Neutrosophic Sets and Systems, Vol. 36, 2020
University of New Mexico

Pentapartitioned neutrosophic set and its properties

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Abstract: The main objective of this paper is to propose a new type of set which we call pentapartitioned neutrosophic set. We also prove some of its basic properties.

Keywords: Neutrosophic set, Single valued neutrosophic set, Pentapartitioned neutrosophic set.

1. Introduction:

Smarandache [1] defined Neutrosophic Set (NS) to deal with uncertainty, indeterminacy and inconsistency involved in mathematical objects. It generalizes fuzzy set [2] and intuitionistic fuzzy set [3] by incorporating degrees of indeterminacy and rejection (falsity or non-membership) as independent components. Wang et al. [4] defined Single Valued Neutrosophic Set (SVNS) in 2010. Chatterjee et al. [5] defined Quadripartitioned SVNS (QSVNS) that involves truth, falsity, unknown and contradiction based on four valued logics [6, 7].

Smarandache [7] split indeterminacy into unknown, contradiction, ignorance and proposed Five Symbol Valued Neutrosophic Logic (FSVNL). In this paper we utilize FSVNL and propose pentapartitioned neutrosophic set. We also establish some basic properties of the proposed set. The proposed structure is generalization of existing theories of SVNS and QSVNS.

The organization of the paper is as follows: Section 1 provides a brief introduction; Section 2 is dedicated to recalling some preliminary results; Section 3 introduces the concept of a pentapartitioned neutrosophic set. Section 4 deals with some basic set-theoretic operations over pentapartitioned neutrosophic sets. Section 5 concludes the paper stating future scope of research.

1. Preliminary:

Definition 1: An NS [1] N on the universe of discourse Q is defined as:

$N=\{<q, T_N(q), I_N(q), F_N(q)>: q\in Q\}$ where $T, I, F: Q \to]^{-}0,1^{+}[$ and $^{-}0 \le T_N(q)+I_N(q)+F_N(q) \le 3^{+}$.

2. Single Valued Pentapartitioned Neutrosophic Sets:

Based on Smarandache FSVNL [7], we define the concept of Pentapartitioned Neutrosophic Set (PNS). The term "pentapartitioned" means something that divided into five characteristic features. The indeterminacy is split into three parts signifying contradiction, ignorance and unknown respectively. We now defined a PNS as follows:

Definition 3: Let P be a non-empty set. A PNS A over P characterizes each element p in P by a truth-membership function T_A, a contradiction membership function C_A, an ignorance membership function G_A, unknown membership function U_A and a falsity membership function F_A such that for each $p \in P$, $T_A, C_A, G_A, U_A, F_A \in [0,1]$ and

$$0 \le T_A(p) + C_A(p) + G_A(p) + U_A(p) + F_A(p) \le 5.$$

Example: Consider the statement: "Is Facebook good for society?".

Suppose, this statement is posed in front of a group of five people, say, $P = \{p_1, p_2, p_3 p_4, p_5\}$ (which constitute the universe under consideration) and they are requested to express their opinion regarding this statement. Now it may so happen that the opinion of the people may vary among the following possible options: "a degree of agreement with the statement", "a degree of both agreement as well as disagreement regarding the statement "a degree of neither agreement nor disagreement regarding the statement", "a degree of ignore agreement and disagreement" and "a degree of disagreement with respect to the statement". According to the response of the people, the available information can be represented in terms of a PNS as follows:

From the above PNS, it is seen that the person p_1 is to great extent, in agreement with the statement whereas, p_5 mostly disagrees with the statement while p_2 opines that the statement is both true as well as false, p_3 is mainly in ignorance regarding the truth of the statement and p_4 totally ignores the truth and false of the statement.

It is to be noted that when Indeterminacy (I) is refined into I1, I2, I3, and together T, I1, I2, I3, F form a pentapartitioned neutrosophic set. It is a special case of the n- valued refined neutrosophic set, introduced by Smarandache [7] in 2013.

