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Neutrosophic Vague Line Graphs

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Abstract: Neutrosophic graphs are employed as a mathematical key to hold an imprecise and unspecified data. Vague sets gives more intuitive graphical notation of vague information, that delicates crucially better analysis in data relationships, incompleteness and similarity measures. In this paper, the neutrosophic vague line graphs are introduced. The necessary and sufficient condition for a line graph to be neutrosophic vague line graph is provided. Further, homomorphism, weak vertex and weak line isomorphism are discussed. The given results are illustrated with suitable example.

Keywords: Neutrosophic vague line graph, Weak isomorphism of neutrosophic vague line graph, Homomorphism.

1. Introduction

The line graph, $L(G)$, of a graph G is the intersection graph of the set of lines of G. Hence the vertices of $L(G)$ are the lines of G with two vertices of $L(G)$ adjacent whenever the corresponding lines of G are adjacent [20]. Vague sets are denoted as a higher-order fuzzy sets which develops the solution process are more complex to obtain the results more accurate than fuzzy but not affecting the complexity on computation time/volume and memory space. Can we see an example, suppose there are 10 patients to check a pandemic during testing. In which, there are four patients having positive, five will have negative and one is undecided or yet to come. In the view of neutrosophic concepts, the mathematical form is represented as $x(0.4,0.1,0.5)$. Thus it is clear that, the neutrosophic field arises to hold the indeterminacy data. It generalizes the fuzzy sets and intuitionistic sets from the philosophical viewpoint. The single-valued neutrosophic set is the generalisation of intuitionistic fuzzy sets and is used expediently to deal with real-world problems, especially in decision support [1, 2, 3]. The computation of believe in that element (truth), the disbelieve in that element (falsehood) and the indeterminacy part of that element with the sum of these three components are strictly less than 1. The neutrosophic set is introduced by the author Smarandache in order to use the inconsistent and indeterminate information, and has been studied extensively (see [28]-[33]). In the definition of neutrosophic set, the indeterminacy value is quantified explicitly and truth-membership, indeterminacy-membership and false-membership are defined completely independent with the sum of these values lies between 0 and 3. Neutrosophic set and related notions paid attention by the researchers in many weird domains. The combination of neutrosophic set and vague set are introduced by Alkhazaleh in 2015 [6]. Single valued neutrosophic graph are established in [11, 12].

The neutrosophic graph is efficiently model the inconsistent information about any real-life problem. Some types of neutrosophic graphs and co-neutrosophic graphs are discussed in [16]. Intuitionistic bipolar neutrosophic set and its application to graphs are established in [25]. Al-Quran and Hassan in [5] introduced a combination of neutrosophic vague set and soft expert set to improving the reason-ability of decision making in real life application. Neutrosophic vague graphs are investigated in [24]. Comparative study of regular and (highly) irregular vague graphs with applications are obtained in [13]. Furthermore, some properties of degree of vague graphs, domination number and regularity properties of vague graphs are established by the author Borzooei in [7, 8, 9]. Neutrosophic vague set theory was extensively studied in [6]. The concept of a single-valued neutrosophic line graph of a single-valued neutrosophic graph is introduced by the authors in [21]. In which, a necessary and sufficient condition for a single-valued neutrosophic graph to be isomorphic to its corresponding single-valued neutrosophic line graph. Further, some remarkable properties of strong neutrosophic vague graphs, complete neutrosophic vague graphs and self-complementary neutrosophic vague graphs are investigated in [24]. Moreover, Cartesian product, lexicographic product, cross product, strong product and composition of neutrosophic vague graphs are investigated in [22]. As far, there exists no research work on the concept of neutrosophic vague line graphs until now. In order to fill this gap in the literature and motivated by papers [6, 21, 24], we put forward a new idea concerning the neutrosophic vague line graphs. The main contributions of this paper are as follows:

- Neutrosophic Vague Line Graphs (NVLGs) are introduced and explained with an example. The obtained neutrosophic vague line graph $L(\mathbb{G})$ is a strong neutrosophic vague graph.
- The necessary and sufficient condition for a line graph to be NVLG is formulated with supporting proofs.
- Furthermore, the results of homomorphism, weak vertex and weak line isomorphism are developed.

The manuscript is organised as follows: The basic definitions and example which are essential for the main results are given in Section 2. The necessary and sufficient condition of NVLG are provided and also the definition of NVLGs, homomorphism and weak isomorphism are given in Section 3. Finally, a conclusion is provided.

2 Preliminaries

In this section, basic definitions and example are given.

