_{\circledcirc} (N^N Cl\ (M_1^*) \cup_{\circledcirc} N^N Cl\ (M_2^*)) \cap_{\circledcirc} (N^N Cl\ ({M_1^*}^C) \cap_{\circledcirc} N^N Cl({M_2^*}^C))$
$= [(N^N Cl\ (M_1^*) \cup_{\circledcirc} N^N Cl\ (M_2^*)\ ) \cap_{\circledcirc} N^N Cl\ ({M_1^*}^C)\ ] \cap_{\circledcirc} [\ (N^N Cl\ (M_1^*) \cup_{\circledcirc} N^N Cl\ (M_2^*)\ ) \cap_{\circledcirc} N^N Cl\ \ ({M_2^*}^C)\ ]$
$= [(N^N Cl\ (M_1^*) \cap_{\circledcirc} N^N Cl\ ({M_1^*}^C)) \cup (\ N^N Cl\ (M_2^*) \cap_{\circledcirc} N^N Cl({M_1^*}^C))] \cap_{\circledcirc} [(\ N^N Cl\ (M_1^*) \cap_{\circledcirc} N^N Cl\ ({M_2^*}^C)) \cup (\ N^N Cl(M_2^*) \cap_{\circledcirc} N^N Cl({M_2^*}^C))]$
Again by Definition 3.1.,
$= [N^N Fr\ (M_1^*) \cup_{\circledcirc} (\ N^N Cl\ (M_2^*) \cap_{\circledcirc} N^N Cl\ ({M_1^*}^C))\ ] \cap_{\circledcirc} [\ (N^N Cl\ (M_1^*) \cap_{\circledcirc} N^N Cl\ ({M_2^*}^C)\ ) \cup_{\circledcirc} N^N Fr(M_2^*)\ ]$
$= (N^N Fr\ (M_1^*) \cup_{\circledcirc} N^N Fr\ (M_2^*)) \cap_{\circledcirc} [\ (N^N Cl\ (M_2^*) \cap_{\circledcirc} N^N Cl\ ({M_1^*}^C)\ )$

$\cup\circledcirc(N^NCl\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^{*C}))]$
$\subseteq\circledcirc N^NFr\ (M_1^*) \cup\circledcirc N^NFr\ (M_2^*)$.
Hence $N^NFr\ (M_1^* \cup\circledcirc M_2^*) \subseteq\circledcirc N^NFr\ (M_1^*) \cup\circledcirc N^NFr\ (M_2^*)$.

Note 3.17.
$N^NFr\ (M_1^* \cap\circledcirc M_2^*) \nsubseteq N^NFr\ (M_1^*) \cap\circledcirc N^NFr\ (M_2^*)$ and
$N^NFr\ (M_1^*) \cap\circledcirc N^NFr\ (M_2^*) \nsubseteq N^NFr\ (M_1^* \cap\circledcirc M_2^*)$.

Theorem 3.18.
For any *NNSs* M_1^* and M_2^* in the *N-N-T-S* U,
$N^NFr\ (M_1^* \cap\circledcirc M_2^*) \subseteq\circledcirc(N^NFr\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^*)) \cup\circledcirc(N^NFr\ (M_2^*) \cap N^NCl\ (M_1^*))$.

Proof : Let M_1^* and M_2^* be the two *NNSs* in the *N-N-T-S* U.
Using Definition 3.1.,
$N^NFr\ (M_1^* \cap\circledcirc M_2^*) = N^NCl\ (M_1^* \cap\circledcirc M_2^*) \cap\circledcirc N^NCl\ ((M_1^* \ \cap M_2^*)^C$
Use Prop., 3.2 (1) [18] ,
$= N^NCl\ (M_1^* \cap\circledcirc M_2^*) \cap\circledcirc N^NCl\ M_1^{*C} \cup\circledcirc M_2^{*C})$
. By Propositon (2.4),
$\subseteq\circledcirc(N^NCl\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^*)) \cap\circledcirc(N^NCl\ (M_1^{*C}) \cup\circledcirc N^NCl\ (M_2^{*C}))$
$= [(N^NCl\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^*)) \cap\circledcirc N^NCl\ (M_1^{*C})] \cup\circledcirc[(N^NCl\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^*)) \cap\circledcirc N^NCl\ (M_2^{*C})]$
Again by Definition 3.1.,
$= (N^NFr\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^*)) \cup\circledcirc(N^NFr\ (M_2^*) \cap\circledcirc N^NCl\ (M_1^*))$
Hence $N^NFr\ (M_1^* \cap\circledcirc M_2^*) \subseteq\circledcirc(N^NFr\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^*)) \cup$
$(N^NFr\ (M_2^*) \cap\circledcirc N^NCl\ (M_1^*))$.

Corollary 3.19.
For any *NNSs* M_1^* and M_2^* in the *N-N-T-S* U,
$N^NFr\ (M_1^* \cap\circledcirc M_2^*) \subseteq\circledcirc N^NFr\ (M_1^*) \cup\circledcirc N^NFr\ (M_2^*)$.

Proof :
Let M_1^* and M_2^* be the two *NNSs* in the *N-N-T-S* U. Using Definition 3.1.,
$N^NFr\ (M_1^* \cap\circledcirc M_2^*) = N^NCl\ (M_1^* \cap\circledcirc M_2^*) \cap\circledcirc N^NCl\ ((M_1^* \ \cap M_2^*)^C$
. By Propositon (2.4),,
$= N^NCl\ (M_1^* \cap\circledcirc M_2^*) \cap\circledcirc N^NCl\ (M_1^{*C} \cup\circledcirc M_2^{*C})$
. By Propositon (2.4),,
$\subseteq\circledcirc(N^NCl\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^*)) \cap\circledcirc(N^NCl\ (M_1^{*C}) \cup\circledcirc N^NCl\ (M_2^{*C}))$
$= (N^NCl\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^*) \cap\circledcirc N^NCl\ (M_1^{*C})) \cup\circledcirc(N^NCl\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^*) \cap\circledcirc N^NCl\ (M_2^{*C}))$
Again by Definition 3.1.,
$= (N^NFr\ (M_1^*) \cap\circledcirc N^NCl\ (M_2^*)) \cup\circledcirc(N^NCl\ (M_1^*) \cap\circledcirc N^NFr\ (M_2^*))$
$\subseteq\circledcirc N^NFr\ (M_1^*) \cup\circledcirc N^NFr\ (M_2^*)$
Hence $N^NFr\ (M_1^* \cap\circledcirc M_2^*) \subseteq\circledcirc N^NFr\ (M_1^*) \cup\circledcirc N^NFr\ (M_2^*)$.