Definition 4: A PNS A is said to be absolute PNS if and only if its truth-membership, contradiction membership, ignorance membership, unknown membership and falsity membership function values are defined as follow,

$$T_A(p) = 1, C_A(p) = 1, G_A(p) = 0, U_A(p) = 0, F_A(p) = 0.$$

Definition 5: A PNS is said to be null θ PNS if and only if its truth-membership, contradiction membership, ignorance membership, unknown membership and falsity membership function values are respectively defined as follows:

$$T_A(p) = 0, C_A(p) = 0, G_A(p) = 1, U_A(p) = 1, F_A(p) = 1.$$

3. Basic properties:

Definition 6: Consider two PNS R_1 and R_2 over P, R_1 is said to be contained in R_2, denoted by $R_1 \subseteq R_2$ iff $T_{R_1}(p) \le T_{R_2}(p), C_{R_1}(p) \le C_{R_2}(p), G_{R_1}(p) \ge G_{R_2}(p), U_{R_1}(p) \ge U_{R_2}(p)$ and $F_{R_1}(p) \ge F_{R_2}(p)$ where $p \in P$.

Definition 7: The complement of PNS R_1 is denoted by R_1^C and is defined as:

$R_1^C = \{(F_{R_1}(p), U_{R_1}(p), 1 - G_{R_1}(p), C_{R_1}(p), T_{R_1}(p)) \mid p \in P\}$ i.e. $T_{R_1}(p) = F_{R_1}(p), C_{R_1}(p) = U_{R_1}(p),$

$G_{R_1}(p) = 1 - G_{R_1}(p), U_{R_1}(p) = C_{R_1}(p)$ and $F_{R_1}(p) = T_{R_1}(p), p \in P$

Definition 8: The union and intersection of any two PNSs R_1 and R_2 is denoted by $R_1 \cup R_2$ and $R_1 \cap R_2$ is defined as:

$$\begin{aligned} R_1 \cup R_2 &= \{(\max(T_{R_1}(p), T_{R_2}(p)), \max(C_{R_1}(p), C_{R_2}(p)), \min(G_{R_1}(p), G_{R_2}(p)), \min(U_{R_1}(p), U_{R_2}(p)), \min(F_{R_1}(p), F_{R_2}(p)) \mid p \in P\} \\ &= \{(T_{R_1}(p), C_{R_1}(p), G_{R_1}(p), U_{R_1}(p), F_{R_1}(p)) \vee (T_{R_2}(p), C_{R_2}(p), G_{R_2}(p), U_{R_2}(p), F_{R_2}(p)) \mid p \in P\} \end{aligned}$$

$$\begin{aligned} R_1 \cap R_2 &= \{(\min(T_{R_1}(p), T_{R_2}(p)), \min(C_{R_1}(p), C_{R_2}(p)), \max(G_{R_1}(p), G_{R_2}(p)), \max(U_{R_1}(p), U_{R_2}(p)), \max(F_{R_1}(p), F_{R_2}(p)) \mid p \in P\} \\ &= \{(T_{R_1}(p), C_{R_1}(p), G_{R_1}(p), U_{R_1}(p), F_{R_1}(p)) \wedge (T_{R_2}(p), C_{R_2}(p), G_{R_2}(p), U_{R_2}(p), F_{R_2}(p)) \mid p \in P\} \end{aligned}$$

Example: Consider any two PNSs defined over P, presented as:

$$E = \langle 0.6, 0.4, 0.3, 0.2, 0.3 \rangle / r_1 + \langle 0.5, 0.3, 0.4, 0.5, 0.4 \rangle / r_2 + \langle 0.3, 0.7, 0.5, 0.2, 0.4 \rangle / r_3$$
$$F = \langle 0.7, 0.2, 0.4, 0.3, 0.5 \rangle / r_1 + \langle 0.7, 0.4, 0.3, 0.4, 0.5 \rangle / r_2 + \langle 0.6, 0.5, 0.6, 0.4, 0.3 \rangle / r_3$$

Then we have,

$$E^C = \langle 0.3, 0.2, 0.7, 0.4, 0.6 \rangle / r_1 + \langle 0.4, 0.5, 0.6, 0.3, 0.5 \rangle / r_2 + \langle 0.4, 0.2, 0.5, 0.7, 0.3 \rangle / r_3$$
$$E \cup F = \langle 0.7, 0.4, 0.4, 0.3, 0.5 \rangle / r_1 + \langle 0.7, 0.4, 0.4, 0.5, 0.5 \rangle / r_2 + \langle 0.6, 0.7, 0.6, 0.4, 0.4 \rangle / r_3$$
$$E \cap F = \langle 0.6, 0.2, 0.3, 0.2, 0.3 \rangle / r_1 + \langle 0.5, 0.3, 0.3, 0.4, 0.4 \rangle / r_2 + \langle 0.3, 0.5, 0.5, 0.2, 0.3 \rangle / r_3$$