Definition 2.1 *[34] A vague set $\mathbb{A}$ on a non empty set $\mathbb{X}$ is a pair $(T_{\mathbb{A}}, F_{\mathbb{A}})$, where $T_{\mathbb{A}}: \mathbb{X} \to [0,1]$ and $F_{\mathbb{A}}: \mathbb{X} \to [0,1]$ are true membership and false membership functions, respectively, such that*
$0 \leq T_{\mathbb{A}}(x) + F_{\mathbb{A}}(x) \leq 1$ for any $x \in \mathbb{X}$.

Let $\mathbb{X}$ and $\mathbb{Y}$ be two non-empty sets. A vague relation $\mathbb{R}$ of $\mathbb{X}$ to $\mathbb{Y}$ is a vague set $\mathbb{R}$ on $\mathbb{X}\times\mathbb{Y}$ that is $\mathbb{R}=(T_{\mathbb{R}},F_{\mathbb{R}})$, where $T_{\mathbb{R}}:\mathbb{X}\times\mathbb{Y}\to[0,1], F_{\mathbb{R}}:\mathbb{X}\times\mathbb{Y}\to[0,1]$ and satisfy the condition: $0\le T_{\mathbb{R}}(x,y)+F_{\mathbb{R}}(x,y)\le 1$ for any $x,y\in\mathbb{X}$.

Definition 2.2 *[7] Let* $\mathbb{G}^*=(\mathbb{V},\mathbb{E})$ *be a graph. A pair* $\mathbb{G}=(\mathbb{J},\mathbb{K})$ *is called a vague graph on* $\mathbb{G}^*$, *where* $\mathbb{J}=(T_{\mathbb{J}},F_{\mathbb{J}})$ *is a vague set on* $\mathbb{V}$ *and* $\mathbb{K}=(T_{\mathbb{K}},F_{\mathbb{K}})$ *is a vague set on* $\mathbb{E}\subseteq\mathbb{V}\times\mathbb{V}$ *such that for each* $xy\in\mathbb{E}$, $T_{\mathbb{K}}(xy)\le\min(T_{\mathbb{J}}(x),T_{\mathbb{J}}(y))$ and $F_{\mathbb{K}}(xy)\ge\max(F_{\mathbb{J}}(x),F_{\mathbb{J}}(y))$.

Definition 2.3 *[28] A Neutrosophic set* $\mathbb{A}$ *is contained in another neutrosophic set* $\mathbb{B}$, *(i.e)* $\mathbb{A}\subseteq\mathbb{B}$ *if* $\forall x\in\mathbb{X}, T_{\mathbb{A}}(x)\le T_{\mathbb{B}}(x), I_{\mathbb{A}}(x)\ge I_{\mathbb{B}}(x)$*and* $F_{\mathbb{A}}(x)\ge F_{\mathbb{B}}(x)$.

Definition 2.4 *[14, 28] Let* $\mathbb{X}$ *be a space of points (objects), with generic elements in* $\mathbb{X}$ *denoted by* x. *A single valued neutrosophic set* $\mathbb{A}$ *in* $\mathbb{X}$ *is characterised by truth-membership function* $T_{\mathbb{A}}(x)$, *indeterminacy-membership function* $I_{\mathbb{A}}(x)$ *and falsity-membership-function* $F_{\mathbb{A}}(x)$,

For each point x in $\mathbb{X}$, $T_{\mathbb{A}}(x),F_{\mathbb{A}}(x),I_{\mathbb{A}}(x)\in[0,1]$. Also $A=\{x,T_{\mathbb{A}}(x),F_{\mathbb{A}}(x),I_{\mathbb{A}}(x)\}$ and

$$0\le T_{\mathbb{A}}(x)+I_{\mathbb{A}}(x)+F_{\mathbb{A}}(x)\le 3.$$

Definition 2.5 *[4, 12] A neutrosophic graph is defined as a pair* $\mathbb{G}^*=(\mathbb{V},\mathbb{E})$ *where*

(i) $\mathbb{V}=\{v_1,v_2,..,v_n\}$ such that $T_1:\mathbb{V}\to[0,1]$, $I_1:\mathbb{V}\to[0,1]$ and $F_1:\mathbb{V}\to[0,1]$ denote the degree of truth-membership function, indeterminacy-function and falsity-membership function, respectively, and

$$0\le T_1(v)+I_1(v)+F_1(v)\le 3,$$

(ii) $\mathbb{E}\subseteq\mathbb{V}\times\mathbb{V}$ where $T_2:\mathbb{E}\to[0,1]$, $I_2:\mathbb{E}\to[0,1]$ and $F_2:\mathbb{E}\to[0,1]$ are such that

$$T_2(uv)\le\min\{T_1(u),T_1(v)\},$$
$$I_2(uv)\le\min\{I_1(u),I_1(v)\},$$
$$F_2(uv)\le\max\{F_1(u),F_1(v)\},$$
$$\text{and } 0\le T_2(uv)+I_2(uv)+F_2(uv)\le 3,\ \forall uv\in\mathbb{E}.$$