Theorem 3.20
For any *NNS* M_1^* in the *N-N-T-S* U,
(1) $N^NFr\ (N^NFr\ (M_1^*)) \subseteq\circledcirc N^NFr\ (M_1^*)$,
(2) $N^NFr\ (N^NFr\ (N^NFr\ (M_1^*))) \subseteq\circledcirc N^NFr\ (N^NFr\ (M_1^*))$.

Proof : (1) Let M_1^* be the *NNS* in the Neutrosophic Nano topological space U. Using Definition 3.1.,
$N^NFr\ (N^NFr\ (M_1^*)) = N^NCl\ (N^NFr\ (M_1^*)) \cap\circledcirc N^NCl\ ((N^NFr\ (M_1^*))^C$ Again by Definition 3.1.,
$= N^NCl\ (N^NCl\ (M_1^*) \cap\circledcirc N^NCl\ (M_1^{*C})) \cap N^NCl\ (\left((N^NCl\ (M_1^*) \ \cap N^NCl\ (M_1^{*C}) \right)^C$
By Propositon (2.4), and by By Propositon (2.4),
$\subseteq\circledcirc(N^NCl\ (N^NCl\ (M_1^*)) \cap\circledcirc N^NCl\ (N^NCl\ (M_1^{*C}))) \cap\circledcirc N^NCl\ (N^NInt\ (M_1^{*C}) \cup\circledcirc N^NInt\ (M_1^*))$
Use Prop., 1.18 (f) [18] ,
$= (N^NCl\ (M_1^*) \cap\circledcirc N^NCl\ (M_1^{*C})) \cap\circledcirc(N^NCl\ (N^NInt\ (M_1^{*C})) \cup\circledcirc N^NCl\ (N^NInt\ (M_1^*))$
$\subseteq\circledcirc N^NCl\ (M_1^*) \cap\circledcirc N^NCl\ (M_1^{*C})$
By Definition 3.1.,

= $N^N Fr\ (M_1^*)$

Therefore $N^N Fr\ (N^N Fr\ (M_1^*)) \subseteq_{\circledcirc} N^N Fr\ (M_1^*)$.

(2) By Definition 3.1.,

$N^N Fr\ (N^N Fr\ (N^N Fr\ (M_1^*)))$ = $N^N Cl\ (N^N Fr\ (N^N Fr\ (M_1^*))) \cap N^N Cl\ ((N^N Fr\ (N^N Fr\ (M_1^*)))^C)$

Use Prop., 1.18 (f) [18] ,

$\subseteq_{\circledcirc}(N^N Fr\ (N^N Fr\ (M_1^*))) \cap_{\circledcirc} N^N Cl((N^N Fr\ (N^N Fr\ (M_1^*))))^C \subseteq_{\circledcirc} N^N Fr\ (N^N Fr\ (M_1^*))$.

Hence $N^N Fr\ (N^N Fr\ (N^N Fr\ (M_1^*))) \subseteq_{\circledcirc} N^N Fr\ (N^N Fr\ (M_1^*))$.

Example 3.21.

Let U and $\mathcal{A}$ be two non-empty finite sets,

where U is the universe and $\mathcal{A}$ the set of attributes

The members of U= {P_1, P_2, P_3,P_4}are patients

Let U/R ={{P_1, P_2, P_3}, {P_4}} be an equivalence relation

$\mathcal{A} = \{\text{Head ache, Temperature}\}$ are two attributes

$P_1= \langle x, \left(\frac{8}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{4}{10},\frac{6}{10},\frac{7}{10}\right)\rangle$

$P_2= \langle x, \left(\frac{4}{10},\frac{2}{10},\frac{9}{10}\right),\left(\frac{1}{10},\frac{4}{10},\frac{9}{10}\right)\rangle$

$P_3= \langle x, \left(\frac{5}{10},\frac{6}{10},\frac{8}{10}\right),\left(\frac{7}{10},\frac{4}{10},\frac{4}{10}\right)\rangle$

$P_4= \langle x, \left(\frac{9}{10},\frac{8}{10},\frac{4}{10}\right),\left(\frac{9}{10},\frac{6}{10},\frac{1}{10}\right)\rangle$

$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(F)}, \overline{N(F)}, B_N(F)\}$

$\underline{N(F)} = \langle x, \left(\frac{4}{10},\frac{2}{10},\frac{9}{10}\right),\left(\frac{1}{10},\frac{4}{10},\frac{9}{10}\right)\rangle$

$\overline{N(F)} = \langle x, \left(\frac{8}{10},\frac{6}{10},\frac{5}{10}\right),\left(\frac{7}{10},\frac{6}{10},\frac{4}{10}\right)\rangle$

$BN(F) = \langle x, \left(\frac{9}{10},\frac{6}{10},\frac{4}{10}\right),\left(\frac{7}{10},\frac{6}{10},\frac{4}{10}\right)\rangle$

$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \langle x, \left(\frac{4}{10},\frac{2}{10},\frac{9}{10}\right),\left(\frac{1}{10},\frac{4}{10},\frac{9}{10}\right)\rangle, \langle\left(\frac{8}{10},\frac{6}{10},\frac{5}{10}\right),\left(\frac{7}{10},\frac{6}{10},\frac{4}{10}\right)\rangle,$
$\langle\left(\frac{9}{10},\frac{6}{10},\frac{4}{10}\right),\left(\frac{7}{10},\frac{6}{10},\frac{4}{10}\right)\rangle, \left(\frac{9}{10},\frac{8}{10},\frac{4}{10}\right),\left(\frac{9}{10},\frac{6}{10},\frac{1}{10}\right)\}$

$M_1^* = \langle x, \left(\frac{6}{10},\frac{7}{10},\frac{8}{10}\right),\left(\frac{5}{10},\frac{4}{10},\frac{5}{10}\right)\rangle$

Then $N^N Fr\ (M_1^*) = \langle x, \left(\frac{9}{10},\frac{8}{10},\frac{4}{10}\right),\left(\frac{9}{10},\frac{6}{10},\frac{1}{10}\right)\rangle$

$N^N Fr\ (N^N Fr\ (M_1^*)) = \langle x, \left(\frac{4}{10},\frac{2}{10},\frac{9}{10}\right),\left(\frac{1}{10},\frac{4}{10},\frac{9}{10}\right)\rangle$.