Proposition 1: PNSs satisfy the following properties under the aforementioned set theoretic operations:

i. Commutative law

$(a) R_1 \cup R_2 = R_2 \cup R_1$
$(b) R_1 \cap R_2 = R_2 \cap R_1$

ii. Associative law

$(c) R_1 \cup (R_2 \cup R_3) = (R_1 \cup R_2) \cup R_3$
$(d) R_1 \cap (R_2 \cap R_3) = (R_1 \cap R_2) \cap R_3$

iii. Distributive law

$(e) R_1 \cup (R_2 \cap R_3) = (R_1 \cup R_2) \cap (R_1 \cup R_3)$

$(f) R_1 \cap (R_2 \cup R_3) = (R_1 \cap R_2) \cup (R_1 \cap R_3)$

iv. Absorption law

$(g) R_1 \cup (R_1 \cap R_2) = R_1$

$(h) R_1 \cap (R_1 \cup R_2) = R_1$

v. Involution law

$(i) (R_1^C)^C = R_1$

vi. Law of contradiction

$(j) R_1 \cap R_1^C = \theta$

vii. De Morgan's law

$(k) (R_1 \cup R_2)^C = R_1^C \cap R_2^C$

$(l) (R_1 \cap R_2)^C = R_1^C \cup R_2^C$

Proof:

(a) $R_1 \cup R_2 = R_2 \cup R_1$

We know that,

$$R_1 \cup R_2 = \{(\max(T_{R_1}(p), T_{R_2}(p)), \max(C_{R_1}(p), C_{R_2}(p)), \min(G_{R_1}(p), G_{R_2}(p)), \min(U_{R_1}(p), U_{R_2}(p)), \min(F_{R_1}(p), F_{R_2}(p)) \mid p \in P\}$$

$$= \{(T_{R_1}(p), C_{R_1}(p), G_{R_1}(p), U_{R_1}(p), F_{R_1}(p)) \vee (T_{R_2}(p), C_{R_2}(p), G_{R_2}(p), U_{R_2}(p), F_{R_2}(p)) \mid p \in P\}$$

$Let, x_i \in R_1 \cup R_2$

$\Rightarrow x_i \in \{\max(T_{R_1}, T_{R_2}), \max(C_{R_1}, C_{R_2}), \min(G_{R_1}, G_{R_2}), \min(U_{R_1}, U_{R_2}), \min(F_{R_1}, F_{R_2})\}$

$\Rightarrow x_i \in \{\max(T_{R_2}, T_{R_1}), \max(C_{R_2}, C_{R_1}), \min(G_{R_2}, G_{R_1}), \min(U_{R_2}, U_{R_1}), \min(F_{R_2}, F_{R_1})\}$

$\Rightarrow x_i \in R_2 \cup R_1$

$\Rightarrow R_1 \cup R_2 \subset R_2 \cup R_1$ (1)

$Let, y_i \in R_2 \cup R_1$

$\Rightarrow y_i \in \{\max(T_{R_2}, T_{R_1}), \max(C_{R_2}, C_{R_1}), \min(G_{R_2}, G_{R_1}), \min(U_{R_2}, U_{R_1}), \min(F_{R_2}, F_{R_1})\}$

$\Rightarrow y_i \in \{(\max(T_{R_1}, T_{R_2}), \max(C_{R_1}, C_{R_2}), \min(G_{R_1}, G_{R_2}), \min(U_{R_1}, U_{R_2}), \min(F_{R_1}, F_{R_2})\}$

$\Rightarrow y_i \in R_1 \cup R_2$

$\Rightarrow R_2 \cup R_1 \subset R_1 \cup R_2$ (2)

Therefore, from (1) and (2) we obtain,

$R_1 \cup R_2 = R_2 \cup R_1$

(b) Similarly, we can prove that

$R_1 \cap R_2 = R_2 \cap R_1$

(c) $R_1 \cup (R_2 \cup R_3) = (R_1 \cup R_2) \cup R_3$

$Assum\,that, x_i \in R_1 \cup (R_2 \cup R_3)$

$=\succ x_i \in R_1 \cup \left\langle \max(T_{R_2}, T_{R_3}), \max(C_{R_2}, C_{R_3}), \min(G_{R_2}, G_{R_3}), \min(U_{R_2}, U_{R_3}), \min(F_{R_2}, F_{R_3}) \right\rangle$