Definition 2.6 *[6] A neutrosophic vague set* $\mathbb{A}_{NV}$ *(NVS in short) on the universe of discourse* $\mathbb{X}$ *be written as* $\mathbb{A}_{NV}=\{\langle x,\hat{T}_{\mathbb{A}_{NV}}(x),\hat{I}_{\mathbb{A}_{NV}}(x),\hat{F}_{\mathbb{A}_{NV}}(x)\rangle,x\in\mathbb{X}\}$, whose truth-membership, indeterminacy-membership and falsity-membership function is defined as

$$\hat{T}_{\mathbb{A}_{NV}}(x)=[T^-(x),T^+(x)],\hat{I}_{\mathbb{A}_{NV}}(x)=[I^-(x),I^+(x)]\text{and}\hat{F}_{\mathbb{A}_{NV}}(x)=[F^-(x),F^+(x)],$$

where $T^+(x)=1-F^-(x),F^+(x)=1-T^-(x)$, and $0\le T^-(x)+I^-(x)+F^-(x)\le 2$.

Definition 2.7 *[6] The complement of NVS* $\mathbb{A}_{NV}$ *is denoted by* $\mathbb{A}^c_{NV}$ *and it is given by*

$$\hat{T}^c_{\mathbb{A}_{NV}}(x)=[1-T^+(x),1-T^-(x)],$$
$$\hat{I}^c_{\mathbb{A}_{NV}}(x)=[1-I^+(x),1-I^-(x)],$$
$$\hat{F}^c_{\mathbb{A}_{NV}}(x)=[1-F^+(x),1-F^-(x)].$$

Definition 2.8 *[6] Let* $\mathbb{A}_{NV}$ *and* $\mathbb{B}_{NV}$ *be two NVSs of the universe* $\mathbb{U}$. *If for all* $u_i\in\mathbb{U}$,

$$\hat{T}_{\mathbb{A}_{NV}}(u_i)\le\hat{T}_{\mathbb{B}_{NV}}(u_i),\hat{I}_{\mathbb{A}_{NV}}(u_i)\ge\hat{I}_{\mathbb{B}_{NV}}(u_i),\hat{F}_{\mathbb{A}_{NV}}(u_i)\ge\hat{F}_{\mathbb{B}_{NV}}(u_i),$$

then the NVS, $\mathbb{A}_{NV}$ are included in $\mathbb{B}_{NV}$, denoted by $\mathbb{A}_{NV}\subseteq\mathbb{B}_{NV}$ where $1\le i\le n$.

Definition 2.9 *[6] The union of two NVSs* $\mathbb{A}_{NV}$ *and* $\mathbb{B}_{NV}$ *is a NVSs,* $\mathbb{C}_{NV}$, *written as* $\mathbb{C}_{NV}=\mathbb{A}_{NV}\cup\mathbb{B}_{NV}$, *whose truth-membership function, indeterminacy-membership function and false-membership function are related to those of* $\mathbb{A}_{NV}$ *and* $\mathbb{B}_{NV}$ *by*

$$\hat{T}_{\mathbb{C}_{NV}}(x)=[\max(T^-_{\mathbb{A}_{NV}}(x),T^-_{\mathbb{B}_{NV}}(x)),\max(T^+_{\mathbb{A}_{NV}}(x),T^+_{\mathbb{B}_{NV}}(x))]$$

$$\hat{I}_{\mathbb{C}_{NV}}(x) = [\min(I^-_{\mathbb{A}_{NV}}(x), I^-_{\mathbb{B}_{NV}}(x)), \min(I^+_{\mathbb{A}_{NV}}(x), I^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{F}_{\mathbb{C}_{NV}}(x) = [\min(F^-_{\mathbb{A}_{NV}}(x), F^-_{\mathbb{B}_{NV}}(x)), \min(F^+_{\mathbb{A}_{NV}}(x), F^+_{\mathbb{B}_{NV}}(x))].$$