$N^N Fr\ (M_1^*) \nsubseteq N^N Fr\ (N^N Fr\ (M_1^*))$

III. NEUTROSOPHIC NANO SEMI-FRONTIER

In this section, we introduce the Neutrosophic Nano semi-frontier and their properties in N-N-T-S s.

Definition 4.1.

Let M_1^* be a *NNS* in the *N-N-T-S* U. Then the Neutrosophic Nano semi-frontier of M_1^* is defined as *NN(S)Fr* (M_1^*) = $N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl({M_1^*}^C)$.

Obviously$N^N(S)Fr\ (M_1^*)$ is a *NN(S)C* set in U.

Theorem 4.2.

Let M_1^* be a *NNS* in the *N-N-T-S* U. Then the following conditions are holds :

(i) $N^N(S)Cl\ (M_1^*) = M_1^* \cup_{\circledcirc} N^N Int\ (N^N Cl\ (M_1^*))$,

(ii) $N^N(S)Int\ (M_1^*) = M_1^* \cap_{\circledcirc} N^N Cl\ (N^N Int\ (M_1^*))$.

Proof : (i) Let M_1^* be a *NNS* in U. Consider

$N^N Int\ (N^N Cl\ (M_1^* \cup_{\circledcirc} N^N Int\ (N^N Cl\ (M_1^*))\ )\ )$

= $N^N Int\ (N^N Cl\ (M_1^*) \cup_{\circledcirc} N^N Cl\ (N^N Int\ (N^N Cl\ (M_1^*))\ )\ )$

= $N^N Int\ (N^N Cl\ (M_1^*))$

$\subseteq_{\circledcirc} M_1^* \cup_{\circledcirc} N^N Int\ (N^N Cl\ (M_1^*))$

It follows that $M_1^* \cup_{\circledcirc} N^N Int\ (N^N Cl\ (M_1^*))$ is a *NN(S)C* set in U.

Hence $N^N(S)Cl\ (M_1^*) \subseteq_{\circledcirc} M_1^* \cup_{\circledcirc} N^N Int\ (N^N Cl\ (M_1^*))$ ... (1)

Use Prop$N^N(S)Cl\ (M_1^*)$ is $N^N(S)C$ set in

U. We have $N^N Int\ (N^N Cl\ (M_1^*)) \subseteq_{\circledcirc} N^N Int\ (N^N Cl\ (N^N(S)Cl\ \ (M_1^*))) \subseteq_{\circledcirc} N^N(S)Cl\ \ (M_1^*)$.
Thus $M_1^* \cup_{\circledcirc} N^N Int\ (N^N Cl\ (M_1^*)) \subseteq_{\circledcirc} N^N(S)Cl\ \ (M_1^*)$ ... (2).
From (1) and (2), $N^N(S)Cl\ \ (M_1^*) = M_1^* \cup_{\circledcirc} N^N Int\ (N^N Cl\ (M_1^*))$.
(ii) This can be proved in a similar manner as (i).

Theorem 4.3.

For a *NNS* M_1^* in the *N-N-T-S* U, $N^N(S)Fr\ (M_1^*) = N^N(S)Fr\ ({M_1^*}^C)$.

Proof : Let M_1^* be the *NNS* in the N-N-T-S U. Using Definition 4.1,

$N^N(S)Fr\ (M_1^*) = N^N(S)Cl\ \ (M_1^*) \cap_{\circledcirc} N^N(S)Cl\ \ ({M_1^*}^C)$
$= N^N(S)Cl\ \ ({M_1^*}^C) \cap_{\circledcirc} N^N(S)Cl\ \ (M_1^*)$
$= N^N(S)Cl\ \ ({M_1^*}^C) \cap_{\circledcirc} N^N(S)Cl\left({M_1^*}^C\right)^C$

Again by Definition 4.1,
$= N^N(S)Fr\ ({M_1^*}^C)$

Hence $N^N(S)Fr\ (M_1^*) = N^N(S)Fr\ ({M_1^*}^C)$.

Theorem 4.4.

If M_1^* is *NN(S)C* set in U, then$N^N(S)Fr\ (M_1^*) \subseteq_{\circledcirc} M_1^*$.

Proof : Let M_1^* be the *NNS* in the N-N-T-S U. Using Definition 4.1,

$N^N(S)Fr\ (M_1^*) = N^N(S)Cl\ \ (M_1^*) \cap_{\circledcirc} N^N(S)Cl\ \ ({M_1^*}^C)$
$\subseteq_{\circledcirc} N^N(S)Cl(M_1^*) = M_1^*$

Hence $N^N(S)Fr\ (M_1^*) \subseteq_{\circledcirc} M_1^*$, if M_1^* is *NN(S)C* in U.

The converse of the above theorem is not true as shown by the following example.

Example 4.5.

Let U and $\mathcal{A}$ be two non-empty finite sets,
where U is the universe and $\mathcal{A}$ the set of attributes
U={F_1,F_2,F_3, F_4}are Fruits
Let U/R ={{ F_1,F_2,F_3}, { F_4}} be an equivalence relation
$\mathcal{A} = \{\text{Proteins, minerals, vitamins}\}$ are three attributes ,its Neutrosphic values are given below

$F_1 = \langle\left(\frac{4}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{3}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\rangle$
$F_2 = \langle\left(\frac{2}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{1}{10},\frac{1}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\rangle$
$F_3 = \langle\left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{4}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\rangle$
$F_4 = \langle\left(\frac{4}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{2}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\rangle$

$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N}(M), \overline{N}(M), B_N(M)\}$

$\underline{N}(F) = \langle\left(\frac{2}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{1}{10},\frac{1}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\rangle$
$\overline{N}(F) = \langle\left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{4}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\rangle$
$B_N(F) = \langle\left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{2}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{8}{10}\right)\rangle$

$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \langle\left(\frac{2}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{1}{10},\frac{1}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\rangle,$
$\langle\left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{4}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{8}{10}\right)\rangle$
$\langle\left(\frac{5}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{2}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{8}{10},\frac{5}{10},\frac{8}{10}\right)\rangle,$
$\langle\left(\frac{4}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{2}{10},\frac{2}{10},\frac{2}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{8}{10}\right)\rangle\}$

$M_1^*\ \langle\left(\frac{2}{10},\frac{5}{10},\frac{5}{10}\right),\left(\frac{2}{10},\frac{7}{10},\frac{2}{10}\right),\left(\frac{7}{10},\frac{5}{10},\frac{5}{10}\right)\rangle$, is Neutrosophic Nano semi-closed set

Then $N^N(S)Fr\ (M_1^*) \subseteq M_1^*$

Theorem 4.6.