$=\succ x_i \in \left\langle \max(T_{R_1}, T_{R_2}, T_{R_3}), \max(C_{R_1}, C_{R_2}, C_{R_3}), \min(G_{R_1}, G_{R_2}, G_{R_3}), \min(U_{R_1}, U_{R_2}, U_{R_3}), \min(F_{R_2}, F_{R_2}, F_{R_3}) \right\rangle$

$=\succ x_i \in \left\langle \max(T_{R_1}, T_{R_2}), \max(C_{R_1}, C_{R_2}), \min(G_{R_1}, G_{R_2}), \min(U_{R_1}, U_{R_2}), \min(F_{R_1}, F_{R_2}) \right\rangle \cup R_3$

$=\succ x_i \in (R_1 \cup R_2) \cup R_3$

$R_1 \cup (R_2 \cup R_3) \subset (R_1 \cup R_2) \cup R_3 \qquad (3)$

$Assum\,that, y_i \in (R_1 \cup R_2) \cup R_3$

$=\succ y_i \in \left\langle \max(T_{R_1}, T_{R_2}), \max(C_{R_1}, C_{R_2}), \min(G_{R_1}, G_{R_2}), \min(U_{R_1}, U_{R_2}), \min(F_{R_1}, F_{R_2}) \right\rangle \cup R$

$=\succ y_i \in \left\langle \max(T_{R_1}, T_{R_2}, T_{R_3}), \max(C_{R_1}, C_{R_2}, C_{R_3}), \min(G_{R_1}, G_{R_2}, G_{R_3}), \min(U_{R_1}, U_{R_2}, U_{R_3}), \min(F_{R_1}, F_{R_2}, F_{R_3}) \right\rangle$

$=\succ y_i \in R_1 \cup \left\langle \max(T_{R_2}, T_{R_3}), \max(C_{R_2}, C_{R_3}), \min(G_{R_2}, G_{R_3}), \min(U_{R_2}, U_{R_3}), \min(F_{R_2}, F_{R_3}) \right\rangle$

$=\succ y_i \in R_1 \cup (R_2 \cup R_3)$

$(R_1 \cup R_2) \cup R_3 \subset R_1 \cup (R_2 \cup R_3) \qquad (4)$

From (3) and (4) we conclude that,

$R_1 \cup (R_2 \cup R_3) = (R_1 \cup R_2) \cup R_3$

(d) Similarly, we can prove that

$R_1 \cap (R_2 \cap R_3) = (R_1 \cap R_2) \cap R_3$

(e) $R_1 \cup (R_2 \cap R_3) = (R_1 \cup R_2) \cap (R_1 \cup R_3)$

$Assume\,that, x_i \in R_1 \cup (R_2 \cap R_3)$

$=\succ x_i \in R_1 \cup \left\langle \min(T_{R_2}, T_{R_3}), \min(C_{R_2}, C_{R_3}), \max(G_{R_2}, G_{R_3}), \max(U_{R_2}, U_{R_3}) \max(F_{R_2}, F_{R_3}) \right\rangle$

$$=\succ x_i \in \left\langle \begin{matrix} \max(T_{R_1}, \min(T_{R_2}, T_{R_3})), \max(C_{R_1}, \min(C_{R_2}, C_{R_3})), \min(G_{R_1}, \max(G_{R_2}, G_{R_3})), \\ \min(U_{R_1}, \max(U_{R_2}, U_{R_3})), \min(F_{R_1}, \max(F_{R_2}, F_{R_3})) \end{matrix} \right\rangle$$

$=\succ x_i \in \left\langle \max(T_{R_1}, T_{R}), \max(C_{R_1}, C_{R_2}), \min(G_{R_1}, G_{R_2}), \min(U_{R_1}, U_{R_2}), \min(F_{R_1}, F_{R_2}) \right\rangle$

$\cap \left\langle \max(T_{R_2}, T_{R_3}), \max(C_{R_2}, C_{R_3}), \min(G_{R_2}, G_{R_3}), \min(U_{R_2}, U_{R_3}), \min(F_{R_2}, F_{R_3}) \right\rangle$