Definition 2.10 *[6] The intersection of two NVSs,* A_{NV} *and* B_{NV} *is a NVSs* C_{NV}*, written as* $C_{NV} = A_{NV} \cap B_{NV}$*, whose truth-membership function, indeterminacy-membership function and false-membership function are related to those of* A_{NV} *and* B_{NV} *by*

$$\hat{T}_{\mathbb{C}_{NV}}(x) = [\min(T^-_{\mathbb{A}_{NV}}(x), T^-_{\mathbb{B}_{NV}}(x)), \min(T^+_{\mathbb{A}_{NV}}(x), T^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{I}_{\mathbb{C}_{NV}}(x) = [\max(I^-_{\mathbb{A}_{NV}}(x), I^-_{\mathbb{B}_{NV}}(x)), \max(I^+_{\mathbb{A}_{NV}}(x), I^+_{\mathbb{B}_{NV}}(x))]$$
$$\hat{F}_{\mathbb{C}_{NV}}(x) = [\max(F^-_{\mathbb{A}_{NV}}(x), F^-_{\mathbb{B}_{NV}}(x)), \max(F^+_{\mathbb{A}_{NV}}(x), F^+_{\mathbb{B}_{NV}}(x))].$$

Definition 2.11 *[24] Let* $\mathbb{G}^* = (\mathbb{R}, \mathbb{S})$ *be a graph. A pair* $\mathbb{G} = (\mathbb{A}, \mathbb{B})$ *is called a neutrosophic vague graph (NVG) on* $\mathbb{G}^*$ *or a neutrosophic vague graph where* $\mathbb{A} = (\hat{T}_{\mathbb{A}}, \hat{I}_{\mathbb{A}}, \hat{F}_{\mathbb{A}})$ *is a neutrosophic vague set on* $\mathbb{R}$ *and* $\mathbb{B} = (\hat{T}_{\mathbb{B}}, \hat{I}_{\mathbb{B}}, \hat{F}_{\mathbb{B}})$ *is a neutrosophic vague set* $\mathbb{S} \subseteq \mathbb{R} \times \mathbb{R}$ *where*

(1) $\mathbb{R} = \{v_1, v_2, \dots, v_n\}$ such that $T^-_{\mathbb{A}}: \mathbb{R} \to [0,1], I^-_{\mathbb{A}}: \mathbb{R} \to [0,1], F^-_{\mathbb{A}}: \mathbb{R} \to [0,1]$ which satisfies the condition $F^-_{\mathbb{A}} = [1 - T^+_{\mathbb{A}}]$

$T^+_{\mathbb{A}}: \mathbb{R} \to [0,1], I^+_{\mathbb{A}}: \mathbb{R} \to [0,1], F^+_{\mathbb{A}}: \mathbb{R} \to [0,1]$ which satisfying the condition $F^+_{\mathbb{A}} = [1 - T^-_{\mathbb{A}}]$

denotes the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i \in \mathbb{R}$, and

$$0 \le T^-_{\mathbb{A}}(v_i) + I^-_{\mathbb{A}}(v_i) + F^-_{\mathbb{A}}(v_i) \le 2$$
$$0 \le T^+_{\mathbb{A}}(v_i) + I^+_{\mathbb{A}}(v_i) + F^+_{\mathbb{A}}(v_i) \le 2.$$

(2) $\mathbb{S} \subseteq \mathbb{R} \times \mathbb{R}$ where

$$T^-_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1],\ I^-_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1],\ F^-_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1]$$
$$T^+_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1],\ I^+_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1],\ F^+_{\mathbb{B}}: \mathbb{R} \times \mathbb{R} \to [0,1]$$

represents the degree of truth membership function, indeterminacy membership and falsity membership of the element $v_i, v_j \in \mathbb{S}$, respectively and such that,

$$0 \le T^-_{\mathbb{B}}(v_i v_j) + I^-_{\mathbb{B}}(v_i v_j) + F^-_{\mathbb{B}}(v_i v_j) \le 2$$
$$0 \le T^+_{\mathbb{B}}(v_i v_j) + I^+_{\mathbb{B}}(v_i v_j) + F^+_{\mathbb{B}}(v_i v_j) \le 2,$$

such that

$$T^-_{\mathbb{B}}(v_i v_j) \le \min\{T^-_{\mathbb{A}}(v_i), T^-_{\mathbb{A}}(v_j)\}$$
$$I^-_{\mathbb{B}}(v_i v_j) \le \min\{I^-_{\mathbb{A}}(v_i), I^-_{\mathbb{A}}(v_j)\}$$
$$F^-_{\mathbb{B}}(v_i v_j) \le \max\{F^-_{\mathbb{A}}(v_i), F^-_{\mathbb{A}}(v_j)\},$$

and similarly

$$T^+_{\mathbb{B}}(v_i v_j) \le \min\{T^+_{\mathbb{A}}(v_i), T^+_{\mathbb{A}}(v_j)\}$$
$$I^+_{\mathbb{B}}(v_i v_j) \le \min\{I^+_{\mathbb{A}}(v_i), I^+_{\mathbb{A}}(v_j)\}$$
$$F^+_{\mathbb{B}}(v_i v_j) \le \max\{F^+_{\mathbb{A}}(v_i), F^+_{\mathbb{A}}(v_j)\}.$$

3 Neutrosophic Vague Line Graphs

In this section, the necessary and sufficient condition of NVLGs are provided. The definition of NVLGs, homomorphism and weak isomorphism are given.