If M_1^* is NNSO set in U, then $N^N(S)Fr\ (M_1^*) \subseteq {M_1^*}^C$

Proof : Let M_1^* be the NNS in the N-N-T-S U. Using Proposition 4.3 [18] ,

M_1^* is NNSO set implies ${M_1^*}^C$ is NN(S)C set in U. By Theorem 3.4, N^N(S)Fr $({M_1^*}^C) \subseteq {M_1^*}^C$ and by Theorem 3.3, we get N^N(S)Fr $(M_1^*) \subseteq {M_1^*}^C$

Theorem 4.7.

Let $M_1^* \subseteq⊚ M_2^*$ and M_2^* ⊚⊚$N^N(S)C$ (U) (resp., M_2^* ⊚⊚$N^N SO$ (U)). Then $N^N(S)Fr$ $(M_1^*) \subseteq⊚ M_2^*$ (resp., $N^N(S)Fr$ $(M_1^*) \subseteq⊚ {M_2^*}^C$, where $N^N(S)C$ (U) (resp., $N^N SO$ (U)) denotes the class of Neutrosophic Nano semi-closed (resp.,Neutrosophic Nano semi-open) sets in U.

Proof : Use Prop., 6.3 (iv) [18] , $M_1^* \subseteq⊚ M_2^*$,

$N^N(S)Cl$ $(M_1^*) \subseteq⊚ N^N(S)Cl$ (M_2^*) -------------------- (1).

By Definition 4.1,

$N^N(S)Fr$ (M_1^*) = $N^N(S)Cl(M_1^*)$ ∩⊚$N^N(S)Cl$ $({M_1^*}^C)$

⊆⊚$N^N(S)Cl(M_2^*)$ ∩⊚$N^N(S)Cl$ $({M_1^*}^C)$ by (1)

⊆⊚$N^N(S)Cl$ (M_2^*)Use Prop., 6.3 (ii) [18] ,= M_2^*

Hence $N^N(S)Fr$ $(M_1^*) \subseteq⊚ M_2^*$.

Theorem 4.8.

Let M_1^* be the NNS in the N-N-T-S U. Then $(N^N(S)Fr\ (M_1^*)\ )^C$= $N^N(S)Int$ (M_1^*) ∪⊚$N^N(S)Int$ $({M_1^*}^C$).

Proof : Let M_1^* be the NNS in the N-N-T-S U. Using Definition 4.1,

$(N^N(S)Fr\ (M_1^*)\ )^C = \left((N^N(S)Cl\ (M_1^*)\ \cap N^N(S)Cl\ ({M_1^*}^C\ ))\right)^C$Use Prop., 3.2 (1) [18] ,

= $(N^N(S)Cl$ $(M_1^*))^C$ ∪⊚$(N^N(S)Cl$ $({M_1^*}^C))^C$Use Prop., 6.2 (ii) [18] ,

= $N^N(S)Int$ $({M_1^*}^C)$ ∪⊚$N^N(S)Int$ (M_1^*)

Hence $(N^N(S)Fr\ (M_1^*)\ )^C$ = $N^N(S)Int$ (M_1^*) ∪⊚$N^N(S)Int$ $({M_1^*}^C)$.

Theorem 4.9.

For a NNS M_1^* in the N-N-T-S U, then $N^N(S)Fr$ $(M_1^*) \subseteq⊚ N^N Fr$ (M_1^*).

Proof : Let M_1^* be the *NNS* in the N-N-T-S U. Using Proposition 6.4 [18] ,

$N^N(S)Cl$ $(M_1^*) \subseteq⊚ N^N Cl$ (M_1^*) and $N^N(S)Cl$ $({M_1^*}^C) \subseteq⊚ N^N Cl$ $({M_1^*}^C)$.Now by Definition 4.1,

$N^N(S)Fr$ (M_1^*) = $N^N(S)Cl$ (M_1^*) ∩⊚$N^N(S)Cl$ $({M_1^*}^C)$ ⊆⊚ $N^N Cl$ (M_1^*) ∩⊚$N^N Cl$ $({M_1^*}^C)$By Definition 3.1.,

= $N^N Fr$ (M_1^*)

Hence $N^N(S)Fr$ $(M_1^*) \subseteq⊚ N^N Fr$ (M_1^*).

Theorem 4.10.

For a *NNS* M_1^* in the *N-N-T-S* U, then $N^N(S)Cl$ $(N^N(S)Fr$ $(M_1^*)) \subseteq N^N Fr$ (M_1^*).

Proof : Let M_1^* be the *NNS* in the N-N-T-S U. Using Definition 4.1,

$N^N(S)Cl$ $(N^N(S)Fr$ $(M_1^*))$ = $N^N(S)Cl$ $(N^N(S)Cl$ (M_1^*) ∩⊚$N^N(S)Cl$ $(({M_1^*}^C)))$

⊆⊚$N^N(S)Cl$ $(N^N(S)Cl$ $(M_1^*))$ ∩⊚$N^N(S)Cl$ $(N^N(S)Cl$ $(({M_1^*}^C)))$Use Prop., 6.3 (iii) [18] ,

= $N^N(S)Cl$ (M_1^*) ∩⊚$N^N(S)Cl$ $(({M_1^*}^C))$By Definition 4.1,

= $N^N(S)Fr$ (M_1^*)By Theorem 3.10,

⊆⊚$N^N Fr$ (M_1^*)

Hence $N^N(S)Cl$ $(N^N(S)Fr$ $(M_1^*)) \subseteq⊚ N^N Fr$ (M_1^*).

Theorem 4.11

Let M_1^* be a $N^N S$ in the *N-N-T-S* U. Then $N^N(S)Fr$ (M_1^*) = $N^N(S)Cl$ (M_1^*) – $N^N(S)Int$ (M_1^*).