$=\succ x_i \in (R_1 \cup R_2) \cap (R_1 \cup R_3) \qquad (5)$

$Assume\,that, y_i \in (R_1 \cup R_2) \cap (R_1 \cup R_3)$

$=\succ y_i \in \left\langle \max(T_{R_1}, T_{R}), \max(C_{R_1}, C_{R_2}), \min(G_{R_1}, G_{R_2}), \min(U_{R_1}, U_{R_2}), \min(F_{R_1}, F_{R_2}) \right\rangle$

$\cap \left\langle \max(T_{R_2}, T_{R_3}), \max(C_{R_2}, C_{R_3}), \min(G_{R_2}, G_{R_3}), \min(U_{R_2}, U_{R_3}), \min(F_{R_2}, F_{R_3}) \right\rangle$

$$=\succ y_i \in \left\langle \begin{matrix} \max(T_{R_1}, \min(T_{R_2}, T_{R_3})), \max(C_{R_1}, \min(C_{R_2}, C_{R_3})), \min(G_{R_1}, \max(G_{R_2}, G_{R_3})), \\ \min(U_{R_1}, \max(U_{R_2}, U_{R_3})), \min(F_{R_1}, \max(F_{R_2}, F_{R_3})) \end{matrix} \right\rangle$$

$=\succ y_i \in < T_{R_1}, C_{R_1}, G_{R_1}, U_{R_1}, F_{R_1} > \cup \left\langle \min(T_{R_2}, T_{R_3}), \min(C_{R_2}, C_{R_3}), \max(G_{R_2}, G_{R_3}), \max(U_{R_2}, U_{R_3}) \max(F_{R_2}, F_{R_3}) \right\rangle$

$=\succ y_i \in R_1 \cup \left\langle \min(T_{R_2}, T_{R_3}), \min(C_{R_2}, C_{R_3}), \max(G_{R_2}, G_{R_3}), \max(U_{R_2}, U_{R_3}) \max(F_{R_2}, F_{R_3}) \right\rangle$

$=\succ y_i \in R_1 \cup (R_2 \cap R_3) \qquad (6)$

From (5) and (6), we conclude that

$R_1 \cup (R_2 \cap R_3) = (R_1 \cup R_2) \cap (R_1 \cup R_3)$

(g) $R_1 \cup (R_1 \cap R_2) = R_1$

$$Assume\,that, x_i \in R_1 \cup (R_1 \cap R_2)$$
$$=\succ x_i \in R_1 \cup \left\langle \min(T_{R_1}, T_{R_2}), \min(C_{R_1}, C_{R_2}), \max(G_{R_1}, G_{R_2}), \max(U_{R_1}, U_{R_2}) \max(F_{R_1}, F_{R_2}) \right\rangle$$
$$=\succ x_i \in \left\langle \begin{matrix} \max(T_{R_1}, \min(T_{R_1}, T_{R_2})), \max(C_{R_1}, \min(C_{R_1}, C_{R_2})), \min(G_{R_1}, \max(G_{R_1}, G_{R_2})), \\ \min(U_{R_1}, \max(U_{R_1}, U_{R_2})), \min(F_{R_1}, \max(F_{R_1}, F_{R_2})) \end{matrix} \right\rangle$$
$$=\succ x_i \in < T_{R_1}, C_{R_1}, G_{R_1}, U_{R_1}, F_{R_1} >$$
$$=\succ x_i \in R_1$$

$$\Rightarrow R_1 \cup (R_1 \cap R_2) \subset R_1 \qquad (7)$$
$$Assume\,that, x_i \in R_1$$
$$=\succ x_i \in < T_{R_1}, C_{R_1}, G_{R_1}, U_{R_1}, F_{R_1} >$$
$$=\succ x_i \in \left\langle \max(T_{R_1}, \min(T_{R_1}, T_{R_2})), \max(C_{R_1}, \min(C_{R_1}, C_{R_2})), \min(G_{R_1}, \max(G_{R_1}, G_{R_2})), \min(U_{R_1}, \max(U_{R_1}, U_{R_2})), \min(F_{R_1}, \max(F_{R_1}, F_{R_2})) \right\rangle$$
$$=\succ x_i \in R_1 \cup \left\langle \min(T_{R_1}, T_{R_2}), \min(C_{R_1}, C_{R_2}), \max(G_{R_1}, G_{R_2}), \max(U_{R_1}, U_{R_2}) \max(F_{R_1}, F_{R_2}) \right\rangle$$
$$=\succ x_i \in R_1 \cup (R_1 \cap R_2)$$
$$=\succ R_1 \subset R_1 \cup (R_1 \cap R_2) \qquad (8)$$