Definition 3.1 *Let* $\Lambda(D) = (D, S)$ *be an intersection graph* $G = (V, E)$ *and let* $\mathbb{G} = (H_1, K_1)$ *be a NVG with underlying set* V*. A NVG of* $\Lambda(D)$ *is a pair* (H_2, K_2)*, where* $H_2 = (T^+_{H_2}, I^+_{H_2}, F^+_{H_2}, T^-_{H_2}, I^-_{H_2}, F^-_{H_2})$ *and* $K_2 = (T^+_{K_2}, I^+_{K_2}, F^+_{K_2}, T^-_{K_2}, I^-_{K_2}, F^-_{K_2})$ *are NVSs of* D *and* S*, respectively, such that*

$$T^+_{H_2}(D_i) = T^+_{H_1}(v_i), I^+_{H_2}(D_i) = I^+_{H_1}(v_i), F^+_{H_2}(D_i) = F^+_{H_1}(v_i),$$

$$T^-_{H_2}(D_i) = T^-_{H_1}(v_i), I^-_{H_2}(D_i) = I^-_{H_1}(v_i), F^-_{H_2}(D_i) = F^-_{H_1}(v_i),$$

for all $D_i,\ D_j \in D$.

$$T^+_{K_2}(D_iD_j) = T^+_{K_1}(v_iv_j), I^+_{K_2}(D_iD_j) = I^+_{K_1}(v_iv_j), F^+_{K_2}(D_iD_j) = F^+_{K_1}(v_iv_j),$$
$$T^-_{K_2}(D_iD_j) = T^-_{K_1}(v_iv_j), I^-_{K_2}(D_iD_j) = I^-_{K_1}(v_iv_j), F^-_{K_2}(D_iD_j) = F^-_{K_1}(v_iv_j)$$

for all $D_iD_j \in S$.

That is, any NVG of intersection graph $\Lambda(D)$ is also a neutrosophic vague intersection graph of $\mathbb{G}$.

Definition 3.2 *Let $L(G) = (M, N)$ be a line graph of a graph $G = (V, E)$. A NVLG of a NVG $\mathbb{G} = (H_1, K_1)$ (with underlying set V) is a pair $L(\mathbb{G}) = (H_2, K_2)$, where $H_2 = (T^+_{H_2}, I^+_{H_2}, F^+_{H_2}, T^-_{H_2}, I^-_{H_2}, F^-_{H_2})$ and $K_2 = (T^+_{K_2}, I^+_{K_2}, F^+_{K_2}, T^-_{K_2}, I^-_{K_2}, F^-_{K_2})$ are NVSs of M and N, respectively such that,*

$$T^+_{H_2}(D_x) = T^+_{K_1}(x) = T^+_{K_1}(u_xv_x)$$
$$I^+_{H_2}(D_x) = I^+_{K_1}(x) = I^+_{K_1}(u_xv_x)$$
$$F^+_{H_2}(D_x) = F^+_{K_1}(x) = F^+_{K_1}(u_xv_x)$$
$$T^-_{H_2}(D_x) = T^-_{K_1}(x) = T^-_{K_1}(u_xv_x)$$
$$I^-_{H_2}(D_x) = I^-_{K_1}(x) = I^-_{K_1}(u_xv_x)$$
$$F^-_{H_2}(D_x) = F^-_{K_1}(x) = F^-_{K_1}(u_xv_x).$$

for all $D_x \in M, u_xv_x \in N$.

$$T^+_{K_2}(D_xD_y) = \min\{T^+_{K_1}(x), T^+_{K_1}(y)\}$$
$$I^+_{K_2}(D_xD_y) = \min\{I^+_{K_1}(x), I^+_{K_1}(y)\}$$
$$F^+_{K_2}(D_xD_y) = \max\{F^+_{K_1}(x), F^+_{K_1}(y)\}$$
$$T^-_{K_2}(D_xD_y) = \min\{T^-_{K_1}(x), T^-_{K_1}(y)\}$$
$$I^-_{K_2}(D_xD_y) = \min\{I^-_{K_1}(x), I^-_{K_1}(y)\}$$
$$F^-_{K_2}(D_xD_y) = \max\{F^-_{K_1}(x), F^-_{K_1}(y)\} \quad \text{for all } D_xD_y \in N.$$