Proof :

Let M_1^* be the *NNS* in the N-N-T-S U. Use Prop., 6.2 (ii) [18] ,

$(N^N(S)Cl$ $({M_1^*}^C)))^C$ = $N^N(S)Int$ (M_1^*) and by Definition 4.1,

$N^N(S)Fr$ (M_1^*) = $N^N(S)Cl$ (M_1^*) ∩ $N^N(S)Cl$ $(\ ({M_1^*}^C))$

= $N^N(S)Cl$ (M_1^*) – $(N^N(S)Cl\ (({M_1^*}^C)))^C$by using $M_1^* - M_2^* = M_1^* \cap ({M_2^*}^C)$Use Prop., 6.2 (ii) [18] ,

= $N^N(S)Cl$ (M_1^*) – $N^N(S)Int$ (M_1^*)

Hence $N^N(S)Fr$ (M_1^*) = $N^N(S)Cl$ (M_1^*) – $N^N(S)Int$ (M_1^*).

Theorem 4.12.

For a *NNS* M_1^* in the *N-N-T-S* U, then $N^N(S)Fr$ $(N^N(S)Int$ $(M_1^*)) \subseteq N^N(S)Fr$ (M_1^*).

Proof : Let M_1^* be the *NNS* in the N-N-T-S U. Using Definition 4.1,

$N^N(S)Fr$ $(N^N Int$ $(M_1^*))$= $N^N(S)Cl$ $(N^N Int(M_1^*))$∩ $N^N(S)Cl$ ($(N^N(S)Int$ $(M_1^*))^C$) Use Prop., 6.2 (i) [18] ,

$=N^N(S)Cl\ (N^N(S)Int\ (M_1^*))\cap\ N^N(S)Cl\ (N^N(S)Cl\ ((M_1^{*C})))$Use Prop., 6.3 (iii) [18] ,

$= N^N(S)Cl\ (N^N(S)Int\ (M_1^*)) \cap N^N(S)Cl\ (M_1^{*C})$Use Prop., 5.2 (ii) [18] ,

$\subseteq N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl\ (M_1^{*C}))$By Definition 4.1,

$= N^N(S)Fr\ (M_1^*)$

Hence $N^N(S)Fr\ (N^N(S)Int\ (M_1^*)) \subseteq N^N(S)Fr\ (M_1^*)$.

Example 4.13.

Let U and $\mathcal{A}$ be two non-empty finite sets,

where U is the universe and $\mathcal{A}$ the set of attributes

U={P1,P2 ,P3, P4}are Patients

Let U/R ={{ P1,P2 ,P3, P4 }, { P5}} be an equivalence relation

$\mathcal{A} = \{\text{Head ache, Temperature, Cold}\}$ are three attributes

its Neutrosphic values are given below

P1 = $\langle\left(\frac{3}{10},\frac{4}{10},\frac{2}{10}\right),\left(\frac{5}{10},\frac{6}{10},\frac{7}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{2}{10}\right)\rangle$

P2 =$\langle\left(\frac{3}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{4}{10},\frac{6}{10},\frac{2}{10}\right),\left(\frac{8}{10},\frac{4}{10},\frac{6}{10}\right)\rangle$

P3 =$\langle\left(\frac{3}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{4}{10},\frac{6}{10},\frac{2}{10}\right),\left(\frac{8}{10},\frac{4}{10},\frac{6}{10}\right)\rangle$

P4 = $\langle\left(\frac{3}{10},\frac{4}{10},\frac{2}{10}\right),\left(\frac{4}{10},\frac{6}{10},\frac{7}{10}\right),\left(\frac{8}{10},\frac{4}{10},\frac{6}{10}\right)\rangle$

P5 = $\langle\left(\frac{5}{10},\frac{6}{10},\frac{1}{10}\right),\left(\frac{6}{10},\frac{7}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{2}{10}\right)\rangle$

$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(M)}, \overline{N}(M), B_N(M)\}$

$\underline{N(F)} = \langle\left(\frac{3}{10},\frac{4}{10},\frac{2}{10}\right),\left(\frac{4}{10},\frac{6}{10},\frac{7}{10}\right),\left(\frac{8}{10},\frac{4}{10},\frac{6}{10}\right)\rangle$

$\overline{N}(F)= \langle\left(\frac{3}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{5}{10},\frac{6}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{2}{10}\right)\rangle$

$B_N(F)= \langle\left(\frac{2}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{5}{10},\frac{6}{10},\frac{4}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{6}{10}\right)\rangle$

$N_N(\tau)= \{0_{N_N}, 1_{N_N}, \langle\left(\frac{3}{10},\frac{4}{10},\frac{2}{10}\right),\left(\frac{4}{10},\frac{6}{10},\frac{7}{10}\right),\left(\frac{8}{10},\frac{4}{10},\frac{6}{10}\right)\rangle, \langle\left(\frac{3}{10},\frac{5}{10},\frac{1}{10}\right),\left(\frac{5}{10},\frac{6}{10},\frac{2}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{2}{10}\right)\rangle$
$\langle\left(\frac{2}{10},\frac{5}{10},\frac{3}{10}\right),\left(\frac{5}{10},\frac{4}{10},\frac{4}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{6}{10}\right)\rangle, \langle\left(\frac{5}{10},\frac{6}{10},\frac{1}{10}\right),\left(\frac{6}{10},\frac{7}{10},\frac{1}{10}\right),\left(\frac{9}{10},\frac{5}{10},\frac{2}{10}\right)\rangle\}$

$M_1^* = \langle\left(\frac{2}{10},\frac{6}{10},\frac{5}{10}\right),\left(\frac{5}{10},\frac{4}{10},\frac{5}{10}\right),\left(\frac{6}{10},\frac{5}{10},\frac{7}{10}\right)\rangle$ is a *NNSO*

Therefore $N^N(S)Fr\ (M_1^*) \nsubseteq N^N(S)Fr\ (N^N(S)Int\ (M_1^*))$.

Theorem 4.14.

For a *NNS* M_1^* in the *N-N-T-S* U, then$N^N(S)Fr\ (N^N(S)Cl\ (M_1^*)) \subseteq N^N(S)Fr\ (M_1^*)$.