From (6) and (8), we conclude that

$$R_1 \cup (R_1 \cap R_2) = R_1$$

(h) Similarly, we can prove that

$$R_1 \cap (R_1 \cup R_2) = R_1$$

(i) $(R_1^{\,C})^C = R_1$

$$Assum\ that, x_i \in (R_1^{\,C})^C$$
$$=\succ x_i \in \left(F_{R_1}, U_{R_1}, 1 - G_{R_1}, C_{R_1}, T_{R_1}\right)^C$$
$$=\succ x_i \in (T_{R_1}, C_{R_1}, G_{R_1}, U_{R_1}, F_{R_1})$$
$$=\succ x_i \in R_1$$
$$=\succ (R_1^{\,C})^C \subset R_1 \qquad (9)$$

$$Assum\ that, y_i \in R_1$$
$$=\succ y_i \in (T_{R_1}, C_{R_1}, G_{R_1}, U_{R_1}, F_{R_1})$$
$$=\succ y_i \in \left(F_{R_1}, U_{R_1}, 1 - G_{R_1}, C_{R_1}, T_{R_1}\right)^C$$
$$=\succ y_i \in (R_1^{\,C})^C$$
$$=\succ R_1 \subset (R_1^{\,C})^C \qquad (10)$$

From (9) and (10), we obtain

$$(R_1^{\,C})^C = R_1$$

(j) $R_1 \cap R_1^{\,C} = \theta$

$Assum\ that, x_i \in R_1 \cap R_1^C$
$\Rightarrow x_i \in (T_{R_1}, C_{R_1}, G_{R_1}, U_{R_1}, F_{R_1}) \cap (F_{R_1}, U_{R_1}, 1-G_{R_1}, C_{R_1}, T_{R_1})$
$\Rightarrow x_i \in <\min(T_{R_1}, F_{R_1}), \min(C_{R_1}, U_{R_1}), \max(G_{R_1}, 1-G_{R_1}), \max\left(U_{R_1}, C_{R_1}\right), \max(F_{R_1}, T_{R_1})>$
$\Rightarrow x_i \in \theta$
$\Rightarrow R_1 \cap R_1^C \subset \theta$ (11)

$Assum\ that, y_i \in \theta$
$\Rightarrow y_i \in <\min(T_{R_1}, F_{R_1}), \min(C_{R_1}, U_{R_1}), \max(G_{R_1}, 1-G_{R_1}), \max\left(U_{R_1}, C_{R_1}\right), \max(F_{R_1}, T_{R_1})>$
$\Rightarrow y_i \in (T_{R_1}, C_{R_1}, G_{R_1}, U_{R_1}, F_{R_1}) \cap (F_{R_1}, U_{R_1}, 1-G_{R_1}, C_{R_1}, T_{R_1})$
$\Rightarrow y_i \in R_1 \cap R_1^C$
$\Rightarrow \theta \subset R_1 \cap R_1^C$ (12)

From (11) and (12), we obtain
$R_1 \cap R_1^C = \theta$

(k) $(R_1 \cup R_2)^C = R_1^C \cap R_2^C$

$Assum\ that, x_i \in (R_1 \cup R_2)^C$
$\Rightarrow x_i \in (\max(T_{R_1}, T_{R_2}), \max(C_{R_1}, C_{R_2}), \min(G_{R_1}, G_{R_2}), \min(U_{R_1}, U_{R_2}), \min(F_{R_1}, F_{R_2}))^C$
$\Rightarrow x_i \in \left\langle \min(F_{R_1}, F_{R_2}), \min(U_{R_1}, U_{R_2}), 1-\min(G_{R_1}, G_{R_2}), \max(C_{R_1}, C_{R_2}), \max(T_{R_1}, T_{R_2}) \right\rangle$
$\Rightarrow x_i \in \left(F_{R_1}, U_{R_1}, 1-G_{R_1}, C_{R_1}, T_{R_1}\right) \cap \left(F_{R_2}, U_{R_2}, 1-G_{R_2}, C_{R_2}, T_{R_2}\right)$
$\Rightarrow x_i \in R_1^C \cap R_2^C$
$\Rightarrow (R_1 \cup R_2)^C \subset R_1^C \cap R_2^C$ (13)