Example 3.3 *Consider $G = (V, E)$, where $V = \{b_1, b_2, b_3, b_4\}$ and $E = \{Q_1 = b_1b_2, Q_2 = b_2b_3, Q_3 = b_3b_4, Q_4 = b_4b_1\}$. Let $\mathbb{G} = (H_1, K_1)$ be a NVG of G as shown in figure 1, defined by*

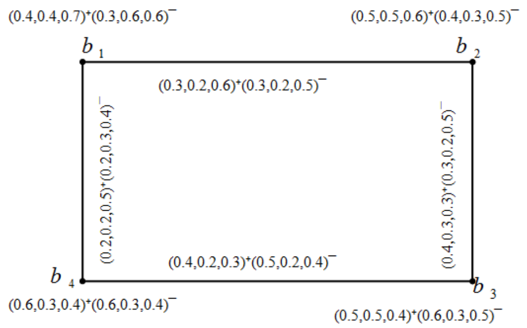
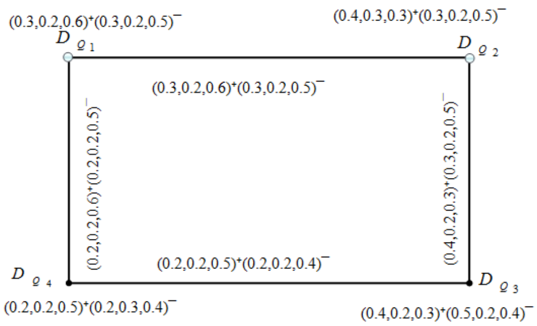


Figure 1

Neutrosophic Vague Graph

consider a line graph $L(G) = (M, N)$ where $M = (D_{Q_1}, D_{Q_2}, D_{Q_3}, D_{Q_4})$ and $N = (D_{Q_1}D_{Q_2}, D_{Q_2}D_{Q_3}, D_{Q_3}D_{Q_4}, D_{Q_4}D_{Q_1})$. Let $L(\mathbb{G})$ be the NVLG, as shown in figure 2.



$Figure$ 2

Neutrosophic Vague Line Graph

$L(\mathbb{G})$

Proposition 3.4 *A NVLG is always a strong NVG.*

Proof. It is obvious from the definition, therefore it is omitted.

Proposition 3.5 *If* $L(\mathbb{G})$ *is NVLG of NVG* $\mathbb{G}$*. Then* $L(G)$ *is the line graph of* G*.*

Proof. Given $\mathbb{G} = (H_1, K_1)$ is NVLG of G and $L(\mathbb{G}) = (H_2, K_2)$ is a NVG of $L(G)$

$$T_{H_2}^{+}(D_x) = T_{K_1}^{+}(x)$$
$$I_{H_2}^{+}(D_x) = I_{K_1}^{+}(x)$$
$$F_{H_2}^{+}(D_x) = F_{K_1}^{+}(x)$$
$$T_{H_2}^{-}(D_x) = T_{K_1}^{-}(x)$$
$$I_{H_2}^{-}(D_x) = I_{K_1}^{-}(x)$$
$$F_{H_2}^{-}(D_x) = F_{K_1}^{-}(x),$$

$\forall x \in E$ and so $D_x \in M$ if and only if for $x \in E$,

$$T_{K_2}^{+}(D_x D_y) = \min\{T_{K_1}^{+}(x), T_{K_1}^{+}(y)\}$$
$$I_{K_2}^{+}(D_x D_y) = \min\{I_{K_1}^{+}(x), I_{K_1}^{+}(y)\}$$
$$F_{K_2}^{+}(D_x D_y) = \max\{F_{K_1}^{+}(x), F_{K_1}^{+}(y)\}$$
$$T_{K_2}^{-}(D_x D_y) = \min\{T_{K_1}^{-}(x), T_{K_1}^{-}(y)\}$$
$$I_{K_2}^{-}(D_x D_y) = \min\{I_{K_1}^{-}(x), I_{K_1}^{-}(y)\}$$
$$F_{K_2}^{-}(D_x D_y) = \max\{F_{K_1}^{-}(x), F_{K_1}^{-}(y)\},$$

for all $D_x D_y \in N$, and so $M = \{D_x D_y | D_x \cup D_y \neq \emptyset, x, y \in E, x \neq y\}$. Hence proved.