Proof :

Let M_1^* be the $N^N S$ in the N-N-T-S U. Using Definition 4.1,

$N^N(S)Fr(N^N(S)Cl\ (M_1^*))= N^N(S)Cl\ (N^N(S)Cl\ (M_1^*))\cap\ N^N(S)Cl((N^N(S)Cl\ (M_1^*)))^C$

Use Prop., 6.3 (iii) and Proposition 6.2 (ii) [18] ,

$= N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl\ (N^N(S)Int\ (M_1^{*C}))$Use Prop., 5.2 (i) [18] ,

$\subseteq N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl\ (M_1^{*C})$By Definition 4.1,

$= N^N(S)Fr\ (M_1^*)$

Hence $N^N(S)Fr\ (N^N(S)Cl\ (M_1^*)) \subseteq N^N(S)Fr\ (M_1^*)$.

Remark 4.15.

In general topology, the following conditions conditions are hold :

$N^N(S)Fr\ (M_1^*) \cap N^N(S)Int\ (M_1^*) = 0_N$,

$N^N(S)Int\ (M_1^*) \cup N^N(S)Fr\ (M_1^*) = N^N(S)Cl\ (M_1^*)$,

$N^N(S)Int\ (M_1^*) \cup N^N(S)Int\ (M_1^{*C}) \cup N^N(S)Fr\ (M_1^*) = 1_{N_N}$.

Theorem 4.16.

Let M_1^* and M_2^* be *NNSs* in the *N-N-T-S* U.

Then $N^N(S)Fr\ (M_1^* \cup M_2^*) \subseteq N^N(S)Fr\ (M_1^*) \cup N^N(S)Fr\ (M_2^*)$.

Proof : Let M_1^* and M_2^* be *NNSs* in the *N-N-T-S* U. Using Definition 4.1,

$N^N(S)Fr\ (M_1^* \cup M_2^*) = N^N(S)Cl\ (M_1^* \cup M_2^*) \cap N^N(S)Cl\ (M_1^* \cup M_2^*)^C$. By Propositon (2.4),
$= N^N(S)Cl\ (M_1^* \cup M_2^*) \cap N^N(S)Cl\ ((M_1^{*C}) \cap (M_2^{*C}))$ Use Prop., 6.5 (i) and (ii) [18] ,
$\subseteq (N^N(S)Cl\ (M_1^*) \cup N^N(S)Cl\ (M_2^*)) \cap (N^N(S)Cl\ (M_1^{*C}) \cap N^N(S)Cl\ (M_2^{*C}))$
$= [\ (N^N(S)Cl(M_1^*) \cup N^N(S)Cl\ (M_2^*)\ ) \cap N^N(S)Cl\ ((M_1^{*C}] \cap [\ (N^N(S)Cl\ (M_1^*) \cup N^N(S)Cl\ (M_2^*)\ ) \cap N^N(S)Cl\ (M_2^{*C})\ ]$
$= [(N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl\ (M_1^{*C})) \cup (N^N(S)Cl\ (M_2^*) \cap N^N(S)Cl\ (M_1^{*C}))]$
$\cap [(N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl\ (M_2^{*C})) \cup (N^N(S)Cl\ (M_2^*) \cap N^N(S)Cl\ (M_2^{*C}))]$ By Definition 4.1,
$= [N^N(S)Fr\ (M_1^*) \cup (N^N(S)Cl\ (M_2^*) \cap N^N(S)Cl\ (M_1^{*C})\ )\ ] \cap [\ (N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl\ (M_2^{*C})\ ) \cup N^N(S)Fr(M_2^*)\ ]$
$= (N^N(S)Fr\ (M_1^*) \cup N^N(S)Fr\ (M_2^*)) \cap [\ (N^N(S)Cl\ (M_2^*) \cap N^N(S)Cl\ (M_1^{*C})\ ) \cup (N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl\ (M_2^{*C})\ )\ ]$
$\subseteq N^N(S)Fr\ (M_1^*) \cup N^N(S)Fr\ (M_2^*)$.
Hence $N^N(S)Fr\ (M_1^* \cup M_2^*) \subseteq N^N(S)Fr\ (M_1^*) \cup N^N(S)Fr\ (M_2^*)$.

Example 4.17.

Let U and $\mathcal{A}$ be two non-empty finite sets,
where U is the universe and $\mathcal{A}$ the set of attributes
U={P1,P2 ,P3, P4}are Patients
Let U/R ={{ P1,P2 ,P3}, { P4}} be an equivalence relation
$\mathcal{A}$ = {Temperature} are one attributes
U/R = {P1}{ P2,P3, P4}
$P_1 = \langle\left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right)\rangle$
$P_2 = \langle\left(\frac{4}{10},\frac{5}{10},\frac{7}{10}\right)\rangle$
$P_3 = \langle\left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right)\rangle$
$P_4 = \langle\left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right)\rangle$.
Then $N_N(\tau) = \{0_{N_N}, 1_{N_N}, \underline{N(M)}, \overline{N(M)}, B_N(M)\}$
$\underline{N(F)} = \langle\left(\frac{4}{10},\frac{5}{10},\frac{7}{10}\right)\rangle$
$\overline{N(F)} = \langle\left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right)\rangle$
$B_N(F) = \langle\left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right)\rangle$
$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \left(\frac{4}{10},\frac{5}{10},\frac{7}{10}\right), \left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right), \left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right), \left(\frac{3}{10},\frac{8}{10},\frac{6}{10}\right)\}$
$N_N(\tau) = \{0_{N_N}, 1_{N_N}, \left(\frac{4}{10},\frac{5}{10},\frac{7}{10}\right), \left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right), \left(\frac{3}{10},\frac{5}{10},\frac{6}{10}\right)\}$
$M_1^* = \left(\frac{4}{10},\frac{5}{10},\frac{6}{10}\right), M_2^* = \left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right)$ are NN(S)C
$M_1^* \cup M_2^* = \left(\frac{6}{10},\frac{5}{10},\frac{4}{10}\right)$
$N^N(S)Fr\ (M_1^* \cup M_2^*) \subseteq N^N(S)Fr\ (M_1^*) \cup N^N(S)Fr\ (M_2^*)$.

Theorem 4.18.

For any *NNSs* M_1^* and M_2^* in the *N-N-T-S* U,
$N^N(S)Fr\ (M_1^* \cap M_2^*) \subseteq (N^N(S)Fr\ (M_1^*) \cap N^N(S)Cl\ (M_2^*)\ ) \cup (N^N(S)Fr\ (M_2^*) \cap N^N(S)Cl\ (M_1^*)\ )$.