$Again,\ Assum\ that, y_i \in R_1^C \cap R_2^C$
$\Rightarrow y_i \in \left(F_{R_1}, U_{R_1}, 1-G_{R_1}, C_{R_1}, T_{R_1}\right) \cap \left(F_{R_2}, U_{R_2}, 1-G_{R_2}, C_{R_2}, T_{R_2}\right)$
$\Rightarrow y_i \in \left\langle \min(F_{R_1}, F_{R_2}), \min(U_{R_1}, U_{R_2}), 1-\min(G_{R_1}, G_{R_2}), \max(C_{R_1}, C_{R_2}), \max(T_{R_1}, T_{R_2}) \right\rangle$
$\Rightarrow y_i \in (\max(T_{R_1}, T_{R_2}), \max(C_{R_1}, C_{R_2}), \min(G_{R_1}, G_{R_2}), \min(U_{R_1}, U_{R_2}), \min(F_{R_1}, F_{R_2}))^C$
$\Rightarrow y_i \in (R_1 \cup R_2)^C$
$\Rightarrow R_1^C \cap R_2^C \subset (R_1 \cup R_2)^C$ (14)

From (13) and (14), we conclude that
$(R_1 \cup R_2)^C = R_1^C \cap R_2^C$

(l) $(R_1 \cap R_2)^C = R_1^C \cup R_2^C$

$Assum\ that, x_i \in (R_1 \cap R_2)^C$
$\Rightarrow x_i \in (\min(T_{R_1}, T_{R_2}), \min(C_{R_1}, C_{R_2}), \max(G_{R_1}, G_{R_2}), \max(U_{R_1}, U_{R_2}), \max(F_{R_1}, F_{R_2}))^C$
$\Rightarrow x_i \in \left\langle \max(F_{R_1}, F_{R_2}), \max(U_{R_1}, U_{R_2}), 1-\max(G_{R_1}, G_{R_2}), \min(C_{R_1}, C_{R_2}), \min(T_{R_1}, T_{R_2}) \right\rangle$
$\Rightarrow x_i \in \left(F_{R_1}, U_{R_1}, 1-G_{R_1}, C_{R_1}, T_{R_1}\right) \cup \left(F_{R_1}, U_{R_1}, 1-G_{R_1}, C_{R_1}, T_{R_1}\right)$
$\Rightarrow x_i \in R_1^C \cup R_2^C$

$=\succ (R_1 \cap R_2)^C \subset R_1^C \cup R_2^C$ (15)

$Again,\ Assum\,that, y_i \in R_1^C \cup R_2^C$

$=\succ y_i \in \langle F_{R_1}, U_{R_1}, 1-G_{R_1}, C_{R_1}, T_{R_1} \rangle \cup \langle F_{R_2}, U_{R_2}), 1-G_{R_2}, C_{R_2}, T_{R_2} \rangle$

$=\succ y_i \in \langle \max(F_{R_1}, F_{R_2}), \max(U_{R_1}, U_{R_2}), 1-\max(G_{R_1}, G_{R_2}), \min(C_{R_1}, C_{R_2}), \min(T_{R_1}, T_{R_2}) \rangle$

$=\succ y_i \in (\min(T_{R_1}, T_{R_2}), \min(C_{R_1}, C_{R_2}), \max(G_{R_1}, G_{R_2}), \max(U_{R_1}, U_{R_2}), \max(F_{R_1}, F_{R_2}))^C$

$=\succ y_i \in (R_1 \cap R_2)^C$

$=\succ R_1^C \cup R_2^C \subset (R_1 \cap R_2)^C$ (16)

From (15) and (16) we conclude that,

$(R_1 \cap R_2)^C = R_1^C \cup R_2^C$

4. Conclusion:

In this article we have develop pentapartitioned neutrosophic set. The pentapartitioned neutrosophic set is extension of SVNS and QSVNS. The concept of complement law, inclusion law, union law, intersection law, commutative law, etc. have been defined on pentapartitioned neutrosophic sets. Future works may comprise of the study of different types of operators on pentapartitioned neutrosophic sets dealing with actual problems and implementing them in decision-making problems [8-13].

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Received: May 1, 2020. Accepted: September 22, 2020