Proposition 3.6 *Let* $L(\mathbb{G}) = (H_2, K_2)$ *be a NVG of* $\mathbb{G}$*. Then* $L(\mathbb{G})$ *is a NVG of some NVG of* G *if and only if*

$$T_{K_2}^{+}(D_x D_y) = \min\{T_{H_2}^{+}(D_x), T_{H_2}^{+}(D_y)\}$$
$$T_{K_2}^{-}(D_x D_y) = \min\{T_{H_2}^{-}(D_x), T_{H_2}^{-}(D_y)\}$$
$$I_{K_2}^{+}(D_x D_y) = \min\{I_{H_2}^{+}(D_x), I_{H_2}^{+}(D_y)\}$$
$$I_{K_2}^{-}(D_x D_y) = \min\{I_{H_2}^{-}(D_x), I_{H_2}^{-}(D_y)\}$$

$$F^{+}_{K_2}(D_xD_y) = \max\{F^{+}_{H_2}(D_x), F^{+}_{H_2}(D_y)\}$$
$$F^{-}_{K_2}(D_xD_y) = \max\{F^{-}_{H_2}(D_x), F^{-}_{H_2}(D_y)\},$$
for all $D_xD_y \in N$.

Proof. Suppose that $T^{P}_{K_2}(D_xD_y) = \min\{T^{+}_{H_2}(D_x), T^{+}_{H_2}(D_y)\}$,
$I^{+}_{K_2}(D_xD_y) = \min\{I^{+}_{H_2}(D_x), I^{+}_{H_2}(D_y)\}$, $F^{+}_{K_2}(D_xD_y) = \max\{F^{+}_{H_2}(D_x), F^{+}_{H_2}(D_y)\}$ for all $D_xD_y \in N$.
Define, $T^{+}_{H_2}(D_x) = T^{+}_{K_1}(x), I^{+}_{H_2}(D_x) = I^{+}_{K_1}(x), F^{+}_{H_2}(D_x) = F^{+}_{K_1}(x)$ for all $x \in E$, then
$$T^{+}_{K_2}(D_xD_y) = \min\{T^{+}_{H_2}(D_x), T^{+}_{H_2}(D_y)\} = \min\{T^{+}_{K_1}(x), T^{+}_{K_1}(x)\},$$
$$I^{+}_{K_2}(D_xD_y) = \min\{I^{+}_{H_2}(D_x), I^{+}_{H_2}(D_y)\} = \min\{I^{+}_{K_1}(x), I^{+}_{K_1}(x)\},$$
$$F^{+}_{K_2}(D_xD_y) = \max\{F^{+}_{H_2}(D_x), F^{+}_{H_2}(D_y)\} = \max\{F^{+}_{K_1}(x), F^{+}_{K_1}(x)\},$$
for all $D_xD_y \in M$.
We know that NVG H_1 yields the properties,

$$T^{+}_{K_1}(uv) \le \min\{T^{+}_{H_1}(u), T^{+}_{H_1}(v)\}$$

$$I^{+}_{K_1}(uv) \le \min\{I^{+}_{H_1}(u), I^{+}_{H_1}(v)\}$$

$$F^{+}_{K_1}(uv) \le \max\{F^{+}_{H_1}(u), F^{+}_{H_1}(v)\}.$$

In the similar way, we prove for the similar part also, The converse part of this theorem is obvious by using the definition of $L(\mathbb{G})$.

Theorem 3.7 $L(\mathbb{G})$ *is a NVLG if and only if* $L(G)$ *is a line graph and*
$$T^{+}_{K_2}(uv) = \min\{T^{+}_{H_2}(u), T^{+}_{H_2}(v)\}$$
$$I^{+}_{K_2}(uv) = \min\{I^{+}_{H_2}(u), I^{+}_{H_2}(v)\}$$
$$F^{+}_{K_2}(uv) = \max\{F^{+}_{H_2}(u), F^{+}_{H_2}(v)\}$$
$$T^{-}_{K_2}(uv) = \min\{T^{-}_{H_2}(u), T^{-}_{H_2}(v)\}$$
$$I^{-}_{K_2}(uv) = \min\{I^{-}_{H_2}(u), I^{-}_{H_2}(v)\}$$
$$F^{-}_{K_2}(uv) = \max\{F^{-}_{H_2}(u), F^{-}_{H_2}(v)\} \quad \forall uv \in M.$$

Proof. The proof follows from the above Proposition 3.5 and Proposition 3.6.

Definition 3.8 *A homomorphism* $\chi: \mathbb{G}_1 \to \mathbb{G}_2$ *of two NVGs* $\mathbb{G}_1 = (H_1, K_1)$ *and* $\mathbb{G}_2 = (H_2, K_2)$ *is mapping* $\chi: V_1 \to V_2$ *such that*
$$(A) T^{+}_{H_1}(x_1) \le T^{+}_{H_2}(\chi(x_1)), T^{-}_{H_1}(x_1) \le T^{-}_{H_2}(\chi(x_1)),$$
$$I^{+}_{H_1}(x_1) \le I^{+}_{H_2}(\chi(x_1)), I^{-}_{H_1}(x_1) \le I^{-}_{H_2}(\chi(x_1)),$$
$$F^{+}_{H_1}(x_1) \le F^{+}_{H_2}(\chi(x_1)), F^{-}_{H_1}(x_1) \le F^{-}_{H_2}(\chi(x_1)), \quad \forall x_1 \in V_1.$$