Proof : Let M_1^* and M_2^* be $N^N Ss$ in the $N^N TS$ U. Using Definition 4.1,
$N^N(S)Fr\ (M_1^* \cap M_2^*) = N^N(S)Cl\ (M_1^* \cap M_2^*) \cap N^N(S)Cl\ (M_1^* \cap M_2^*)^C$ Use Prop., 3.2 (1) [18] ,
$= N^N(S)Cl\ (M_1^* \cap M_2^*) \cap N^N(S)Cl\ ((M_1^{*C}) \cup (M_2^{*C})\ )$ Use Prop., 6.5 (ii) and (i) [18] ,
$\subseteq (N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl\ (M_2^*)) \cap (N^N(S)Cl\ ((M_1^{*C})) \cup N^N(S)Cl\ ((M_2^{*C})))$
$= [\ (N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl\ (M_2^*)\ ) \cap N^N(S)Cl\ ((M_1^{*C}] \cup [\ (N^N(S)Cl\ (M_1^*) \cap N^N(S)Cl\ (M_2^*)\ ) \cap N^N(S)Cl\ (M_2^{*C})\ ]$ By Definition 4.1,
$= (N^N(S)Fr\ (M_1^*) \cap N^N(S)Cl\ (M_2^*)\ ) \cup (N^N(S)Fr\ (M_2^*) \cap N^N(S)Cl\ (M_1^*))$
Hence $N^N(S)Fr\ (M_1^* \cap M_2^*) \subseteq (N^N(S)Fr\ (M_1^*) \cap N^N(S)Cl\ (M_2^*)\ ) \cup (N^N(S)Fr\ (M_2^*) \cap N^N(S)Cl\ (M_1^*)\ )$.

Corollary 4.19

For any *NNSs* M_1^* and M_2^* in the *N-N-T-S* U,

$N^N(S)Fr\ (M_1^* \cap⊚ M_2^*) \subseteq⊚ N^N(S)Fr\ (M_1^*) \cup⊚ N^N(S)Fr\ (M_2^*)$.

Proof : Let M_1^* and M_2^* be *NNSs* in the *N-N-T-S* U. Using Definition 4.1,

$N^N(S)Fr\ (M_1^* \cap⊚ M_2^*) = N^N(S)Cl\ \ (M_1^* \cap⊚ M_2^*) \cap⊚ N^N(S)Cl\ \ (M_1^*\ \cap M_2^*)^C$ Use Prop., 3.2 (1) [18] ,

$= N^N(S)Cl\ \ (M_1^* \cap⊚ M_2^*) \cap⊚ N^N(S)Cl\ \ (\ ({M_1^*}^C) \cup⊚ ({M_2^*}^C)\ )$,

$\subseteq⊚ (N^N(S)Cl\ (M_1^*) \cap\ \ N^N(S)Cl\ (M_2^*)) \cap (\ N^N(S)Cl\ (({M_1^*}^C \cup⊚ N^N(S)Cl\ (({M_2^*}^C)))$

$=\ (N^N(S)Cl\ \ \ (M_1^*) \cap⊚ N^N(S)Cl\ \ \ (M_2^*) \cap⊚ N^N(S)Cl\ \ (({M_1^*}^C) \cup⊚ (\ N^N(S)Cl(M_1^*) \cap⊚ N^N(S)Cl\ \ \ (M_2^*) \cap⊚ N^N(S)Cl$ $(({M_2^*}^C))\ )$

By Definition 4.1,

$=\ (N^N(S)Fr\ \ (M_1^*) \cap⊚ N^N(S)Cl\ \ \ (M_2^*)\ \ ) \cup⊚ (\ N^N(S)Cl\ \ \ (M_1^*) \cap⊚ N^N(S)Fr\ \ (M_2^*)) \subseteq⊚\ N^N(S)Fr\ \ (M_1^*) \cup⊚ N^N(S)Fr$ (M_2^*).

Hence $N^N(S)Fr\ (M_1^* \cap⊚ M_2^*) \subseteq⊚ N^N(S)Fr\ (M_1^*) \cup⊚ N^N(S)Fr\ (M_2^*)$.

Theorem 4.20

For any *NNS* M_1^* in the *N-N-T-S* U,

(1) $N^N(S)Fr\ (N^N(S)Fr\ (M_1^*)) \subseteq⊚ N^N(S)Fr\ (M_1^*)$,

(2) $N^N(S)Fr\ (N^N(S)Fr\ (N^N(S)Fr\ (M_1^*))) \subseteq⊚ N^N(S)Fr\ (N^N(S)Fr\ (M_1^*))$.

Proof : (1) Let M_1^* be the *NNS* in the N-N-T-S U. Using Definition 4.1,

$N^N(S)Fr\ (N^N(S)Fr\ (M_1^*)) = N^N(S)Cl\ \ (N^N(S)Fr\ (M_1^*)) \cap⊚ N^N(S)Cl\ \ ((N^N(S)Fr\ (M_1^*)\ )^C\ (N^N(S)Fr\ (M_1^*)\ ))$

By Definition 4.1,

$= N^N(S)Cl\ \ (N^N(S)Cl\ \ (M_1^*) \cap⊚ N^N(S)Cl\ \ \ ({M_1^*}^C)) \cap N^N(S)Cl\ \ (\left((N^N(S)Cl\ (M_1^*)\ \ \cap N^N(S)Cl\ (({M_1^*}^C))\right)^C))$

Use Prop., 6.3 (iii) and 6.2 (ii) [18] ,

$\subseteq⊚ (\ N^N(S)Cl\ \ \ (N^N(S)Cl\ \ \ (M_1^*)) \cap⊚ N^N(S)Cl\ \ \ (N^N(S)Cl\ \ \ (({M_1^*}^C)))\ \ ) \cap N^N(S)Cl\ \ \ (N^N(S)Int\ \ \ (({M_1^*}^C))$ $\cup⊚ N^N(S)Int\ \ (M_1^*)\ )$Use Prop., 6.3 (iii) [18] ,

$= (N^N(S)Cl\ \ (M_1^*) \cap⊚ N^N(S)Cl\ \ \ ({M_1^*}^C))) \cap⊚ (N^N(S)Cl\ \ (N^N(S)Int\ ({M_1^*}^C)) \cup⊚\ N^N(S)Cl\ \ (N^N(S)Int\ \ (M_1^*))$

$\subseteq⊚ N^N(S)Cl\ \ (M_1^*) \cap⊚ N^N(S)Cl\ \ (\ {M_1^*}^C))$ By Definition 4.1,