$$(B) T^{+}_{K_1}(x_1y_1) \le T^{+}_{K_2}(\chi(x_1)\chi(y_1)), T^{-}_{K_1}(x_1y_1) \le T^{-}_{K_2}(\chi(x_1)\chi(y_1)),$$
$$I^{+}_{K_1}(x_1y_1) \le I^{+}_{K_2}(\chi(x_1)\chi(y_1)), I^{-}_{K_1}(x_1y_1) \le I^{-}_{K_2}(\chi(x_1)\chi(y_1)),$$
$$F^{+}_{K_1}(x_1y_1) \le F^{+}_{K_2}(\chi(x_1)\chi(y_1)), F^{-}_{K_1}(x_1y_1) \le F^{-}_{K_2}(\chi(x_1)\chi(y_1)), \quad \forall x_1y_1 \in E_1.$$

Definition 3.9 *A (weak) vertex-isomorphism is a bijective homomorphism* $\chi: \mathbb{G}_1 \to \mathbb{G}_2$ *such that*

$$(A) T^{+}_{H_1}(x_1) = T^{+}_{H_2}(\chi(x_1)),$$
$$T^{-}_{H_1}(x_1) = T^{-}_{H_2}(\chi(x_1)),$$
$$I^{+}_{H_1}(x_1) = I^{+}_{H_2}(\chi(x_1)),$$
$$I^{-}_{H_1}(x_1) = I^{-}_{H_2}(\chi(x_1)),$$
$$F^{+}_{H_1}(x_1) = F^{+}_{H_2}(\chi(x_1)),$$
$$F^{-}_{H_1}(x_1) = F^{-}_{H_2}(\chi(x_1)), \quad \forall x_1 \in V_1.$$

A (weak) line-isomorphism is bijective homomorphism $\chi: \mathbb{G}_1 \to \mathbb{G}_2$ such that

$$(B) T^{+}_{K_1}(x_1 y_1) = T^{+}_{K_2}(\chi(x_1)\chi(y_1)),$$
$$T^{-}_{K_1}(x_1 y_1) = T^{-}_{K_2}(\chi(x_1)\chi(y_1)),$$
$$I^{+}_{K_1}(x_1 y_1) = I^{+}_{K_2}(\chi(x_1)\chi(y_1)),$$
$$I^{-}_{K_1}(x_1 y_1) = I^{-}_{K_2}(\chi(x_1)\chi(y_1)),$$
$$F^{+}_{K_1}(x_1 y_1) = F^{+}_{K_2}(\chi(x_1)\chi(y_1)),$$
$$F^{-}_{K_1}(x_1 y_1) = F^{-}_{K_2}(\chi(x_1)\chi(y_1)), \quad \forall x_1 y_1 \in E_1.$$

If $\chi: \mathbb{G}_1 \to \mathbb{G}_2$ is a weak-vertex isomorphism and a (weak) line-isomorphism, then χ is called a (weak) isomorphism.

Proposition 3.10 *Let* $\mathbb{G} = (H_1, K_1)$ *be a NVG with underlying set* V. *Then* (H_2, K_2) is a NVG of $\Lambda(D)$ and $(H_1, K_1) \cong (H_2, K_2)$

Proposition 3.11 *Let* $\mathbb{G}$ *and* $\mathbb{G}'$ *be NVGs of* G *and* G' *respectively, if* $\chi: \mathbb{G} \to \mathbb{G}'$ *is a weak isomorphism then* $\chi: \mathbb{G} \to \mathbb{G}'$ *is an isomorphism.*

Proof. Let $\chi: \mathbb{G} \to \mathbb{G}'$ be a weak isomorphism, then $u \in V$ if and only if $\chi(u) \in V'$ and $uv \in E$ if and only if $\chi(u)\chi(v) \in E'$. Hence proved.

Conclusion

A neutrosophic graph is very useful to interpret the real-life situations and it is regarded as a generalisation of intuitionistic fuzzy graph. Neutrosophic vague graphs are represented as a context-dependent generalized fuzzy graphs which holds the indeterminate and inconsistent information. This paper dealt with the necessary and sufficient condition for NVLG to be a line graph are also derived. The properties of homomorphism, weak vertex and weak line isomorphism are established. Further we are able to extend by investigating the regular and isomorphic properties of the interval valued neutrosophic vague line graph.

Conflict of Interest: The authors declare that they have no conflict of interest.

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