$= N^N(S)Fr\ (M_1^*)$

Therefore $N^N(S)Fr\ (N^N(S)Fr\ (M_1^*)) \subseteq⊚ N^N(S)Fr\ (M_1^*)$.(2) By Definition 4.1,

$N^N(S)Fr\ \ (N^N(S)Fr\ \ (N^N(S)Fr\ \ (M_1^*)))\ =\ N^N(S)Cl\ \ \ (N^N(S)Fr\ \ (N^N(S)Fr\ \ (M_1^*))) \cap⊚ N^N(S)Cl\ \ \ (\ \ (N^N(S)Fr$ $(N^N(S)Fr\ (M_1^*)))^C)$

Use Prop., 6.3 (iii) [18] ,

$\subseteq⊚ (N^N(S)Fr\ (N^N(S)Fr\ (M_1^*))) \cap⊚ N^N(S)Cl\ \ \ ((N^N(S)Fr\ (N^N(S)Fr\ (M_1^*)))^C)\ (N^N(S)Fr\ (N^N(S)Fr\ (M_1^*))))$

$\subseteq⊚ N^N(S)Fr\ (N^N(S)Fr\ (M_1^*))$.

Hence $N^N(S)Fr\ (N^N(S)Fr\ (N^N(S)Fr\ (M_1^*))) \subseteq⊚ N^N(S)Fr\ (N^N(S)Fr\ (M_1^*))$.

Conclusion

This research article shared some fundmental properties of introduce the Neutrosophic Nano semi-frontier .This concepts for further research will be on elaborating the structure of Neutrosophic Nano topology to more new classes of weak and strong forms of nano-open sets, new classes of generalized sets and new classes of continuous functions.There is further scope of launching into wider applications of Neutrosophic nano topology in different branches of Sciences and Humanities.

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References

[1] Abdel-Basset, M., Gamal, A., Son, L. H., & Smarandache, F. (2020). A Bipolar Neutrosophic Multi Criteria Decision Making Framework for Professional Selection. Applied Sciences, 10(4), 1202.

[2] Abdel-Basset, M., Mohamed, R., Zaied, A. E. N. H., Gamal, A., & Smarandache, F. (2020). Solving the supply chain problem using the best-worst method based on a novel Plithogenic model. In Optimization Theory Based on Neutrosophic and Plithogenic Sets (pp. 1-19). Academic Press.

[3] Abdel-Basset, Mohamed, et al. "An integrated plithogenic MCDM approach for financial performance evaluation of manufacturing industries." Risk Management (2020): 1-27.

[4] Abdel-Basset, M., Mohamed, R., Sallam, K., & Elhoseny, M. (2020). A novel decision-making model for sustainable supply chain finance under uncertainty environment. Journal of Cleaner Production, 122324.

[5] Abdel-Basst, M., Mohamed, R., & Elhoseny, M. (2020). A novel framework to evaluate innovation value proposition for smart product-service systems. Environmental Technology & Innovation, 101036.

[6] V.Banu priya, S.Chandrasekar, Neutrosophic α generalized semi closed set, Neutrosophic Sets and Systems, Vol. 28, 2019,162-170.

[7] Maheswari, C., Sathyabama, M., Chandrasekar, S., Neutrosophic generalized b-closed Sets Neutrosophic Topological Spaces,Journal of physics Conf. Series, 1139 (2018) 012065.

[8] Maheswari, C. and S. Chandrasekar. "Neutrosophic gb-closed Sets and Neutrosophic gb-Continuity." Neutrosophic Sets and Systems 29, 1(2020). https://digitalrepository.unm.edu/nss_journal/vol29/iss1/8

[9] T Rajesh kannan , S.Chandrasekar,Neutosophic $\omega\alpha$-closed sets in Neutrosophic topological space, Journal Of Computer And Mathematical Sciences,vol.9(10),1400-1408 Octobe2018.

[10] T Rajesh kannan ,S.Chandrasekar,Neutosophic α-continuous multifunction in Neutrosophic topological space, The International Journal of Analytical and Experimental Modal Analysis, Volume XI,IssueIX,September 2019,1360-9367

[11] T.RajeshKannan, and S.Chandrasekar, Neutrosophic α-Irresolute Multifunction In Neutrosophic Topological Spaces, " Neutrosophic Sets and Systems 32, 1 (2020),390-400. https://digitalrepository.unm.edu/ nss_journal/vol32/iss1/25

[12] V.K.Shanthi ,S.Chandrasekar, K.SafinaBegam, Neutrosophic Generalized Semi Closed Sets In Neutrosophic Topological Spaces, International Journal of Research in Advent Technology, Vol.6, No.7, July 2018, 1739-1743

[13] Salama.A and Alblowi.S.A., Generalized neutrosophic set and generalized neutrosophictopological spaces, Journal Computer (S)Ci. Engineering, Vol.2 No.7, (2012),129-132.

[14] Smarandache.F, "A unifying field in logics neutrosophy neutrosophic probability,set and Logic", Rehoboth American Research Press 1999.

[15] Lellis Thivagar.M and Carmel Richard.,On nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, Vol.1 No.1,(2013),31-37.

[16] M. Lellis Thivagar, Saeid Jafari V. Sutha Devi V. Antonysamy., A novel approach to nano topology via neutrosophic sets., Neutrosophic Sets and Systems, Vol. 20, 2018.,86-93

[17] M.Parimala, R.Jeevitha, S.Jafari, F.Smarandache, M.Karthika, Neutrosophic Nano Aψ Closed Sets In Neutrosophic Nano Topological Spaces, Jour of Adv Research in Dynamical & Control Systems, Vol. 10, 10-Special Issue, 2018.,523-531

[18] R. Vijayalakshmi , Mookambika.A.P., Properties Of Neutrosophic Nano Semi Open Sets (Communicated)

[19] R. Vijayalakshmi, A. Savitha Mary and S. Anjalmose: Neutrosophic Semi-Baire Spaces, Neutrosophic Sets and Systems, vol. 30, 2019, pp. 132-142. DOI: 10.5281/zenodo.3569673

[20] I. Mohammed Ali Jaffer and K. Ramesh: Neutrosophic Generalized Pre Regular Closed Sets, Neutrosophic Sets and Systems, vol. 30, 2019, pp. 171-181. DOI: 10.5281/zenodo.3569681

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